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Tracer transport within abyssal mixing layers

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ABSTRACT

12 Mixing layers near sloped topography in the abyss are thought to play a
13 critical role in the global overturning circulation. Yet the behavior of passive
14 tracers within sloping boundary layer systems has received little attention,
15 despite the extensive use of tracer observations to understand abyssal circu-
16 lation. Here, we investigate the behavior of a passive tracer released near a
17 sloping boundary within a flow governed by one-dimensional boundary layer
18 theory. The spreading rate of the tracer across isopycnals is influenced by a
19 number of factors including the bottom-intensification of mixing, the dipole
20 of upwelling (in the boundary layer) and downwelling (in the outer mixing
21 layer) and along-isopycnal diffusion. For isolated near-boundary tracer re-
22 leases the bulk diffusivity, proportional to the rate of increase of the variance
23 of the tracer distribution in buoyancy space, is much less than what would be
24 expected from averaging the diapycnal diffusivity over the tracer patch. This
25 stems from the presence of the bottom boundary that prevents tracer diffu-
26 sion through it. Furthermore, when along-isopycnal diffusion is weak, the
27 boundary tends to drive the tracer up the slope toward less dense fluid on av-
28 erage due to asymmetries between boundary layer and interior flows. With
29 strong along-isopycnal diffusion this upslope movement is reduced, while at
30 the same time the average diapycnal spreading rate is increased due to a re-
31 duced influence of the bottom boundary. These results have implications for
32 what can be learned about the characteristics of mixing near sloping bound-
33 aries from past and future tracer-release experiments.

34 **1. Introduction**

35 The transport of tracers within the ocean plays an important role not only in ocean dynamics,
36 thermodynamics and biogeochemistry, but also as a method with which to observe the ocean, and
37 infer circulation properties. Because, by its nature, tracer transport integrates over both spatial
38 and temporal scales, it allows us to measure the large-scale, integral impact of a range of smaller
39 scale processes that are difficult to observe directly. However, in order to correctly interpret and
40 use tracer measurements, a good understanding of tracer transport and its relation to circulation
41 properties is needed.

42 Tracer transport has been particularly useful for understanding the dynamics of the ocean's deep
43 overturning circulation, where small-scale turbulent mixing plays a key role (Watson and Ledwell
44 2000). Since the pioneering study of Munk (1966) and the realization that mixing in the interior
45 is generally weak (e.g. Ledwell et al. 1993), mixing near the ocean's boundaries has been thought
46 to play a critical role in closing the global diapycnal circulation (Armi 1978; Ivey 1987b; Wunsch
47 1970; Phillips 1970; Thorpe 1987; Garrett 1991). Observations suggest that turbulent mixing
48 is bottom intensified due to internal wave breaking within the stratified fluid above the boundary
49 layer (Toole et al. 1994; Polzin et al. 1997; Ledwell et al. 2000; St. Laurent et al. 2012; Waterhouse
50 et al. 2014). The bottom-intensification of mixing within so called "abyssal mixing layers" implies
51 a downward mass transport across isopycnals in what some authors have termed the Stratified
52 Mixing Layer (SML, McDougall and Ferrari 2017). In order to balance the formation of dense
53 waters at high-latitudes there must therefore be a somewhat larger upwelling transport within
54 thin bottom boundary layers (BBLs) along the sloping seafloor where the turbulent buoyancy flux
55 converges (de Lavergne et al. 2016, 2017; Ferrari et al. 2016; McDougall and Ferrari 2017). The
56 requirement for net upwelling and the near-compensation between the net diapycnal transports

57 in the BBL and SML imply a complex balance between factors such as topographic geometry
58 (McDougall and Ferrari 2017; Holmes et al. 2018), variations in stratification (Ferrari et al. 2016;
59 Callies and Ferrari 2018; Banyte et al. 2018) and the lateral structure in the intensity of turbulent
60 mixing near the ocean floor (Kunze 2017a,b). How this balance is achieved at both global and
61 regional scales remains an open question that observations of tracer behavior may help to answer.

62 Field tracer release experiments (TREs) have provided many insights into the dynamics of di-
63 apycnal ocean circulation. TREs have shown that mixing away from boundaries is weak (Ledwell
64 et al. 1993; Ledwell 1998), and highlighted the importance of strong boundary mixing for closing
65 basin-scale budgets (Goudsmit et al. 1997; Inall 2009; Ledwell and Bratkovich 1995). The Brazil
66 Basin TRE (BBTRE, Ledwell et al. 2000) in particular demonstrated the presence of intensified
67 mixing in the abyss above the rough seafloor of the Mid-Atlantic Ridge, corroborating microstruc-
68 ture measurements (Polzin et al. 1997). The BBTRE tracer was released well above the ocean
69 bottom, and its centroid showed a tendency to descend across isopycnals in the eastern basin con-
70 sistent with the expected diapycnal downwelling in the SML. However, the observations close to
71 the boundary suggested that in ridge canyons the tracer moves upslope, likely toward less dense
72 waters. These observations pointed to the importance of the near boundary region, though BBTRE
73 was not designed to study this region.

74 The importance of intense boundary mixing was further highlighted by the Diapycnal and Isopy-
75 cnal Mixing Experiment in the Southern Ocean (DIMES, Ledwell et al. 2011; Watson et al. 2013).
76 DIMES also exposed a common discrepancy whereby diffusivities inferred from tracer measure-
77 ments often exceed those estimated from microstructure surveys. These discrepancies are often
78 attributed to sampling issues (e.g. Wüest et al. 1996; Voet et al. 2015) and can be reconciled if the
79 full temporal and spatial distribution of the tracer is taken into account (Mashayek et al. 2017).
80 Studies such as Mashayek et al. (2017) and Ledwell et al. (2000) highlight the importance of

81 tracer-boundary interactions, but do not discuss the details of tracer transport within any particular
82 boundary-driven flow. Idealized studies have examined near-boundary tracer dispersion above a
83 horizontal boundary (e.g. Saffman 1962; Csanady 1969). Here we examine a similar problem near
84 a sloping boundary.

85 Our tracer study will be conducted in the context of one-dimensional boundary layer theory (e.g.
86 Wunsch 1970; Phillips et al. 1986; Thorpe 1987; Garrett 1990). The theory considers the one-
87 dimensional problem of flow over a uniform slope driven by an isotropic diffusivity. To satisfy
88 the no-flux boundary condition, isopycnals slope down as they approach the boundary, leading to
89 a buoyancy-driven upslope flow in a weakly-stratified BBL where friction is important (Garrett
90 et al. 1993). When the diffusivity is bottom-intensified a corresponding downslope flow appears
91 in the outer portion of the abyssal mixing layer (the SML). Recently, Callies (2018) has shown that
92 for realistic abyssal ocean parameters boundary layer theory predicts a much weaker stratification
93 than typically observed. He suggests that restratification by submesoscale eddies, generated by
94 baroclinic instability of the resulting flow field (Wenegrat et al. 2018), is necessary to maintain the
95 stratification and therefore permit significant near-boundary water-mass transformation.

96 Eddies, along with tides, intrusions and other processes, can also drive strong along-isopycnal
97 tracer transports (e.g. Ivey 1987a; McPhee-Shaw 2006; Wain and Rehmann 2010; Winters 2015;
98 Dell and Pratt 2015). This along-isopycnal exchange (in this article captured at first-order by an
99 along-isopycnal tracer diffusivity) provides one means in which the strong boundary mixing can
100 communicate with the interior.

101 In this article we will aim to address the following questions; 1) what processes within abyssal
102 mixing layers are most important for determining tracer transport? 2) Will a tracer be transported
103 toward lighter (upslope) or heavier (downslope) density classes, on average, when released near

104 a sloping boundary? 3) What can be inferred about the properties of mixing and circulation from
105 bulk measurements of the tracer cloud dispersion?

106 To approach these questions we analyze the behavior of a passive tracer released within a flow
107 governed by one-dimensional boundary layer theory (described in Section 2). The tracer evolves
108 in two dimensions due to the effects of diapycnal diffusion, isopycnal diffusion and advection
109 (Section 3). We introduce an analytical framework based around the tracer moments in buoyancy
110 space (the tracer center-of-mass and variance) to understand the contribution of different processes
111 to tracer dispersion (Sections 4-6). We find that the presence of the boundary can slow the rate
112 of diapycnal tracer dispersion below that expected from averaging the diapycnal diffusivity over
113 the tracer patch. Due to asymmetries between the upslope BBL and downslope SML flows (the
114 BBL upwelling being strong and narrow compared to the more diffuse SML downwelling), the
115 tracer tends to move upslope on average. The extent of upslope movement and tracer dispersion
116 depends on the initial release position of the tracer, as well as a number of other factors such as the
117 decay scale of bottom-intensified mixing and along-isopycnal diffusion (Section 7). Our results
118 have implications not only for our understanding of tracer behavior near the ocean floor (Section
119 9), but also for what can be learned about ocean dynamics from field TREs (Section 8).

120 **2. Boundary layer theory**

121 The idealized two-dimensional tracer release experiments will be performed within a flow gov-
122 erned by one-dimensional boundary layer theory. In this section we briefly review the key results
123 required for our tracer study. For a more detailed derivation the reader is referred to Garrett (1990,
124 1991) and Callies (2018). We consider steady flow above a bottom with uniform slope $\tan \theta$ and
125 use a coordinate system aligned with the bottom (i.e. z is bottom-normal with origin at the bound-
126 ary, y is upslope and x is along-slope, Fig. 1). Everything is uniform in the upslope and along-slope

127 directions except pressure and buoyancy (and in the next section the tracer). Buoyancy is charac-
 128 terized by a constant far-field vertical stratification N^2 ,

$$129 \quad \frac{\partial b}{\partial z} \rightarrow N^2 \cos \theta, \quad \text{as } z \rightarrow \infty \quad (1)$$

$$130 \quad \frac{\partial b}{\partial y} = N^2 \sin \theta, \quad \text{all } z, \quad (2)$$

132 where b is the buoyancy field relative to a reference density ρ_0 . The steady, rotating equations of
 133 motion for this system are (e.g. Garrett et al. 1993),

$$134 \quad -fV \cos \theta = \frac{d}{dz} \left(v_u \frac{dU}{dz} \right), \quad (3)$$

$$135 \quad fU \cos \theta = -\frac{1}{\rho_0} \frac{\partial P}{\partial y} + b \sin \theta + \frac{d}{dz} \left(v_v \frac{dV}{dz} \right), \quad (4)$$

$$136 \quad 0 = -\frac{1}{\rho_0} \frac{\partial P}{\partial z} + b \cos \theta, \quad (5)$$

$$137 \quad N^2 V \sin \theta = \frac{d}{dz} \left(\kappa \frac{\partial b}{\partial z} \right), \quad (6)$$

139 where $U(z), V(z)$ are the along-slope and upslope velocities, $v_u(z), v_v(z)$ are along-slope and ups-
 140 lobe eddy viscosities (which may differ for reasons discussed shortly), $\kappa(z)$ is an eddy diffusivity,
 141 $P(y, z)$ is the pressure field and f is the (vertical) Coriolis parameter. These equations can be
 142 combined into (e.g. Garrett 1991),

$$143 \quad \frac{d^2}{dz^2} \left(v_v \frac{d^2 \Psi}{dz^2} \right) + \left(\frac{f^2 \cos^2 \theta}{v_u} + \frac{N^2 \sin^2 \theta}{\kappa} \right) \Psi = N^2 \sin \theta \cos \theta + \frac{f^2 \cos^2 \theta}{v_u} \kappa_\infty \cot \theta, \quad (7)$$

144 where κ_∞ is the far-field diffusivity and the scalar streamfunction $\Psi(z)$ is given by $\frac{d\Psi}{dz} = V$ with
 145 $\Psi(0) = 0$ and,

$$146 \quad \Psi \rightarrow \kappa_\infty \cot \theta, \quad \text{as } z \rightarrow \infty. \quad (8)$$

147 For constant diffusion, $v_u(z) = v_{u0}$, $v_v(z) = v_{v0}$, $\kappa(z) = \kappa_0 (= \kappa_\infty)$, the solution with no-slip
 148 boundary conditions is characterized by a BBL of reduced stratification and upslope flow (Wunsch
 149 1970; Thorpe 1987, dashed black lines in Fig. 2). There is an along-slope flow in the interior in the

150 direction opposite to Kelvin wave propagation, or upwelling-favorable in a bottom-Ekman sense
 151 (Garrett et al. 1993). The BBL width is $\mathcal{O}(q_0^{-1})$ where,

$$152 \quad q_0^4 = \frac{N^2 \sin^2 \theta}{4Pr_{v0} \kappa_0^2} (1 + (SPr_{u0})^{-1}), \quad (9)$$

153 $S^{-1} = f^2 \cos^2 \theta / N^2 \sin^2 \theta$ is the inverse slope Burger number and $Pr_{u0} = v_{u0} / \kappa_0$, $Pr_{v0} = v_{v0} / \kappa_0$
 154 are Prandtl numbers. For typical abyssal parameters of $N^2 = 10^{-6} \text{s}^{-2}$, $f = 10^{-4} \text{s}^{-1}$, $v_{u0} = v_{v0} =$
 155 $\kappa_0 = 10^{-3} \text{m}^2 \text{s}^{-1}$, $\tan \theta = 1/400$ then $S^{-1} = 1600$ is large and the BBL thickness corresponds to
 156 an Ekman layer thickness of $q_0^{-1} = 4.5 \text{m}$.

157 In this article we will instead focus on the case where the diffusivity is bottom intensified with
 158 form,

$$159 \quad \kappa = \kappa_\infty + (\kappa_0 - \kappa_\infty) e^{-z/d}, \quad (10)$$

160 where κ_0 is the diffusivity near the boundary and d is a decay scale. For the parameter space of
 161 interest here the BBL thickness q_0^{-1} is much smaller than the diffusivity decay scale d . Following
 162 Callies (2018), for $q_0 d \gg 1$ we can construct an approximate analytic solution for the bottom-
 163 intensified case by patching together a solution in the BBL, where the mixing coefficients can be
 164 assumed to be constant and equal to their near-boundary values v_{u0} , v_{v0} and κ_0 , to a solution in
 165 the interior, where the influence of friction through the fourth derivative term in Eq. (7) can be
 166 neglected. This procedure is presented in Appendix A.

167 However, as shown by Callies (2018) (see Eqs. (68) and (71) in Appendix A), for the typi-
 168 cal large inverse slope Burger number S^{-1} and order one Prandtl number regime in the abyssal
 169 ocean such a solution predicts very weak stratification over the abyssal mixing layer. Callies
 170 (2018) attributes this weak stratification to the lack of representation of baroclinic instability and
 171 its associated eddy-driven restratification in the one-dimensional system. A realistically stratified
 172 one-dimensional solution requires small values of the parameter $(SPr_{u0})^{-1}$ (see Appendix A for

173 more details) and can therefore be recovered using a large Pr_{u0} . Such a choice of large vertical
174 momentum mixing in the along-slope momentum equation can be physically interpreted as a pa-
175 rameterization for restratification by baroclinic eddies based on the thickness-weighted average
176 formalism (Rhines and Young 1982; Greatbatch and Lamb 1990; Gent and McWilliams 1990;
177 Gent et al. 1995). For simplicity, and to avoid the need to resolve eddies or include an explicit
178 Gent and McWilliams (1990) type parameterization, throughout most of this article we will con-
179 sider the limit $(SPr_{u0})^{-1} \rightarrow 0$ obtained for large Pr_{u0} . This choice assumes that eddies maintain
180 the stratification in the SML at its far-field value¹. In Section 7c we briefly discuss the impact of
181 reduced stratification in the SML through a non-zero $(SPr_{u0})^{-1}$.

182 Note that the representation of eddy-driven restratification through enhanced vertical momentum
183 mixing should have less impact on the BBL solution, which is determined by non-geostrophic fric-
184 tional turbulent boundary layer physics. The eddy-driven restratification and frictional boundary
185 layer processes can be conveniently isolated (at least in our two-dimensional context) by enhancing
186 only the along-slope viscosity. Thus we maintain the choice $Pr_{v0} = 1$ in the upslope momentum
187 equation.

188 With these parameter choices, the full one-dimensional solution is given by (taking the limit
189 $(SPr_{u0})^{-1} \rightarrow 0$ in Eqs. (68) and (71) in Appendix A),

$$190 \quad \Psi = \kappa \cot \theta \left(1 - e^{-q_0 z} (\cos q_0 z + \sin q_0 z) \right), \quad (11)$$

$$191 \quad \frac{\partial b}{\partial z} = N^2 \cos \theta \left(1 - e^{-q_0 z} (\cos q_0 z + \sin q_0 z) \right). \quad (12)$$

193 As for a constant κ solution there is a BBL of thickness $\sim q_0^{-1}$ with weak stratification and upslope
194 flow (solid lines with circles in Fig. 2)². Outside the BBL the vertical stratification is equal to the

¹This parameter choice is equivalent to using non-rotating boundary layer theory due to the parameter dependence $(SPr_{u0})^{-1} \sim f^2/v_{u0}$.

²Note that due to the choice $(SPr_{u0})^{-1} \rightarrow 0$ the BBL thickness is modestly increased to $q_0^{-1} \sim 28\text{m}$ for the typical abyssal parameters considered above.

195 far-field value N^2 and there is a weak downslope flow which largely compensates the upwelling
 196 within the BBL (the imbalance between SML and BBL transports is governed by Eq. (8)). These
 197 compensating upwelling and downwelling flows are equivalent to the water-mass transformation
 198 dipole discussed recently by a number of authors (e.g. de Lavergne et al. 2016; Ferrari et al. 2016;
 199 McDougall and Ferrari 2017). It is this flow that will be used to advect and diffuse a passive tracer
 200 in a two-dimensional y - z plane, as discussed in the next section.

201 3. Two-dimensional tracer dispersion in a one-dimensional slope flow

202 a. The tracer conservation equation

203 We set up a two-dimensional tracer advection-diffusion problem in the one-dimensional bound-
 204 ary layer flow discussed in the previous section. That is, we look for the distribution of a tracer
 205 $\mathcal{C}(y, z, t)$ (where \mathcal{C} has concentration units of tracer m^{-2}) which varies in the upslope and slope-
 206 normal directions (Fig. 1) given an initial distribution $\mathcal{C}(y, z, 0)$ and a tracer conservation equation,

$$207 \quad \frac{\partial \mathcal{C}}{\partial t} = -\nabla \cdot \mathbf{F}_{\mathcal{C}}, \quad (13)$$

208 where the tracer flux,

$$209 \quad \mathbf{F}_{\mathcal{C}} = \mathbf{V}\mathcal{C} - \kappa\nabla\mathcal{C} - A_H\mathbf{K}_I \cdot \nabla\mathcal{C}. \quad (14)$$

210 $\mathbf{F}_{\mathcal{C}}$ has three-components; an advective flux associated with the velocity $\mathbf{V} = (V(z), 0)$ from
 211 boundary layer theory ($V(z)$ is given by the z derivative of Eq. (11)) and diffusive fluxes asso-
 212 ciated with a small-scale isotropic diffusivity $\kappa(z)$ and an along-isopycnal diffusivity A_H . The
 213 along-isopycnal diffusion is specified using the symmetric second rank tensor (Redi 1982),

$$214 \quad \mathbf{K}_I = \frac{1}{|\nabla b|^2} \begin{bmatrix} b_z^2 & -b_y b_z \\ -b_y b_z & b_y^2 \end{bmatrix}, \quad (15)$$

215 where the subscript on b indicates differentiation. The coefficients of \mathbf{K}_I are determined using the
 216 buoyancy gradients $b_z = B_z + b'_z = N^2 \cos \theta + b'_z$ and $b_y = B_y = N^2 \sin \theta$ split into background (B_z
 217 and B_y) and perturbation components where from Eq. (12),

$$218 \quad b'_z = -N^2 \cos \theta e^{-q_0 z} (\cos q_0 z + \sin q_0 z). \quad (16)$$

219 This formulation of the tracer equation is consistent with boundary layer theory (i.e. if b replaces
 220 \mathcal{C} then Eq. (13) reduces to Eq. (6)).

221 *b. Numerical model setup*

222 The tracer conservation Eq. (13) cannot be solved analytically in the general case (although
 223 we will consider simplified cases that can be solved analytically) and so we resort to numerical
 224 simulations. We use the spectral code Dedalus (Burns et al. 2019, <http://dedalus-project.org/>) in
 225 the rotated y, z coordinates (Fig. 1) with periodic boundary conditions in y . We consider a control
 226 parameter set with a y -domain length $L_y = 1500\text{km}$, a slope of $\alpha = 1/400$ (roughly the western
 227 side of the Mid-Atlantic Ridge) and a z -domain height of $L_z = 3000\text{m}$. The far-field stratification
 228 $N^2 = 10^{-6}\text{m}^2\text{s}^{-1}$. The diffusivity will either be bottom-intensified with an exponential profile
 229 [Eq. (10)] or constant, with base parameters $d = 500\text{m}$, $\kappa_0 = 10^{-3}\text{m}^2\text{s}^{-1}$ and $\kappa_\infty = 10^{-5}\text{m}^2\text{s}^{-1}$.
 230 For those simulations with no along-isopycnal diffusion ($A_H = 0$) we use 384 Fourier modes in y
 231 and 192 Chebyshev modes in z , corresponding to an average $\Delta y \sim 4\text{km}$ and an average $\Delta z = 15\text{m}$.
 232 The Chebyshev basis allows finer resolution of the small-scale z gradients in the BBL (see Fig.
 233 2). Eq. (13) is solved implicitly in time using a time step of 8 days. However, because of small-
 234 scale variations in the along-isopycnal tensor coefficients [Eq. (15)] near the boundary, those
 235 simulations with non-zero along-isopycnal diffusion were instead performed with 576 modes in
 236 y and 768 modes in z and a time step of 4 days. The simulations match analytic results almost

237 perfectly where such results are obtainable (e.g. Section 5). The results are robust to numerical
 238 choices, as confirmed by doubling the number of modes in z and running with a time step four
 239 times smaller, which gave almost identical results (not shown).

240 4. Bulk diffusivity and tracer moments

241 The simplest and most common method to quantify the overall tracer dispersion rate in different
 242 coordinates is through the rate of increase of the variance of the tracer distribution. A Gaussian
 243 distribution of tracer spreading in one-dimension (s) due to a constant diffusivity κ evolves as,

$$244 \quad \mathcal{C}(s, t) = A \frac{\sigma_0}{\sigma} e^{-(s-\mu)^2/2\sigma^2}, \quad (17)$$

$$245 \quad \sigma^2(t) = \sigma_0^2 + 2\kappa t, \quad (18)$$

246 where t is time, A is a constant, σ^2 is the variance with initial value σ_0^2 and μ is the centroid
 247 or center-of-mass. The variance increases linearly with time at a rate of 2κ . Thus for any tracer
 248 distribution, which may be spreading due to complex advection-diffusion processes in multiple
 249 dimensions, we can define an equivalent or *bulk diffusivity* in any dimension s in terms of the rate
 250 of increase of the variance σ_s (also see Wüest et al. 1996; Goudsmit et al. 1997),

$$251 \quad \kappa_{bk}^s = \frac{1}{2} \frac{\partial \sigma_s^2}{\partial t}. \quad (19)$$

252 Here, by defining a domain averaging operator over the dimension s ,

$$253 \quad \langle * \rangle^s \equiv \int_{-\infty}^{\infty} * ds, \quad (20)$$

254 we can write both the variance σ_s and the center-of-mass μ_s in terms of the s -moments of the tracer
 255 distribution,

$$256 \quad \mu_s \equiv \frac{\langle s\mathcal{C} \rangle^s}{\langle \mathcal{C} \rangle^s}, \quad (21)$$

$$257 \quad \sigma_s^2 \equiv \frac{\langle s^2\mathcal{C} \rangle^s}{\langle \mathcal{C} \rangle^s} - \mu_s^2, \quad (22)$$

258
259

260 where $\langle s\mathcal{C} \rangle^s$ and $\langle s^2\mathcal{C} \rangle^s$ are the first and second moments, and $\langle \mathcal{C} \rangle^s = A\sqrt{2\pi}\sigma_s$ is the zeroth mo-
 261 ment that quantifies the (conserved) total amount of tracer. The moments provide a useful frame-
 262 work that can be used to understand aspects of the tracer dispersion analytically (e.g., Saffman
 263 1962; Young et al. 1982). They will be used in later sections to derive expressions for the evolu-
 264 tion of the tracer center-of-mass and bulk diffusivity in terms of the various advective and diffusive
 265 tracer fluxes.

266 A similar calculation can also be applied in buoyancy space, to obtain a *bulk diapycnal diffusivity*
 267 κ_{bk}^b (or just κ_{bk}). In this case a mean stratification profile N^2 , which in our two-dimensional context
 268 will be the interior stratification which (in the limit $(SPr_{u0})^{-1} \rightarrow 0$) is independent of height and
 269 time, is also required in order to convert the spreading rate into the units of a diffusivity. We define,

$$270 \quad \kappa_{bk} \equiv \frac{1}{N^4} \frac{1}{2} \frac{\partial \sigma_b^2}{\partial t} \quad (23)$$

271 where,

$$272 \quad \mu_b = \frac{\langle b\mathcal{C} \rangle}{\langle \mathcal{C} \rangle}, \quad (24)$$

$$273 \quad \sigma_b^2 = \frac{\langle b^2\mathcal{C} \rangle}{\langle \mathcal{C} \rangle} - \mu_b^2, \quad (25)$$

274

275 and the averaging operation in our general two-dimensional context occurs over the full tracer
 276 distribution in the two spatial dimensions y and z ,

$$277 \quad \langle * \rangle \equiv \langle \langle * \rangle^y \rangle^z = \int_0^\infty \int_{-\infty}^\infty * \, dydz. \quad (26)$$

278

279 The bulk diffusivity [Eq. (23)] will be used to characterize the overall rate of spreading of the
 280 tracer across buoyancy surfaces. However, it should be emphasized that κ_{bk} should not necessarily
 281 be interpreted directly as a diffusivity, as such an interpretation masks the potentially complex
 282 processes that are leading to the spreading of that tracer distribution in the first place (here, two-
 283 dimensional advection and diffusion). Comparisons of κ_{bk} to the actual isotropic diffusivity κ

284 will prove useful, as is commonly done for field TREs where microstructure measurements are
 285 also available (e.g. Ledwell et al. 2000; Watson et al. 2013; Mashayek et al. 2017). We will also
 286 compare κ_{bk} to the diffusivity calculated using a one-dimensional model commonly applied to
 287 field data (Ledwell and Watson 1991; Ledwell 1998, see Appendix B).

288 **5. Boundary restriction: tracer behavior in z**

289 We begin by examining the behavior of the tracer in the slope-normal coordinate z . Integrating
 290 the two-dimensional tracer conservation equation [Eq. (13)] across all y yields,

$$291 \quad \frac{\partial \langle C \rangle^y}{\partial t} = \frac{\partial}{\partial z} \left(K^z \frac{\partial \langle C \rangle^y}{\partial z} \right), \quad (27)$$

292 where $K^z = \kappa + A_H b_y^2 / |\nabla b|^2$ and the boundary conditions are $K^z \frac{\partial \langle C \rangle^y}{\partial z} = 0$ at $z = 0$ and $\langle C \rangle^y \rightarrow 0$ as
 293 $z \rightarrow \infty$. Thus in the slope-normal coordinate z the tracer diffuses according to a one-dimensional
 294 diffusion equation that does not depend on the upslope velocity V . We are not aware of any
 295 analytic solutions to Eq. (27) for the case where κ is exponential, A_H is non-zero and there is a
 296 boundary at $z = 0$ (see Zamani and Bombardelli 2014, and references therein). However, when K^z
 297 is a constant (where here for simplicity we set $K^z = \kappa$ with κ constant) a solution is easy to obtain
 298 and, despite the simple governing equation, shows some interesting behavior as a consequence of
 299 the boundary at $z = 0$.

300 *a. Constant diffusivity*

301 When κ is constant and $A_H = 0$ the tracer evolution for an initial Gaussian distribution with
 302 center-of-mass at $z = \mu_0$ and spread σ_0 (provided the initial tracer distribution is isolated from the

303 boundary, $\sigma_0 \ll \mu_0$) is given by the sum of two Gaussians centered at $z = \mu_0$ and $z = -\mu_0$,

$$304 \quad \langle C \rangle^y(z, t) = A \frac{\sigma_0}{\sigma} \left(e^{-(z-\mu_0)^2/2\sigma^2} + e^{-(z+\mu_0)^2/2\sigma^2} \right), \quad (28)$$

$$305 \quad \text{where } \sigma^2(t) = \sigma_0^2 + 2\kappa t. \quad (29)$$

307 Due to the symmetry about $z = 0$, this solution satisfies the no flux boundary condition at $z = 0$,
308 while the individual Gaussians satisfy the one-dimensional diffusion equation without a boundary.

309 This analytic solution matches the numerical two-dimensional solution averaged in y (shown at
310 three different times by the orange curves in Fig. 3a).

311 The center-of-mass, variance and bulk diffusivity derived from the z -moments of this solution
312 (defined for $z > 0$ using Eq. (26) for the domain average) are,

$$313 \quad \mu_z = \frac{\langle zC \rangle}{\langle C \rangle} = \frac{2\sigma}{\sqrt{2\pi}} e^{-\mu_0^2/2\sigma^2} + \mu_0 \text{Erf}\left(\frac{\mu_0}{\sqrt{2\sigma^2}}\right) \quad (30)$$

$$314 \quad \sigma_z^2 = \frac{\langle z^2C \rangle}{\langle C \rangle} = \mu_0^2 + \sigma^2 - \mu_z^2 \quad (31)$$

$$315 \quad \kappa_{bk}^z = \frac{1}{2} \frac{\partial \sigma_z^2}{\partial t} \\ 316 \quad = \kappa \left(1 - \frac{2}{\pi} e^{-\mu_0^2/\sigma^2} - \frac{2\mu_0}{\sqrt{2\pi\sigma^2}} \text{Erf}\left(\frac{\mu_0}{\sqrt{2\sigma^2}}\right) e^{-\mu_0^2/2\sigma^2} \right) \quad (32)$$

318 At early times, when $\mu_z \gg \sigma$ the center-of-mass is stationary $\mu_z = \mu_0$, $\sigma_z^2 = \sigma^2$ and the bulk
319 diffusivity is equal to the actual diffusivity $\kappa_{bk}^z = \kappa$. However, as time increases, namely once
320 the tracer encounters the boundary such that $\sigma \sim \mu_z$, the center-of-mass moves away from the
321 boundary, eventually increasing like the square-root of time $\mu_z \sim \sqrt{t}$ (orange line in Fig. 3b). As
322 a result, the rate of increase of variance σ_z reduces (due to the μ_z^2 term in Eq. (31), orange line in
323 Fig. 3c), and the bulk diffusivity is reduced below κ (compare orange solid and dashed lines in
324 Fig. 3c). Hence the effect of the boundary is to reduce the bulk diffusivity by limiting downward
325 tracer spreading. In the limit of long time,

$$326 \quad \kappa_{bk}^z \rightarrow \left(1 - \frac{2}{\pi}\right) \kappa \approx 0.36\kappa, \quad \text{as } t \rightarrow \infty, \quad (33)$$

327 meaning that the bulk diffusivity is reduced by more than a factor of two. This is perhaps sur-
 328 prising, since it may be thought that the boundary prevents the spreading of half of the Gaussian
 329 tracer distribution. However, once the tracer encounters the boundary it no longer spreads like a
 330 half-Gaussian because its ‘reflection spreads into the region $z > 0$, resulting in an effective accu-
 331 mulation of tracer near the boundary.

332 *b. Bottom-intensified diffusivity*

333 When instead the diffusivity is bottom intensified ($\kappa_\infty \neq \kappa_0$, with $A_H = 0$) the solution cannot be
 334 obtained analytically. However, the influence of the boundary on the tracer diffusion can still be
 335 understood by deriving equations for the evolution of the z -moments directly from the tracer con-
 336 servation equation (this technique will also be used in later sections to analyze the tracer behavior
 337 in buoyancy coordinates). Multiplying Eq. (27) by z and integrating over the domain,

$$338 \quad \langle \mathcal{C} \rangle \frac{\partial \mu_z}{\partial t} = \frac{\partial \langle z \mathcal{C} \rangle}{\partial t} = \langle z \frac{\partial}{\partial z} (\kappa \mathcal{C}_z) \rangle, \quad (34)$$

$$339 \quad = \langle \frac{\partial}{\partial z} (z \kappa \mathcal{C}_z) \rangle + \langle -\kappa \mathcal{C}_z \rangle \quad (35)$$

$$340 \quad = \langle -\kappa \mathcal{C}_z \rangle, \quad (36)$$

341

342 where in the second line we have used the chain rule and the third line we have used the boundary
 343 condition at $z = 0$ to eliminate the first term. Eq. (36) simply states that the tracer center-of-mass
 344 will move according to the domain-averaged tracer flux. Furthermore, using the chain rule to move
 345 the z -derivative from \mathcal{C} to κ and the boundary condition to rewrite $\langle \partial_z(\kappa \mathcal{C}) \rangle$ in terms of the tracer
 346 concentration on the boundary yields,

$$347 \quad \langle \mathcal{C} \rangle \frac{\partial \mu_z}{\partial t} = \kappa_0 \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) dy + \langle \kappa_z \mathcal{C} \rangle. \quad (37)$$

348 The first term in Eq. (37) exposes the ‘‘boundary effect’’ discussed above: the boundary prevents
 349 the tracer flux in the negative z direction (once tracer accumulates there) and thus the flux in the

350 positive z direction dominates, driving the center-of-mass away from the boundary. A z -dependent
 351 diffusivity can also drive net tracer movement through the second term in Eq. (37). For a bottom-
 352 intensified diffusivity, this term enhances the downward tracer diffusion below the initial tracer
 353 patch driving the center-of-mass toward the boundary (dotted blue line in Fig. 3b). However, this
 354 downward motion is transient and the boundary effect dominates once enough tracer accumulates
 355 on the boundary (solid blue line in Fig. 3b).

356 An equation for the second z tracer moment can be derived using a procedure analogous to that
 357 employed for Eq. (36),

$$358 \quad \frac{\partial \langle z^2 \mathcal{C} \rangle}{\partial t} = 2 \langle -\kappa \mathcal{C}_z z \rangle. \quad (38)$$

360 The bulk diffusivity or rate of change of the variance is then,

$$361 \quad \kappa_{bk}^z = \frac{\langle -\kappa \mathcal{C}_z (z - \mu_z) \rangle}{\langle \mathcal{C} \rangle}, \quad (39)$$

363 I.e. κ_{bk}^z is given by the domain-averaged tracer fluxes weighted by their distance from the center-
 364 of-mass $z - \mu_z$. Furthermore, by shifting the z -derivative from the \mathcal{C} to κ , and absorbing the factor
 365 $(z - \mu_z)$,

$$366 \quad \kappa_{bk}^z = \frac{\langle \kappa \mathcal{C} \rangle}{\langle \mathcal{C} \rangle} - \frac{\mu_z \kappa_0}{\langle \mathcal{C} \rangle} \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) dy + \frac{\langle (z - \mu_z) \mathcal{C} \kappa_z \rangle}{\langle \mathcal{C} \rangle}. \quad (40)$$

368 Eq. (40) clearly shows that the bulk diffusivity κ_{bk}^z is not what would be expected from simply
 369 averaging the isotropic diffusivity κ over the tracer patch (or the *in-situ diffusivity* $\langle \kappa \mathcal{C} \rangle / \langle \mathcal{C} \rangle$,
 370 dashed lines in Fig. 3c, also see Mashayek et al. 2017). Instead, the boundary significantly *reduces*
 371 this spreading rate, once tracer encounters the boundary (compare solid and dashed lines in Fig.
 372 3c). This occurs because the boundary prevents tracer spreading in the negative z direction, and
 373 constricts the tracer to remain closer to its center-of-mass [Eq. (39)]. On the other hand, the
 374 vertical diffusivity gradient, when present, drives a modest enhancement of the bulk diffusivity

375 through the last term in Eq. (40) (dotted blue line in Fig. 3c). It should be noted that the terms in
 376 Eq. (40) [and Eq. (37)] are not independent; each term affects the tracer distribution \mathcal{C} on which
 377 they are all dependent.

378 **6. Diapycnal spreading: tracer behavior in b**

379 Though the upslope velocity V does not influence the spreading of the tracer in the slope normal
 380 direction z , it does affect the tracer spreading rate in the upslope direction y and across buoyancy
 381 surfaces. The two-dimensional evolution of tracer patches released at $z_0 = d/2 = 250\text{m}$ in flows
 382 with constant (Fig. 4a-c) and bottom-intensified (Fig. 4d-f) diffusivities clearly show the impacts
 383 of the boundary layer flow. With a constant diffusivity there is only upslope flow within the BBL.
 384 This flow drives tracer up the slope after which it diffuses vertically out of the BBL, forming
 385 a characteristic upslope tracer tongue (Fig. 4a-c). The center-of-mass (with upslope position
 386 $\mu_y = \langle y\mathcal{C} \rangle / \langle \mathcal{C} \rangle$) moves up the slope, at a rate determined by the tracer-weighted velocity,

$$387 \quad \frac{\partial \mu_y}{\partial t} = \frac{\langle V\mathcal{C} \rangle}{\langle \mathcal{C} \rangle}, \quad (41)$$

388
 389 (obtained by multiplying Eq. (13) by y and integrating over the domain), and slightly away from
 390 the boundary (μ_z increases, see closed circles in Fig. 4a-c). Thus the advective tracer transport in
 391 the BBL is significant despite its thinness. The expanding tracer patch continues to supply tracer to
 392 the BBL through downward diffusion, accounting for the large-amount of tracer advected upward
 393 within the BBL.

394 When the diffusivity is instead bottom-intensified (Fig. 4d-f), the upslope spreading of tracer is
 395 reduced due to both the restriction of diffusion to the region near the boundary and the presence
 396 of downslope transport in the SML (compare closed circles in Figs. 4c and 4f). Due to the shear
 397 in the SML velocity, the expanding interior tracer patch acquires a tilt.

398 The evolution of the tracer concentrations binned into buoyancy classes also indicates a strong
 399 upward net motion of the tracer distribution, in both cases (orange and blue distributions in Fig.
 400 5a), reflected in the upward motion of the center-of-mass (solid lines in Fig. 5b). In the following
 401 subsections we develop a framework based on the tracer moments to evaluate the contributions of
 402 various advective and diffusive tracer fluxes to the diapycnal tracer spreading.

403 *a. Advective and diffusive tracer fluxes and the buoyancy budget constraint*

404 Following the same procedure used in Section 5 for the one-dimensional case, we obtain the
 405 following equations for the buoyancy-moments of the tracer,

$$406 \quad \frac{\partial \langle bC \rangle}{\partial t} = \langle \mathbf{F}_C \cdot \nabla b \rangle, \quad (42)$$

$$407 \quad \frac{\partial \langle b^2 C \rangle}{\partial t} = 2 \langle b \mathbf{F}_C \cdot \nabla b \rangle, \quad (43)$$

$$408 \quad \kappa_{bk} = \frac{1}{N^2 \langle C \rangle} \langle (b - \mu_b) \mathbf{F}_C \cdot \nabla b \rangle, \quad (44)$$

409

410 where $\mu_b = \langle bC \rangle / \langle C \rangle$. Eq. (42) shows that the tracer center-of-mass moves across isopycnals if
 411 the domain-averaged diapycnal tracer flux is non-zero. The rate of change of the tracer variance
 412 in buoyancy space, proportional to κ_{bk} [Eq. (44)], is positive if the diapycnal fluxes move tracer
 413 on average away from the tracer center-of-mass μ_b . Noting that the along-isopycnal flux drops out
 414 yields contributions from diapycnal advection and isotropic diffusion,

$$415 \quad \frac{\partial \langle bC \rangle}{\partial t} = \langle C \mathbf{V} \cdot \nabla b \rangle - \langle \kappa \nabla C \cdot \nabla b \rangle, \quad (45)$$

$$416 \quad \kappa_{bk} = \frac{1}{N^4 \langle C \rangle} \langle (b - \mu_b) C \mathbf{V} \cdot \nabla b \rangle - \frac{1}{N^4 \langle C \rangle} \langle (b - \mu_b) \kappa \nabla C \cdot \nabla b \rangle. \quad (46)$$

417

418 Eqs. (45)-(46) can be further simplified by noting that from the buoyancy equation (vector form
 419 of Eq. (6)),

$$420 \quad \mathbf{V} \cdot \nabla b = -\nabla \cdot (-\kappa \nabla b). \quad (47)$$

421 Multiplying by \mathcal{C} , integrating over the domain and using the chain rule and boundary conditions,

$$422 \quad \langle \mathcal{C} \mathbf{V} \cdot \nabla b \rangle = -\langle \kappa \nabla \mathcal{C} \cdot \nabla b \rangle. \quad (48)$$

423 Thus the effects of advection and diffusion on the center-of-mass are identical such that Eq. (45)
424 can be written,

$$425 \quad \frac{\partial \langle b \mathcal{C} \rangle}{\partial t} = -2 \langle \kappa \nabla \mathcal{C} \cdot \nabla b \rangle. \quad (49)$$

426 This surprising result, that advection and diffusion both drive tracer on average in the same direc-
427 tion (despite their influence on buoyancy being exactly opposite), comes about because the tracer
428 is localized. This localization implies that the tracer gradient $\nabla \mathcal{C}$ must change sign, weighting the
429 diapycnal diffusive flux such that tracer is diffused diapycnally on average in the same direction
430 as it is advected diapycnally.

431 Similarly, multiplying Eq. (47) by $\mathcal{C}(b - \mu_b)$ and integrating over the domain yields,

$$432 \quad \langle (b - \mu_b) \mathcal{C} \mathbf{V} \cdot \nabla b \rangle = \langle (b - \mu_b) \mathcal{C} \nabla \cdot (\kappa \nabla b) \rangle, \quad (50)$$

$$433 \quad = -\langle \kappa \nabla b \cdot \nabla (\mathcal{C}(b - \mu_b)) \rangle, \quad (51)$$

$$434 \quad = -\langle (b - \mu_b) \kappa \nabla b \cdot \nabla \mathcal{C} \rangle - \langle \kappa \mathcal{C} |\nabla b|^2 \rangle. \quad (52)$$

435

436 Using Eq. (52), Eq. (46) for the bulk diffusivity can be rewritten as,

$$437 \quad \kappa_{bk} = -\frac{2}{N^4 \langle \mathcal{C} \rangle} \langle \kappa \nabla b \cdot \nabla (\mathcal{C}(b - \mu_b)) \rangle + \frac{1}{\langle \mathcal{C} \rangle} \langle \kappa \mathcal{C} \frac{|\nabla b|^2}{N^4} \rangle. \quad (53)$$

438 If the tracer is mostly outside of the BBL where $|\nabla b|^2 \approx N^4$ then the additional second term is the
439 in-situ diffusivity $\langle \kappa \mathcal{C} \rangle / \langle \mathcal{C} \rangle$. Eqs. (42)-(53) hold generally for any tracer when integrated over
440 a region with zero sources or boundary tracer fluxes. Eqs. (49) and (53) also hold even if the
441 buoyancy field is changing in time.

442 *b. Boundary and diffusivity gradient contributions to center-of-mass motion*

443 We now split the buoyancy gradient field from boundary layer theory [Eqs. (1) and (2)] into its
 444 background $\nabla B = (B_y, B_z) = N^2(\sin \theta, \cos \theta)$ and perturbation $\nabla b' = (0, b'_z)$ [Eq. (16)] compo-
 445 nents. Then, using the chain rule and the boundary conditions, the center-of-mass tendency [Eq.
 446 (45)] can be expressed as,

$$447 \quad \langle \mathcal{C} \rangle \frac{\partial \mu_b}{\partial t} = B_y \langle V \mathcal{C} \rangle + \kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) dy + B_z \langle \mathcal{C} \kappa_z \rangle - \langle \kappa \mathcal{C}_z b'_z \rangle, \quad (54)$$

448 exposing the influence of the boundary (second term on the RHS, which will also be referred to
 449 as the “boundary effect”) and the diffusivity gradient (third term on the RHS) discussed earlier in
 450 the one-dimensional context (Eq. (37), except here multiplied by B_z). The final term is associated
 451 with variations in the buoyancy gradient in the BBL and is negligible in our context (not shown).
 452 The first advective term, as discussed above, is equal to the sum of the three other diffusive terms.
 453 We can go even further by identifying the advective tracer fluxes in the BBL and SML with the
 454 boundary and diffusivity gradient terms respectively.

455 *c. BBL and SML contributions to center-of-mass motion*

456 Recall that the approximate analytic solution derived from one-dimensional boundary layer the-
 457 ory is obtained by combining two solutions (see Appendix A); one for the SML where the dif-
 458 fusivity varies (in the limit $(SPr_{u0})^{-1} \rightarrow 0$, Eqs. (67) and (68)), and one for the BBL where the
 459 diffusivity is constant and equal to κ_0 [Eq. (70)]. Therefore,

$$460 \quad \mathbf{V}_{BBL} \cdot \nabla b = \kappa_0 \nabla^2 b, \quad (55)$$

461 where $\mathbf{V}_{BBL} = (\partial \Psi^{BBL} / \partial z) \hat{\mathbf{y}}$. Multiplying by \mathcal{C} and integrating as for the derivation of Eq. (48),

$$462 \quad \langle \mathcal{C} \mathbf{V}_{BBL} \cdot \nabla b \rangle = -\kappa_0 \langle \nabla \mathcal{C} \cdot \nabla b \rangle = \kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) dy - \kappa_0 \langle \mathcal{C}_z b'_z \rangle. \quad (56)$$

463 Thus, the influence of diapycnal tracer advection in the BBL on μ_b is equivalent to the influence
 464 of the boundary term plus a minor correction due to the BBL buoyancy perturbation. In turn, this
 465 implies that tracer diapycnal advection in the SML is equivalent to the diffusivity gradient term
 466 plus an even smaller correction,

$$467 \quad \langle C \mathbf{V}_{SML} \cdot \nabla b \rangle = B_z \langle C \kappa_z \rangle - \langle (\kappa - \kappa_0) C_z b'_z \rangle. \quad (57)$$

468 Both of these results match intuition: the boundary effect drives the tracer center-of-mass upward
 469 in b whenever there is tracer within the BBL (by limiting tracer diffusion toward denser fluid), as
 470 does the upslope BBL diapycnal flow. Similarly, the diffusivity gradient drives the tracer center-
 471 of-mass toward denser fluid (κ_z is negative), as does the downslope SML diapycnal flow.

472 For a constant diffusivity there is no SML flow or diffusivity gradient, and so the center-of-mass
 473 moves toward lighter water due solely to the influence of BBL advection and the boundary effect,
 474 which contribute equally (orange dashed line in Fig. 5b, which when multiplied by 2 provides the
 475 full center-of-mass evolution μ_b , the solid line). In the bottom-intensified case, both the diffusivity
 476 gradient and the downslope SML transport drive the tracer center-of-mass downward initially
 477 (dotted blue line in Fig. 5b, which again should be multiplied by 2 to account for influence of
 478 the SML transport), but the boundary effect and advection within the BBL quickly dominates
 479 driving the tracer toward less dense fluid on average (solid blue line in Fig. 5b).

480 *d. BBL and SML contributions to the bulk diffusivity*

481 Following the same procedures as for the center-of-mass, the bulk diffusivity [Eq. (46)] can be
 482 split into contributions from a number of different processes,

$$\begin{aligned}
 \kappa_{bk} = & \frac{\langle \kappa \mathcal{C} \rangle}{\langle \mathcal{C} \rangle} + \frac{1}{N^4 \langle \mathcal{C} \rangle} \left[B_y \langle (b - \mu_b) V \mathcal{C} \rangle \right. \\
 & + \kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) (b(y, 0) - \mu_b) dy + B_z \langle \mathcal{C} (b - \mu_b) \kappa_z \rangle \\
 & \left. - \langle (b - \mu_b) \kappa \mathcal{C}_z b'_z \rangle + B_z \langle \mathcal{C} \kappa b'_z \rangle \right]. \tag{58}
 \end{aligned}$$

487 As for the simple one-dimensional case considered in Section 5, this equation clearly shows that
 488 the bulk diffusivity characterizing the overall spreading rate of tracer across isopycnals is not
 489 simply equal to the tracer-weighted isotropic diffusivity or in-situ diffusivity (first term on the
 490 RHS, compare solid with dashed lines in Fig. 5c). Instead, there are contributions from advection,
 491 from the accumulation of tracer on the boundary, from the vertical gradient in the diffusivity and
 492 from the BBL buoyancy perturbation (these last two terms in Eq. (58) related to b'_z are negligible,
 493 less than $5 \times 10^{-5} \text{m}^2 \text{s}^{-1}$, in all cases considered here). Once again, the advective term can be
 494 split into contributions from the BBL and SML, which identify with the diffusive boundary term
 495 and the diffusivity gradient term in Eq. (58) respectively³. Eq. (58) provides a useful diagnostic
 496 for examining the tracer dispersion. However, it should be noted that the terms are not mutually
 497 independent as they all affect and depend on the tracer concentration \mathcal{C} .

³Using Eq. (55) in the advective flux term in Eq. (46),

$$\langle (b - \mu_b) \mathcal{C} \mathbf{V}_{BBL} \cdot \nabla b \rangle = \kappa_0 \langle (b - \mu_b) \mathcal{C} \nabla^2 b \rangle \tag{59}$$

$$= -\kappa_0 \langle \nabla (\mathcal{C} (b - \mu_b)) \cdot \nabla b \rangle \tag{60}$$

$$= \kappa_0 B_z \int_{-\infty}^{\infty} \mathcal{C}(y, 0, t) (b(y, 0, t) - \mu_b) dy - \kappa_0 \langle \frac{\partial}{\partial z} (\mathcal{C} (b - \mu_b)) b'_z \rangle, \tag{61}$$

where we have once again used the chain rule and the boundary conditions. The last term is a minor correction dependent on b'_z . In turn Eq. (61) implies that the influence of SML advection is equal to the diffusivity gradient effect (plus another minor correction dependent on b'_z).

498 In the case of a constant diffusivity (orange line in Fig. 5c) the main factor influencing the
 499 spreading rate of the tracer across isopycnals, apart from the in-situ diffusivity, is the boundary
 500 effect (and equivalently BBL advection). While the influence of the boundary on the center-of-
 501 mass motion is always to push the center-of-mass toward less dense fluid (the second RHS term in
 502 Eq. (54) is always positive), its influence on the tracer dispersion rate κ_{bk} (second term inside the
 503 square brackets in Eq. (58)) can have either sign due to the $(b - \mu_b)$ weighting factor. In fact, this
 504 term can be further manipulated to yield,

$$505 \quad \frac{\kappa_0 B_z}{N^4 \langle C \rangle} \left(\int_{-\infty}^{\infty} C(0)b(0)dy - \mu_b \int_{-\infty}^{\infty} C(0)dy \right) = \frac{\kappa_0 B_z}{N^4 \langle C \rangle} \langle C(0) \rangle^y (\mu_b^0 - \mu_b), \quad (62)$$

506 where $C(0)$ and $b(0)$ are shorthand for the tracer and buoyancy on the boundary $C(y,0,t)$ and
 507 $b(y,0)$, and $\mu_b^0 = \langle b(0)C(0) \rangle^y / \langle C(0) \rangle^y$ is the centroid buoyancy of the tracer on the boundary.
 508 Thus the boundary effect will generally reduce κ_{bk} if μ_b^0 is at a denser level than μ_b (the usual case,
 509 compare closed circles and crosses in Fig. 4)⁴. Its magnitude depends on the separation between
 510 μ_b^0 and μ_b and the amount of tracer on the boundary $\langle C(0) \rangle^y$. Once again, this boundary effect
 511 is equivalent to the influence of BBL advection and can also be understood from this perspective;
 512 BBL advection will generally reduce the diapycnal spreading rate as it moves more tracer toward
 513 the center-of-mass than away from the center-of-mass (e.g. see Fig. 4a,d).

514 The reduction in the bulk diffusivity κ_{bk} below the in-situ diffusivity (compare dashed and solid
 515 lines in Fig. 5c) due to the boundary effect occurs as soon as the tracer encounters the boundary.
 516 The behavior of the constant and bottom-intensified diffusivity cases are similar, apart from the
 517 magnitude of the initial, and in-situ, diffusivities (compare blue and orange lines in Fig. 5c). This
 518 is because the influence of SML advection and the diffusivity gradient are weak (dotted blue line
 519 in Fig. 5c). These terms act to drive a modest enhancement of the diapycnal spreading rate as

⁴Of course bizarre tracer distributions where the tracer is in contact with the boundary only above the tracer center-of-mass μ_b are possible, if unlikely, in which case the boundary effect would enhance κ_{bk} .

520 the diapycnal SML velocity is *divergent* around the tracer patch due to the positive curvature in
521 κ (since N^2 is constant). It is the curvature of κ that matters, despite the appearance of only the
522 first-derivative of κ in Eq. (58), because of the weighting factor $(b - \mu_b)$.

523 *e. Summary*

524 To summarize, the main effect highlighted by these near-boundary point-release tracer experi-
525 ments is a strong reduction in the diapycnal spreading rate, below that which would be expected
526 from in-situ measurements of the isotropic diffusivity averaged over the tracer patch⁵, due to the
527 presence of the boundary which limits tracer diffusion toward denser water. Accompanying this
528 reduced spreading rate is a tendency for the overall tracer patch, quantified by its center-of-mass,
529 to move up the slope toward less dense fluid. We also showed that due to the buoyancy budget
530 constraint the advective and diffusive fluxes have equivalent influences on the tracer dispersion
531 [Eqs. (49) and (53)], as do their individual BBL and SML contributions. We will discuss these
532 results in the context of field TREs in Section 8. However, in the next section we first examine the
533 sensitivity of the tracer dispersion to several other parameters.

534 **7. Sensitivity to other parameters**

535 *a. Release Location*

536 In the previous two sections we showed that the presence of the boundary significantly reduces
537 the diapycnal spreading rate of the tracer, and alters the tracer center-of-mass motion. The time-
538 scale over which these effects appear depends on where the tracer is released. When released
539 further from the boundary (e.g. at $z_0 = 3d/4$, Fig. 6a-c) the tracer initially spreads according

⁵Note that this conclusion is not altered if alternative methods of estimating the in-situ diffusivity are considered. For example, the *tracer-gradient* weighted diffusivity $\langle \kappa |C_z| \rangle / \langle |C_z| \rangle$ gives a similar estimate (not shown) consistent with Mashayek et al. (2017).

540 to the isotropic diffusivity, with enhanced downward diffusion due to bottom-intensification and
541 the downslope SML flow (orange solid line in Fig. 7b). There is an initial slight increase in the
542 bulk diffusivity (solid orange line in Fig. 7c before day ~ 250), due to the fact that the SML
543 flow is divergent (dotted orange line in Fig. 7c, which represents the equivalent influence of the
544 diffusivity gradient). However, once the tracer encounters the boundary the diapycnal spreading
545 rate is significantly reduced.

546 The reduction in the diapycnal spreading rate occurs earlier for tracers released closer to the
547 boundary (e.g. at $z_0 = 62.5\text{m}$, Figs. 6d-f and blue lines in Fig. 7). While tracers released very
548 close to the boundary visually appear to experience more overall dispersion (compare Figs. 6f and
549 6c), most of this dispersion manifests in the lateral direction (despite the absence of any along-
550 isopycnal diffusion) with less spreading across buoyancy surfaces (compare solid blue and orange
551 lines in Fig. 7c). Furthermore, when released close to the boundary the tracer experiences a large
552 upward movement of both its center-of-mass (blue line in Fig. 7b) and overall distribution, such
553 that most of the tracer lies at buoyancies well above its initial buoyancy after 320 days (blue lines
554 in Fig. 7a). This net movement toward less dense fluid is a consequence of the upslope BBL flow
555 and the asymmetric distribution of tracer between the BBL and SML. The latter asymmetry is due
556 to the boundary, which tends to trap tracer in the BBL. The net upward movement of tracer is
557 strongest when the tracer is released next to the boundary, while tracers released away from the
558 boundary in the SML experience a net downward movement (Fig. 8). Field TREs conducted very
559 close to a sloping boundary may be expected to show a similar net diapycnal upward movement of
560 tracer (e.g. Inall 2009). However, in Section 7e we show that the introduction of along-isopycnal
561 diffusion can limit this upward movement.

562 *b. Topographic Slope*

563 Though the position of the initial release point of the tracer clearly impacts the initial behavior
564 of the tracer, the slope of the boundary does not in these point release experiments. Altering the
565 slope of the boundary has a number of effects on the boundary layer flow. Firstly, it influences the
566 thickness of the BBL q_0^{-1} , with steeper slopes having thinner BBLs (a relatively minor effect due
567 to the fourth-power exponent in Eq. (9), also see Callies and Ferrari 2018). Secondly, it alters the
568 strength of the upslope and downslope boundary layer flows (linearly, due to the factor of $\cot \theta$ in
569 Eq. (11), also compare orange and blue solid lines in Fig. 2a and see Dell and Pratt 2015). The
570 change in the upslope and downslope flow strength arises because the local diapycnal advective
571 flux of buoyancy must remain the same (as the diffusion has not changed), but the separation of
572 buoyancy surfaces in the direction parallel to the boundary changes. Comparing tracer releases
573 performed over a slope of 1/100 versus 1/400 shows significant visual differences in the rate of
574 tracer dispersal (Fig. 9). However, the rate of tracer spreading across buoyancy surfaces is in fact
575 almost identical (not shown), because the magnitude of the diapycnal velocity does not change,
576 being set by diffusion. Instead, the spacing of buoyancy surfaces in the y direction ($B_y = N^2 \sin \theta$)
577 increases in proportion to the decrease in upslope flow ($V \sim \cot \theta$), such that the total advective
578 flux of tracer $\langle CV \cdot \nabla b \rangle = B_y \langle VC \rangle$ remains the same. However, this result only holds for cases
579 where the tracer is released at a single point. When the tracer is instead released as a horizontal
580 layer, a case considered in Appendix C, the slope of the boundary does influence the spreading
581 rate as it influences the width of the SML and thus the extent to which the tracer is exposed to
582 strong boundary mixing.

583 *c. Reduced mixing layer stratification*

584 Throughout most of this article we have taken the limit $SPr_{u0}^{-1} \rightarrow 0$, where eddies are assumed
585 to flatten isopycnals in the SML and maintain the stratification there. However, observations from
586 the Brazil Basin suggest that isopycnals are often sloped in the active mixing layer and the strat-
587 ification is reduced (St. Laurent et al. 2001; Ledwell et al. 2000). Callies (2018) found that the
588 observed Brazil Basin stratification was well fitted by the one-dimensional boundary layer solu-
589 tion for a Prandtl number of 230, yielding $SPr_{u0}^{-1} \approx 2$. This corresponds to a three-fold reduction
590 in the SML stratification⁶, with a similar reduction in the upslope transport Ψ (Eq. (71) in Ap-
591 pendix A)⁷. This reduction in upslope transport reduces the upslope spreading of tracer (compare
592 Figs. 9d-f and 9g-i) and the net movement of tracer toward lighter fluid (as the upslope buoyancy
593 gradient b_y remains the same). The rate of increase of the variance of the tracer in buoyancy space
594 also reduces. However, the bulk diffusivity includes a stratification normalization factor [Eq. (23)]
595 which, as the tracer does not extend beyond the SML, in this case should be chosen as the SML
596 stratification $N^2/(1 + SPr_{u0}^{-1})$. With this normalization factor the bulk diffusivity and its contri-
597 butions from the boundary effect, the diffusivity gradient and BBL and SML advection remain
598 almost compared with the $SPr_{u0}^{-1} = 0$ case (not shown). This implies that the slope of the isopyc-
599 nals (independent of the change in N^2) has only a minor impact on the tracer dispersion. However,
600 this may no longer be true for tracer clouds that extend across the entire SML into the far-field.

601 *d. Decay scale of bottom-intensified mixing*

602 The tracer dispersion is also sensitive to the decay scale used for the isotropic diffusivity [d in
603 Eq. (10)]. When the tracer is released at the same distance from the boundary but this d scale is

⁶Note that all our tracer experiments are conducted within the SML and do not extend into the far-field, defined by the distance $d \log(\kappa_0/\kappa_\infty)$

(Callies 2018) where the stratification does eventually return to N^2 .

⁷There is also a small factor of $3^{1/4}$ modification to the BBL scale q_0 (Eq. (9)).

604 reduced to 200m (a value that may be more representative of the Brazil Basin, St. Laurent et al.
605 2001; Callies 2018) then the tracer remains restricted closer to the boundary and the net upslope
606 movement of the tracer toward lighter buoyancy is reduced (compare Figs. 9j-l with 9a-c). The
607 reduction in the center-of-mass movement in buoyancy space is due to the increase in the vertical
608 gradient of the isotropic diffusivity κ_z , or equivalently an increase in the SML transport, that
609 drives more tracer downward (compare blue and orange dotted and dashed lines in Fig. 10b). This
610 increase in the vertical gradient of the diffusivity also acts to enhance the tracer spreading rate in
611 buoyancy space such that the bulk diffusivity does not reduce as much as would be expected from
612 the reduced in-situ diffusivity (compare orange and blue lines in Fig. 10c).

613 *e. Influence of along-isopycnal diffusion*

614 Finally we consider the impact of along-isopycnal diffusion on the tracer behavior. Large along-
615 isopycnal diffusion may be expected in abyssal mixing layers due to processes such as intrusions
616 (e.g. McPhee-Shaw 2006) or baroclinic instability setup by boundary mixing (e.g. Callies 2018;
617 Wenegrat et al. 2018). Along-isopycnal diffusion can rapidly mix tracer between the SML and
618 BBL, an effect which is clear when comparing TREs with large ($A_H = 100\text{m}^2\text{s}^{-1}$, Fig. 11d-f) and
619 small ($A_H = 10\text{m}^2\text{s}^{-1}$, Fig. 11a-c) along-isopycnal diffusivities. The along-isopycnal diffusivity
620 also appears to influence the spreading of the tracer in buoyancy space, despite the inability of
621 along-isopycnal diffusion to directly flux tracer across isopycnals (Fig. 12). At early times, the
622 enhanced along-isopycnal diffusion limits the initial downward motion of the center-of-mass by
623 mixing tracer more rapidly into the BBL, enhancing the boundary effect (compare orange and
624 blue solid and dashed lines in Fig. 12b before day 500). However, at later times along-isopycnal
625 diffusion limits the motion of the center-of-mass up the slope toward less dense fluid as the tracer
626 is mixed between the regions of upslope and downslope motion in the BBL and SML respectively.

627 In terms of the framework introduced in Section 6, the limited upslope motion of the center-of-
628 mass arises because of a reduction in the influence of BBL advection (or equivalently the boundary
629 effect, compare dashed lines in Fig. 12b after day 1000), as there is less tracer in contact with the
630 boundary.

631 Similarly, the reduction of the influence of the boundary, or BBL advection, on the tracer distri-
632 bution under strong along-isopycnal diffusion results in an enhanced rate of diapycnal spreading
633 as quantified by κ_{bk} (compare solid blue and orange lines in Fig. 12c). The magnitude of the
634 boundary effect responsible for the reduction in κ_{bk} is dependent on two factors [Eq. (62)]; 1) the
635 amount of tracer on the boundary $\langle C(0) \rangle^y$ and 2) the buoyancy spacing between the total tracer
636 center-of-mass μ_b and that on the boundary μ_b^0 . Large along-isopycnal diffusion decreases both
637 of these factors by moving tracer away from the boundary on average and by homogenizing the
638 tracer between the boundary and SML such that $\mu_b^0 - \mu_b$ is reduced (compare positions of closed
639 circles and crosses in Fig. 11). The second of these two factors generally dominates (not shown).

640 A secondary effect of along-isopycnal diffusion is to reduce the in-situ diffusivity averaged
641 over the tracer patch (compare dashed blue and orange lines in Fig. 12c) by driving a net lateral
642 movement of tracer toward the weakly mixing interior. This net lateral movement toward the far-
643 field with along-isopycnal diffusion is once again due to the presence of the boundary that prevents
644 tracer transport through it.

645 The results of these experiments are summarized in Fig. 13, where three measures of the time-
646 averaged or cumulative diffusivity up to day 800 are shown for a range of along-isopycnal dif-
647 fusivities⁸. For all values of A_H the total diapycnal spreading rate of tracer across-isopycnals, as
648 quantified by the bulk diffusivity, is smaller than the in-situ diffusivity (compare circles and crosses

⁸We do not include a similar summary figure for the dependence of the center-of-mass motion on A_H as this depends strongly on the time-scale of interest (compare blue and orange solid lines in Fig. 12b).

649 in Fig. 13). As along-isopycnal diffusion increases the diapycnal spreading rate increases and the
650 in-situ diffusivity decreases. The enhancement of diapycnal tracer fluxes by along-isopycnal diffu-
651 sion (which has no influence on buoyancy) is a salient result which stems from the presence of the
652 boundary and interactions between the tracer gradients and diapycnal diffusion. Finally, a more
653 sophisticated estimate of the tracer derived diffusivity discussed in Appendix B, based on a least-
654 squares fit to the results of a one-dimensional advection-diffusion equation, gives similar results
655 to the bulk diffusivity (compare triangles and circles in Fig. 13), meaning that if this estimate was
656 interpreted as a measure of the in-situ diffusivity it would be an under-estimate.

657 **8. Relation to field experiments**

658 Our study has focused on tracer behavior in the immediate boundary region, as opposed to past
659 field TREs such as BBTRE and DIMES. Nevertheless, it is useful to discuss our results in the
660 context of these past experiments. The BBTRE tracer (Ledwell et al. 2000) was released along
661 a target isopycnal that sat about 1000m above the bottom of a fracture zone trough. Much of
662 the tracer cloud thus lay in the outer SML, although because of topographic variability some
663 tracer was located much closer to the boundary (Fig. 4 of Ledwell et al. 2000). The initial tracer
664 dispersion was well modeled with a one-dimensional advection-diffusion equation and showed a
665 tendency to move downward toward denser water (Fig. 2a of Ledwell et al. 2000), consistent with
666 our results for bottom-intensified mixing (e.g. Fig. 6a-c). A smaller portion of the tracer was
667 also drawn eastward and more strongly downward toward the MAR boundary. Once significant
668 amounts of tracer came into contact with the boundary Ledwell et al. (2000) point out that the
669 tracer tended to get mixed back toward lighter density levels, consistent with the boundary effect
670 presented in this paper. During this later time period Ledwell et al. (2000) do not attempt to use
671 a one-dimensional model for these later periods of BBTRE, as it would likely underestimate the

672 diffusivity. We have applied such a one-dimensional model to our idealized experiments and found
673 that it provides a similar estimate of the diapycnal tracer spreading rate to the bulk diffusivity κ_{bk}
674 (see Appendix B), and therefore indeed underestimates the in-situ diffusivity $\langle \kappa C \rangle / \langle C \rangle$. In our
675 case this underestimate is roughly a factor of 3 (Fig. A3). However, in the BBTRE case (all else
676 being equal) we would expect a more accurate result as a larger proportion of the tracer lies in
677 the interior where the boundary has less impact. In Appendix C we examine tracers released as a
678 strip extending further into the interior where we indeed find that the boundary has less influence.
679 However, we also find that a one-dimensional model has some difficulty modeling this case due to
680 the spatial variation in the isotropic diffusivity across the tracer patch.

681 Our results have less immediate applicability to DIMES, given the topographic complexity and
682 strong mean flows characterizing the Southern Ocean. In particular, we note that the order-of-
683 magnitude mismatch between diffusivities estimated from the DIMES tracer and microstructure
684 observations cannot be explained using the physics discussed here. The boundary effect discussed
685 in this study would suggest that in-situ estimates should exceed those obtained from the tracer
686 measurements rather than the other way around, and by less than an order-of-magnitude.

687 While neither BBTRE or DIMES were specifically designed to study near boundary tracer dis-
688 persion, future experiments are planned with this aim. The relations derived in this article may help
689 in the analysis of these experiments. Firstly, the realization that there is some equivalence between
690 the effects of diapycnal advection and diffusion on the tracer moments in buoyancy space [e.g.
691 Eqs. (49) and (53), which hold generally providing boundary tracer fluxes and interior sources can
692 be eliminated] may allow some calculations to be simplified. For example, if the change in the
693 center-of-mass density of the tracer patch $\Delta \langle bC \rangle$ over a specified time period is measured, then by

694 time-integrating Eq. (49), using the chain rule and eliminating the boundary term,

$$695 \quad \overline{\langle \mathcal{C} \nabla \cdot (\kappa \nabla b) \rangle} = \frac{\Delta \langle b \mathcal{C} \rangle}{2}, \quad (63)$$

696 where the over line indicates a time-integral. Eq. (63) allows the convergence of the diffusive
697 buoyancy flux, or net buoyancy source Db/Dt , averaged over the tracer patch to be easily obtained.

698 Secondly, if detailed information is available (perhaps as part of a series of surveys of the initial
699 spreading of tracer) then it may be possible to utilize some of our relations to better infer properties
700 of the underlying turbulent diffusivity. For example, if a structure $f(\tilde{z})$ of the turbulent diffusivity
701 as a function of distance above bottom \tilde{z} was assumed (such that $\kappa(\tilde{z}) = \kappa_0 f(\tilde{z})$ with $f(0) = 1$), then
702 substitution into Eq. (58) and some rearrangement (where we have also, for the point of argument,
703 ignored the terms dependent on b'_z , and removed the advective term in favor of multiplying the
704 diffusive terms by 2 as shown in Eq. (53)) yields,

$$705 \quad \kappa_0 \approx \langle \mathcal{C} \rangle \kappa_{bk} \left\{ \langle f \mathcal{C} \rangle + 2N^{-2} \cos \theta \left[\langle \mathcal{C}(0) (b(0) - \mu_b) \rangle^y + \langle \mathcal{C}(b - \mu_b) f_z \rangle \right] \right\}^{-1}. \quad (64)$$

706 If the three-dimensional tracer moments $\langle b \mathcal{C} \rangle$, $\langle b^2 \mathcal{C} \rangle$, κ_{bk} , along with the shape function cor-
707 relations $\langle f \mathcal{C} \rangle$, $\langle \mathcal{C}(b - \mu_b) f_z \rangle$ and the concentrations of tracer near the boundary $\langle \mathcal{C}(0) \rangle^y$ and
708 $\langle \mathcal{C}(0) b(0) \rangle^y$ can be measured, then Eq. (64) provides an estimate of the peak near-bottom diffu-
709 sivity κ_0 taking into account the boundary and κ_z effects. Nevertheless, it remains unclear whether
710 these time- and space-averaged tracer correlations can be estimated with sufficient accuracy given
711 sampling limitations in the field.

712 Finally, it may also be possible to use the knowledge and relations gained from this study to build
713 an intermediate complexity prognostic model of the tracer dispersion that is simplified relative to
714 the full 2D or 3D problem but takes into account some of the boundary effects neglected by the 1D
715 advection-diffusion model considered in Appendix B. A more comprehensive inverse model along
716 these lines may allow better estimates of the properties of the small-scale turbulent diffusivity to

717 be obtained from the sparsely sampled tracer data. The development of such a model is, however,
718 outside the scope of this article.

719 **9. Summary**

720 We have examined the behavior of a passive tracer released near a sloping boundary within a
721 flow governed by one-dimensional boundary layer theory. Results can be summarized as follows:

722 1. For isolated near-boundary tracer releases the presence of the *boundary reduces the net di-*
723 *apycnal tracer spreading rate* below that which would be expected from averaging the in-situ
724 diffusivity over the tracer patch (Figs. 5c and 13).

725 2. The dipole of diapycnal flow, upward in the BBL and downward in the SML, also influences
726 the tracer dispersion. In particular, when the tracer is released close to the boundary it tends
727 to move *upslope toward less dense fluid on average* (e.g. Fig. 4d-f) due to the asymmetric
728 distribution of diapycnal flow between the BBL and SML (the SML flow being spread over
729 a much wider region). However, the extent of this upslope motion depends on the proximity
730 of the tracer release point to the boundary (Fig. 6) with tracers released further from the
731 boundary in the outer SML experiencing a downward net diapycnal motion (as summarized
732 by Fig. 8).

733 3. As a consequence of the advection-diffusion balance in the buoyancy equation, there is an
734 *equivalence between the diffusive and advective tracer fluxes*; they both contribute equally to
735 the domain-averaged diapycnal tracer flux [Eq. (48)]. Further, the slowing of the diapycnal
736 tracer spreading rate due to the presence of the boundary is equivalent to the influence of tracer
737 advection within the BBL, while the modest enhancement in tracer spreading associated with

738 the divergent diapycnal flow in the SML is equivalent to the diffusive influence of the gradient
739 in the isotropic diffusivity.

740 4. For point release experiments the boundary slope has little effect on the diapycnal spreading
741 rate.

742 5. The introduction of reduced stratification and sloping isopycnals in the SML (Callies 2018)
743 reduces the rate at which tracer spreads across isopycnals. However, as a diffusivity measures
744 the rate of tracer spreading in physical space the bulk diffusivity does not change.

745 6. When the decay scale of bottom-intensified mixing is decreased then the overall tracer spread-
746 ing across isopycnals decreases. However, this decrease is not as much as would be expected
747 from the change in in-situ diffusivity because the SML transport becomes more divergent.

748 7. *Along-isopycnal diffusion enhances the diapycnal tracer spreading rate* for near-boundary
749 point releases by reducing the damping influence of the boundary and BBL advection (Fig.
750 13). Along-isopycnal diffusion also drives more tracer away from the boundary to where
751 mixing is weak, reducing the in-situ diffusivity averaged over the tracer patch.

752 While the differences in diffusivity arising from the additional boundary effects exposed in this
753 study are generally less than an order of magnitude, our results nevertheless highlight the complex-
754 ity of tracer behavior near sloping boundaries. Additional complexities present in the observational
755 context likely make matters worse. Variations in topographic slope, mixing intensity and stratifi-
756 cation are likely to drive additional tracer transports. Submeso- to large-scale currents will also
757 drive interactions between the tracer patch and different boundaries at a range of depths, buoyan-
758 cies and times. It is possible that averaging over these complexities will result in smooth Gaussian
759 tracer spreading across buoyancy surfaces with a well-defined central diffusivity. However, our
760 results suggest that relating this tracer-derived diffusivity to the small-scale turbulent diffusivity

761 important for buoyancy and mass transport must be done with care. In Section 8 we discussed
762 several avenues that may help with the analysis of future near-boundary TREs.

763 Further modeling work is also needed to assess the impact of additional complexities. For exam-
764 ple, a large *net* up- or down-slope mass transport associated with upslope variations in topography,
765 stratification or mixing may influence the tracer behavior. The role of intrusions of boundary layer
766 fluid into the interior (e.g., Gloor et al. 2000) is another area that deserves further attention. In this
767 article, the influence of such intrusions on tracer transport was captured at first-order through an
768 along-isopycnal diffusivity. Such an approximation may be appropriate for the statistical average
769 tracer behavior, but is less applicable to a tracer released, for example, at a particular phase of a
770 passing eddy. Finally, the relationship between the along-isopycnal diffusion and the eddy-driven
771 overturning required to maintain a stratified SML in rotating boundary layer theory (e.g. Callies
772 2018), and whether this influences the tracer transport, is worthy of further exploration. A deep
773 understanding of these processes is needed in order to represent them appropriately in general
774 circulation models.

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780 **Appendix A: The approximate analytic solution**

781 Here we present the derivation of the approximate analytic solution to one-dimensional bound-
782 ary layer theory with a bottom-intensified diffusivity [Eq. (10)] following Callies (2018) (note
783 that our derivation differs from Callies (2018) through the use of distinct viscosities in the x and y

784 directions). We non-dimensionalize the boundary layer equations using,

$$785 \quad \kappa = \kappa_0 \hat{\kappa}, \quad v_u = v_{u0} \hat{\kappa}, \quad v_v = v_{v0} \hat{\kappa}, \quad z = d \hat{z}, \quad \Psi = \kappa_0 \cot \theta \hat{\Psi}, \quad (65)$$

786 where κ_0 , v_{u0} and v_{v0} represent the near-boundary diffusivity and viscosity. Eq. (7) then becomes,

$$787 \quad \frac{d^2}{d\hat{z}^2} \left(\hat{\kappa} \frac{d^2 \hat{\Psi}}{d\hat{z}^2} \right) + 4(dq_0)^4 \left(\frac{\hat{\Psi}}{\hat{\kappa}} - \frac{1 + (SPr_{u0})^{-1} \frac{r}{\hat{\kappa}}}{1 + (SPr_{u0})^{-1}} \right) = 0, \quad (66)$$

788 where $r = \kappa_\infty / \kappa_0$, q_0 is the BBL width parameter based on the mixing coefficients near the bound-
 789 ary [Eq. (9)] and we have assumed constant Prandtl numbers Pr_{u0} and Pr_{v0} . For reasonable deep
 790 ocean parameters $q_0^{-1} \sim 4.5\text{m}$ (or $q_0^{-1} = 28\text{m}$ for small $(SPr_{u0})^{-1}$ where q_0 is determined by up-
 791 slope friction only, see Eq. (9)). Therefore, with $d \sim 500\text{m}$, $dq_0 \sim 20$ the fourth derivative term
 792 in Eq. (66) can be ignored outside the thin BBL. Following Garrett (2001), a solution outside the
 793 BBL is therefore (restoring the dimensions),

$$794 \quad \Psi^{SML} = \frac{\cot \theta}{1 + (SPr_{u0})^{-1}} \left(\kappa + (SPr_{u0})^{-1} \kappa_\infty \right) \quad (67)$$

$$795 \quad \frac{\partial b^{SML}}{\partial z} = N^2 \frac{\cos \theta}{1 + (SPr_{u0})^{-1}} \left(1 + (SPr_{u0})^{-1} \frac{\kappa_\infty}{\kappa} \right) \quad (68)$$

797 For large $(SPr_{u0})^{-1}$, this gives much reduced stratification over the entire SML until κ approaches
 798 κ_∞ (Callies 2018).

799 Eqs. (67) and (68) provide a solution in the SML, but do not satisfy the boundary conditions
 800 $\Psi = 0$ and $\partial \Psi / \partial z = 0$ at $z = 0$. To satisfy these boundary conditions we introduce an inner BBL
 801 solution where the fourth derivative term in Eq. (66) becomes important. For this inner solution
 802 we can make the assumption that the mixing coefficients are constant $\kappa \sim \kappa_0$, $v_u \sim v_{u0}$, $v_v \sim v_{v0}$.
 803 With $\Psi = \Psi^{BBL} + \Psi^{SML}$, where Ψ^{SML} is assumed constant in the BBL, substitution into Eq. (7)
 804 yields,

$$805 \quad \frac{d^4 \Psi^{BBL}}{dz^4} + 4q_0^4 \Psi^{BBL} = 0, \quad (69)$$

806 The solution that satisfies the boundary condition $\Psi^{BBL}(0) = -\Psi^{SML}(0)$ and $\partial\Psi^{BBL}/\partial z(0) = 0$
 807 (neglecting the small $\partial\Psi^{SML}/\partial z$ at $z = 0$) is given by,

$$808 \quad \Psi^{BBL} = -\frac{\cot\theta}{1 + (SPr_{u0})^{-1}} (\kappa_0 + (SPr_{u0})^{-1}\kappa_\infty) e^{-q_0 z} (\cos q_0 z + \sin q_0 z), \quad (70)$$

809 so that the full approximate solution is,

$$810 \quad \Psi = \frac{\cot\theta}{1 + (SPr_{u0})^{-1}} (\kappa + (SPr_{u0})^{-1}\kappa_\infty) (1 - e^{-q_0 z} (\cos q_0 z + \sin q_0 z)), \quad (71)$$

811 where we have also used the fact that $\kappa_0 e^{-q_0 z} (\cos q_0 z + \sin q_0 z) \approx \kappa e^{-q_0 z} (\cos q_0 z + \sin q_0 z)$. In the
 812 limit $(SPr_{u0})^{-1} \rightarrow 0$ Eq. (71) reduces to Eq. (11), which will be used to determine the upslope
 813 velocity $V = \partial\Psi/\partial z$ and buoyancy field $b_z = N^2 \sin\theta \Psi/\kappa$ for the 2D tracer release experiments.

814 **Appendix B: A one-dimensional model of the tracer dispersion across isopycnals**

815 In this appendix we apply a one-dimensional model, commonly used in field TREs (e.g. Ledwell
 816 and Watson 1991; Ledwell 1998) to the evolution of the tracer distribution in buoyancy space. The
 817 tracer evolution is modeled using the one-dimensional equation,

$$818 \quad \frac{\partial\bar{C}}{\partial t} + (\bar{w} - \bar{\kappa}_h) \frac{\partial\bar{C}}{\partial h} = \bar{\kappa} \frac{\partial^2\bar{C}}{\partial h^2}, \quad (72)$$

819 where the over line denotes an area-average on isopycnals at a given height h above a target
 820 buoyancy surface. A mean stratification profile is used to convert between h and b . The area-
 821 averaged diffusivity is assumed to be a linear function of h , $\bar{\kappa} = \bar{\kappa}_0 + h\bar{\kappa}_h$. Following Ledwell and
 822 Watson (1991) and Ledwell (1998), we discretize Eq. (72) using a forward-in-time, centered-in-
 823 space discretization and minimize the sum of squares,

$$824 \quad \sum_h \left(C_n^{obs} - L^n C_0 \right), \quad (73)$$

825 with respect to the three parameters $\bar{\kappa}_0$, \bar{w} and $\bar{\kappa}_h$. The sum is performed over the discretized
 826 height h . Here C_n^{obs} is the distribution of tracer in buoyancy space from the full 2D simulation at

827 a given time t_n , L is a matrix operator representing the centered-in-space spatial discretization of
 828 Eq. (72) and \mathcal{C}_0 is the initial tracer distribution⁹.

829 The model matches the evolution of the buoyancy tracer profiles from the full two-dimensional
 830 simulations very well for both constant (not shown) and bottom-intensified (compare solid lines
 831 and open circles in Fig. A1a) diffusivity cases. The least-squares fit provides an estimate of a
 832 single diffusivity and its linear gradient over the full evolution of the tracer up to a given time.
 833 This diffusivity, at the center-of-mass position of the fitted tracer distribution, compares well with
 834 the time-averaged bulk diffusivity measured from the tracer moments (compare solid blue and
 835 orange lines in Fig. A1b). In particular, the fitting method reproduces the rapid initial decrease in
 836 the tracer spreading rate below that expected from the in-situ diffusivity due to the presence of the
 837 boundary¹⁰.

838 The least-squares fit also provides estimates of a mean diapycnal velocity \bar{w} and diffusivity
 839 gradient (solid and dashed lines in Fig. A1c respectively). The fit predicts negative values for
 840 \bar{w} and $\bar{\kappa}_h$ at initial times before the tracer contacts the boundary, consistent with what would be
 841 expected within the SML. However, at later times $\bar{\kappa}_h$ is predicted as positive. Integrating Eq. (72)
 842 over all h shows that the rate of change of the center-of-mass μ_h and the one-dimensional bulk
 843 diffusivity in the one-dimensional model are given by,

$$\frac{\partial \mu_h}{\partial t} = \bar{w} + \bar{\kappa}_h \quad (74)$$

$$\kappa_{bk}^{1D} = \frac{1}{2} \frac{\partial \sigma_h^2}{\partial t} = \frac{\langle \bar{\kappa} \mathcal{C} \rangle^h}{\langle \mathcal{C} \rangle} = \bar{\kappa}_h \mu_h + \bar{\kappa}_0 \quad (75)$$

⁹We use a non-linear Levenberg-Marquardt least-squares minimization algorithm (Moré 1978). The spatial discretization uses a regular height grid with 100-points converted from buoyancy space using the far-field stratification N^2 . The time step is 1 day.

¹⁰However, if the presence of a boundary at a particular value of b was known, then this could be included in the boundary conditions of the one-dimensional model and a better estimate of the in-situ diffusivity could be obtained. In a field TRE boundaries are likely present at varying depths and buoyancies over the history of the experiment, and thus this knowledge is unlikely.

847 Thus, the positive \bar{w} and $\bar{\kappa}_h$ at later times are present in order to drive the center-of-mass toward
848 less dense fluid [Eq. (74)]. In the full two-dimensional case this center-of-mass movement is
849 achieved by the boundary effect and BBL advection, that are not captured by the one-dimensional
850 model [they do not appear in Eq. (74)]. Further, in the one-dimensional model the bulk diffu-
851 sivity is equal to the one-dimensional in-situ diffusivity $\langle \bar{\kappa} \mathcal{C} \rangle^h / \langle \mathcal{C} \rangle$, which in turn is equal to the
852 diffusivity at the tracer center-of-mass position μ_h [Eq. (75)]. Comparing Eq. (75) to Eq. (58),
853 the boundary effect, the advective term and the diffusivity gradient term in Eq. (58) all drop
854 out because \bar{w} and κ_h are constants and can be removed from the averaging leaving the factor
855 $\langle (h - \mu_h) \mathcal{C} \rangle = 0$. Thus, the one-dimensional model, while reproducing the evolution of the tracer
856 variance in buoyancy space, assigns all of that spreading rate to the *in-situ* diffusivity $\bar{\kappa}$. Because
857 we found the spreading rate to be less than expected from the in-situ diffusivity, the 1D method
858 would thus yield a much reduced in-situ diffusivity compared to that actually acting on the tracer
859 $\langle \kappa \mathcal{C} \rangle / \langle \mathcal{C} \rangle$.

860 **Appendix C: Tracer released as a layer**

861 We have investigated the spreading of a tracer released at a single location near the boundary,
862 within the actively mixing layer. Different behavior may be expected in the case where a tracer is
863 released outside the mixing layer, is homogenized along-isopycnals, and only later interacts with
864 the boundary. In this appendix we examine an idealized example of such a case where the tracer
865 is released as a layer spread horizontally and centered around a particular buoyancy surface (see
866 Fig. A2: the tracer is released within the dashed white box). To emphasize the boundary effects
867 we release the tracer as a strip extending only across the SML.

868 Enhanced vertical diffusion of the tracer patch occurs in the region close to the boundary (Fig.
869 A2c). The near-boundary tracer spreads further up than down the slope as a consequence of the

870 BBL flow as for the point-release experiments (e.g. Fig. 4). However, the center-of-mass motion
871 is much weaker here, because the majority of the tracer patch experiences only weak diffusion
872 (solid lines in Fig. A3b).

873 When a larger proportion of tracer is located further from the boundary the bulk diffusivity
874 is more comparable to the in-situ diffusivity (compare dashed and solid lines in Fig. A3c, and
875 compare to the point-release cases in Fig. 5). This is because the damping effect of the boundary
876 depends on the ratio of tracer near the boundary versus away from the boundary, $\langle C(0) \rangle^y / \langle C \rangle$ [see
877 Eq. (62)]. As the initial tracer patch is extended further into the far-field the in-situ and bulk
878 diffusivities weaken and converge (compare orange and blue lines in Fig. A3c). The addition of
879 along-isopycnal diffusion also drives more tracer into the far-field where mixing is weak, acting
880 to reduce both the in-situ and bulk diffusivities (Fig. A4). This contrasts with the near-boundary
881 point-release experiments where along-isopycnal diffusion enhanced the diapycnal spreading rate
882 by reducing the boundary effect (Fig. 13). In the layer cases the boundary effect is already weak
883 and so the enhancement of tracer movement away from the boundary dominates.

884 Finally, when the tracer is released as a layer the tracer profiles in buoyancy space evolve in a
885 manner which is not as well captured by the one-dimensional advection-diffusion equation con-
886 sidered in Appendix B (compare solid and dotted lines in Fig. A3a), compared to the point-release
887 cases (e.g. Fig. A1a). The tracer buoyancy profiles in the layer case exhibit more rapid spreading
888 of the tails of the distribution and slower spreading of the core of the distribution (associated with
889 the near-boundary and weak-diffusion far-field regions respectively, Fig. A2). The 1D advection-
890 diffusion equation with constant parameters \bar{w} , $\bar{\kappa}_h$ and κ_0 [e.g. Eq. (72)] cannot capture this
891 behavior, as the constant velocity and linear diffusivity can drive the development of skewness
892 but not of kurtosis in the distribution. It is possible that a 1D advection-diffusion equation with a
893 diffusivity that was quadratically dependent on h , for example, may be able to capture this behav-

ior. However, such a model is not supported by the setting. Due to this difficulty the diffusivity predicted by the one-dimensional method does not provide as good an estimate of the overall spreading rate of the tracer, predicting a weaker value (compare triangles to open circles in Fig. A4). It is possible that a similar problem may manifest in field experiments when a tracer encounters a boundary where mixing is enhanced, or in any situation where the tracer patch spreads over two regions with very different mixing characteristics. Providing that the tracer is sampled in both regions, then this effect can be identified from the non-standard behavior of the tracer distributions in buoyancy space. However, if the near-boundary regions where diffusion is strong are not sampled then the diffusivity may be underestimated.

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- 1042 **Fig. 1.** A schematic illustrating the two-dimensional slope-normal coordinate system used in this
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 1044 slope such that z is normal to the slope, y points up the slope and x is along-slope. The density
 1045 field is characterized by a constant stratification N^2 within the SML, while isopycnals slope
 1046 down within the weakly stratified BBL. One-dimensional boundary layer theory yields an
 1047 upslope velocity profile $V(z)$ that is characterized by strong upwelling across isopycnals in
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1125		distribution as a function of buoyancy at days 0, 192 and 1600. (b) Time series of the center	
1126		of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends	
1127		(dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines).	
1128		(c) Time series of the instantaneous bulk diffusivity (solid lines), in-situ diffusivity (dashed	
1129		lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).	63
1130	Fig. 13.	A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from all	
1131		the point-release experiments with a slope of $1/400$, bottom-intensified mixing and a release	

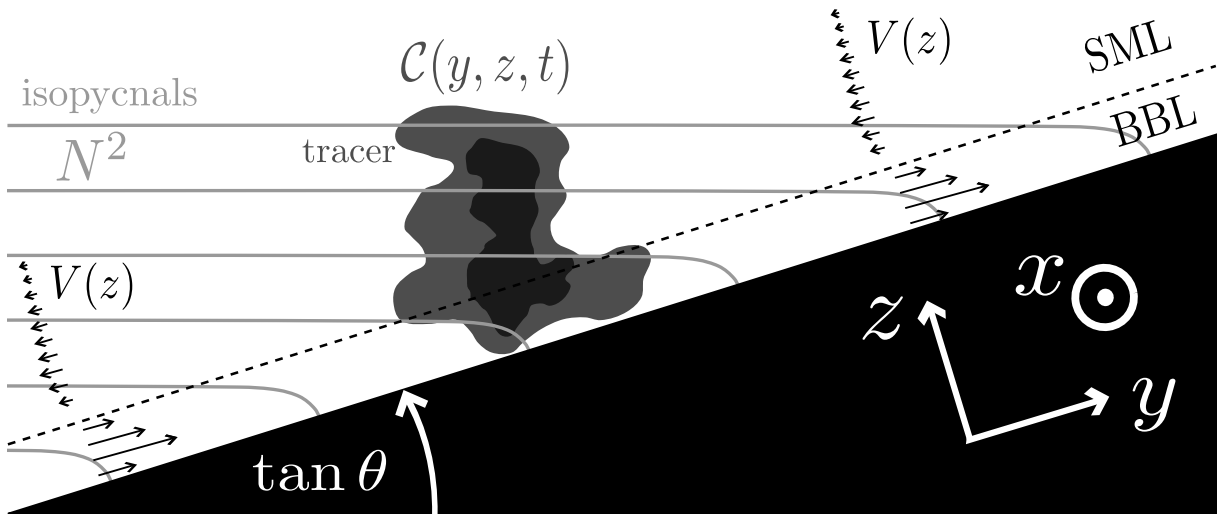
1132 point at $z_0 = d/2$. Shown are the time-averaged bulk (open circles) and in-situ (crosses) dif-
 1133 fusivities as a function of the along-isopycnal diffusivity A_H . Also shown (black triangles) is
 1134 the diffusivity estimated from a three-parameter least-squares fit to a 1D advection-diffusion
 1135 equation (Ledwell and Watson 1991; Ledwell 1998, see Appendix B). 64

1136 **Fig. A1.** (a) Evolution of the tracer buoyancy profiles for the exponential isotropic diffusivity two-
 1137 dimensional case shown in Fig. 5 (solid lines) and the center-of-mass time series (dot-
 1138 ted lines). The open circles are profiles from a three-parameter least-squares fit to a one-
 1139 dimensional advection-diffusion equation [Eq. (72)]. (b) Time series of the time-averaged
 1140 bulk diffusivity (solid orange line), the in-situ diffusivity (dashed orange line) and the dif-
 1141 fusivities estimated from the three-parameter least-squares fit at $h = 0$ (dotted blue line)
 1142 and at $h = \mu_h$ (solid blue line). (c) Time series of the diffusivity gradient (dashed line) and
 1143 diapycnal velocity (solid line) from the three-parameter fit. 65

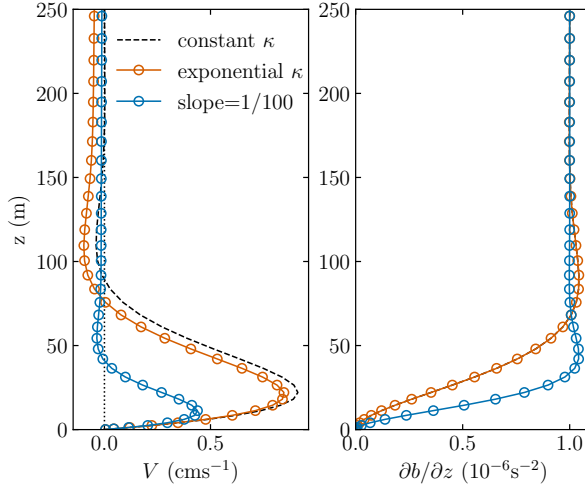
1144 **Fig. A2.** Tracer distributions for experiments where the initial tracer distribution is a narrow Gaus-
 1145 sian centered about a particular buoyancy (whose initial extent is indicated by the dashed
 1146 white box), the along-isopycnal diffusivity is $A_H = 10\text{m}^2\text{s}^{-1}$ and the slope is $1/200$. The
 1147 initial width of the tracer distribution extends out to a distance (a-c) $2d = 1000\text{m}$ above the
 1148 bottom, or a horizontal distance of 200km from the boundary and (d-f) $3d = 1500\text{m}$ above
 1149 the bottom, or a horizontal distance of 300km from the boundary. Distributions are shown
 1150 at day (a,d) 400, (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass (μ_y, μ_z)
 1151 is shown with a white circle and the center-of-mass on the boundary is shown with a cross. 66

1152 **Fig. A3.** Buoyancy-space tracer behavior for the cases considered in Fig. A2. (a) The tracer distri-
 1153 bution as a function of buoyancy at days 192 and 1600. Also shown are the predicted tracer
 1154 distributions from the one-dimensional least-squares fit discussed in Appendix B (dotted
 1155 lines). (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffu-
 1156 sivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL advection
 1157 to center-of-mass trends (dashed lines). (c) Time series of the instantaneous bulk diffusivity
 1158 (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient
 1159 to the bulk diffusivity (dotted lines). 67

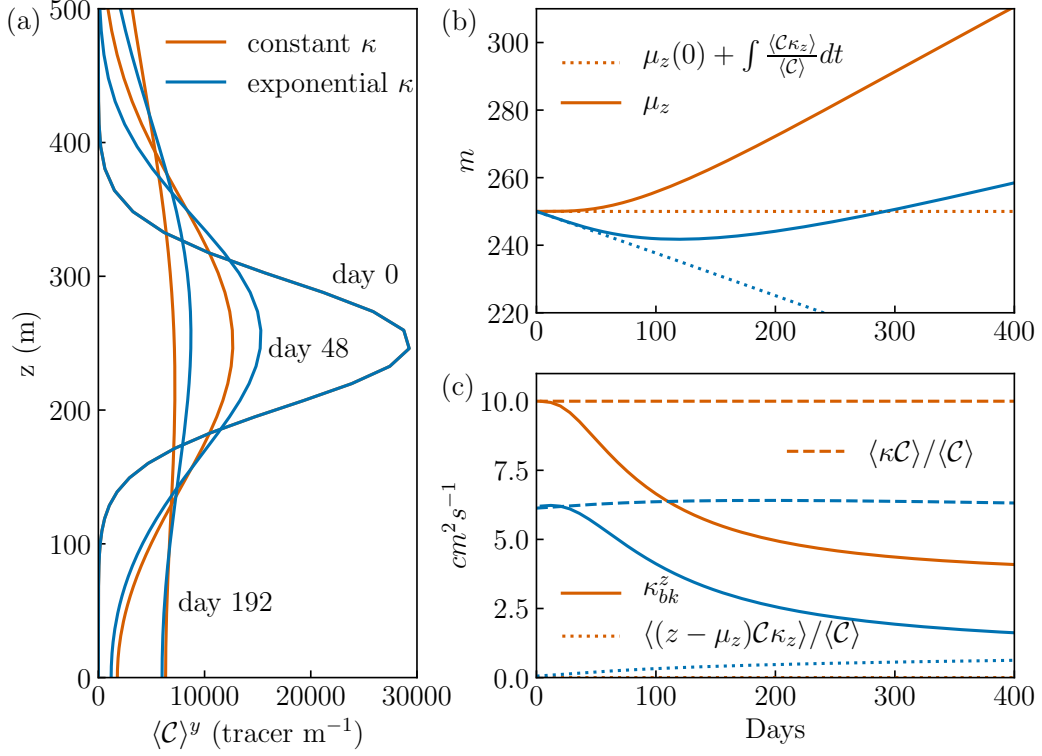
1160 **Fig. A4.** A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from
 1161 all the layer-release experiments with a slope of $1/200$ and where the layer has an initial
 1162 width of 200km , or extends out to a distance of $2d = 1000\text{m}$ above the bottom. Shown
 1163 are the time-averaged bulk (open circles) and in-situ (crosses) diffusivities as a function of
 1164 the along-isopycnal diffusivity A_H . Also shown (black triangles) is the diffusivity estimated
 1165 from a three-parameter least-squares fit to a 1D advection-diffusion equation (Ledwell and
 1166 Watson 1991; Ledwell 1998, see Appendix B). 68



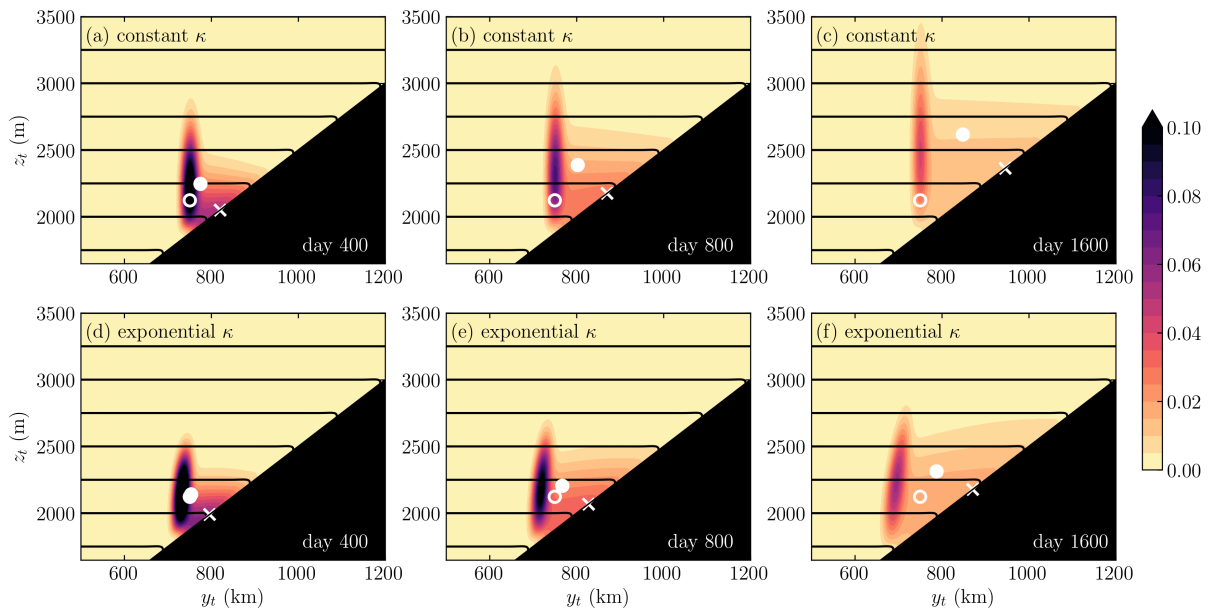
1167 FIG. 1. A schematic illustrating the two-dimensional slope-normal coordinate system used in this article. The
 1168 boundary has a constant slope of $\tan \theta$. The coordinate system is aligned with the slope such that z is normal to
 1169 the slope, y points up the slope and x is along-slope. The density field is characterized by a constant stratification
 1170 N^2 within the SML, while isopycnals slope down within the weakly stratified BBL. One-dimensional boundary
 1171 layer theory yields an upslope velocity profile $V(z)$ that is characterized by strong upwelling across isopycnals
 1172 in the BBL with weaker downwelling across isopycnals in the SML. The tracer \mathcal{C} is released at some point in
 1173 the domain and evolves in the two spatial dimensions y, z and in time t .



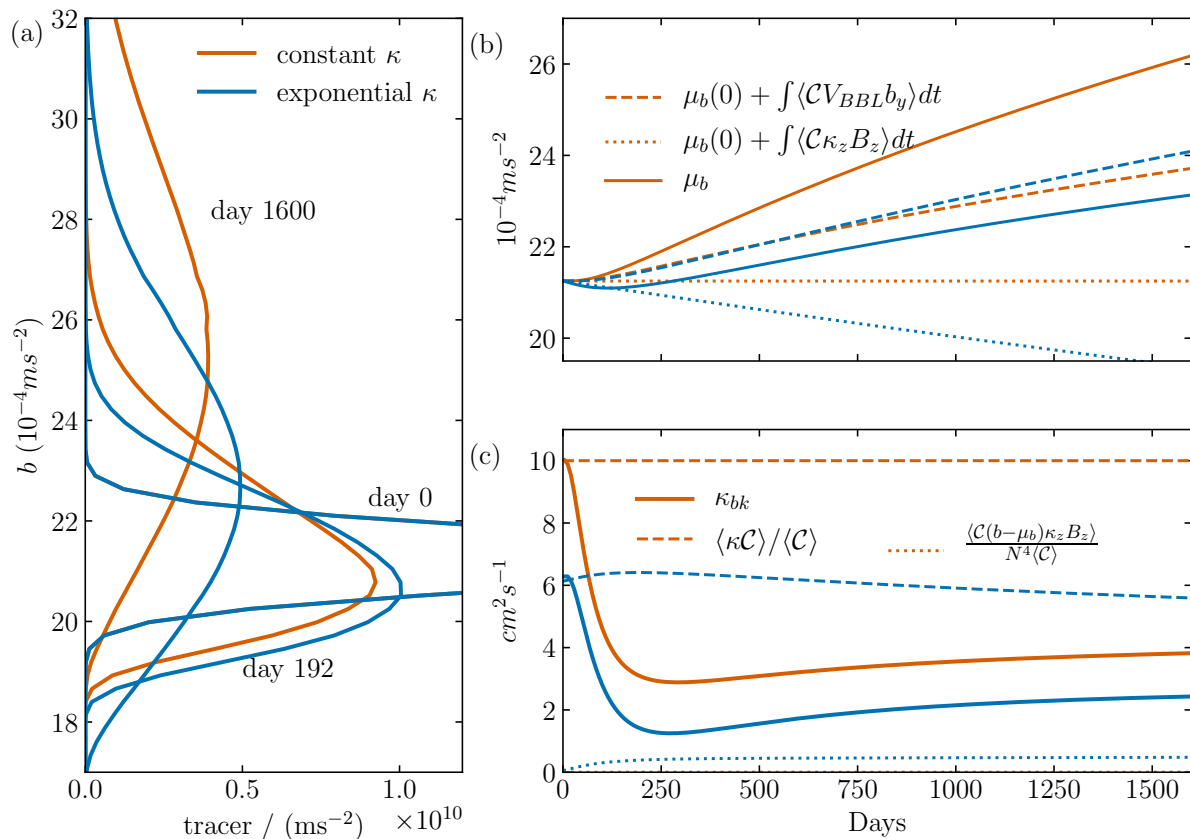
1174 FIG. 2. Upslope velocity V (left) and buoyancy frequency (right) $\frac{\partial b}{\partial z}$ from the approximate analytic solution
 1175 [Eq. (11)] for a constant diffusivity $\kappa = \kappa_0$ (dashed black) and an exponentially-decaying isotropic diffusivity
 1176 $\kappa = \kappa_\infty + (\kappa_0 - \kappa_\infty)e^{-z/d}$ (colored). Parameters are $N^2 = 10^{-6} \text{s}^{-2}$, $\kappa_0 = 10^{-3} \text{m}^2 \text{s}^{-1}$, $\kappa_\infty = 10^{-5} \text{m}^2 \text{s}^{-1}$ and
 1177 $d = 500 \text{m}$. The orange curves and the constant κ case use a slope of $\tan \theta = 1/400$, where the BBL width
 1178 $q_0^{-1} = 28.3 \text{m}$, while the blue curves use a slope of $1/100$, where $q_0^{-1} = 14.1 \text{m}$. Stratification in the constant κ
 1179 case is identical to that in the exponential case. The open circles indicate the effective vertical grid resolution of
 1180 the Dedalus simulations for 192 Chebyshev modes.



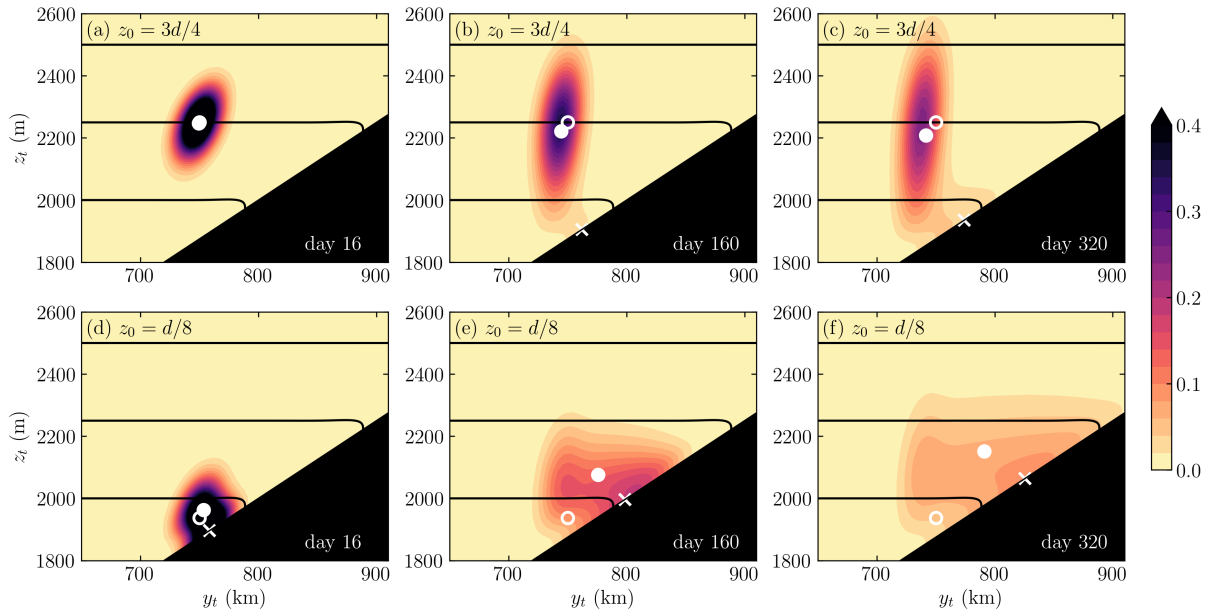
1181 FIG. 3. Behavior of the tracer in z for a constant diffusivity $\kappa = \kappa_0$ (orange lines) and an exponential diffusivity
 1182 (blue lines). (a) The tracer distribution as a function of z integrated in y , $\langle \mathcal{C} \rangle^y$, at days 0, 48 and 192. (b) Time
 1183 series of the center of mass μ_z (solid lines) and the time-integrated contribution of the diffusivity gradient to
 1184 center-of-mass changes (dotted lines, obtained by time-integrating the second term on the RHS of Eq. (37), and
 1185 adding the initial center-of-mass $\mu_z(0)$). Note that the contribution of the boundary term (first term on RHS of
 1186 Eq. (37)) accounts for the increase in μ_z beyond the effect of the diffusivity gradient. (c) The bulk diffusivity
 1187 κ_{bk}^z from the tracer variance (solid lines), the in-situ diffusivity (dashed lines) and the contribution to κ_{bk}^z
 1188 from the diffusivity gradient (dotted lines). Note again that the boundary term (second term in Eq. (40)) is not shown
 1189 and accounts for the down-turn in κ_{bk}^z .



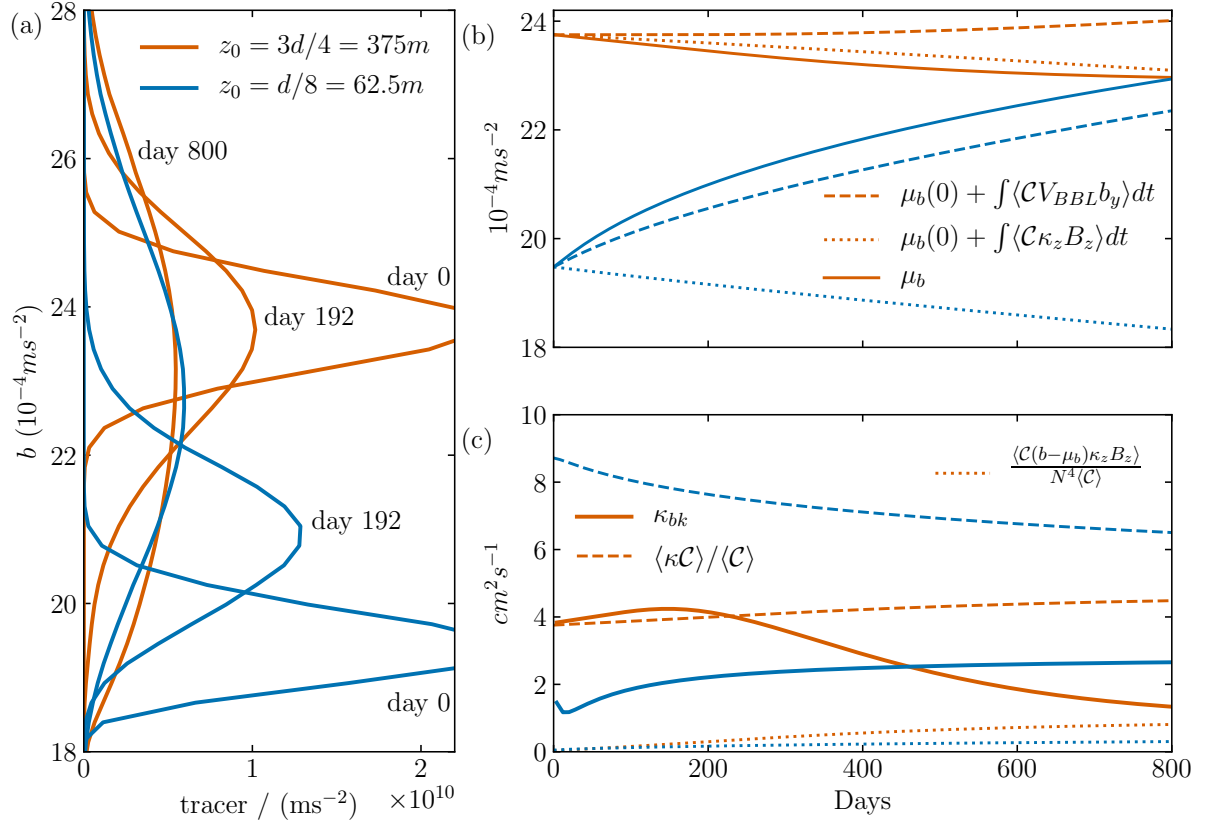
1190 FIG. 4. The evolution of a tracer patch released $z_0 = d/2 = 250\text{m}$ above the bottom (open white circle) with
 1191 (a-c) a constant diffusivity and (d-f) a bottom-intensified diffusivity. Tracer concentrations are shown at days
 1192 (a,d) 400 , (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass (μ_y, μ_z) is shown with a solid circle
 1193 and the position of the center-of-mass of tracer located only on the boundary is shown with a cross. Note that
 1194 the aspect ratio is strongly exaggerated.



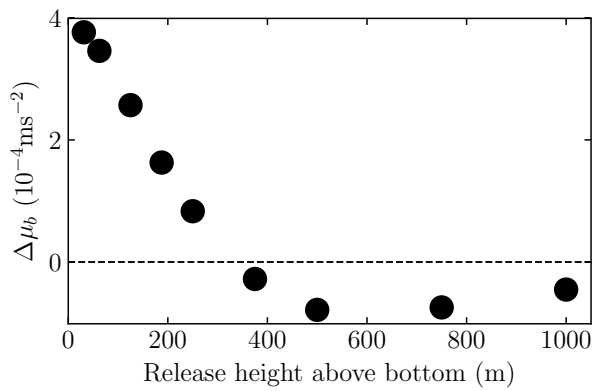
1195 FIG. 5. Buoyancy space tracer behavior for the two runs considered in Fig. 4 with constant diffusivity (orange
 1196 lines) and exponential diffusivity (blue lines). (a) The tracer distribution as a function of b at days 0, 192 and
 1197 1600. (b) Time series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-
 1198 of-mass trends (dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines).
 1199 As discussed in Section 6a, advection and diffusion each contribute half of the total trend, with the boundary
 1200 term and BBL advection also making equal contributions. (c) Time series of the bulk diffusivity (solid lines),
 1201 in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted
 1202 line). As discussed in Section 6d, the influence of SML advection is equivalent to the diffusivity gradient effect
 1203 (dotted line), while BBL advection and the boundary effect account, in equal parts, for the reduction in the bulk
 1204 diffusivity well below the in-situ diffusivity.



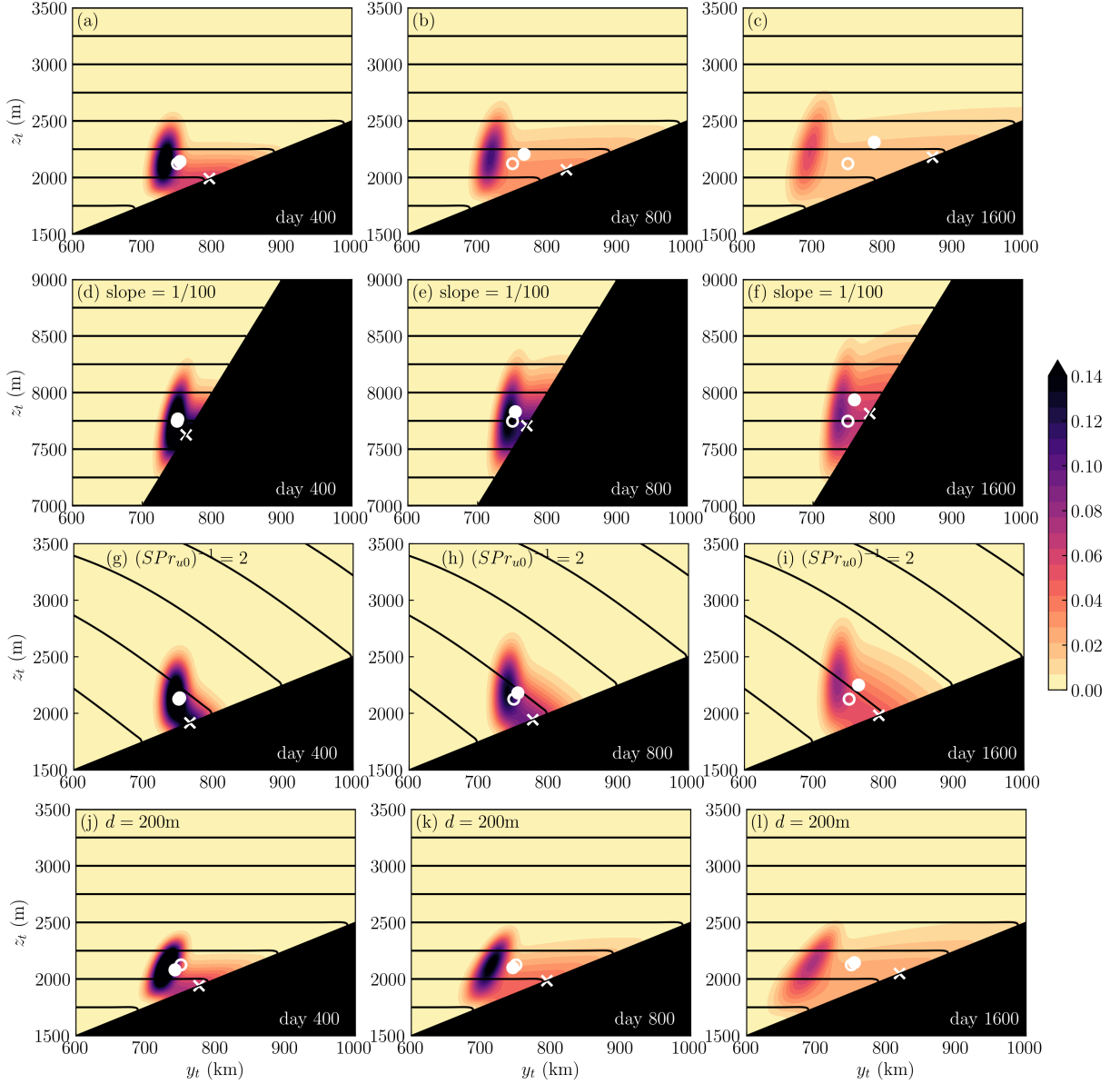
1205 FIG. 6. The evolution of tracer patches released (a-c) $z_0 = 3d/4 = 375\text{m}$ and (d-f) $z_0 = d/8 = 62.5\text{m}$ above
 1206 the bottom (open white circle) with a bottom-intensified isotropic diffusivity and no along-isopycnal diffusivity.
 1207 The position of the tracer center-of-mass (μ_y, μ_z) is shown with a closed white circle and the center-of-mass on
 1208 the boundary is shown with a cross.



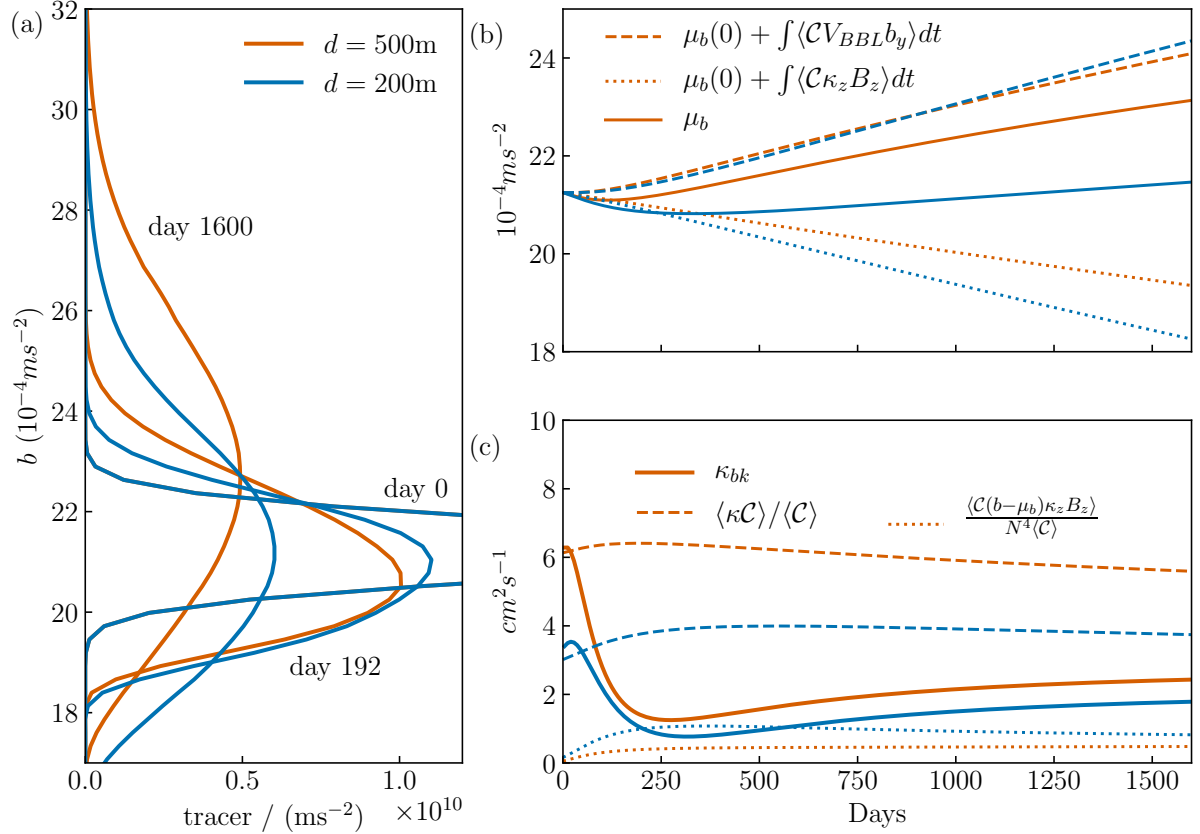
1209 FIG. 7. Buoyancy space tracer behavior for the two runs considered in Fig. 6, where the tracer is released
 1210 $z_0 = 3d/4 = 375\text{m}$ (orange lines) and $z_0 = d/8 = 62.5\text{m}$ (blue lines) above the bottom. (a) The tracer distribution
 1211 as a function of buoyancy at days 0, 192 and 800. (b) Time series of the center of mass μ_b (solid lines),
 1212 the contribution of the diffusivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL
 1213 advection to center-of-mass trends (dashed lines). (c) Time series of the bulk diffusivity (solid lines), in-situ
 1214 diffusivity (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).



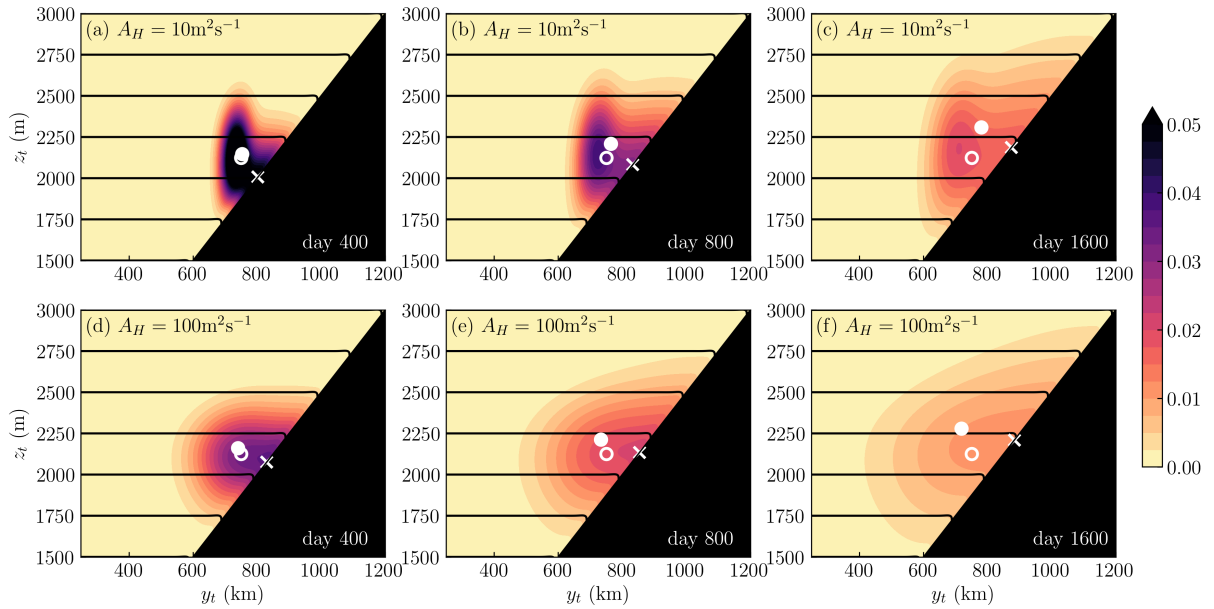
1215 FIG. 8. The change in the tracer center-of-mass in buoyancy space from its initial buoyancy $\Delta\mu_b = \mu_b(t) -$
 1216 $\mu_b(0)$ at day 800 as a function of the initial release height of the tracer above the bottom boundary.



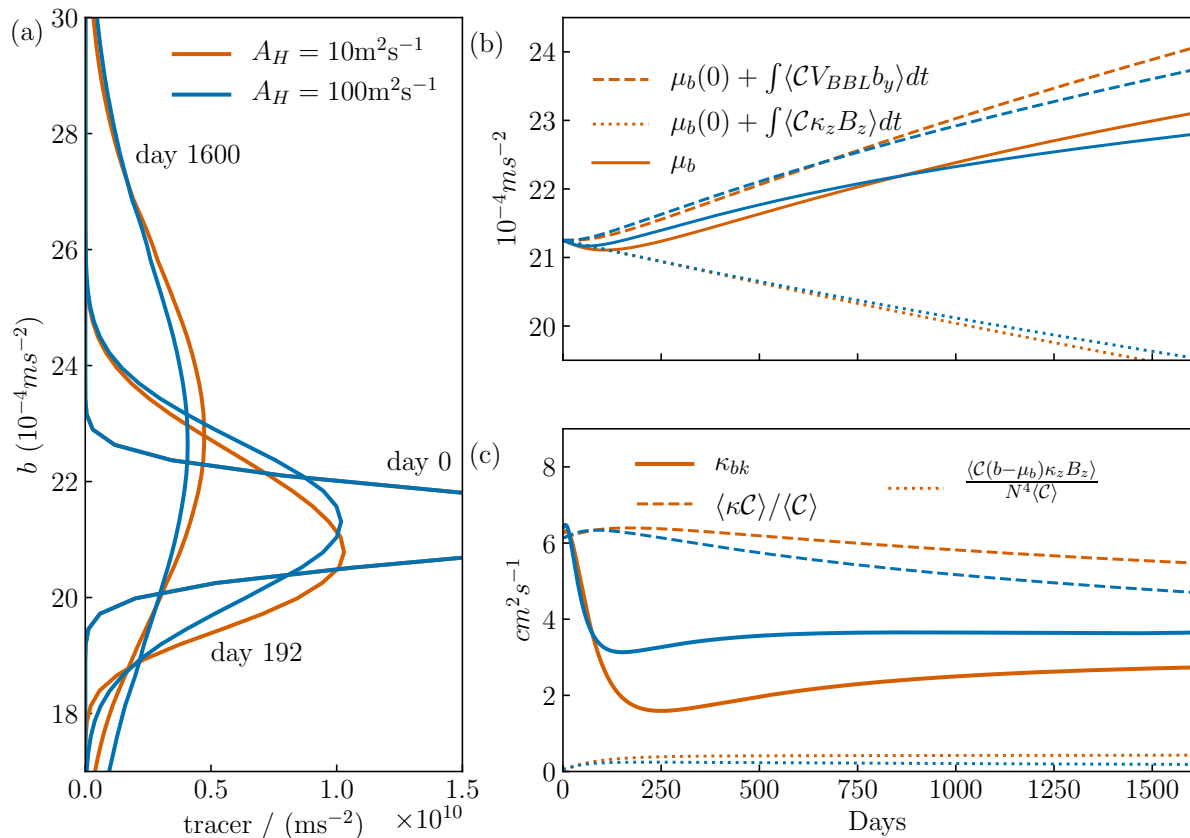
1217 FIG. 9. (a-c) The evolution of tracer patches released $z_0 = d/2 = 250\text{m}$ above the bottom (open white circle)
 1218 as for Fig. 4d-f (with different axis limits) with a slope of $1/400$, $d = 500\text{m}$ and $SPr_{u0}^{-1} = 0$. (d-f) As for a-c
 1219 except with a slope of $1/100$. (g-i) As for a-c except with the vertical stratification in the SML reduced by a
 1220 factor of three through the choice $SPr_{u0}^{-1} = 2$. (j-l) As for a-c except with the diffusivity decay scale reduced
 1221 to $d = 200\text{m}$. Plots are shown at day (a,d,g,j) 400, (b,e,h,k) 800 and (c,f,i,l) 1600. The position of the tracer
 1222 center-of-mass (μ_y, μ_z) is shown with a solid white circle and the center-of-mass on the boundary is shown with
 1223 a cross.



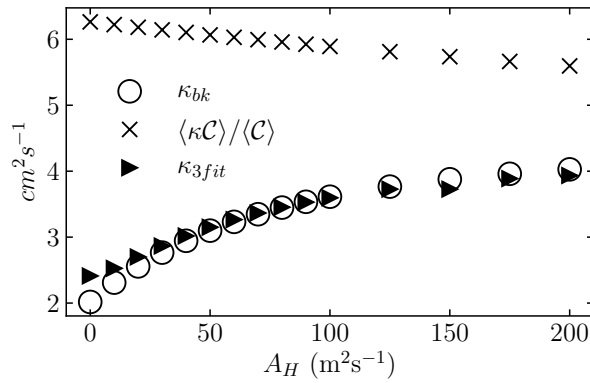
1224 FIG. 10. Buoyancy space tracer behavior for the runs considered in Fig. 9a-c and Fig. 9j-l where the tracer is
 1225 released 250m above the bottom and the vertical decay scale of the diffusivity is $d = 500\text{m}$ (orange lines) and
 1226 $d = 200\text{m}$ (blue lines). (a) The tracer distribution as a function of buoyancy at days 0, 192 and 1600. (b) Time
 1227 series of the center of mass μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends
 1228 (dotted lines) and the contribution of BBL advection to center-of-mass trends (dashed lines). (c) Time series of
 1229 the bulk diffusivity (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient
 1230 to the bulk diffusivity (dotted lines).



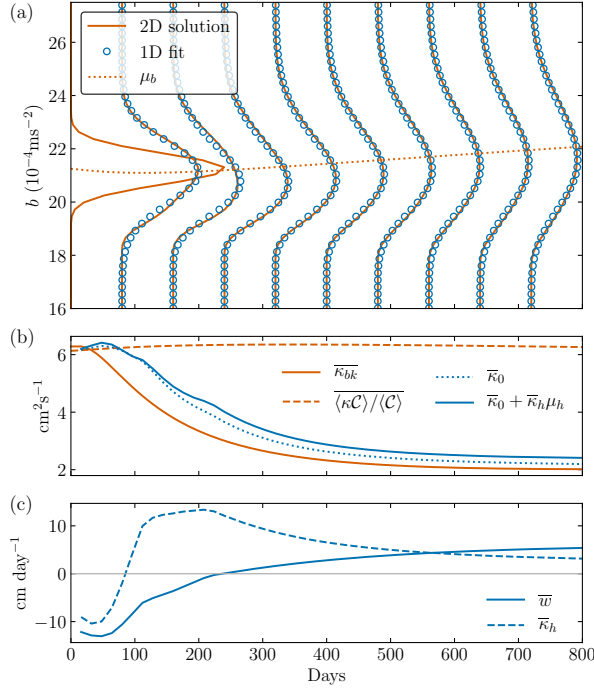
1231 FIG. 11. The evolution of tracer patches released $z_0 = d/2 = 250\text{m}$ above the bottom (open white circle)
 1232 with an exponential diffusivity for two values of along-isopycnal diffusivity (a-c) $A_H = 10\text{m}^2\text{s}^{-1}$ and (d-f)
 1233 $A_H = 100\text{m}^2\text{s}^{-1}$ at day (a,d) 400 , (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass (μ_y, μ_z) is
 1234 shown with a solid white circle and the center-of-mass on the boundary is shown with a cross.



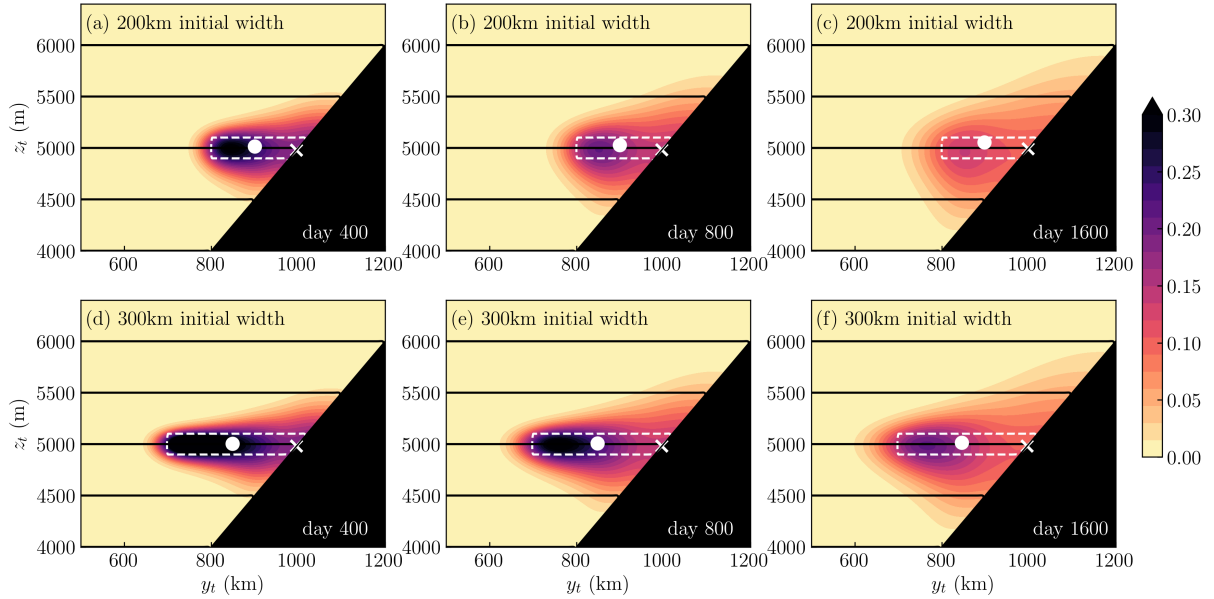
1235 FIG. 12. Buoyancy-space tracer behavior for the two runs considered in Fig. 11 with along-isopycnal diffu-
 1236 sivity $A_H = 10 \text{ m}^2 \text{ s}^{-1}$ (orange lines) and $A_H = 100 \text{ m}^2 \text{ s}^{-1}$ (blue lines). (a) The tracer distribution as a function of
 1237 buoyancy at days 0, 192 and 1600. (b) Time series of the center of mass μ_b (solid lines), the contribution of the
 1238 diffusivity gradient to center-of-mass trends (dotted lines) and the contribution of BBL advection to center-of-
 1239 mass trends (dashed lines). (c) Time series of the instantaneous bulk diffusivity (solid lines), in-situ diffusivity
 1240 (dashed lines) and the contribution of the diffusivity gradient to the bulk diffusivity (dotted lines).



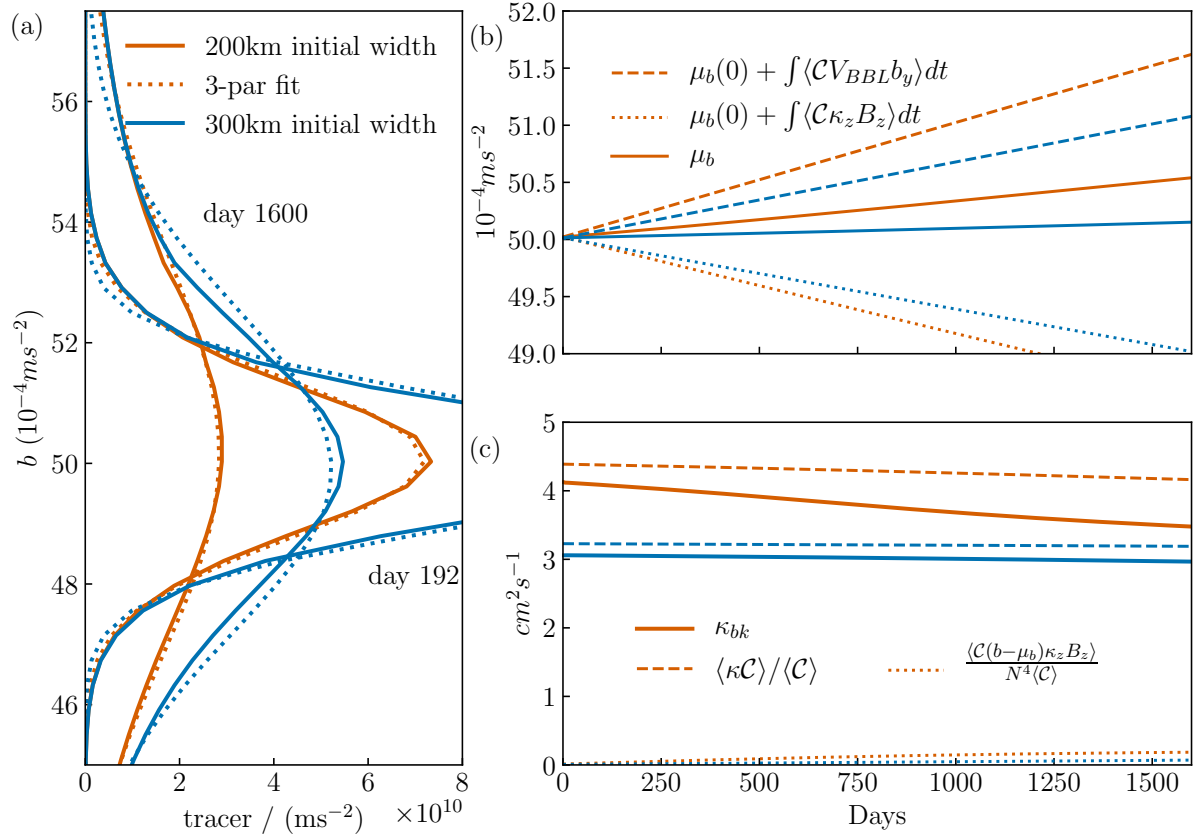
1241 FIG. 13. A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from all the
 1242 point-release experiments with a slope of $1/400$, bottom-intensified mixing and a release point at $z_0 = d/2$.
 1243 Shown are the time-averaged bulk (open circles) and in-situ (crosses) diffusivities as a function of the along-
 1244 isopycnal diffusivity A_H . Also shown (black triangles) is the diffusivity estimated from a three-parameter least-
 1245 squares fit to a 1D advection-diffusion equation (Ledwell and Watson 1991; Ledwell 1998, see Appendix B).



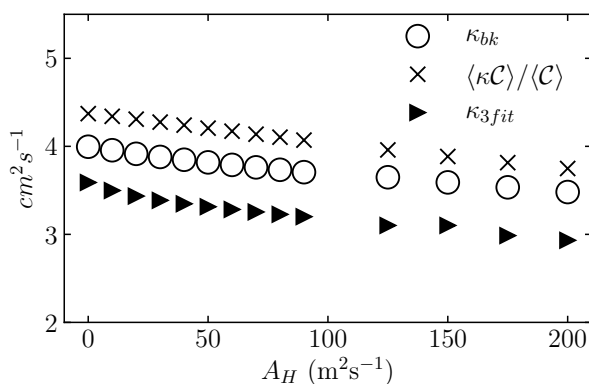
1246 Fig. A1. (a) Evolution of the tracer buoyancy profiles for the exponential isotropic diffusivity
 1247 two-dimensional case shown in Fig. 5 (solid lines) and the center-of-mass time series (dotted lines). The open
 1248 circles are profiles from a three-parameter least-squares fit to a one-dimensional advection-diffusion equation
 1249 [Eq. (72)]. (b) Time series of the time-averaged bulk diffusivity (solid orange line), the in-situ diffusivity
 1250 (dashed orange line) and the diffusivities estimated from the three-parameter least-squares fit at $h = 0$ (dotted
 1251 blue line) and at $h = \mu_h$ (solid blue line). (c) Time series of the diffusivity gradient (dashed line) and diapycnal
 1252 velocity (solid line) from the three-parameter fit.



1253 Fig. A2. Tracer distributions for experiments where the initial tracer distribution is a narrow Gaussian
 1254 centered about a particular buoyancy (whose initial extent is indicated by the dashed white box), the
 1255 along-isopycnal diffusivity is $A_H = 10\text{m}^2\text{s}^{-1}$ and the slope is $1/200$. The initial width of the tracer distribution
 1256 extends out to a distance (a-c) $2d = 1000\text{m}$ above the bottom, or a horizontal distance of 200km from the
 1257 boundary and (d-f) $3d = 1500\text{m}$ above the bottom, or a horizontal distance of 300km from the boundary.
 1258 Distributions are shown at day (a,d) 400, (b,e) 800 and (c,f) 1600. The position of the tracer center-of-mass
 1259 (μ_y, μ_z) is shown with a white circle and the center-of-mass on the boundary is shown with a cross.



1260 Fig. A3. Buoyancy-space tracer behavior for the cases considered in Fig. A2. (a) The tracer distribution as a
1261 function of buoyancy at days 192 and 1600. Also shown are the predicted tracer distributions from the
1262 one-dimensional least-squares fit discussed in Appendix B (dotted lines). (b) Time series of the center of mass
1263 μ_b (solid lines), the contribution of the diffusivity gradient to center-of-mass trends (dotted lines) and the
1264 contribution of BBL advection to center-of-mass trends (dashed lines). (c) Time series of the instantaneous
1265 bulk diffusivity (solid lines), in-situ diffusivity (dashed lines) and the contribution of the diffusivity gradient to
1266 the bulk diffusivity (dotted lines).



1267 Fig. A4. A summary of the time-averaged (or cumulative) diffusivity between days 0 and 800 from all the
 1268 layer-release experiments with a slope of 1/200 and where the layer has an initial width of 200km, or extends
 1269 out to a distance of $2d = 1000\text{m}$ above the bottom. Shown are the time-averaged bulk (open circles) and in-situ
 1270 (crosses) diffusivities as a function of the along-isopycnal diffusivity A_H . Also shown (black triangles) is the
 1271 diffusivity estimated from a three-parameter least-squares fit to a 1D advection-diffusion equation (Ledwell and
 1272 Watson 1991; Ledwell 1998, see Appendix B).