

1 **Intra-annual Rossby waves destabilization as a potential driver of Low**

2 **Latitudes Zonal Jets – Barotropic dynamics**

3 Audrey Delpech*

4 *LEGOS - Université de Toulouse, CNRS, CNES, IRD, Toulouse, France*

5 Claire Ménesguen

6 *LOPS - Université de Bretagne Occidentale, Ifremer, CNRS, IRD, Brest, France*

7 Yves Morel

8 *LEGOS - Université de Toulouse, CNRS, CNES, IRD, Toulouse, France*

9 Leif Thomas

10 *Department of Earth System Science, Stanford University, Stanford, California*

11 Frédéric Marin, Sophie Cravatte

12 *LEGOS - Université de Toulouse, CNRS, CNES, IRD, Toulouse, France*

13 Sylvie Le Gentil

14 *LOPS - Université de Bretagne Occidentale, Ifremer, CNRS, IRD, Brest, France*

15 *Corresponding author: Audrey Delpech, audrey.delpech@legos.obs-mip.fr

ABSTRACT

16 At low latitudes in the ocean, the deep currents are shaped into narrow jets flowing eastward
17 and westward, reversing periodically with latitude between 15°S and 15°N . These jets are present
18 from the thermocline to the bottom. The energy sources and the physical mechanisms responsible
19 for their formation are still debated and poorly understood. This study explores the role of the
20 destabilization of intra-annual equatorial waves in the jets formation process, as these waves are
21 known to be an important energy source at low latitudes. The study focuses particularly on the role
22 of barotropic Rossby waves as a first step towards understanding the relevant physical mechanisms.
23 It is shown from a set of idealized numerical simulations and analytical solutions that Non-Linear
24 Triad Interactions (NLTI) play a crucial role in the transfer of energy towards jet-like structures
25 (long waves with short meridional wavelengths) that induce a zonal residual mean circulation. The
26 sensitivity of the instability emergence and the scale selection of the jet-like secondary wave to the
27 forced primary wave is analyzed. For realistic amplitudes around $5\text{-}20\text{ cm s}^{-1}$, the primary waves
28 that produce the most realistic jet-like structures are zonally-propagating intra-annual waves with
29 periods between 60 and 130 days and wavelengths between 200 and 300 km. The NLTI mechanism
30 is a first step towards the generation of a permanent jet-structured circulation, and is discussed in
31 the context of turbulent cascade theories.

32 **1. Introduction**

33 The deep equatorial and tropical circulation is organized into systems of alternating eastward and
34 westward jets (Firing 1987; Firing et al. 1998; Johnson et al. 2002; Ollitrault et al. 2006; Ascani
35 et al. 2010; Cravatte et al. 2012; Ollitrault and Colin de Verdière 2014; Qiu et al. 2013; Cravatte
36 et al. 2017). We distinguish in particular: (i) meridionally alternating off-equatorial jets with a
37 meridional scale of $\sim 3^\circ$ within the 15°S - 15°N latitude range, which have a large vertical scale
38 (quasi-barotropic), called Extra Equatorial Jets (EEJs); and (ii) vertically alternating equatorial jets
39 trapped equatorward of $\sim 2^\circ$, which have a small vertical scale of $\sim 350\text{m}$, called Equatorial Deep
40 Jets (EDJs). These jets are present from below the thermocline down to at least 2000 m, with some
41 evidence of jets extending to the ocean bottom (Firing et al. 1998; Delpech et al. 2020a).

42 The underlying physical mechanisms generating these systems of jets are still poorly understood.
43 Different physical mechanisms have been proposed to explain their formation, relying on a cascade
44 of mechanisms transferring energy from a deep energy source (generally generated through the
45 propagation at depth of atmospheric variability or currents instabilities) to the mean jet-structured
46 circulation (see Fig. 2 of Ménesguen et al. 2019).

47 Earlier studies have shown that two-dimensional turbulence induces an inverse cascade, with
48 energy transferred towards larger scales. On a rotating planet, this cascade results in the emergence
49 of zonal structures due to the anisotropy of the Coriolis parameter (i.e. the beta effect) (Rhines
50 1975; Vallis and Maltrud 1993). Most of these studies have however been conducted in a theoretical
51 framework and have not addressed the source of the turbulence. The initiation of an inverse cascade
52 at depth at low latitude in the oceans would indeed require the presence of strong turbulence which
53 has not been documented yet.

54 One of the major energy sources present in the ocean at low latitude is associated with planetary
55 waves. It has been shown that in the Atlantic and Pacific ocean, a large source of energy in the
56 deep is associated with annual and semi-annual Rossby waves, as well as intra-annual waves:
57 30-day, 1000-km mixed Rossby-gravity waves, associated with surface Tropical Instability Waves;
58 and short-scale variability, with periods around 70 days and wavelengths around 500 km (Bunge
59 et al. 2008; Von Schuckmann et al. 2008; Tuchen et al. 2018; Lyman et al. 2005, 2007; Kessler
60 and McCreary 1993; Eriksen and Richman 1988; Farrar and Weller 2006; Farrar 2011; Farrar and
61 Durland 2012; Lee et al. 2017; Delpech et al. 2020b) (See Fig. 1 for an overview of the spectral
62 characteristics of the observed variability). Equatorial waves, when generated at the surface,
63 propagate to depth. In the intra-annual range in particular, waves have steep propagation ray paths
64 (Cox 1980) and are thus very efficient at transferring energy to the deep ocean.

65 These waves can develop instabilities and the destabilization of some particular planetary waves
66 have been shown to be a potential mechanism for the formation of a jet-structured circulation (Gill
67 1974; Hua et al. 2008; Connaughton et al. 2010; Qiu et al. 2013; d'Orgeville et al. 2007; Ménesguen
68 et al. 2009; Ascani et al. 2010). In an idealized model configuration forced with different intra-
69 annual baroclinic Yanai waves (Fig. 1 grey dots), d'Orgeville et al. (2007) and Hua et al. (2008)
70 were able to reproduce the EDJs-like structures, whose vertical scale was mainly dependent on
71 the initial wave period. In a similar configuration forced with baroclinic Yanai waves and short
72 barotropic Rossby waves at 50-day period (Fig. 1 magenta dots), Ménesguen et al. (2009) were
73 able to reproduce the EDJs and the first meridionally-alternating EEJs at 2°S and 2°N in equatorial
74 dedicated experiments. Ascani et al. (2010) were able to reproduce meridionally-alternating EEJs
75 in the 5°S - 5°N latitude band, with a vertically propagating beam of a Yanai wave with a period of
76 30 days and a wavelength of 1000 km (Fig. 1 red dot). Finally, in an idealized reduced gravity model
77 forced with a baroclinic mode 1 annual Rossby wave (Fig. 1 blue dot), Qiu et al. (2013) were able

78 to reproduce meridionally-alternating EEJs in the 5°N-40°N latitude band. The configurations and
79 characteristics of the wave forcing as well as the physical interpretation of the processes involved
80 in these numerical experiments differ, however, from study to study. A general framework for
81 interpreting the effects of planetary waves instability on the mean circulation is still missing. In the
82 framework of quasi-geostrophic dynamics, Gill (1974) studied the instability of barotropic Rossby
83 waves on a beta plane, focusing on two particular asymptotic cases: the case of very weak non-
84 linearities (corresponding to resonant triad interactions) and the case of very strong non-linearities.
85 He showed that in the limit of strong non-linearities, Rossby waves can transfer their energy towards
86 a zero zonal wavenumber, a non-zero meridional wavenumber and zero frequency mode, the so-
87 called zonal jet-like structures. Hua et al. (2008) adapted the barotropic theory of Gill (1974) to
88 baroclinic equatorially-trapped waves. They showed that equatorially-trapped baroclinic waves can
89 also be destabilized into vertically-alternating and meridionally-alternating jet-like structures, thus
90 extending Gill's results to lower latitudes. Ménesguen et al. (2009) interpreted the destabilization
91 of the barotropic Rossby waves into EEJs-like structures in their simulation using the theory of Gill
92 (1974) in the strong non-linear limit, while Qiu et al. (2013) interpreted the formation of EEJs in
93 their simulations as the destabilization of annual Rossby waves through resonant triad interactions
94 (corresponding to the weak non-linear limit of Gill's theory) and to further non-linear adjustments
95 involving potential vorticity fluxes. In addition, all the numerical studies exploring the formation of
96 EEJs from waves have focused on a limited set of wave periods. A general continuous framework
97 to analyze the potential contribution of all planetary waves in generating the mean jet-structured
98 circulation as a function of their characteristics has yet to be formulated.

99 In this study, our goal is to provide new insights into the mechanisms that drive the deep
100 tropical circulation and its jet-structuring, focusing on the formation of EEJs-like structures by the
101 destabilization of planetary waves. In particular, we aim to extend previous numerical experiments

102 to other type of waves, with a focus on the intra-annual waves, and to create a general theoretical
103 framework to compare and interpret the different experiments. This study focuses on the early
104 stage of jet formation. We thus emphasize the mechanism of energy transfer from a primary
105 energy source to EEJs-like structures with quasi-barotropic zonal velocities that reverse over a
106 meridional scale of ~ 350 km and that have a long temporal coherence. We leave for further
107 studies the long-term equilibration of these jet-like structures. We will in particular address the
108 following questions: (1) What is the potential for intra-annual waves to create off-equatorial,
109 meridionally-alternating, jet-like structures ? (2) Is there a preferential frequency and wavenumber
110 of the waves for reproducing realistic jets ? (3) What are the processes responsible for the transfer
111 of energy from waves to jet-like structures ?

112
113 To answer these questions, we perform idealized numerical experiments of the equatorial ocean
114 forced by an oscillating wind-stress that acts as a wave maker and generates waves with predeter-
115 mined characteristics. The idealized configuration is designed to evaluate the sensitivity of the
116 response of each forced wave to the period and wavenumber of the forcing. As a first step, we will
117 study the destabilization of barotropic waves using two-dimensional numerical simulations, leaving
118 the generalization to fully three-dimensional dynamics for subsequent work. The barotropic mode
119 offers the advantage that it is not equatorially-trapped, which makes the problem invariant with
120 latitude. It also radiate energy rapidly away from forcing regions, making it easier to simulate freely
121 propagating waves. Evidence for purely barotropic waves in the equatorial oceans are limited to a
122 few studies (Farrar 2011; Rohith et al. 2019; Farrar et al. 2020). The approach described in this
123 study can however be generalized to baroclinic modes.

124 We show that the formation of jet-like structures is triggered by the non-linear terms in the
125 momentum equations, which allow for wave-wave interactions. A full description of the non-linear

126 physics is not possible, due to its intrinsic chaotic nature. We thus investigate how the waves can
127 destabilize and produce jet-like structures within the framework of a truncated non-linear wave
128 interaction theory. This theory, which can be viewed as a generalization of Gill (1974), has been
129 fully described and investigated with applications in geophysics and plasma physics (Connaughton
130 et al. 2010) and is here adapted to study the instability of planetary waves. It allows for solving
131 analytically the non-linearity of the system, assuming a limited number of wave interactions, with
132 a continuity from a weakly non-linear regime to a strongly non-linear regime, for any primary wave
133 spectral characteristics.

134 The remainder of this paper is organized as follows. The numerical model configuration, forcing
135 and wavefield used in the experiments is described in Section 2. The spectral method using
136 wavelets transform that is used to analyze the waves in the simulations is described in Section 3.
137 The results of the numerical experiments are described in Section 4. The instability that develops
138 in the non-linear simulations is compared to the prediction from the 3-mode truncation Non-Linear
139 Triad Interaction (NLTI) analytical model in Section 5. The sensitivity of the creation of jet-like
140 structures to the wavenumber and frequency of the forced wave is investigated in Section 6. We
141 end the article with a discussion of how the results support and complement previous studies, and
142 summarize the main results and provide perspectives for future work.

143 **2. Numerical Experiments**

144 *a. Model description*

145 The idealized simulations performed in this study are run with the Coastal and Regional Com-
146 munity model (CROCO). The CROCO model solves the primitive equations (Shchepetkin and
147 McWilliams 2005, 2009) with a time splitting method between the fast barotropic mode and

148 the slow baroclinic modes. The model is used in its barotropic configuration, which solves
 149 the vertically-integrated momentum equations (Shchepetkin and O'Brien 1996; Shchepetkin and
 150 McWilliams 2009).

$$(hu)_t + (hu^2)_x + (huv)_y - fhv + gh\eta_x = \mathcal{F}^u + \mathcal{D}^u \quad (1a)$$

$$(hv)_t + (huv)_x + (hv^2)_y + fhu + gh\eta_y = \mathcal{F}^v + \mathcal{D}^v \quad (1b)$$

$$h_t + (hu)_x + (hv)_y = 0 \quad (1c)$$

151 where (u, v) is the barotropic velocity vector, $h(x, y, t) = H + \eta(x, y, t)$ is the total water depth, η is
 152 the free surface elevation, H is the water depth at rest, g is the gravity constant and f the Coriolis
 153 parameter. \mathcal{F} and \mathcal{D} represent forcing and frictional terms, respectively.

154 *b. Model configuration*

155 The basin configuration is meant to represent an idealized ocean at low latitudes. Its size is
 156 $140^\circ \times 70^\circ \times 5000$ m (longitude \times latitude \times depth) and it is centered on the equator. The horizontal
 157 resolution is $0.25^\circ \times 0.25^\circ$. The Coriolis parameter follows the variations of an equatorial beta-
 158 plane, $f = \beta y$ where $\beta = 2.3 \times 10^{-11}$ (ms) $^{-1}$ is the planetary vorticity gradient and y the meridional
 159 position from the equator.

160 The model is forced with a surface stress $\tau/\rho_0 = (\tau^x, \tau^y)$, which is incorporated in the model
 161 equations (Eq. 1) as a surface momentum flux (SMF) $\mathcal{F}^u = \tau^x/\rho_0$, $\mathcal{F}^v = \tau^y/\rho_0$. As our objective
 162 is to generate freely propagating intra-annual waves, the SMF is localized inside a specific region
 163 and takes the form of a wavemaker (Eq. 2).

$$\tau^x = -\tau_0 X(x)Y(y) \sin(\mathbf{k}_0 \cdot \mathbf{x} - \omega_0 t) (k_{y0}/k_{x0}) \quad (2a)$$

$$\tau^y = \tau_0 X(x)Y(y) \sin(\mathbf{k}_0 \cdot \mathbf{x} - \omega_0 t) \quad (2b)$$

164 where $X(x)$ and $Y(y)$ are envelope profiles. With this choice of wavemaker, τ will generate a plane
165 wave of streamfunction $\psi = \psi_0 \cos(\mathbf{k}_0 \cdot \mathbf{x} - \omega_0 t)$. Unless specified differently, we take $k_{y,0} = 0$ for all
166 experiments, which leads to $\tau^x = 0$. The generated waves thus have a strong meridional component
167 of the velocity and propagate zonally, as are the characteristics of the observed intra-annual waves.
168 The SMF wavenumber $k_{x,0}$, frequency ω_0 and amplitude τ_0 differ for each experiment (Table 1).
169 The containment of the wavemaker inside a localized forcing region $X(x)Y(y)$ is essential to have
170 an unforced region where the different waves can freely propagate without interference by the
171 forcing. It is also more realistic as most intra-annual waves observed in the ocean are found to be
172 locally generated and vary spatially (e.g., Tuchen et al. 2018; Delpech et al. 2020b). The envelope
173 of this forcing region is defined following Eq. 3, it is similar to a Blackman window of size (x_w, y_w)
174 centered on (x_0, y_0) .

$$X(x) = \tanh\left(\frac{x - x_0 - 0.5x_w}{x_t}\right) - \tanh\left(\frac{x - x_0 + 0.5x_w}{x_t}\right) \quad (3a)$$

$$Y(y) = \tanh\left(\frac{y - y_0 - 0.5y_w}{y_t}\right) - \tanh\left(\frac{y - y_0 + 0.5y_w}{y_t}\right) \quad (3b)$$

175 where (x_0, y_0) is the position of the center of the forcing region, x_w and y_w are the widths of
176 the forcing region, and x_t and y_t are the tapering extent of the forcing region. Unless specified
177 differently, the forcing region is centered in the middle of the basin $(x_0, y_0) = (70^\circ, 0^\circ)$. The other
178 parameters are taken by default as $x_w = 20^\circ$, $y_w = 15^\circ$, $x_t = 5^\circ$, $y_t = 2^\circ$. Note that the exact location
179 of the forcing region does not influence the outcome of the simulations. As the barotropic Rossby
180 deformation radius is very large, the propagation of barotropic Rossby waves does not depend on
181 latitude and are not subject to beta dispersion (Schopf et al. 1981). Test simulations have been
182 conducted with $y_0 = 15^\circ$, without substantial modifications to the results (not shown). Similarly, the
183 conclusions of this study are not sensitive to the extension of the forcing region. Test experiments
184 have been conducted with $y_w = 5^\circ$ and $y_w = 25^\circ$ (not shown). The size of the forcing region

185 controls the energy input into the ocean and changes the amplitude of the wave response. This
 186 sensitivity is thus similar to the sensitivity to the SMF amplitude (τ_0), which is fully investigated
 187 in Section 6. The characterization of the oceanic response to the forcing is investigated in Section
 188 2c and examples are given in Section 4 and illustrate that for the range of parameters considered,
 189 the envelope does not affect the wave response.

190 All lateral boundaries are closed with free slip boundary conditions. A sponge layer is im-
 191 plemented on the northern and southern boundaries to avoid the propagation of artificial coastal
 192 waves. In the sponge layers, the lateral viscosity is increased towards the northern and southern
 193 boundaries. The bottom is flat and a linear bottom drag law is applied to balance the energy input
 194 from the forcing. Therefore, the frictional terms in the equations are: $\mathcal{D}_u = -\alpha u + D_0(\tanh((y -$
 195 $y_s)/dy_s) + \tanh((y + y_s)/dy_s))/2$, $\mathcal{D}_v = -\alpha v + D_0(\tanh((y - y_s)/dy_s) + \tanh((y + y_s)/dy_s))/2$,
 196 with $\alpha = 1.5 \times 10^{-4} \text{ ms}^{-1}$ and $D_0 = 5000 \text{ m}^2\text{s}^{-1}$, $y_s = 34^\circ$ and $dy_s = 1^\circ$. No explicit horizontal
 197 viscosity is included in the basin interior. The horizontal momentum equations are discretized
 198 using a third-order upstream scheme (UP3), which has implicit diffusion.

199 The initial condition is a state at rest with zero velocity and no free surface elevation. The model
 200 is used in non-linear and linear configurations. In the case of linear simulations, the non-linear
 201 advection terms $\mathbf{u} \cdot \nabla \mathbf{u}$ are discarded from Eq. 1.

202 *c. Wave generation*

203 To study the instability of barotropic Rossby waves and their role in the formation of jet-like
 204 structures, we aim to create freely propagating waves using the surface momentum flux described
 205 in Eq. 2. Because the direct response to the SMF is not in geostrophic equilibrium, an adjustment
 206 occurs by radiating many different types of waves. The objective is to ensure that the wave with the
 207 characteristics of the forcing (i.e. with wavevector \mathbf{k}_0 and frequency ω_0) is the wave that contains

208 the most of the energy, which we will call hereafter the primary wave. If the wave vector that
 209 contains most energy and the forcing frequency ω_0 satisfy the dispersion relation for barotropic
 210 Rossby waves

$$\omega = \frac{-\beta k_x}{k_x^2 + k_y^2 + F} \quad (4)$$

211 then this wave will be resonantly forced, where (k_x, k_y) is the wave vector, ω the wave frequency,
 212 and F the inverse of the Rossby deformation radius squared. In this study, we focus on the barotropic
 213 mode, $F = (1/R_{d,0}^2)$, with $R_{d,0}$ the barotropic Rossby deformation radius. In the equatorial regions,
 214 $R_{d,0}$ is very large (i.e. $R_{d,0} \sim 4500\text{km}$ at 20° , the poleward limit of the domain we are using),
 215 thus the upper limit of F in this region is $\sim 10^{-14} \text{ m}^{-2}$, which is smaller than the wavenumbers
 216 considered (typically $|\mathbf{k}|^2 \sim 10^{-10} \text{ m}^{-2}$). In the following, we thus consider $F \sim 0$.

217 Because the SMF wave maker is modulated by the envelope, its spatial spectral footprint is spread
 218 over a range of wavenumber (Fig. 2). Its energy maximum in wavenumber space is however found
 219 close to \mathbf{k}_0 , and intersects the Rossby wave dispersion relation for the frequency ω_0 , thus satisfying
 220 the resonant condition. It is ensured that all the experiments conducted in this study (Table 1)
 221 satisfy this condition.

222 In the case of resonant SMF, the primary wave in the simulations is a barotropic wave with the
 223 characteristics of the forcing function (i.e. wavevector \mathbf{k}_0 and frequency ω_0) that propagates along
 224 the ray paths defined by the integration of the group velocity vector (C_{gx}, C_{gy}) .

$$C_{gx} = \frac{\beta(k_x^2 - k_y^2)}{(k_x^2 + k_y^2)^2} \quad (5a)$$

$$C_{gy} = \frac{2\beta k_x k_y}{(k_x^2 + k_y^2)^2} \quad (5b)$$

225 Because the initial state of the simulations is at rest, there is a spin up time which corresponds to
 226 the time it takes for the forcing to transfer energy to the oceanic response and balance the different

227 dissipation terms. The spin up time and the amplitude of the response can vary for the different
 228 experiments. It is important to evaluate both quantities to get the best possible description of the
 229 generated primary wave, since the characteristics of the primary wave determine the properties of
 230 the secondary waves (Sections 5, 6).

231 We define the spin up time for a wave forced at \mathbf{k}_0 and ω_0 (Eq. 2) as $S_{\mathbf{k}_0, \omega_0}$ the time at which the
 232 energy reaches 60% of its maximal value.

$$S_{\mathbf{k}_0, \omega_0} = \min_{t \in \chi} \left(\overline{\text{KE}}(t) - 0.6 \times \overline{\text{KE}}_{\text{max}} > 0 \right) \quad (6)$$

233 where $\overline{\text{KE}}$ is the spatial average of the kinetic energy over the spatial domain of the simulation
 234 (Ω), χ is the total duration of the simulation. The spin up time decreases slightly towards higher
 235 forcing frequencies but the values remain around 100-150 days (Fig. 3a).

236 We define the amplitude for a wave forced at \mathbf{k}_0 and ω_0 (Eq. 2) as $A_{\mathbf{k}, \omega}$, the average of the highest
 237 velocities values in the forcing region.

$$A_{\mathbf{k}_0, \omega_0} = \frac{1}{\Omega S_{\mathbf{k}_0, \omega_0}} \int_0^{S_{\mathbf{k}_0, \omega_0}} \iint_{\Omega} v_{\mathbf{k}_0, \omega_0}(x, y, t) H(v_{\mathbf{k}_0, \omega_0}(x, y, t) - v_{\mathbf{k}_0, \omega_0}^*) dx dy dt \quad (7)$$

238 where $v_{\mathbf{k}_0, \omega_0}$ is the meridional velocity in a simulation forced by \mathbf{k}_0 and ω_0 (Eq.2), $v_{\mathbf{k}_0, \omega_0}^*$ is the
 239 95th percentile of the distribution of v in the forcing region $X(x)Y(y)$ (Eq. 3), H is the Heaviside
 240 function and Ω the simulation domain. The amplitude of the forced wave varies linearly with the
 241 magnitude of the SMF (Fig. 3b).

242 *d. Experimental Design*

243 Table 1 summarizes all the experiments used and described in this study. The simulations are
 244 chosen to span the intra-annual period range from 30 to 130 days with primary waves that follow the
 245 dispersion relation for barotropic Rossby waves with zero meridional wavenumber (experiments a
 246 to l in Fig. 1). These $k_y = 0$ primary waves are found to be the optimal waves to produce jet-like

247 structures, as will be shown in Section 6b. For this reason, our simulations focus on this particular
 248 wave type. All experiments (a to l) are run for a nominal SMF amplitude of $\tau_0/\rho_0 = 5 \times 10^{-5}$
 249 m^2s^{-2} (simulations 1). Experiments a, c, e, g and i are run with for $\tau_0/\rho_0 = 5 \times 10^{-6} \text{m}^2\text{s}^{-2}$
 250 (simulations 2), $\tau_0/\rho_0 = 2.5 \times 10^{-5} \text{m}^2\text{s}^{-2}$ (simulations 3), $\tau_0/\rho_0 = 7.5 \times 10^{-5} \text{m}^2\text{s}^{-2}$ (simulations
 251 4), $\tau_0/\rho_0 = 1.0 \times 10^{-4} \text{m}^2\text{s}^{-2}$ (simulations 5) and $\tau_0/\rho_0 = 1.5 \times 10^{-4} \text{m}^2\text{s}^{-2}$ (simulations 6). All
 252 simulations are run with a non-linear physics (NL) and simulations 1a and 1l are repeated using a
 253 linear physics (L). The simulations are run for 1800 days (5 years) with output every 10 days.

254 3. Spectral Analysis Method

255 One objective of the analysis of the simulations is to quantify the primary wave destabilization
 256 and transfer of energy between wave modes by NLTI. Spectral analyses are the ideal tool for this
 257 task. As the properties of the waves and their spectral content are expected to change in space and
 258 time, we compute local spectra using wavelet transforms (Graps 1995; Gargour et al. 2009).

259 a. Wavelet transform

260 A continuous wavelet transform using a complex Morlet wavelet (Lee et al. 2019) is applied to
 261 the velocity field (u, v) . This method results in an estimation of the amplitude of the velocity signal
 262 contained in each frequency $(\widehat{v}^t(\omega; x, y, t))$ and wavenumber $(\widehat{v}^x(k_x; x, y, t), \widehat{v}^y(k_y; x, y, t))$ at each
 263 location $(x, y) \in \Omega$ of the domain Ω and each time $t \in \chi$ of the simulation, over the time interval χ .

264 We define the spatially-averaged one-dimensional spectral kinetic energy as:

$$\widehat{\text{KE}}^{x_i}(\widehat{x}_i; x, y, t) = \frac{1}{\Omega} \iint_{\Omega} \left(|\widehat{u}^{x_i}(\widehat{x}_i; x, y, t)|^2 + |\widehat{v}^{x_i}(\widehat{x}_i; x, y, t)|^2 \right) dx dy \quad (8)$$

265 where $x_i \in \{x, y, t\}$ and \widehat{x}_i is the spectral variable along x_i dimension (including frequency). Note
 266 that this definition of the one-dimensional spectral kinetic energy differs from the wavelet transform
 267 of the kinetic energy $((u^2 + v^2)/2)$, and has the advantage of retaining the spectral scales of the

268 velocity field. The spectral dimensions and their resolution are chosen as follows: we estimate the
 269 temporal wavelet transform for periods from 10 days to 350 days with a 3.3 days interval, and the
 270 spatial wavelet transform for wavelengths from 90 km to 3700 km with a 18.5 km interval. The
 271 wavelet transform amplitude cannot be accurately estimated for locations close to the boundaries of
 272 the simulation domain and for times close to the beginning and the end of the simulations (especially
 273 for long wavelengths and periods). This is known as the edge cone of influence (Torrence and
 274 Compo 1998). Estimates that lie within the cone of influence are thus discarded.

275 *b. Two-dimensional spectra from wavelet transform*

276 A wavelet analysis provides a one-dimensional spectrum as the spectral transform is computed
 277 on each physical dimension separately. In order to correlate this information, we construct cross-
 278 product spectra $\widehat{q}^{x_i x_j}$ of a quantity q along the spectral dimension $\widehat{x}_i \widehat{x}_j$ as defined by Eq. 9.

$$\widehat{q}^{x_i x_j}(\widehat{x}_i, \widehat{x}_j; t) = \frac{1}{\Omega} \iint_{\Omega} |\widehat{q}^{x_i}(\widehat{x}_i; x, y, t)|^2 |\widehat{q}^{x_j}(\widehat{x}_j; x, y, t)|^2 dx dy \quad (9)$$

279 where $x_i, x_j \in \{x, y, t\}$ and \widehat{x}_i is the spectral variable along the x_i dimension. Note that $\widehat{q}^{x_i x_j}$ is
 280 equivalent to the two-dimensional Fourier transform, but also varies in time.

281 We define the two-dimensional spectral kinetic energy as:

$$\widehat{\text{KE}}^{x_i x_j}(\widehat{x}_i, \widehat{x}_j; t) = \widehat{u}^{x_i x_j}(\widehat{x}_i, \widehat{x}_j; t) + \widehat{v}^{x_i x_j}(\widehat{x}_i, \widehat{x}_j; t) \quad (10)$$

282 **4. Numerical solutions**

283 *a. Propagation of the forced waves in the linear simulations*

284 The linear simulations illustrate the key features of each experiment and help to ensure that the
 285 dominant signal generated by the SMF is associated with the forced primary wave (Section c).
 286 In simulations 1d and 1l (Fig. 4), waves with a strong signature in meridional velocity propagate

287 eastward of the forcing region. These waves have the characteristics (i.e. wavenumber and
288 frequency) of the forcing (not shown). Their energy propagation follows the theoretical ray
289 paths predicted from the dispersion relation. The propagation is much faster for high-frequency,
290 long-wavelength forcing (Fig. 4d-f) than for low-frequency, short-wavelength forcing (Fig. 4a-
291 c). Note that there is also a slight spreading of the energy in the meridional direction when
292 the wave propagates eastward. Barotropic waves are plane waves and are thus invariant in the
293 direction perpendicular to their propagation direction (the meridional direction in the case of the
294 two examples). The relaxation of the forced solution (which is not an exact plane wave due to the
295 non-zero k_y associated with the envelope $X(x)Y(y)$) towards a free plane wave solution outside
296 of the forcing region can thus explain the meridional spread of the wave signature. These linear
297 simulations confirm that the SMF excites principally a barotropic Rossby wave with the forcing
298 characteristics, as expected from the chosen idealized setup (Section b).

299 *b. Propagation of the forced wave and secondary waves in non-linear simulations*

300 When the non-linear terms are taken into account in the model (Eq. 1), the key features of some
301 simulations is modified. For example, in simulation 1d, in addition to the eastward propagation
302 of energy associated with the forced primary wave (Section a), there is also westward energy
303 propagation (Fig. 5a-i). This westward propagation is associated with a wave with quasi-zonal
304 phase lines and has a dominant signature in the zonal velocity and sea surface height. This suggests
305 that an additional wave (aside from the forced wave) is also present in the simulation. Conversely,
306 for other simulations forced by different wavenumbers and frequencies, such as simulation 1l, the
307 non-linear response does not differ much from the linear response (Fig. 5j-r and Fig. 4d-f).

308 *c. Resulting jet-like mean flows in the non-linear simulations*

309 The secondary wave that develops in simulation 1d generates zonal velocities that are coherent
310 over a large zonal extension (i.e. from 0° to 60°) and that change signs over short meridional
311 scales (i.e. every 2.5°) (Fig.5f). When temporally-averaged, this wave results in a jet-like mean
312 flow (Fig. 6). By jet-like mean flow, we specify a circulation that shares common characteristics
313 with the observed jet-structured circulation in the equatorial and tropical oceans (e.g., Ollitrault
314 et al. 2006; Cravatte et al. 2012; Maximenko et al. 2005), that has meridionally-alternating zonal
315 velocities with amplitudes around 0.03 to 0.05 m s^{-1} , sea surface height anomalies of around 2
316 to 3 cm that alternate sign on short meridional scales (typically 1 to 2°) and that have zonal and
317 temporal coherence over thousand kilometers and several years respectively (see also Section 1).

318 The average over the last three years of the simulations shows that simulation 1d develops jet-like
319 mean flow (Fig. 6a), even if its characteristics are not fully realistic (Fig. 6c); namely the meridional
320 scale (2°) is slightly larger than what is observed in the Pacific (1.5°) and the amplitude (0.01 m
321 s^{-1}) is weaker than in the observations (0.06 m s^{-1}). In contrast, simulation 1l does not develop
322 a jet-like mean flow (Fig. 6b). The generation of jet-like mean flows and their characteristics
323 (meridional scale, amplitude) thus strongly depend on the characteristics of the forced waves.

324 The aim of the remainder of this paper is to understand the origin of the secondary wave that
325 generates jet-like mean flows, and to understand in what conditions they develop and what are their
326 characteristics. We investigate the physical mechanisms in simulation 1d in depth before examining
327 the sensitivity of this mechanism to the primary wave characteristics using multiple experiments
328 (Section 2d).

329 5. Evidence for non-linear triadic instability in the numerical simulations

330 In this section, we focus on one particular experiment, simulation 1d (Table 1), to describe
 331 the secondary wave and explain it in the framework of a 3-mode truncation NLTI model. An
 332 analytical solution can be derived for the evolution of the primary wave streamfunction, by limiting
 333 its non-linear interactions to a single triad, with three waves exchanging energy. The growth rate
 334 (σ) of this instability, which determines the two secondary waves with wavevectors \mathbf{q} and $\mathbf{p} - \mathbf{q}$
 335 that interact with the primary wave with wavevector \mathbf{p} in a triad ($\mathbf{p} = \mathbf{q} + \mathbf{p} - \mathbf{q}$), is given by Eq. 11
 336 and is derived analytically in Appendix A, following Connaughton et al. (2010),

$$\sigma = \text{Im} \left(\frac{1}{2} (\Delta \pm \sqrt{\Delta^2 - 4|\psi_{\mathbf{p}}|^2 T(\mathbf{q}, \mathbf{p}, \mathbf{q} - \mathbf{p}) T(\mathbf{p} - \mathbf{q}, \mathbf{p}, -\mathbf{q})}) \right), \quad (11)$$

337 where $\Delta = \omega_{\mathbf{p}} - \omega_{\mathbf{q}} - \omega_{\mathbf{p} - \mathbf{q}}$ is the resonant triad condition (the triad is resonant if $\Delta = 0$), $T(\mathbf{k}, \mathbf{k}_i, \mathbf{k}_j) =$
 338 $-\frac{(\mathbf{k}_i \times \mathbf{k}_j)_z (k_i^2 - k_j^2)}{k^2}$ and Im designates the imaginary part. The growth rate depends on the wavevector
 339 (\mathbf{p}), frequency ($\omega_{\mathbf{p}}$) and amplitude of the streamfunction ($|\psi_{\mathbf{p}}|$) of the primary wave. Note that the
 340 triad instability described by this model is more general than the resonant triad instability (e.g.,
 341 Pedlosky 2013). The approach is very similar to that of Gill (1974). But while Gill (1974) explored
 342 the weakly non-linear limit $|\psi_{\mathbf{p}}| p^3 / \beta \ll 1$ (leading to triad resonance) and the strongly non-linear
 343 limit $|\psi_{\mathbf{p}}| p^3 / \beta \gg 1$ as two separate instabilities, the analytical solution used here is a continuous
 344 function of the primary wave amplitude $|\psi_{\mathbf{p}}|$. Given the characteristics of the primary wave, Eq. 11
 345 allows one to determine the secondary waves wavevectors, \mathbf{q} and $\mathbf{p} - \mathbf{q}$, that yield the maximum
 346 growth rate. These waves are the ones most likely to emerge from non-linear interactions.

347 *a. Theoretical prediction for instability growth rate and secondary waves*

348 We compute the growth rate given by Eq. 11 for the primary wave \mathbf{p} of simulation 1d (Fig. 7).
 349 The secondary waves \mathbf{q} and $\mathbf{p} - \mathbf{q}$ that will develop as predicted by theory are determined by the

350 maxima of this growth rate. The growth rate maximum is found numerically using a wavevector
 351 grid spanning -5×10^{-5} to $5 \times 10^{-5} \text{ m}^{-1}$ in both the k_x and k_y directions with a resolution of
 352 10^{-7} m^{-1} . The two secondary waves resulting in the fastest growth have different characteristics.
 353 The first one has a long zonal wavelength ($\sim 5000 \text{ km}$), a short meridional wavelength ($\sim 430 \text{ km}$)
 354 (Fig. 7a), and a long period (~ 1.5 years) (Fig. 7b), called hereafter the long secondary wave. The
 355 second one has a short zonal wavelength ($\sim 340 \text{ km}$, similar to the one of the primary wave), a
 356 short meridional wavelength ($\sim 430 \text{ km}$) (Fig. 7a), and a short period (~ 4 months) (Fig. 7b), called
 357 hereafter the short secondary wave. The exact wavenumbers and frequencies associated with each
 358 of these waves are listed in Table 2. The long secondary wave thus has characteristics similar to
 359 the jet-like structures.

360 The growth rate of the secondary waves is $\sigma_{\max} = 0.032 \text{ day}^{-1}$ (Fig. 7). The primary wave is
 361 very unstable to NLTI because σ_{\max}^{-1} is of the same order of magnitude as its period. We define the
 362 time of emergence of the instability as:

$$T_{\mathbf{p}}^e = \frac{5}{\sigma_{\max}}; \quad \text{such that} \quad \psi_{\mathbf{q}}(T_{\mathbf{p}}^e) = e^5 \psi_{\mathbf{q}}(0) \simeq 10^2 \psi_{\mathbf{q}}(0) \quad (12)$$

363 where σ_{\max} is the maximum growth rate of the instability (Eq. 11). $T_{\mathbf{p}}^e$ corresponds to the time it
 364 takes for the amplitude of the secondary wave \mathbf{q} (the fastest growing perturbation of the primary
 365 wave \mathbf{p}), which has initially a small amplitude $\psi_{\mathbf{q}}(0)$, to increase by a factor $e^5 \simeq 10^2$ since the
 366 secondary waves grow as $e^{t\sigma_{\max}}$ (Appendix A). The factor of 5 that has been chosen will be justified
 367 a posteriori. We thus expect to detect the secondary waves after $5\sigma_{\max}^{-1} \simeq 150$ days. The group
 368 velocity, which determines the direction of energy propagation, is also different for the three waves
 369 (Fig. 8). It is eastward for the primary wave, westward for the long secondary wave and poleward
 370 for the short secondary wave. We thus expect to detect the signature of the primary wave eastward
 371 of the forcing region, the signature of the long secondary wave westward of the destabilization

372 region (which corresponds approximately to the forcing region in this case since the primary wave
373 has a slow group velocity and the secondary wave has a fast growth rate), and the signature of the
374 short secondary wave poleward of the forcing region.

375 *b. Comparison with the spectral analysis of the non-linear simulation*

376 We perform spatial and temporal wavelet analyses of the velocity in the simulations to infer the
377 spectral content at the time and location where we expect the primary and secondary waves to be
378 strongest.

379 The primary wave is observable after a few wave periods east of the forcing region (see ray
380 tracing Fig. 4 and group velocity Fig. 8). The wavelet transform in this region (red square in
381 Fig. 5c) shows a spectral peak at a period of 79 days (Fig. 9a; solid red curve), a zonal wavelength
382 of 250 km (Fig. 9b; solid red curve) and a long meridional wavelength of 3000 km (Fig. 9b; dashed
383 red curve). This matches the characteristics of the expected forced primary wave (Table 2). The
384 long secondary wave is detected west of the forcing region, the spectral characteristics measured
385 from the wavelet transforms in this region (green square in Fig. 5f) show a long period and a
386 long zonal wavelength (not precisely quantifiable within the spanned spectral range (Fig. 9; solid
387 green curve)). The meridional wavelength is around 500 km (Fig. 9b; dashed green curve). This
388 wave has spectral characteristics consistent with the prediction from the NLTI theory (Table 2).
389 The short secondary wave is difficult to identify in the simulations because its period and zonal
390 wavenumber are similar to the primary wave's (Table 2). It also has a slower group velocity than
391 the long secondary wave (Fig. 8), and its energy propagation is directed poleward, in a region where
392 the primary wave is also present. There is thus no region where the short secondary wave can be
393 observed alone. Its presence can however be identified in the two-dimensional spatial spectrum
394 (Fig. 11c).

395 The time at which the instability occurs in this simulation is inferred from the wavelet scalogram,
 396 which shows the evolution of the mean spectral characteristics averaged in the basin $\overline{\text{KE}}^f(t)$,
 397 $\overline{\text{KE}}^x(t)$, $\overline{\text{KE}}^y(t)$ (Fig. 10). The instability occurs when there is a shift in the spectral characteristics
 398 from the forced frequency and wavelengths to the emerging secondary wave, which is around
 399 $t = 275$ days in the simulation (dashed white line in Fig. 10). The shift is particularly visible
 400 for $\overline{\text{KE}}^y(t)$ (Fig. 10c) where the energy goes from a large to small meridional wavelength. The
 401 energy in the forced, small period and small zonal wavelength is concurrently strongly attenuated
 402 (Fig. 10a,b). The theoretical growth rate corresponds to an instability emergence time of ~ 150
 403 days (Section a). Based on Fig. 10, the spin up period, which corresponds to the time before the
 404 detection of the spectral characteristics of the primary wave, is around 130 days. The instability
 405 emergence time in the simulation (275 days) corresponds thus approximately to the sum of the
 406 spin up period and theoretical instability emergence time. The spatially-averaged two-dimensional
 407 spectral kinetic energy shows that basin wide, the velocity field is dominated by the forced wave
 408 before the instability occurs (Fig. 11 a,b) and by the two secondary waves after the instability occurs
 409 (Fig. 11 c,d). Note that the two-dimensional spectrum $\widehat{\text{KE}}^{xy}$ allows one to separate the contribution
 410 of the short secondary wave (Fig. 11 c), and reveals that it is less energetic than the long secondary
 411 wave that is dominant in both $\widehat{\text{KE}}^{xy}$ and $\widehat{\text{KE}}^{xt}$.

412 We have thus shown that the secondary waves that develop in the non-linear simulations are
 413 consistent with a non-linear triad instability. The growth rate and wavenumbers of the instability
 414 are well predicted using a non-linear truncated theory. In the example shown, the long secondary
 415 wave has jet-like characteristics.

416 **6. Application to the generation of jet-like structures**

417 The objective of this section is to explore the sensitivity of the formation of jet-like long secondary
 418 waves to the characteristics of the primary wave (i.e. the frequency, wavevector and amplitude).
 419 From the simulations that have been performed, it appears that not all primary waves are able to
 420 trigger instability, nor to create waves that resemble jet-like structures.

421 *a. Instability threshold and emergence time*

422 Some non-linear simulations are not subject to instability, like for example simulation 11, in
 423 which the forced primary wave propagates without spectral modification (Fig. 5j-r). Theoretically,
 424 all primary waves are subject to triad instability (Appendix A). But the growth rate can be close to
 425 zero, in which case it would take a very long time for the instability to develop, and thus it might
 426 not be realizable in numerical simulations. To determine whether a wave will be readily unstable
 427 or not, we define an instability emergence ratio (Eq. 13) following Qiu et al. (2013), which is a
 428 combination of the instability growth rate and the primary wave energy propagation velocity. We
 429 define the instability emergence ratio as:

$$R_{\mathbf{p}}^e = \frac{T_{\mathbf{p}}^a}{T_{\mathbf{p}}^e}; \quad \text{with} \quad T_{\mathbf{k}}^a = \frac{2\pi}{|\mathbf{k}||\mathbf{C}_{\mathbf{gk}}|} \quad (\text{for any wave } \mathbf{k}) \quad (13)$$

430 where $|\mathbf{k}|$ is the modulus of the wavevector ($|\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$) and $|\mathbf{C}_{\mathbf{gk}}|$ the modulus of the wave group
 431 velocity vector ($|\mathbf{C}_{\mathbf{gk}}| = \sqrt{C_{gx}^2 + C_{gy}^2}$). Following Qiu et al. (2013), $T_{\mathbf{p}}^a$ represents the propagation
 432 time scale of the primary wave \mathbf{p} (i.e. the time needed for a wavepacket to transit of one wavelength).
 433 $R_{\mathbf{p}}^e$ corresponds thus to the ratio of the propagation time scale to the instability emergence time.

434 The instability emergence time depends on the primary wavenumber, frequency and amplitude
 435 (Fig. 12a). As predicted by the theoretical instability emergence ratio (Eq. 13), simulations 1a,
 436 1d and 1f, should undergo triad instability (Fig. 12a). These simulations have a primary wave

437 amplitude around 0.07 m s^{-1} (see Fig. 3b for the relation between the forcing strength and the
 438 primary wave amplitude). The emergence of the instability in these simulations is visible from
 439 the switch of spectral properties in $\widetilde{\text{KE}}^y(t)$ (Fig. 12b-d). In contrast, the primary wave in the
 440 simulation 1h remains stable and $\widetilde{\text{KE}}^y(t)$ shows a dominant meridional scale that is constant in
 441 time (Fig. 12e). This shows that the truncated NLTI theory describes well what happens in the
 442 different simulations and can be used to predict the stability or instability of the different barotropic
 443 waves in the ocean. It is interesting to note that some waves will not destabilize unless they have
 444 an unrealistic amplitude (Fig. 13). This is the case for barotropic waves with a long wavelength.
 445 For example, a wave with a TIW-like spectral signature, with a 30-day period and a 1000-km zonal
 446 wavelength destabilizes only for amplitude greater than 75 cm s^{-1} (Fig. 13). For other waves, and
 447 in particular short wavelength primary waves, the amplitude threshold needed to reach $R_p^e \geq 1$ is
 448 very low.

449 The time at which the instability occurs in the simulation can be approximated by the time at
 450 which the energy in the forced spectral range reaches a maximum and starts to decrease. For
 451 all unstable simulations ($R_p^e \geq 1$) this time is found to be inversely proportional to the theoretical
 452 growth rate expected from the 3-mode truncation triad interaction theory (Fig. 14). Primary
 453 waves with a larger amplitude (big dots in Fig. 14) have a higher instability growth rate and
 454 thus a lower instability emergence time than primary waves with a smaller amplitude. Similarly,
 455 simulations with a higher primary wave number (blue dots in Fig. 14) have a higher growth rate
 456 and lower instability emergence time than simulations with lower primary wavenumber (orange
 457 dots in Fig. 14). The coefficient of proportionality between the inverse of the growth rate and the
 458 emergence time is approximately equal to 5, which justifies the definition of the emergence time
 459 (Eq. 12).

460 *b. Selection for the secondary waves' spatial and temporal scales*

461 The non-linear simulations show that the secondary waves resulting from the triad instability
462 differ from one simulation to another, for example, in their meridional wavelengths (Fig. 12b-d).
463 The characteristics of the long secondary wave depend on the forced primary wave's wavenumbers
464 and frequency, as inferred from the growth rate computation at a fixed amplitude (Fig. 15). The
465 correspondence between the scale for the long secondary wave in the numerical simulations and
466 the theoretical prediction in Fig. 15 shows that the dominant instabilities in the simulations are
467 consistent with NLTI theory.

468 The long secondary wave meridional wavelength varies with the period of the primary wave
469 ($2\pi/\omega_p$). It is shorter (~ 100 - 300 km) for long periods ($2\pi/\omega_p > 75$ days) and much longer
470 (up to 1000 - 3000 km) for shorter periods ($2\pi/\omega_p < 50$ days). At fixed ω_p , the secondary wave
471 meridional wavelength also increases with decreasing p_x (Fig. 15a). The long secondary wave
472 zonal wavelength varies non-monotonically with along the primary wave's spectral characteristics.
473 At fixed ω_p , it is large for short and long primary waves. It reaches a maximum for short
474 intra-annual primary waves, with periods between 50 and 145 days, along the dispersion relation
475 curves corresponding to low meridional wavenumbers (Fig. 15b). The long secondary wave's
476 period is longer for short primary waves (large p_x) and shorter for long primary waves (small
477 p_x). Similar to the zonal wavelength, the secondary wave's period reaches a maximum for short
478 intra-annual primary waves with periods comprised between 50 and 145 days, along the dispersion
479 relation curves for low meridional wavenumbers (Fig. 15c). The emergence time of the instability
480 (defined by Eq. 12) increases from about a hundred days to several years from high to low p_x
481 respectively. Short primary waves are thus much more unstable. The short intra-annual waves

482 described in the paragraphs above in particular have very short emergence times (Fig. 15d).

483

484 Realistic jet-like structures have a meridional wavelength of about 350 km, a long zonal
485 wavelength (greater than 8000 km) and a long period (more than 10 years). Given the dependence
486 of the secondary wave's properties on the primary wave's characteristics (Fig. 15a-c), the
487 "optimal" primary waves that yield realistic jet-like secondary waves have periods between
488 70 and 90 days, zonal wavelengths from 200 to 300 km, and low meridional wavenumber
489 (as indicated by the squared box in Fig. 15). From the simulations that have been performed,
490 simulation 1c seems to have the most realistic jet-like secondary wave. This is consistent with Fig. 6.

491

492 The analysis of the dependence of the scales of the long secondary wave to the primary wave's
493 properties described above assumed a constant primary wave amplitude of 0.07 m s^{-1} , correspond-
494 ing to the amplitude of the primary wave used in the numerical simulations. The optimal primary
495 wave spectral characteristics shown in Fig. 15 are fairly robust, regardless of the primary wave's
496 amplitude. Indeed, Fig. 16a shows that the long secondary wave's meridional wavelength is not
497 very sensitive to the amplitude of the primary wave, in particular for the range of simulations that
498 have been performed here. The variation coefficient, defined as the ratio of the standard deviation
499 of the secondary wave's properties across an amplitude range to the mean of these properties, is
500 close to zero for most of primary waves, except for waves with very high meridional wavenum-
501 bers (Fig. 16d). The secondary wave zonal wavelength varies with the amplitude of the primary
502 wave (Fig. 16 b,e). These variations are however not very large with respect to the values of the
503 wavelengths: even if they vary by one or two thousands of kilometers, they are still long waves
504 (i.e. with wavelengths larger than 3000 km). The variation coefficient remains less than 0.5 almost
505 everywhere in the spectrum (Fig. 16e). The period of the long secondary wave is the most sensitive

506 to the amplitude of the primary wave. Especially along the first barotropic dispersion relation
507 ($k_y = 0$), where the variation coefficient is close to 0.8 (Fig. 16f) and the secondary waves' period
508 can go from 200 days to a few years (Fig. 16c). In the simulations, the secondary wave meridional
509 wavelength is also found to be not sensitive to the primary wave amplitude (Fig. 16a), illustrating
510 again the similarity between the numerical simulations and the NLTI theory in complement of
511 Fig. 15a.

512 **7. Discussion, Summary and Perspectives**

513 *a. Beyond the 3-mode truncation theory*

514 This study shows that NLTI theory can predict the instabilities that can form jet-like structures in
515 the equatorial oceans. The theoretical framework was however developed under strong assumptions,
516 in particular non-linear interactions were limited to three waves. In the model, the dynamics is not
517 truncated and the non-linear interactions may involve additional triads of waves. However, it is
518 interesting to note that the 3-mode truncation captures the early stages of the simulations well and
519 provides a simple description of the instability process. However, on longer time scales, secondary
520 instabilities may arise, which involve additional waves. The spectra show evidence of this (e.g.,
521 Fig. 11d) as they indicate that more than 3 waves are clearly present in the simulations. Spectral
522 analysis of the simulation 1d on long time scales shows that energy is present at various scales
523 and is exchanged through a broad continuum of waves, similar to a turbulent cascade, rather than
524 being contained in a few discrete waves (Fig. 17). Energy tends however to accumulate in regions
525 where $k_x = 0$, which correspond to jet-like structures. This might be explained by the stability
526 of jet-like waves to NLTI. Namely, the growth rate for any jet-like primary wave, $\mathbf{p} = (0, p_y)$, is
527 identically equal to zero (Fig. 18), such that jet-like secondary waves are very stable and will likely

528 not undergo further triad interactions. The jet-like waves thus act as an attractor of this "wave
529 cascade", and every wave undergoing NLTI might eventually cascade towards jet-like structures.
530 This is analogous to turbulent cascades on a beta plane, where the energy cascades from being
531 isotropically distributed in spectral space towards accumulation zones at low frequencies and small
532 zonal wavenumbers, which correspond to zonal jets (Rhines 1975; Vallis and Maltrud 1993).

533 *b. Beyond barotropic dynamics*

534 Because we are only considering a two-dimensional dynamics, we neglect the possible in-
535 teractions between different vertical modes. A full description of the non-linear dynamics of
536 equatorial waves should consider baroclinic modes and their possible non-linear interactions. The
537 two-dimensional problem investigated in this paper is well posed for the barotropic mode only.
538 Non-linear interactions of barotropic waves indeed remain barotropic (not shown). In this case
539 the two-dimensional reduction of the problem is a good approximation and 2D simulations are a
540 good tool to study the NLTI. This is not the case for baroclinic waves. Non-linear interactions of
541 baroclinic modes project mainly on the barotropic mode (not shown). The barotropic mode thus
542 acts also as an attractor in the "wave cascade".

543 *c. Towards a realistic mean circulation*

544 Intra-annual waves with large wavenumbers such as the ones described in this study are not
545 unrealistic as they have been observed in the tropical ocean (Farrar 2011; Tuchen et al. 2018;
546 Delpech et al. 2020b). Farrar (2011) evidenced barotropic intra-annual waves in the northern
547 tropical Pacific ocean, radiating away from the Tropical Instability Waves with a 30-day period and
548 a 1000-km wavelength. Tuchen et al. (2018) described evidence of 50-day Yanai waves with a
549 baroclinic structure at the equator. Delpech et al. (2020b) described observations of waves at 1000

m in the northern tropical Pacific with a 70-day period and a 500-km wavelength with properties consistent with barotropic Rossby waves. It is interesting to note that the waves observed by Farrar (2011) are found to be stable to NTLI (e.g., Fig. 13). This could explain why they are seen propagating away from their generation site. On the contrary, waves unstable to NTLI have only sporadically been observed, their small wavelengths make them challenging to capture with in-situ data and as they are associated with a weak signature in sea surface height (typically less than 2 cm at low latitudes), it is also difficult to capture them with the current generation Nadir altimetry. The new generation swath satellite altimeter designed to observe submesoscale features (e.g., SWOT Fu and Ubelmann (2014)) may open new opportunities in this regard. The jet-like structures produced by the unstable intra-annual waves through NTLI are long waves with zonal wavelengths that can reach several thousands of kilometers (Fig. 16b) and with periods of several years (Fig. 16c). However, their time averaged velocities generally result in a weak mean circulation (e.g., Fig. 6a). This mechanism is therefore not sufficient to explain the observed mean deep circulation. Other mechanisms may intervene in the long term equilibration of these jet-like structures to produce the jet-structured mean circulation as has been observed in 10-year averages of Argo float displacements (Ollitrault et al. 2006; Cravatte et al. 2012). Equilibration processes may involve modification of the potential vorticity by the zonal jets combined with positive feedback mechanisms such as eddy-driven jet-sharpening (Dritschel and Scott 2011; Greatbatch et al. 2018; Ménesguen et al. 2019). In addition, the observed zonal jets are not purely barotropic and have a complex vertical structure (Qiu et al. 2013; Cravatte et al. 2017). Therefore baroclinic dynamics could come into play and should be considered in future studies. Finally, bottom topography could have an influence on the dynamics of barotropic Rossby waves (such as causing scattering, refraction and trapping) through the topographic beta effect (e.g., Wang and Kobalinsky 1994; Durland and Farrar 2020) and the modification of classical vertical modes (Lacasse 2017). The

574 topographic beta effect scales as the Coriolis parameter and the bottom slope. We therefore expect
575 it to be of lesser importance at low latitudes, away from steep continental margins or seamounts. It
576 is however interesting to note that the influence of topography could transform a stable large scale
577 long period Rossby wave forced by the atmosphere, into a small scale long period Rossby wave,
578 subject to NLTI (Rhines and Bretherton 1973).

579 *d. Summary and Perspectives*

580 Deep intra-annual waves, which have been frequently observed at low latitudes in the ocean are
581 prone to instability. Focusing on the barotropic mode, we demonstrated that the energy transfer
582 mechanism of this instability is well described by a 3-mode truncation NLTI theory. The growth
583 rate and characteristics of the instability are a function of the wavenumber, frequency and amplitude
584 of the primary wave. In particular cases, the secondary waves have jet-like structures. In this study,
585 we showed that intra-annual waves can destabilize into jet-like structures using idealized barotropic
586 numerical simulations of an equatorial basin forced by an oscillatory wind-stress. We determined
587 that the primary waves that are optimal to reproduce realistic jet-like structures are intra-annual
588 zonally-propagating waves with period between 60 to 130 days, zonal wavelengths between 200
589 to 300 km and amplitude from 2 to 10 cm s⁻¹. The jet-like structures produced in the simulations
590 and predicted by the theory do not capture all the properties of the time-mean deep circulation as it
591 has been observed in low-latitude oceans (in particular the jets are too weak). Future work should
592 address the potential role of three-dimensional dynamics in the generation of these jets and include
593 baroclinic modes into NLTI theory. The effects of bottom topography and variable stratification
594 should be investigated as they can modify the properties and propagation of Rossby waves. Future
595 work will also involve running realistic numerical simulations to study the role of NLTI and other

596 equilibration processes in the formation of the jet-structured mean circulation within a realistic
597 equatorial wave field.

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606 APPENDIX

607 **Analytical solutions for waves non-linear triad interactions**

608 The aim of this section is to understand the dynamics of the forced wave and in particular its
609 stability. We describe its stability (or instability) using a theoretical truncated non-linear model
610 which allows to resolve analytically the first order of non linearity. We derive this problem in a
611 quasi-geostrophic framework, which corresponds to the limit of the primitive equation model for
612 small amplitudes (a reasonable assumption when working with waves).

613 The two dimensional quasi-geostrophic equation for the stream function ψ is:

$$\frac{\partial}{\partial t}(\nabla^2\psi - F\psi) + \beta\frac{\partial\psi}{\partial x} + J(\psi, \nabla^2\psi) = 0 \quad (\text{A1})$$

614 where J is the Jacobian operator and F is the inverse of the squared deformation radius. The
615 deformation radius are defined as $L_{R,0}$ for the barotropic deformation radius and $L_{R,n}$ for the n^{th}

616 baroclinic deformation radius.

$$L_{R,0} = \frac{\sqrt{gH}}{f_0}; \quad L_{R,n} = \frac{NH}{n\pi f_0} \quad (\text{A2})$$

617 A plane monochromatic wave, $\psi = \psi_0 e^{i(k_x x + k_y y - \omega t)}$, is solution of the equations if it satisfies the
618 relation.

$$\omega = \frac{-\beta k_x}{k_x^2 + k_y^2 + F} \quad (\text{A3})$$

619 which is known as the Rossby wave dispersion relation. Plane monochromatic Rossby waves are
620 thus exact solutions of the non-linear quasi-geostrophic equation (Eq. A1).

621 However, a superposition of waves, $\psi = \sum_n \psi_n = \sum_n \psi_{0n}(t) e^{i(k_{xn}x + k_{yn}y)}$, is not an exact solution
622 of the Eq. A1.

$$\frac{\partial}{\partial t} \left(\sum_n -(k_{xn}^2 + k_{yn}^2 + F) \psi_n \right) + \beta \sum_n i k_{xn} \psi_n + \frac{1}{2} \sum_{i,j} (k_{xi} k_{yj} - k_{xj} k_{yi}) (k_i^2 - k_j^2) \psi_i \psi_j = 0 \quad (\text{A4})$$

623 Because of the quadratic nature of non linear Jacobian term in Eq. A1, the waves interaction is
624 triadic: the waves i, j and n interact together and their amplitude ψ_0 depend on time.

625 The equation for the evolution of a wave of streamfunction $\psi_{\mathbf{k}}$ and wavenumber \mathbf{k} is then given
626 by:

$$\frac{\partial}{\partial t} \psi_{\mathbf{k}} + i\omega_{\mathbf{k}} + \frac{1}{2} \sum_{i,j|\mathbf{k}_i+\mathbf{k}_j=\mathbf{k}} T(\mathbf{k}, \mathbf{k}_i, \mathbf{k}_j) \psi_{\mathbf{k}_i} \psi_{\mathbf{k}_j} = 0 \quad (\text{A5})$$

627 where

$$\omega_{\mathbf{k}} = -\frac{\beta k_x}{k_x^2 + k_y^2 + F}; \quad T(\mathbf{k}, \mathbf{k}_i, \mathbf{k}_j) = -\frac{(\mathbf{k}_i \times \mathbf{k}_j)_z (k_i^2 - k_j^2)}{k^2 + F} \quad (\text{A6})$$

628 T is the triadic interaction term.

629 For simplicity and in order to keep the problem analytically solvable, we consider in a first time
630 a reduced problem for which only one triad (three waves) is retained. We note \mathbf{p} , \mathbf{q} and $\mathbf{p} - \mathbf{q}$ the
631 wave vectors of these three waves, such that $\mathbf{p} = \mathbf{q} + \mathbf{p} - \mathbf{q}$. Following Eq. A5, we can derive an

632 equation for each of these waves.

$$\begin{cases} \partial_t \psi_{0\mathbf{p}} + i\omega_{\mathbf{p}} \psi_{0\mathbf{p}} + T(\mathbf{p}, \mathbf{q}, \mathbf{p}_-) \psi_{0\mathbf{q}} \psi_{0\mathbf{p}_-} = 0 \\ \partial_t \psi_{0\mathbf{q}} + i\omega_{\mathbf{q}} \psi_{0\mathbf{q}} + T(\mathbf{q}, \mathbf{p}, -\mathbf{p}_-) \psi_{0\mathbf{p}} \psi_{0\mathbf{p}_-}^* = 0 \\ \partial_t \psi_{0\mathbf{p}_-} + i\omega_{\mathbf{p}_-} \psi_{0\mathbf{p}_-} + T(\mathbf{p}_-, \mathbf{p}, -\mathbf{q}) \psi_{0\mathbf{p}} \psi_{0\mathbf{q}}^* = 0 \end{cases} \quad (\text{A7})$$

633 where the star denotes the conjugate of a complex. $\psi_{0\mathbf{k}}^* = \psi_{0-\mathbf{k}}$. Introducing $\psi_{\mathbf{k}}(t) = \psi_{0\mathbf{k}}(t) e^{-i\omega_{\mathbf{k}} t}$

634 Eq. A7 becomes:

$$\begin{cases} \partial_t \psi_{\mathbf{p}} + T(\mathbf{p}, \mathbf{q}, \mathbf{p}_-) \psi_{\mathbf{q}} \psi_{\mathbf{p}_-} e^{i\Delta t} = 0 \\ \partial_t \psi_{\mathbf{q}} + T(\mathbf{q}, \mathbf{p}, -\mathbf{p}_-) \psi_{\mathbf{p}} \psi_{\mathbf{p}_-}^* e^{-i\Delta t} = 0 \\ \partial_t \psi_{\mathbf{p}_-} + T(\mathbf{p}_-, \mathbf{p}, -\mathbf{q}) \psi_{\mathbf{p}} \psi_{\mathbf{q}}^* e^{-i\Delta t} = 0 \end{cases} \quad (\text{A8})$$

635 where $\Delta = \omega_{\mathbf{p}} - \omega_{\mathbf{q}} - \omega_{\mathbf{p}_-}$.

636 The process we want to study in the framework of our numerical simulations is the instability
637 of a primary wave of wavevector \mathbf{p} . We linearize the Eq. A8 around the basic state $\psi_{\mathbf{p}}$, with the
638 perturbations $\tilde{\psi}_{\mathbf{q}}$ and $\tilde{\psi}_{\mathbf{p}_-}$.

$$\begin{cases} \partial_t \tilde{\psi}_{\mathbf{q}} + T(\mathbf{q}, \mathbf{p}, -\mathbf{p}_-) \psi_{\mathbf{p}} \tilde{\psi}_{\mathbf{p}_-}^* e^{-i\Delta t} = 0 \\ \partial_t \tilde{\psi}_{\mathbf{p}_-}^* + T(\mathbf{p}_-, \mathbf{p}, -\mathbf{q}) \psi_{\mathbf{p}}^* \tilde{\psi}_{\mathbf{q}} e^{i\Delta t} = 0 \end{cases} \quad (\text{A9})$$

639 Solutions of these equations are harmonics:

$$\begin{cases} \tilde{\psi}_{\mathbf{q}}(t) = A_{\mathbf{q}} e^{-i\sigma_{\mathbf{q}} t} \\ \tilde{\psi}_{\mathbf{p}_-}(t) = A_{\mathbf{p}_-} e^{-i\sigma_{\mathbf{p}_-} t} \end{cases} \quad (\text{A10})$$

640 The instability occurs if $\text{Im}(\sigma_{\mathbf{q}}) > 0$ and $\text{Im}(\sigma_{\mathbf{p}_-}) > 0$. $\text{Im}(\sigma)$ is the growth rate of the instability.

641 The secondary waves \mathbf{q} and \mathbf{p}_- are determined as the waves that give the maximum growth rate of
642 instability.

643 The linear system in Eq. A9 then becomes, using $\sigma_q + \sigma_{p-}^* - \Delta = 0$

$$\begin{pmatrix} -i\sigma_q & T(\mathbf{q}, \mathbf{p}, -\mathbf{p}_-) \psi_{\mathbf{p}} \\ T(\mathbf{p}_-, \mathbf{p}, -\mathbf{q}) \psi_{\mathbf{p}}^* & i\sigma_{p-}^* \end{pmatrix} \begin{pmatrix} A_q \\ A_{p-}^* \end{pmatrix} = 0 \quad (\text{A11})$$

644 Non trivial solutions exist if the determinant of the matrix is equal to 0.

$$\sigma_q^2 - \Delta\sigma_q + T(\mathbf{q}, \mathbf{p}, -\mathbf{p}_-)T(\mathbf{p}_-, \mathbf{p}, -\mathbf{q})|\psi_{\mathbf{p}}|^2 = 0 \quad (\text{A12})$$

645 Which gives solutions for the growth rate:

$$\sigma_q = \frac{1}{2}(\Delta \pm \sqrt{\Delta^2 - 4|\psi_{\mathbf{p}}|^2 T(\mathbf{q}, \mathbf{p}, \mathbf{q} - \mathbf{p})T(\mathbf{p} - \mathbf{q}, \mathbf{p}, -\mathbf{q})}) \quad (\text{A13})$$

646 Note that the two secondary waves \mathbf{q} and \mathbf{p}_- have the same growth rate as $\text{Im}(\sigma_q) = \text{Im}(\sigma_{p-})$.

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 759 zonal wavenumber and amplitude of the forcing (Eq. 2). The experiments are
 760 2D and are either linear (L) or non-linear (NL). The letter of the experiment
 761 label indicates its position on the wavenumber-frequency spectrum (Fig. 1), and
 762 the number indicates the forcing amplitude. 39

763 **Table 2.** The waves involved in non-linear triad interactions. The primary wave is the
 764 forced wave, $\omega_{\mathbf{p}}$, p_x , p_y thus are fixed by the forcing. As the experiment are
 765 resonantly forced for the barotropic mode, they satisfy the barotropic Rossby
 766 wave dispersion relation. The properties of the two secondary waves (\mathbf{q} and
 767 $\mathbf{p}^- = \mathbf{p} - \mathbf{q}$) are determined from where the growth rate is maximum in spectral
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769 **TABLE 1. Experiment Table.** Summary of all numerical experiments used for this study, where T_0 , ω_0 , λ_{x0} ,
770 k_{x0} , and τ_0 are the period, frequency, zonal wavelength, zonal wavenumber and amplitude of the forcing (Eq. 2).
771 The experiments are 2D and are either linear (L) or non-linear (NL). The letter of the experiment label indicates
772 its position on the wavenumber-frequency spectrum (Fig. 1), and the number indicates the forcing amplitude.

Exp.	T_0 (days)	ω_0 (s^{-1})	λ_{x0} (km)	k_{x0} (m^{-1})	τ_0 (m^2s^{-2})	Physics	
1a	132	5.49e-7	150	4.19e-5	5e-5	2D	NL
1b	114	6.41e-7	175	3.59e-5	5e-5	2D	NL
1c	99	7.32e-7	200	3.14e-5	5e-5	2D	NL
1d	79	9.15e-7	250	2.51e-5	5e-5	2D	NL
1e	66	1.10e-6	300	2.09e-5	5e-5	2D	NL
1f	57	1.28e-6	350	1.80e-5	5e-5	2D	NL
1g	50	1.46e-6	400	1.57e-5	5e-5	2D	NL
1h	44	1.65e-6	450	1.40e-6	5e-5	2D	NL
1i	40	1.83e-6	500	1.26e-5	5e-5	2D	NL
1j	36	2.01e-6	550	1.14e-5	5e-5	2D	NL
1k	33	2.2e-6	600	1.0e-5	5e-5	2D	NL
1l	30	2.4e-6	650	9.7e-6	5e-5	2D	NL
2a	132	5.49e-7	150	4.19e-5	5e-6	2D	NL
2c	99	7.32e-7	200	3.14e-5	5e-6	2D	NL
2e	66	1.10e-6	300	2.09e-5	5e-6	2D	NL
2g	50	1.46e-6	400	1.57e-5	5e-6	2D	NL
2i	40	1.83e-6	500	1.26e-5	5e-6	2D	NL
3a	132	5.49e-7	150	4.19e-5	2.5e-5	2D	NL
3c	99	7.32e-7	200	3.14e-5	2.5e-5	2D	NL
3e	66	1.10e-6	300	2.09e-5	2.5e-5	2D	NL
3g	50	1.46e-6	400	1.57e-5	2.5e-5	2D	NL
3i	40	1.83e-6	500	1.26e-5	2.5e-5	2D	NL

Exp.	T_0 (days)	ω_0 (s^{-1})	λ_{x0} (km)	k_{x0} (m^{-1})	τ_0 (m^2s^{-2})	Physics	
4a	132	5.49e-7	150	4.19e-5	7.5e-5	2D	NL
4c	99	7.32e-7	200	3.14e-5	7.5e-5	2D	NL
4e	66	1.10e-6	300	2.09e-5	7.5e-5	2D	NL
4g	50	1.46e-6	400	1.57e-5	7.5e-5	2D	NL
4i	40	1.83e-6	500	1.26e-5	7.5e-5	2D	NL
5a	132	5.49e-7	150	4.19e-5	1e-4	2D	NL
5c	99	7.32e-7	200	3.14e-5	1e-4	2D	NL
5e	66	1.10e-6	300	2.09e-5	1e-4	2D	NL
5g	50	1.46e-6	400	1.57e-5	1e-4	2D	NL
5i	40	1.83e-6	500	1.26e-5	1e-4	2D	NL
6a	132	5.49e-7	150	4.19e-5	1.5e-4	2D	NL
6c	99	7.32e-7	200	3.14e-5	1.5e-4	2D	NL
6e	66	1.10e-6	300	2.09e-5	1.5e-4	2D	NL
6g	50	1.46e-6	400	1.57e-5	1.5e-4	2D	NL
6i	40	1.83e-6	500	1.26e-5	1.5e-4	2D	NL
1d-L	79	9.15e-7	250	2.51e-5	5e-5	2D	L
11-L	30	2.4e-6	650	9.7e-6	5e-5	2D	L

773 TABLE 2. The waves involved in non-linear triad interactions. The primary wave is the forced wave, $\omega_{\mathbf{p}}$, p_x ,
774 p_y thus are fixed by the forcing. As the experiment are resonantly forced for the barotropic mode, they satisfy
775 the barotropic Rossby wave dispersion relation. The properties of the two secondary waves (\mathbf{q} and $\mathbf{p}^- = \mathbf{p} - \mathbf{q}$)
776 are determined from where the growth rate is maximum in spectral space (Eq. 11).

	Forced wave (primary wave)	Long secondary wave	Short secondary wave
k_x (m^{-1})	$p_x = -2.3 \times 10^{-5}$	$q_x = -2.1 \times 10^{-6}$	$p_x^- = -2.0 \times 10^{-5}$
λ_x (km)	$\lambda_{\mathbf{p}x} = 250$ km	$\lambda_{\mathbf{q}x} = 4958$ km	$\lambda_{\mathbf{p}^-x} = 238$ km
k_y (m^{-1})	$p_y = 0$	$q_y = 1.4 \times 10^{-5}$	$p_y^- = 1.7 \times 10^{-5}$
λ_y (km)	$\lambda_{\mathbf{p}y} \sim \infty$	$\lambda_{\mathbf{q}y} = 429$ km	$\lambda_{\mathbf{p}^-y} = 429$ km
ω (s^{-1})	$\omega_{\mathbf{p}} = 9.8 \times 10^{-7}$	$\omega_{\mathbf{q}} = 2.6 \times 10^{-7}$	$\omega_{\mathbf{p}^-} = 7.2 \times 10^{-7}$
T (days)	$T_{\mathbf{p}} = 79$ days	$T_{\mathbf{q}} = 540$ days	$T_{\mathbf{p}^-} = 110$ days

777 **LIST OF FIGURES**

778 **Fig. 1.** Wavenumber frequency diagram with the dispersion relations for equatorial Kelvin waves
779 (green), Yanai waves (red), and baroclinic Rossby waves (blue). Only the first baroclinic
780 mode is represented. The dispersion relation for barotropic Rossby waves with different
781 meridional wavenumbers ($0, 1 \times 10^{-6}, 2 \times 10^{-6}, 4 \times 10^{-6} \text{ m}^{-1}$ from top to bottom) are in
782 black. The frequency and wavenumbers of the forcing used in the experiments conducted
783 in this study are represented by the yellow dots. Wave instability simulations conducted
784 in previous studies are indicated with colored dots along with their references. The square
785 boxes represent the spectral range of waves that have been observed at the surface (solid
786 lines) and at depth (dashed lines). 45

787 **Fig. 2.** Two-dimensional spectrum of the wind stress function $F(x, y, t) = X(x)Y(y)\sin(k_{x0}x +$
788 $k_{y0}y - \omega_0 t)$ (grey) for simulation 1d (see Table 1). Black straight lines correspond to
789 the forced wavenumber $\pm k_{x0}$ and $k_{y0} = 0$. Black circles correspond to the barotropic Rossby
790 wave dispersion relation for the forced frequency ω_0 (T=79 days) and a non-resonant fre-
791 quency (T=180 days). If the dispersion relation intersects a location in wavenumbers space
792 where there is a maximum in energy, the forced wave is resonantly forced. The example
793 shows that only the period corresponding to the barotropic Rossby wave dispersion relation
794 for k_{x0} and k_{y0} satisfies the resonant condition. 46

795 **Fig. 3.** (a) Experiment spin-up time (computed as in Eq. 6) as a function of the forcing frequency
796 (ω_0). (b) Amplitude of the wave response (computed as in Eq. 7) as a function of the strength
797 of the forcing (τ_0) for experiments a-l (Fig. 1). The best linear fit (line) corresponds to a line
798 $y = ax + b$ and r is the correlation coefficient. 47

799 **Fig. 4.** Evolution of the meridional barotropic velocity (v) in the linear simulations (a-c) 1d-L and
800 (d-f) 1l-L at times (a,d) $t = 100$ days, (b,e) $t = 400$ days, (c,f) $t = 800$ days. Black lines
801 indicate the ray path originating from 70° longitude (the center of the forcing region) and
802 $-5^\circ, 0^\circ$ and 5° latitude respectively and integrated from the initial time of the simulation to
803 the time of the snapshot (Eq. 5). Position of the ray at each wave period is indicated by the
804 dots. The zonal barotropic velocity (u) is almost zero in the whole basin and is not shown. 48

805 **Fig. 5.** Evolution of the velocity and sea surface height (ssh) in the non-linear simulations (a-i) 1d
806 and (j-r) 1l, at times: (a,d,g,j) $t = 100$ days, (b,e,h,k) $t = 400$ days and (c,f,i,l) $t = 800$ days for
807 (a-c, j-l) v (meridional velocity); (d-f, m-o) u (zonal velocity); (g-i, p-r). The red and green
808 squares correspond to the locations where the one-dimensional spectra are computed (Fig. 9). 49

809 **Fig. 6.** u averaged over years 2 to 5 of the simulations from 10° to 40° of longitude (i.e. west of the
810 forcing region) for simulations (a) 1d (b) 1l (c) and simulation with optimal primary wave
811 characteristics, corresponding to experiment 1c from Table 1, but with a shifted wind-stress
812 position ($y_0=3^\circ\text{S}$) (d) Observed zonal velocity at 1000 m depth in the tropical Pacific from
813 Argo floats YoMaHA database (Lebedev et al. 2007) reproduced after (Delpech et al. 2020b). 50

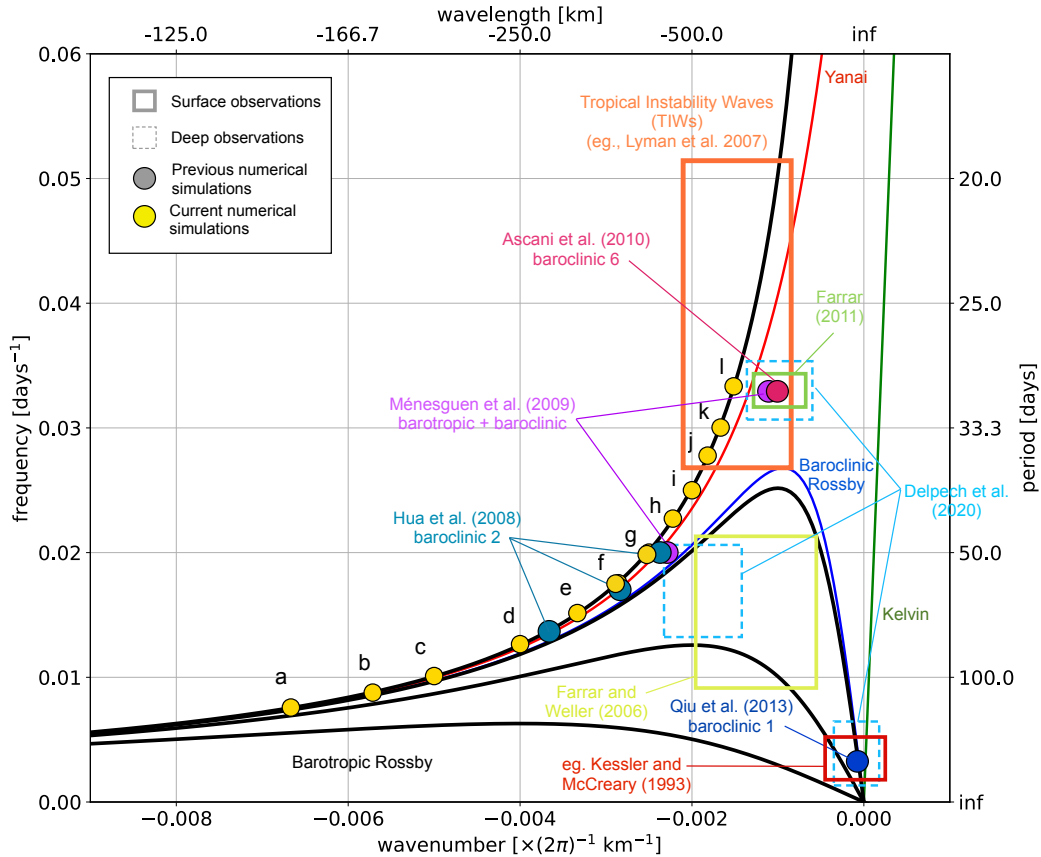
814 **Fig. 7.** The growth rate for the 3-mode truncated non-linear triad interaction (Eq. 11) (a) as a function
815 of the secondary wavenumbers q_x and q_y . The red arrow indicates the primary wave \mathbf{p} and
816 the green arrows the most unstable secondary waves \mathbf{q} and $\mathbf{p} - \mathbf{q}$. The circles represent
817 the magnitude of the wavevectors. (b) The growth rate as a function of the secondary
818 wavenumber q_x and period ω_q . The red dot indicates the position of the primary wave and
819 the green dots the position of the secondary waves in $\omega - q_x$ space. Black lines indicate the
820 dispersion relations for barotropic Rossby waves with $k_y = 0$ and $k_y = 2 \times 10^{-6} \text{ m}^{-1}$. The
821 two secondary waves have the same growth rate. Note that we use the convention of positive

822	frequency for waves, such that the wavevector with $q_x > 0$ (and thus a theoretical negative	
823	frequency) in panel (a) is represented with a $q_x < 0$ and a positive frequency in panel (b).	51
824	Fig. 8. Primary and secondary waves' phase lines orientation (dashed lines) and group velocity	
825	vectors (arrows) in a polar diagram. 0° represents the eastward direction, and the radial axis	
826	indicates the magnitude of the group velocity in m s^{-1} . The zonal wavelength and period of	
827	the wave is indicated in the legend.	52
828	Fig. 9. (red curves) Spectrum of the meridional velocity at the location where the primary wave \mathbf{p}	
829	is expected (red marker in Fig. 5c) and (green curves) Spectrum of the zonal velocity at the	
830	location where the long secondary wave \mathbf{q} is expected (green marker in Fig. 5f). (a) The time	
831	wavelet transform and (b) the space wavelet transform in the zonal direction (solid line) and	
832	in the meridional direction (dashed line). The wavelet transforms are averaged over a $2^\circ \times 2^\circ$	
833	horizontal domain and a 20 day time interval. Vertical lines indicate the characteristics that	
834	are predicted for the primary and secondary waves (Table 2).	53
835	Fig. 10. Wavelet scalogram averaged in the interior of the domain (i.e the region away from 5° of the	
836	boundary) as a function of (a) time and period $\widetilde{\text{KE}}^t$ (Eq. 8); (b) time and zonal wavelength	
837	$\widetilde{\text{KE}}^x$; (c) time and meridional wavelength $\widetilde{\text{KE}}^y$. The spin up period is indicated by the arrow.	
838	The time when the instability occurs is indicated with the dashed line, and corresponds to a	
839	shift in the spectra.	54
840	Fig. 11. Two-dimensional spectra computed from the wavelet transforms following the method de-	
841	scribed in Section 3. (a,c) $\widetilde{\text{KE}}^{xy}$; (b,d) $\widetilde{\text{KE}}^{xt}$; (a,b) before the instability emerges ($t = 150$	
842	days); and (c,d) after the instability emerges ($t = 680$ days). The red (green) dots and ar-	
843	rows indicate the theoretical frequency and wavenumbers of the primary (secondary) waves	
844	respectively. White lines (b,d) indicate the dispersion relation for barotropic Rossby waves	
845	with $k_y = 0$ and $k_y = 2 \times 10^{-6}$.	55
846	Fig. 12. (a) Theoretical instability emergence ratio as a function of the amplitude of the primary wave	
847	\mathbf{p} for four waves with periods and zonal wavelengths indicated in the legend. The instability	
848	emergence ratio (Eq. 13) is defined as the ratio between the time it takes a wavepacket to	
849	travel one wavelength and the emergence time of the secondary wave, illustrating how the	
850	instability is triggered in the simulations when this ratio exceeds 1 (i.e. for times to the right	
851	of the dashed line). (right panels) $\widetilde{\text{KE}}^y(t)$ for simulations (b) 1a; (c) 1d; (d) 1f and (e) 1h.	56
852	Fig. 13. Theoretical amplitude threshold to reach $R_p^e = 1$ (Eq. 13) as a function of the primary wave	
853	zonal wavenumber (k_x) and frequency ω . Above this amplitude, barotropic Rossby primary	
854	waves are unstable to NLTI.	57
855	Fig. 14. The emergence time of the instability in the simulations as a function of the theoretical growth	
856	rate for the primary waves of the different simulations. Each dot represents one simulation,	
857	with size and color indicating the amplitude and zonal wavenumber of the primary wave	
858	respectively. The black curve is proportional to the inverse of the growth rate, with a	
859	proportionality constant of 5.5.	58
860	Fig. 15. Properties of the long secondary wave as a function of the primary wave's wavelength and	
861	period with a fixed amplitude of 0.07 m/s. (a) λ_y , (b) λ_x , (c) period, (d) time of emergence of	
862	the secondary wave. The dispersion relation for barotropic Rossby waves with different k_y	
863	is indicated by solid lines. The hatched regions correspond to regions where no instabilities	
864	grow.	59

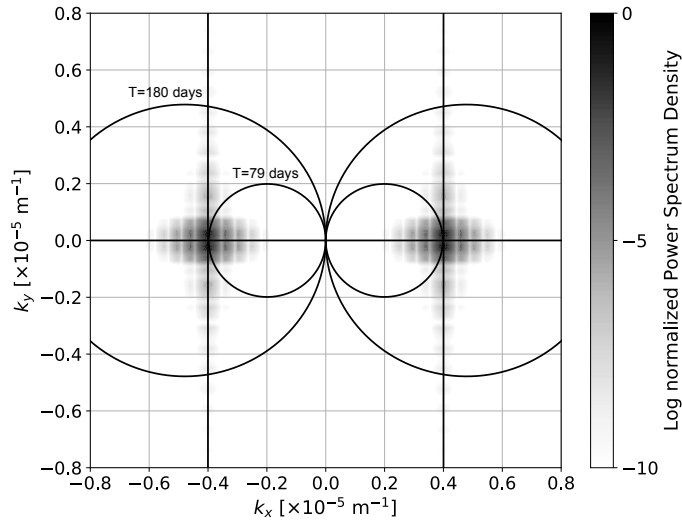
865 **Fig. 16.** (upper panel) Sensitivity of the long secondary waves' properties: (a) meridional wavelength
 866 λ_y , (b) zonal wavelength λ_x , (c) period, to the amplitude of the primary wave. The sensitivity
 867 is computed for primary waves with zonal wavelengths 150 km, 200 km, 300 km, and 400
 868 km and for amplitudes ranging from 0.01 m s^{-1} to 0.3 m s^{-1} with a 0.01 m s^{-1} increment.
 869 The corresponding experiments and the secondary waves' meridional wavelengths computed
 870 using a wavelet analysis are indicated by the stars with their labels referring to Table 1. (lower
 871 panel) The variation coefficient, namely the ratio of the standard deviation of the properties
 872 for the different amplitude to the mean of this properties are computed for the secondary
 873 wave meridional wavelength λ_y (d), zonal wavelength λ_x (e), and period (f). 60

874 **Fig. 17.** Evolution of the energy spectrum in wavenumber space for simulation 1d at times (a) $t = 120$
 875 days, (b) $t = 1000$ days, (c) $t = 1200$ days. 61

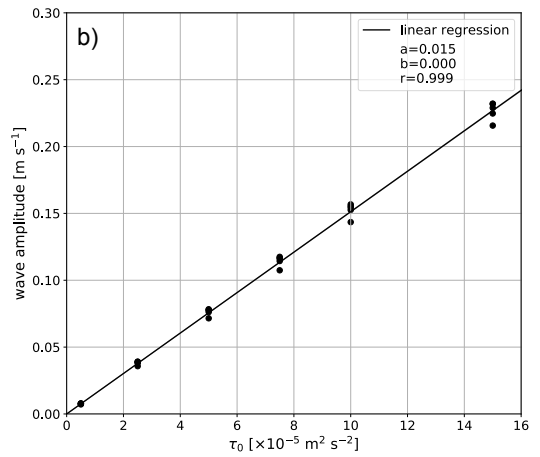
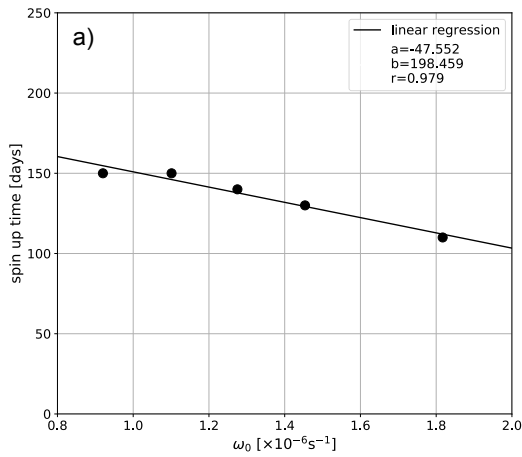
876 **Fig. 18.** The maximum growth rate (Eq. 11) for $|p_x|$ ranging between 0 and $5e-5 \text{ m}^{-1}$ for different
 877 meridional primary wavenumbers p_y (line styles) and amplitude $\psi_0|p_x|$ (line widths). Jet-
 878 like waves correspond to $p_x = 0$ and always have a growth rate equals to zero, illustrating
 879 their particular stability. 62



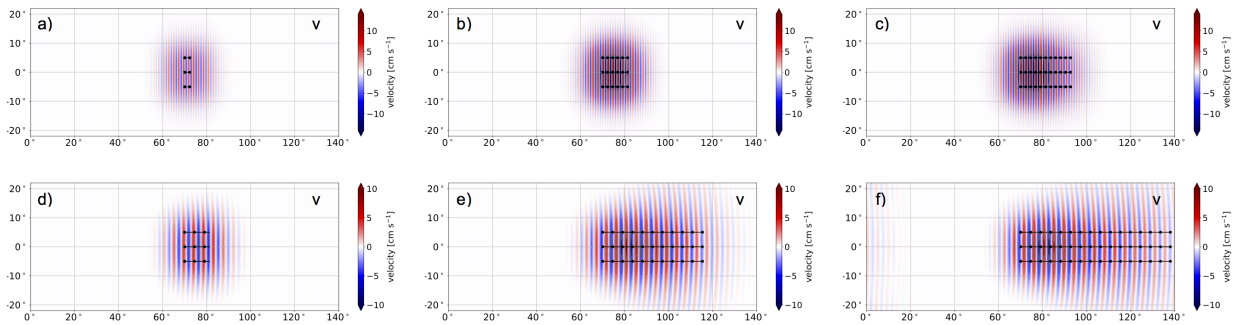
880 FIG. 1. Wavenumber frequency diagram with the dispersion relations for equatorial Kelvin waves (green),
 881 Yanai waves (red), and baroclinic Rossby waves (blue). Only the first baroclinic mode is represented. The
 882 dispersion relation for barotropic Rossby waves with different meridional wavenumbers ($0, 1 \times 10^{-6}, 2 \times 10^{-6},$
 883 $4 \times 10^{-6} \text{ m}^{-1}$ from top to bottom) are in black. The frequency and wavenumbers of the forcing used in the
 884 experiments conducted in this study are represented by the yellow dots. Wave instability simulations conducted
 885 in previous studies are indicated with colored dots along with their references. The square boxes represent the
 886 spectral range of waves that have been observed at the surface (solid lines) and at depth (dashed lines).



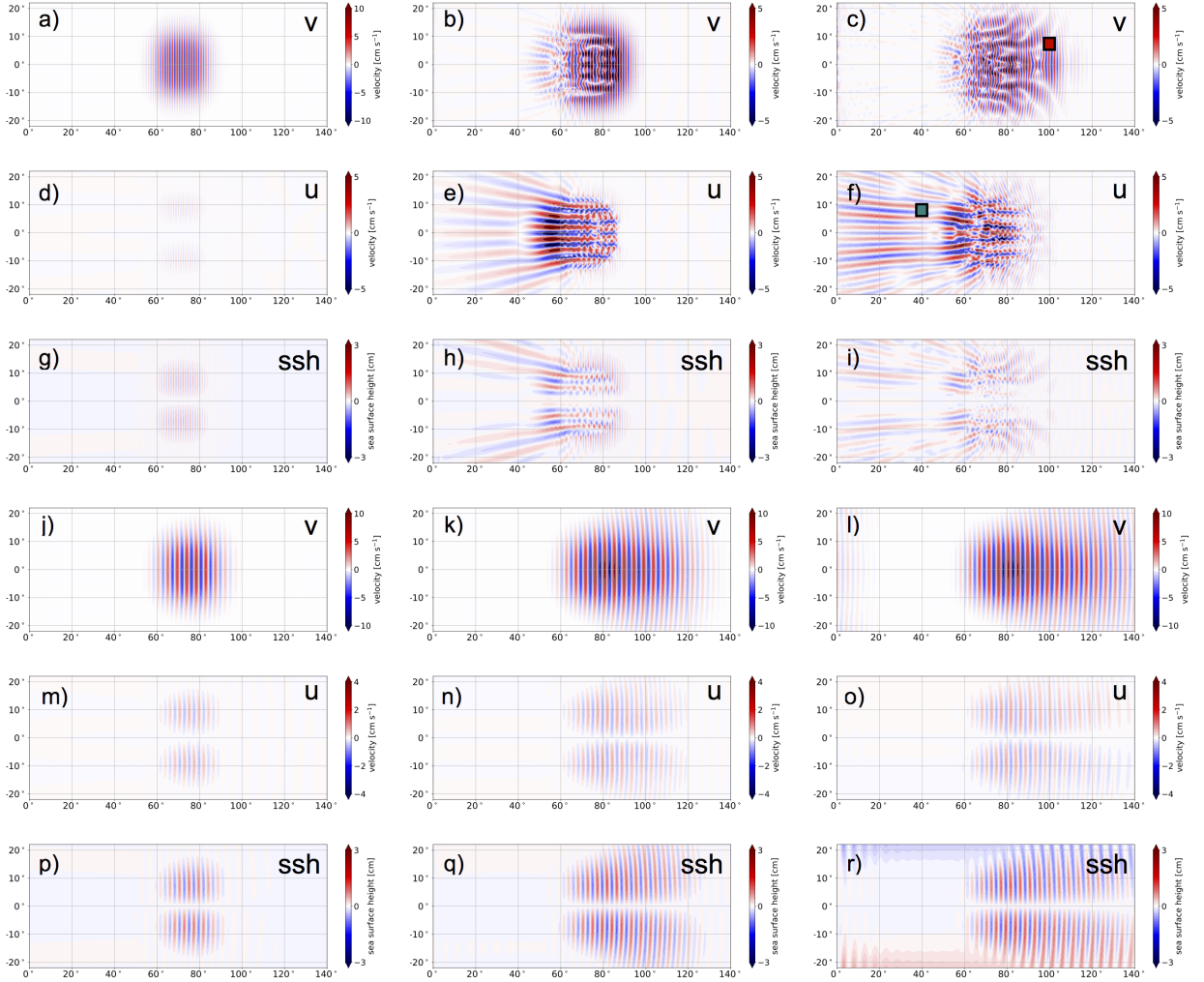
887 FIG. 2. Two-dimensional spectrum of the wind stress function $F(x, y, t) = X(x)Y(y)\sin(k_{x0}x + k_{y0}y - \omega_0 t)$
 888 (grey) for simulation 1d (see Table 1). Black straight lines correspond to the forced wavenumber $\pm k_{x0}$ and
 889 $k_{y0} = 0$. Black circles correspond to the barotropic Rossby wave dispersion relation for the forced frequency
 890 ω_0 (T=79 days) and a non-resonant frequency (T=180 days). If the dispersion relation intersects a location in
 891 wavenumbers space where there is a maximum in energy, the forced wave is resonantly forced. The example
 892 shows that only the period corresponding to the barotropic Rossby wave dispersion relation for k_{x0} and k_{y0}
 893 satisfies the resonant condition.



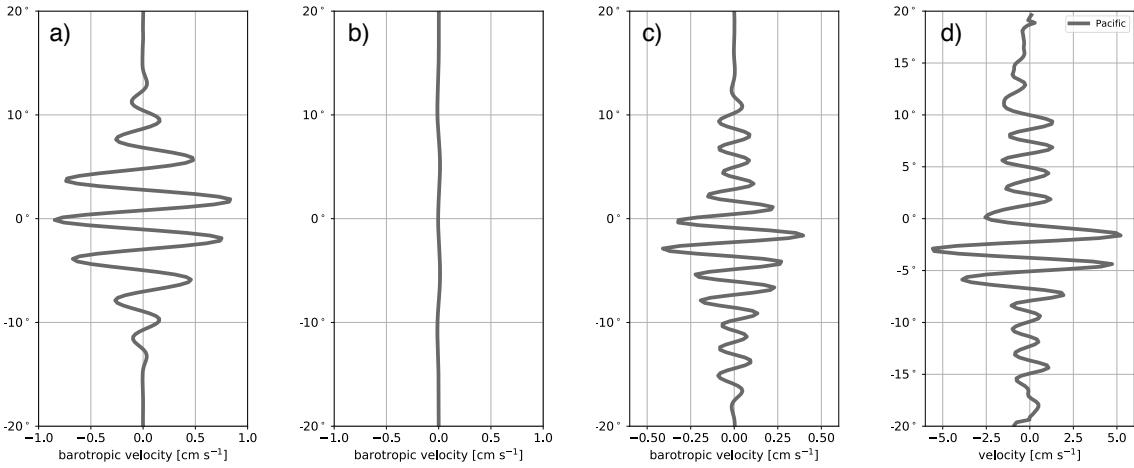
894 FIG. 3. (a) Experiment spin-up time (computed as in Eq. 6) as a function of the forcing frequency (ω_0).
 895 (b) Amplitude of the wave response (computed as in Eq. 7) as a function of the strength of the forcing (τ_0)
 896 for experiments a-1 (Fig. 1). The best linear fit (line) corresponds to a line $y = ax + b$ and r is the correlation
 897 coefficient.



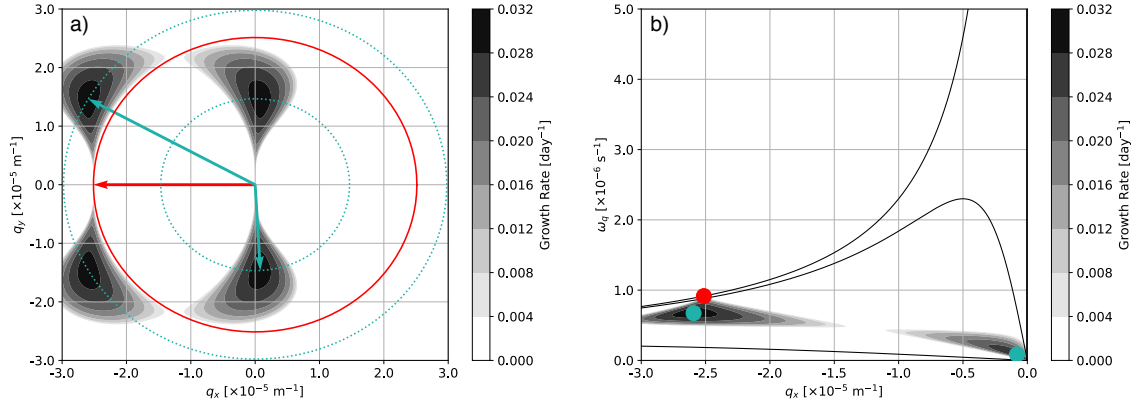
898 FIG. 4. Evolution of the meridional barotropic velocity (v) in the linear simulations (a-c) 1d-L and (d-f) 1l-L
 899 at times (a,d) $t = 100$ days, (b,e) $t = 400$ days, (c,f) $t = 800$ days. Black lines indicate the ray path originating
 900 from 70° longitude (the center of the forcing region) and -5° , 0° and 5° latitude respectively and integrated from
 901 the initial time of the simulation to the time of the snapshot (Eq. 5). Position of the ray at each wave period is
 902 indicated by the dots. The zonal barotropic velocity (u) is almost zero in the whole basin and is not shown.



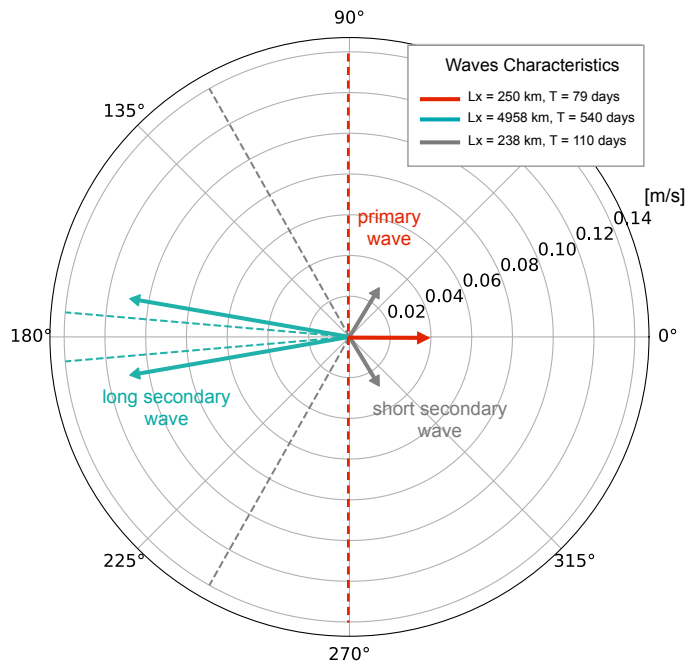
903 FIG. 5. Evolution of the velocity and sea surface height (ssh) in the non-linear simulations (a-i) 1d and (j-r)
 904 1l, at times: (a,d,g,j) $t = 100$ days, (b,e,h,k) $t = 400$ days and (c,f,i,l) $t = 800$ days for (a-c, j-l) v (meridional
 905 velocity); (d-f, m-o) u (zonal velocity); (g-i, p-r). The red and green squares correspond to the locations where
 906 the one-dimensional spectra are computed (Fig. 9).



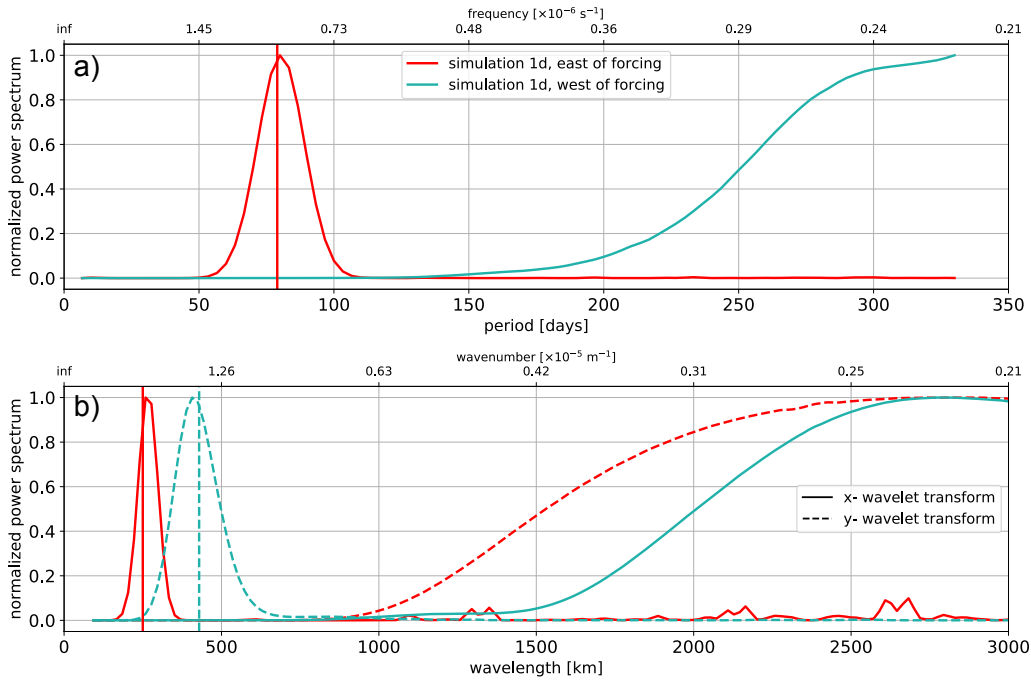
907 FIG. 6. u averaged over years 2 to 5 of the simulations from 10° to 40° of longitude (i.e. west of the forcing
 908 region) for simulations (a) 1d (b) 1l (c) and simulation with optimal primary wave characteristics, corresponding
 909 to experiment 1c from Table 1, but with a shifted wind-stress position ($y_0=3^\circ\text{S}$) (d) Observed zonal velocity at
 910 1000 m depth in the tropical Pacific from Argo floats YoMaHA database (Lebedev et al. 2007) reproduced after
 911 (Delpech et al. 2020b).



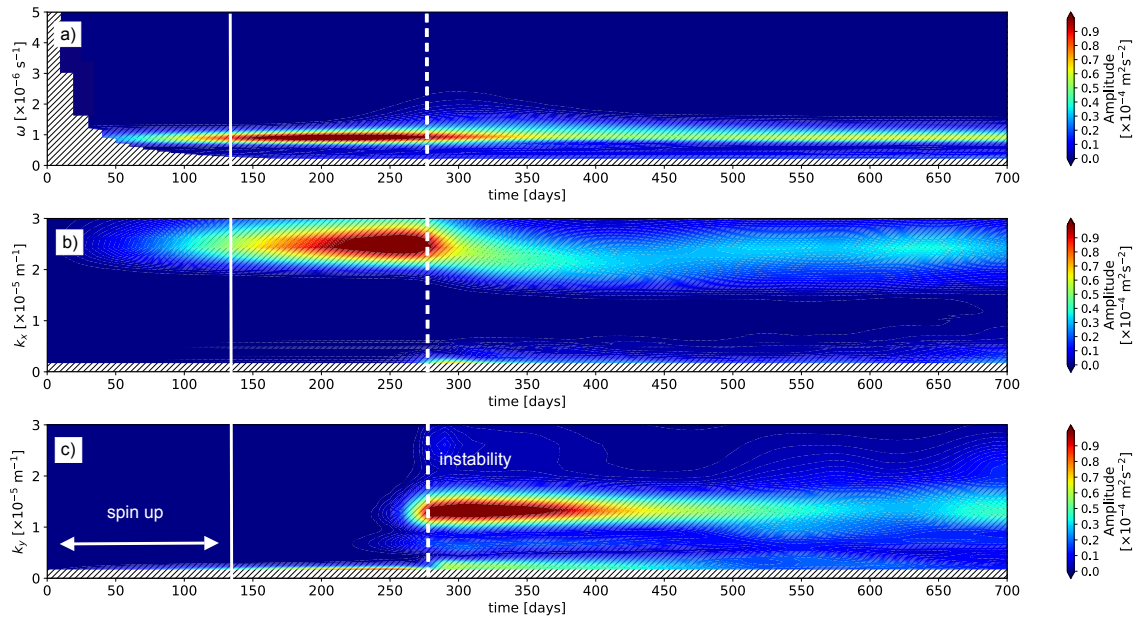
912 FIG. 7. The growth rate for the 3-mode truncated non-linear triad interaction (Eq. 11) (a) as a function of
 913 the secondary wavenumbers q_x and q_y . The red arrow indicates the primary wave \mathbf{p} and the green arrows the
 914 most unstable secondary waves \mathbf{q} and $\mathbf{p} - \mathbf{q}$. The circles represent the magnitude of the wavevectors. (b) The
 915 growth rate as a function of the secondary wavenumber q_x and period ω_q . The red dot indicates the position of
 916 the primary wave and the green dots the position of the secondary waves in $\omega - q_x$ space. Black lines indicate
 917 the dispersion relations for barotropic Rossby waves with $k_y = 0$ and $k_y = 2 \times 10^{-6} \text{ m}^{-1}$. The two secondary
 918 waves have the same growth rate. Note that we use the convention of positive frequency for waves, such that the
 919 wavevector with $q_x > 0$ (and thus a theoretical negative frequency) in panel (a) is represented with a $q_x < 0$ and
 920 a positive frequency in panel (b).



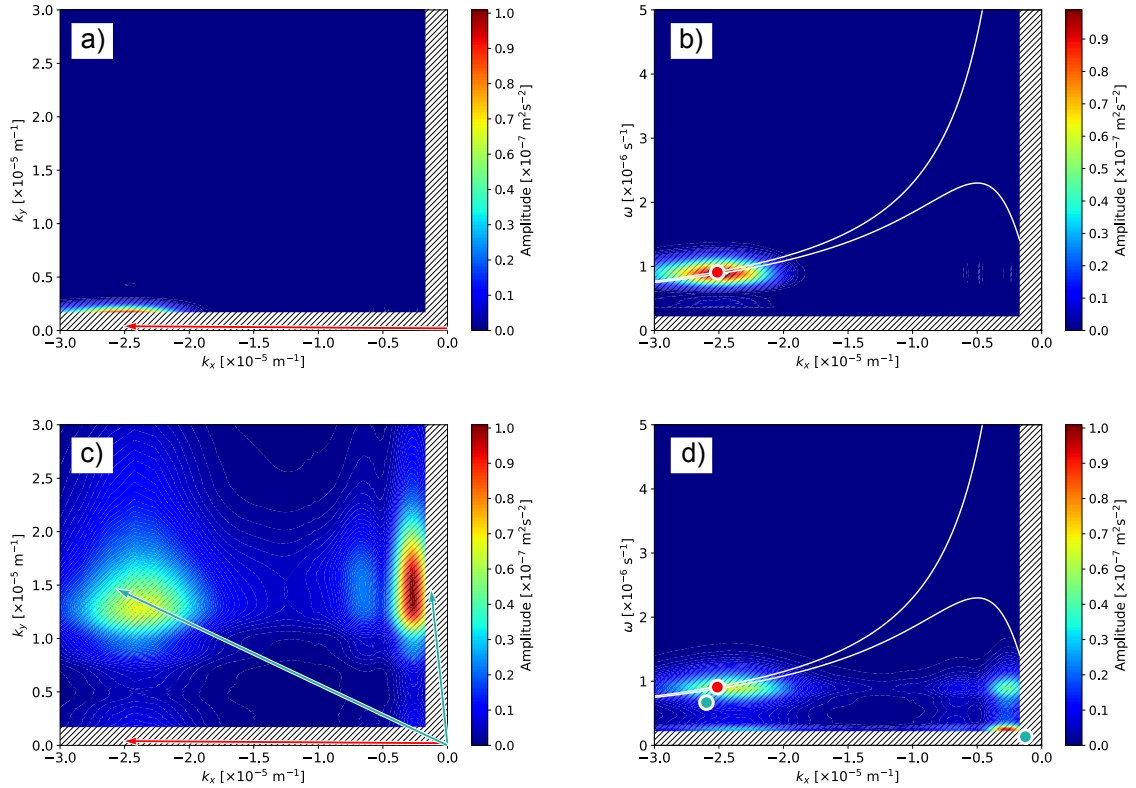
921 FIG. 8. Primary and secondary waves' phase lines orientation (dashed lines) and group velocity vectors
 922 (arrows) in a polar diagram. 0° represents the eastward direction, and the radial axis indicates the magnitude of
 923 the group velocity in m s^{-1} . The zonal wavelength and period of the wave is indicated in the legend.



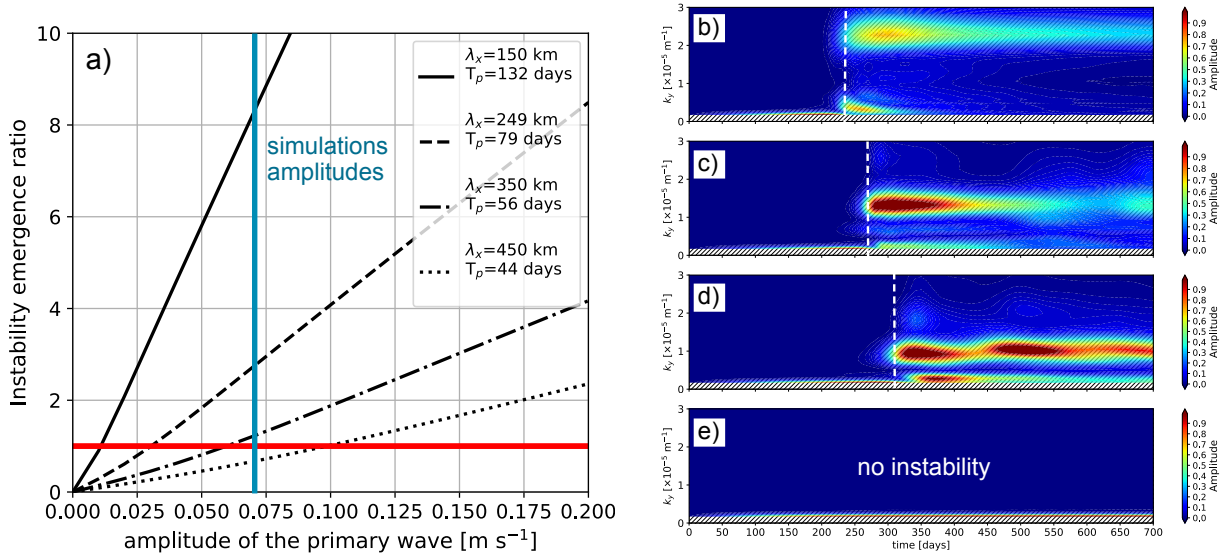
924 FIG. 9. (red curves) Spectrum of the meridional velocity at the location where the primary wave **p** is expected
 925 (red marker in Fig. 5c) and (green curves) Spectrum of the zonal velocity at the location where the long secondary
 926 wave **q** is expected (green marker in Fig. 5f). (a) The time wavelet transform and (b) the space wavelet transform
 927 in the zonal direction (solid line) and in the meridional direction (dashed line). The wavelet transforms are
 928 averaged over a $2^\circ \times 2^\circ$ horizontal domain and a 20 day time interval. Vertical lines indicate the characteristics
 929 that are predicted for the primary and secondary waves (Table 2).



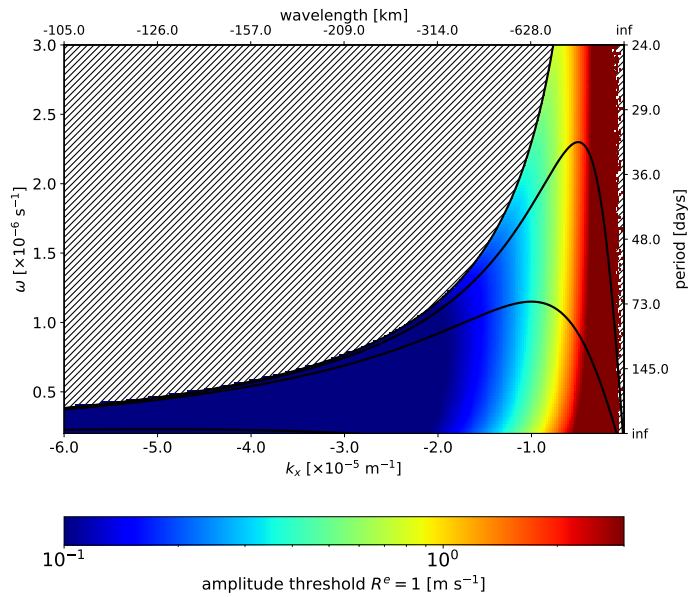
930 FIG. 10. Wavelet scalogram averaged in the interior of the domain (i.e the region away from 5° of the boundary)
 931 as a function of (a) time and period \widetilde{KE}^t (Eq. 8); (b) time and zonal wavelength \widetilde{KE}^x ;
 932 (c) time and meridional wavelength \widetilde{KE}^y . The spin up period is indicated by the arrow. The time when the instability occurs is indicated
 933 with the dashed line, and corresponds to a shift in the spectra.



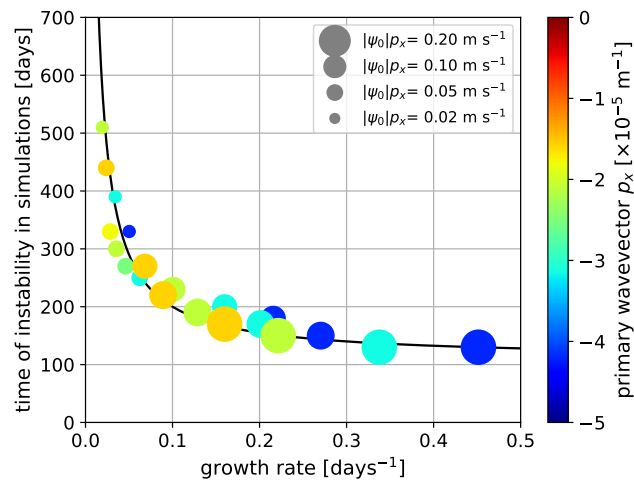
934 FIG. 11. Two-dimensional spectra computed from the wavelet transforms following the method described in
 935 Section 3. (a,c) \widehat{KE}^{xy} ; (b,d) \widehat{KE}^{xt} ; (a,b) before the instability emerges ($t = 150$ days); and (c,d) after the instability
 936 emerges ($t = 680$ days). The red (green) dots and arrows indicate the theoretical frequency and wavenumbers
 937 of the primary (secondary) waves respectively. White lines (b,d) indicate the dispersion relation for barotropic
 938 Rossby waves with $k_y = 0$ and $k_y = 2 \times 10^{-6}$



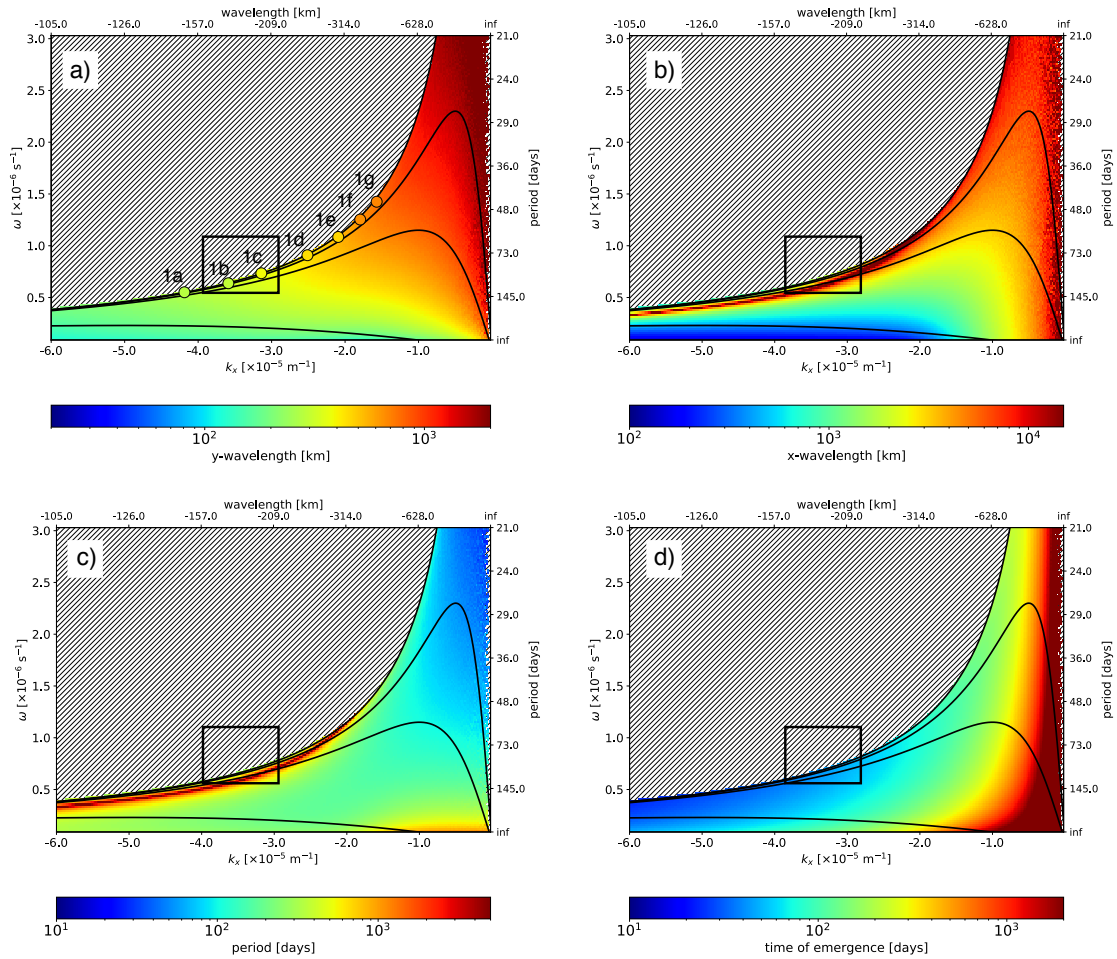
939 FIG. 12. (a) Theoretical instability emergence ratio as a function of the amplitude of the primary wave \mathbf{p} for
 940 four waves with periods and zonal wavelengths indicated in the legend. The instability emergence ratio (Eq. 13)
 941 is defined as the ratio between the time it takes a wavepacket to travel one wavelength and the emergence time of
 942 the secondary wave, illustrating how the instability is triggered in the simulations when this ratio exceeds 1 (i.e.
 943 for times to the right of the dashed line). (right panels) $\widetilde{KE}^y(t)$ for simulations (b) 1a; (c) 1d; (d) 1f and (e) 1h.



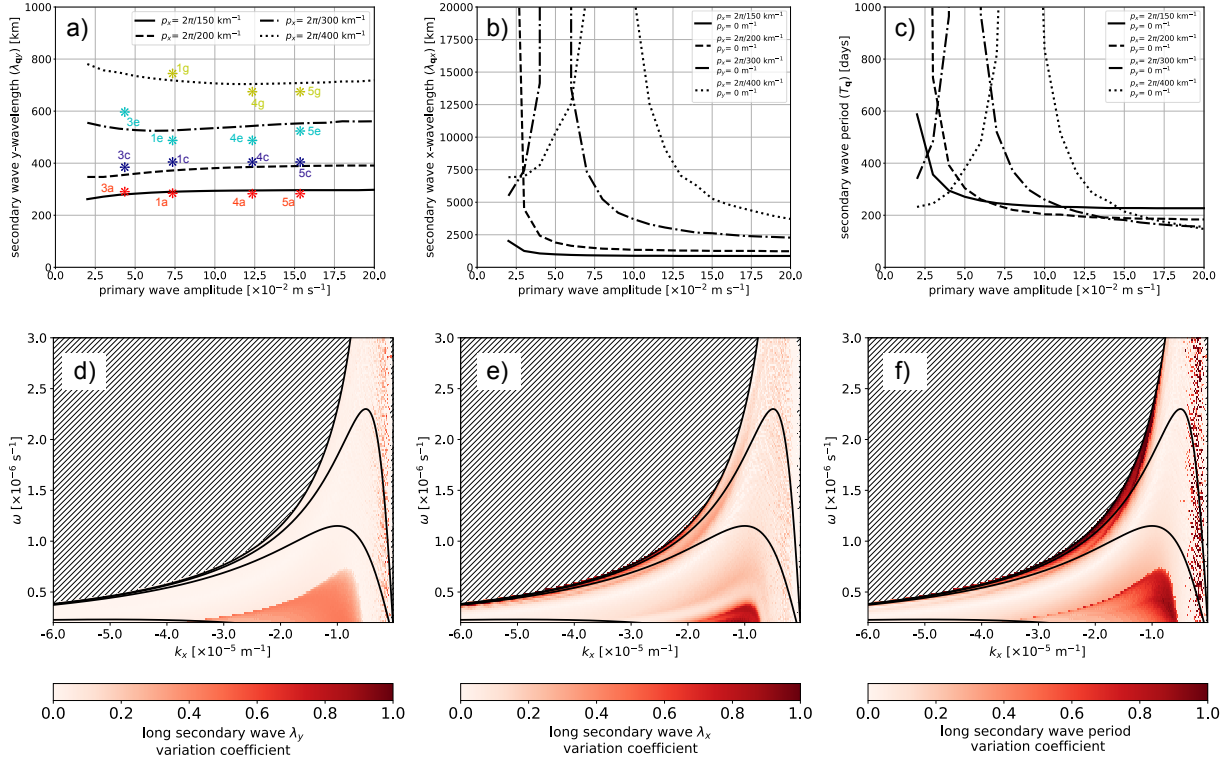
944 FIG. 13. Theoretical amplitude threshold to reach $R_p^e = 1$ (Eq. 13) as a function of the primary wave zonal
 945 wavenumber (k_x) and frequency ω . Above this amplitude, barotropic Rossby primary waves are unstable to
 946 NLTI.



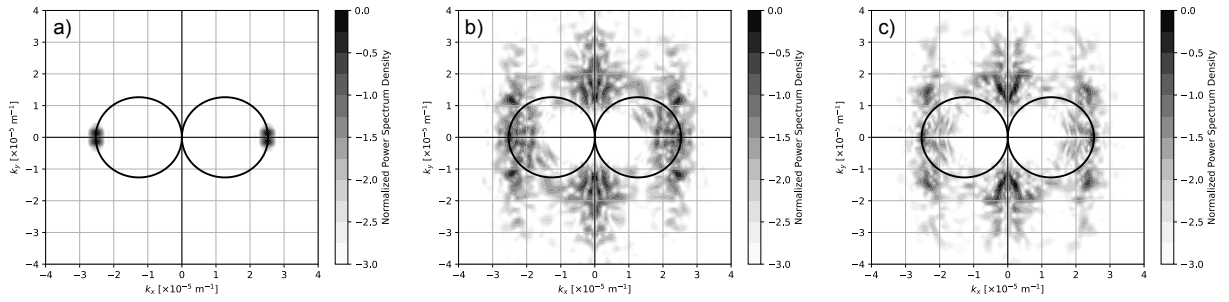
947 FIG. 14. The emergence time of the instability in the simulations as a function of the theoretical growth rate for
 948 the primary waves of the different simulations. Each dot represents one simulation, with size and color indicating
 949 the amplitude and zonal wavenumber of the primary wave respectively. The black curve is proportional to the
 950 inverse of the growth rate, with a proportionality constant of 5.5.



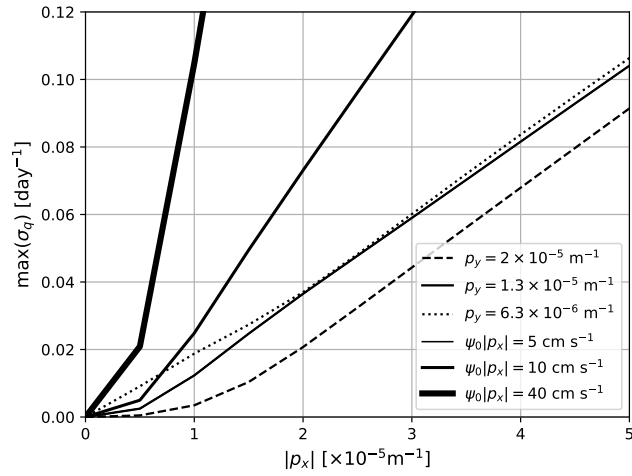
951 FIG. 15. Properties of the long secondary wave as a function of the primary wave's wavelength and period
 952 with a fixed amplitude of 0.07 m/s. (a) λ_y , (b) λ_x , (c) period, (d) time of emergence of the secondary wave. The
 953 dispersion relation for barotropic Rossby waves with different k_y is indicated by solid lines. The hatched regions
 954 correspond to regions where no instabilities grow.



955 FIG. 16. (upper panel) Sensitivity of the long secondary waves' properties: (a) meridional wavelength λ_y , (b)
 956 zonal wavelength λ_x , (c) period, to the amplitude of the primary wave. The sensitivity is computed for primary
 957 waves with zonal wavelengths 150 km, 200 km, 300 km, and 400 km and for amplitudes ranging from 0.01 m s^{-1}
 958 to 0.3 m s^{-1} with a 0.01 m s^{-1} increment. The corresponding experiments and the secondary waves' meridional
 959 wavelengths computed using a wavelet analysis are indicated by the stars with their labels referring to Table
 960 1. (lower panel) The variation coefficient, namely the ratio of the standard deviation of the properties for the
 961 different amplitude to the mean of this properties are computed for the secondary wave meridional wavelength
 962 λ_y (d), zonal wavelength λ_x (e), and period (f).



963 FIG. 17. Evolution of the energy spectrum in wavenumber space for simulation 1d at times (a) $t = 120$ days,
 964 (b) $t = 1000$ days, (c) $t = 1200$ days.



965 FIG. 18. The maximum growth rate (Eq. 11) for $|p_x|$ ranging between 0 and $5 \times 10^{-5} \text{ m}^{-1}$ for different meridional
 966 primary wavenumbers p_y (line styles) and amplitude $\psi_0|p_x|$ (line widths). Jet-like waves correspond to $p_x = 0$
 967 and always have a growth rate equals to zero, illustrating their particular stability.