# Integrating fixed and demand-responsive transportation for flexible transit network design

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# INTEGRATING FIXED AND DEMAND-RESPONSIVE TRANSPORTATION FOR FLEXIBLE TRANSIT NETWORK DESIGN

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**ABSTRACT**

In cities around the world, transit is currently provided with fixed route transportation only, whence the inherent limited Quality of Service (QoS) for travelers in sub-urban areas and during off-peak. On the other hand, it has been shown that completely replacing fixed-route with demand-responsive transit fails to serve the high transportation demand during peak hours. Therefore, it is still unclear how we can maximize the potential of demand-responsive transit by satisfying the complicated demand pattern varying with time and space.

In this paper we propose *Flexible Transit*, a transit system design which gets the best from fixed-route and demand-responsive transit, depending on the demand observed in each sub-region of the urban conurbation and time-of-day. The goal is to provide high transportation capacity while guaranteeing high QoS, two objectives that are instead conflicting with classic fixed-schedule transportation.

To this end, we first resort to microsimulation to show the limits of using either only fixed-route buses or only demand-responsive buses. This motivates the need of alternating between them instead, which we do in Flexible Transit. We then resort to Continuous Approximation to find the optimal design of flexible transit. We show that the flexible transit can significantly improve the user costs, in particular in suburban areas, while also reducing the overall cost of user and operator. We believe our findings suggest important policy insights in designing and planning of future transit systems, to take full advantage of demand-adaptive transportation modes.

*Keywords*: Transit Network Design, Continuous Approximation, Demand-Responsive Transportation, Microsimulation, Mobility on Demand

## INTRODUCTION

In recent years, new demand-responsive mobility services (e.g., ride sharing, dynamic buses, and microtransit) have emerged in the ecosystem of urban transportation. They are generally provided by private companies (e.g. Uber, Lyft) and their increasing success is considered as detrimental for public transit, which has witnessed a decrease in ridership and income (*1*). On the other hand, the service offered by transit has basically not evolved in recent decades, and still consists in fixed routes and fixed schedules, with some exceptions, or pilots or services for a specific targeted population (elders or handicapped). The contrast between "user-centric" demand-responsive services offered by private companies and "fixed-schedule-centric" public transit strongly penalizes the latter, as users often prefer the former (*1*).

The main limitation of fixed schedules is in the areas where demand is sparse (*2*), i.e., few trip requests per km2 and per hour: in order to keep a good QoS, it would be necessary to operate too many lines at a too high frequency, which would result in an excessive agency cost, with respect to the few users served per-vehicle. This problem is evident in suburbs and is one of the reasons for geographical inequity in modern society (*3*). On the other hand, demand-responsive transportation is not the solution to all mobility needs, as it is not suitable to serve dense demand (*2, 4, 5*), which it would result in tortuous vehicle routes (*6*) and thus high operational cost.

Therefore, a combination of fixed-schedule and demand-responsive services is needed in order to guarantee high capacity for dense demand areas and, at the same time, good QoS in sparse-demand areas. For these reasons, in recent years public authorities have launched pilots to experiment with different ways to complement their offers with on-demand services, by subsidizing ride-sharing companies. However, there is no systematic methodology to guide the design of future-generation transit systems integrating fixed and demand-responsive transportation. With this preliminary paper, we aim to fill this gap.

The contribution of this paper can be summarized as follows:

- We show with agent-based simulation in a small demonstrative case the limits of current transit where both rail Mass Rapid Transit (MRT) and bus services have a fixed schedule. We then show that replacing all fixed-schedule buses with demand-responsive transportation is infeasible, in particular in peak-hours.
- To overcome the aforementioned limitations, we propose *Flexible Transit*, which consists of a fixed-schedule backbone Mass Rapid Transit (MRT), with fixed-schedule, and a feeder service provided by bus. Depending on the sub-region of the conurbation and period of the day, the system changes the feeder operation, between fixed-schedule and demand-responsive (DR), in order to adapt to the geographical and temporal variation of the demand density.
- We model our Flexible Transit via Continuous Approximation (CA) and we contrast it with the model of current transit design, where the feeder always has a fixed-schedule.
- Numerical results show that our Flexible Transit improves the user QoS in particular during off-peak hours and in suburban areas, while keeping the overall cost (which includes the agency cost) under a reasonable level, even slightly reducing it.
- We show what are the differences between the design of current transit and Flexible Transit, in terms of line spacing, headway values, etc. In particular, we show how Flexible Transit alternates between fixed and demand-responsive feeder over the time and space.
- For the sake reproducibility, we make the Matlab code to compute the CA models open

source.[1]

The paper is structured as follows: we discuss the related work (pag.4), we then show in simulation that transit based on fixed-only or demand-responsive-only feeder is inefficient (pag. 4). This motivates our proposal of Flexible Transit, which we present in pag. 5, together with classic transit schemes. We then present a Continuous Approximation model of such schemes and the optimization procedure to compute their optimal structure (pag. 11). We finally contrast the performance of Flexible Transit with current schemes in numerical results (pag. 7) and conclude.

## RELATED WORK

During last decades, transit network design has been studied via several optimization problems, based on different objectives (user and/or operator cost minimization, total welfare maximization, protection of the environment, etc.), parameters and decision variables (network structure, demand patterns, fleet characteristics, headway, route and stop spacing, etc.) and solution methodologies (analytical, heuristics or meta-heuristics). An extensive review on Transit Route Network Design Problems is provided by (*7*). The "weak demand areas" (i.e. areas with by low residential density and high motorization rate) are the most critical for conventional public transportation, which is unable to ensure at the same time coverage, ridership and cost-efficiency. In these cases, an effective design of fixed (*8*) or Demand-Responsive (*2*) feeder bus lines connecting weak demand areas with mass rapid transit nodes could therefore help to shift passenger's mode of transport from individual to collective mobility, thus enhancing the accessibility to urban facilities and services.

The advantages of mass transit corridors in the metropolitan transport supply have been shown in (*9*, *10*). In particular, Gshwender et al. (*9*) compared the feeder-trunk scheme against different direct lines structures (where no transfers are required) showing that the first structure performs better when the demand is quite low and dispersed and the distances to travel are high, but also underlining that results are strongly related to the penalty value assigned to the transfers. Mohaymany and Gholami (*10*) demonstrate that feeder lines increase the use of high-capacity mass transit because the travel demand for a more extended area can be satisfied.

While transit network design problems at a strategic level are mostly addressed by discrete models, which are often unable to face large-scale instances, in Continuous Approximation (CA) models demand and supply are density functions over time and space. Such models are simple but powerful tools for the strategic stage of a transit plan. The key idea, as reported by Ansari et al. (*11*), is to construct an objective function, based on the integration of localized functions of x,y coordinates, which can be analytically optimized without huge computational efforts. Results obtained via CA provide general insights about the performance of whole transit systems, only dependent on the choice of model's parameters. It must be noted that CA models are very approximated and lack realism. For instance, they cannot include transportation topologies, traveler behavior and vehicle routing. However, CA methodology can provide useful insights in understanding at high level the impact of different design choices.

An and Lo (*12*) solve the transit network design problem under demand uncertainty trough robust optimization for rapid transit and dial-a-ride services. However passenger waiting time are not included in the model and travel costs proportional to distance and not to travel time. Edwards and Watkins (*13*) expand the comparison of gridded-based fixed-route and demand-responsive transport network to all types of street networks, transit schedules and passenger demand levels;

[1] https://github.com/giovanni-cal/future-transit

they found that DRT feeder services could provide a less expensive alternative for handling trip requests for stations with relatively low demand at off-peak hours. Aldaihani et al. (*14*) propose an analytical model to design a hybrid grid network of two modes of transportation by finding the optimal number of zones in an area, each served by a number of on-demand vehicles, which transfer passengers to a fixed route line.

Daganzo (*15*) proposed a CA model of a transit network with a "hybrid" structure, in the sense that it combines the advantage of both the grid (double transit routes coverage in the central area and the hub-and-spoke (radial routes branching to the periphery) concepts. The transit is described by only three decision variables: stop spacing, vehicle headway and ratio between the side of the downtown district and the side of the city boundary. However, only fixed transit modes are considered, i.e., Bus Rapid Transit (BRT) and rail (metro). Such model is reformulated in (*16*) and applied to a radial routes layout. Among the different outcomes, the authors showed that the radial layout is suitable for a centripetal demand pattern, in which the central area is the major attractor and generator of trips. In (*17*), two different city-wide transit structures are compared, showing that the ring-radial layout is more favorable to transit (in terms of costs) than the grid design. However, the demand density is assumed to be spatially uniform over the entire urban area, which is not realistic. The aforementioned work shows that BRT outperforms a Metro system for a wide range of demand density and coverage areas. However, since the former require quite large streets, it appears unrealistic to imagine that such systems can entirely replace underground transit in the big cities' dense urban fabric. With regard to demand-responsive transit, CA was used to evaluate the performance of both pure door-to-door services (*18*) and dial-a-ride systems with checkpoints able to cluster much of the demand (*19, 20*). While these first studies assumed a many-to-many demand pattern for their flexible transit services, many further works focused on flexible transit as the First Mile/Last Mile (FMLM) solution of a feeder-trunk scheme. On this account, Quadrifoglio and Li (*2*) used CA to estimate the demand density threshold, for a feeder transit service, below which demand-responsive (DR) buses are more efficient than fixed schedule buses. In their model, a feeder services operates in a residential area of rectangular shape and homogeneous density, on the side of the major fixed transit line. Since such services are thought for commuters, no intrazonal trips were supposed (Many-to-One demand pattern). However, the authors just focus on such single local area, while we aim to devise a design for the entire urban area. As in Daganzo (*15*), Nourbakhsh and Ouyang (*21*) consider a hybrid transit network, but they propose the optimal design of a flexible route transit system where individual buses operate without fixed routes or predetermined stops, but rather covering a tube shaped predetermined area where to pick up or drop off passengers while sweeping longitudinally back and forth through the tube. The solution, solved by a simple constrained nonlinear optimization problem, showed that under low-to-moderate passenger demand the system incurs lower cost than other conventional counterparts such as the fixed-route transit system and the chartered taxi system. Chen and Nie (*22*) study a grid and a radial network with fixed route transit lines paired with demand adaptive lines connecting passengers to the stops of the fixed route. Optimal design is formulated as a mixed integer program. The results show that the paired lines design outperforms the other two systems under a wide range of scenario configurations. Luo and Nie (*23*) examine six distinctive transit systems using the CA approach, most of them already studied in the previously cited works. They combine conventional fixed route transit (*15*), line-based hybrid transit (*22*), zone-based hybrid transit (*14*) in two city layouts: grid and ring radial. They found that demand responsive feeder services may have a rather limited impact on the economy of scale in mass transit systems. However, they tilt the balance of

trade-off considerably in the user's favor, at the operator's expense.

None of the previous work has considered combining both fixed-schedule and demand-responsive services, to better adapt to the geographical and temporal demand variations. We introduce such concept under the name of *Flexible Transit*.

## SIMULATION OF THE MOTIVATING SCENARIO

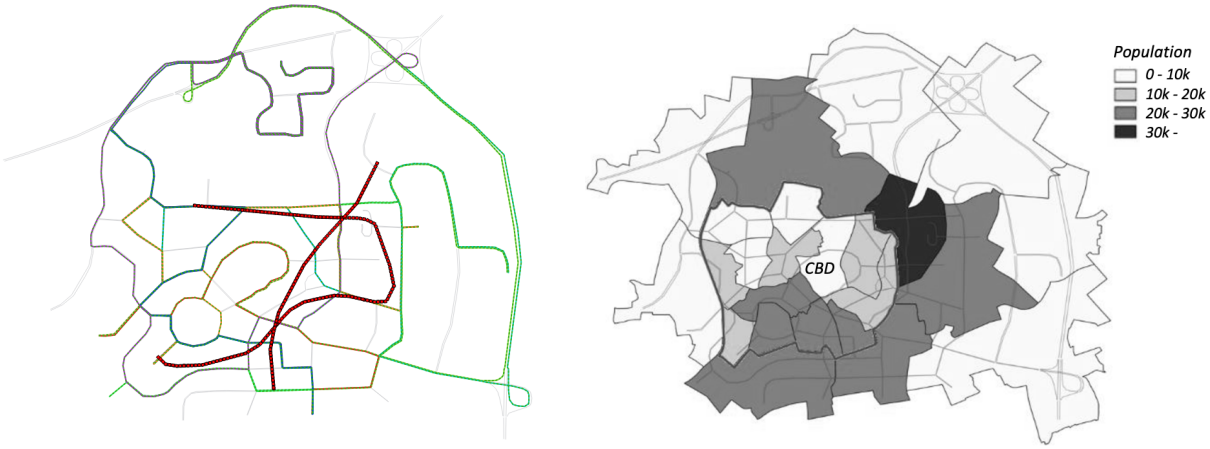
In order to motivate the need for Flexible Transit, which alternates fixed and DR feeder services in the First Mile / Last Mile (FMLM), we show the limitations of always deploying either fixed or DR feeder policy. For this, we utilize a microsimulation platform called SimMobility (*24*) that replicates individual mobility and travel decisions over different timescales (*Short-term*, *Mid-term*, *Long-term*).

Specifically, in this study, we utilize the *Mid-term* simulator (*25*) that models daily travel demand, supply and their interactions on an urban network through three modules: *Pre-day*, *Within-day*, and *Supply*. The *Pre-day* module determines daily activity patterns for the entire population (in the form of a day activity schedule) through an activity-based model (ABM) system (*26*). The ABM system in SimMobility is a herarchical set of discrete choice models organized into three levels, the day pattern level, tour level and intermediate stop level. In the *Within-day* module, planned activities and trips from the *Pre-day* are transformed into actions through models of departure time, route choice and activity re-scheduling (*27*). Following this, the trajectories of all individuals are simulated on a multimodal network in the *Supply* module, utilizing macroscopic speed-density relationships combined with deterministic queuing models. Within the *Supply*, public transit operations are handled by the bus and rail controllers that can simulate both frequency-based and schedule-based services and model the dwelling of individual vehicles explicitly. Recently, SimMobility has been enhanced with the capability to model the fleet operations of demand-responsive services such as mobility-on-demand, automated mobility-on-demand and demand responsive transit (*28*, *29*).

We use a prototypical urban network presented in Figure 1, which is moderately sized and consists of land use patterns and household socio-demographics that resemble Singapore (more details may be found in (*4*)). Specifically, the synthetic population of the prototype city is generated by sampling households and individuals from a synthetic population of Singapore (*30*). The total population of the prototype city is 350,000, or 7% of Singapore. The network consists of 95 nodes connected by 254 links (which in turn are composed of 286 segments) across 24 traffic analysis zones. There are 12 bus lines (which are coded with different colors in the figure) covering 86 bus stops, and 4 MRT lines over the 20 MRT stations (displayed in red in Figure 1a). The central business district is also demarcated in Figure 1b.

As mentioned previously we consider two scenarios:

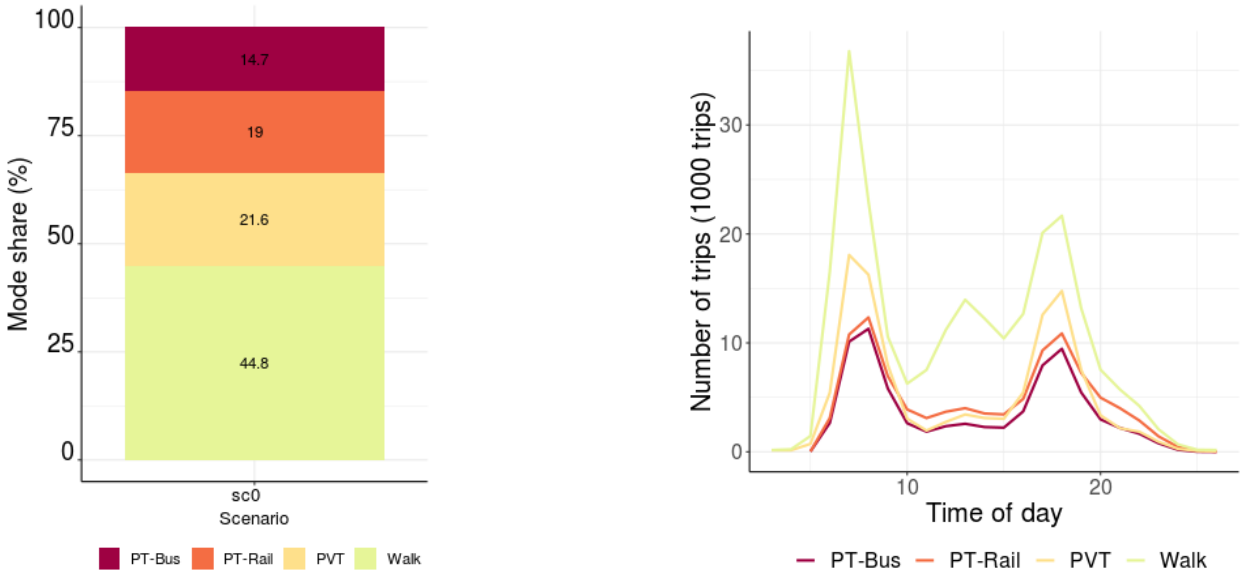
- Scenario **SC0** represents the **classic all-fixed route** transit wherein the following modes are available to travelers: private modes (denoted by PVT and including car, car-pooling (2-3 people within a household), motorcycle, taxi, and private bus), public transit - bus (bus with walk access; denoted by PT-Bus), public transit - rail (rail with either bus access or walk access, denoted by PT-Rail).
- In the second scenario (**SC1**), a fleet of **demand-responsive** (DR) vehicles replace entirely the fixed-schedule buses, and thus, transit users have to now choose between either rail with walk access or with access by the demand responsive service. Note that we assume the demand for private and walk trips does not change across the two scenarios.




(a) Road and transit network (b) Population distribution

**FIGURE 1**: Network of prototypical city

Figure 2 shows the demand pattern including mode shares and the temporal distribution of trips by each mode in the scenario SC0. There are in total around 532,000 trips for the entire 24 hour period. Buses and trains are operated on a fixed schedule for the given routes with a pre-defined dispatching headway over the time-of-day: 5.21 min during the peak (07:00-09:00 for AM, 17:00-20:00 for PM peak) and 12.66 min during the off-peak on average. This results in 642 and 820 bus departures from the terminal during the on- and off-peak respectively. Note also that there are a total of 120 departures for each train line during a day.



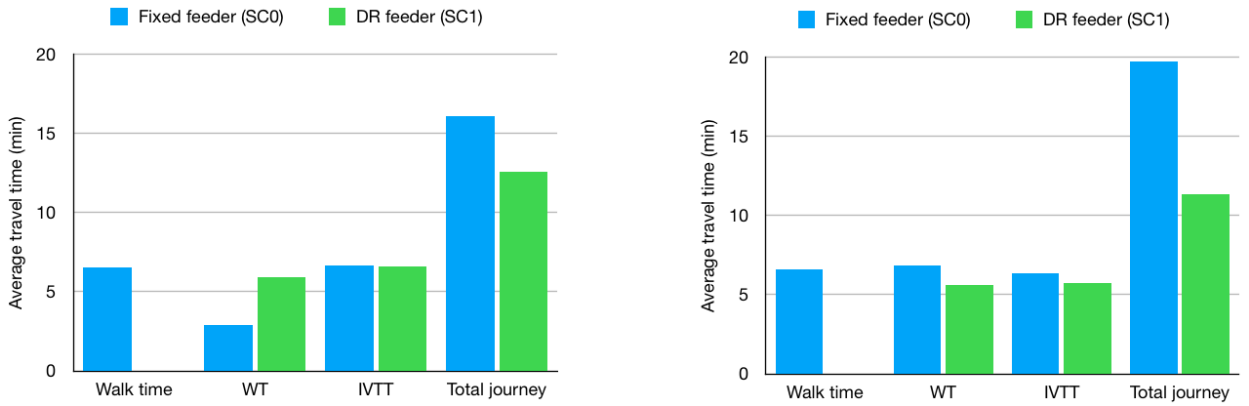

(a) Mode share (b) Demand distribution

**FIGURE 2**: *Pre-day* demand pattern

In Figure 4a, the number of trips served by each mode is summarized. Regarding public transit, in SC0, there are around 164,800 trips by bus and 50,800 trips by rail (note that the rail

mode indicates the use of rail with either walk access or bus access). Most of the rail passengers walk to the rail station, with only 4,460 and 4,330 bus trip legs for access/egress to/from rail stations respectively. As shown in Fig. 3, for the bus trips, the average waiting time (WT) at the bus stop shows larger during the the peak (2.90min) than off-peak(6.82min) which attributed to the smaller bus headway (higher frequency) during the peak. The average in-vehicle travel-time (IVTT) for bus trips is also listed in the figure. In addition to the waiting and in-vehicle travel-time, the agents are required to walk to bus stop from the doorstep.

The number of rail trips in scenario SC1 is around 217,000 trips. As noted previously, fixed schedule bus services in this scenario are substituted with the DR feeder service using a fleet size of 1,000 vehicles. The details of the DR fleet operation can be found in prior work (*28*, *29*). Accordingly, the DR feeder serves around 52,000 rides for access/egress to/from rail stations. The waiting time and in-vehicle travel-time range from 5.6 to 5.9min and 5.7 to 6.6min respectively for the whole day. A comparison between fixed and DR feeder is depicted in Figure 3. During the peak, SC0 yields a larger average journey time despite the relatively small wait time at bus stops, as agents are required to walk to/from the bus stop. The total journey time is higher in case of the fixed feeder than the DR feeder with a significant increase in wait times of passengers due to the larger bus headway during the off-peak period.

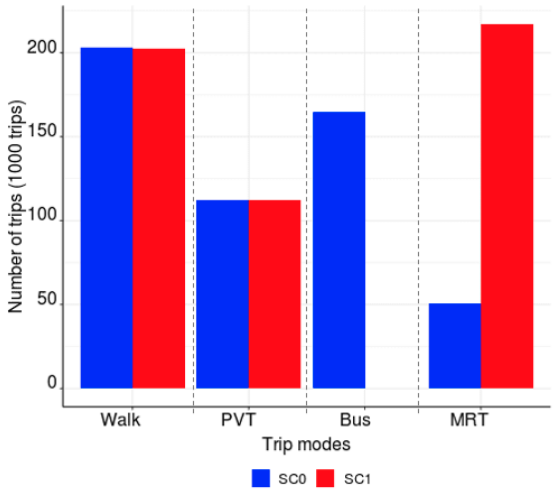



(a) Peak (b) Off peak

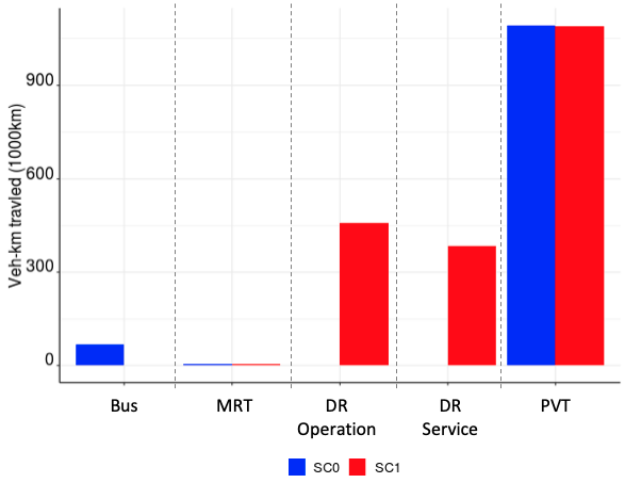
**FIGURE 3**: Journey time for FMLM with feeder service

As for the operating cost, Figure 4b shows the VKT (Vehicle-Km Traveled) by each mode for the two scenarios. The private trips (PVT) involve a total VKT of more than 540,000km, while the VKT of fixed-schedule public transit is around 43,000km for bus (in SC0) and 4,500km for rail (in both SC0 and SC1). The VKT generated by the DR service in SC1 involves both dead-heading (e.g. empty trips for cruising and parking, pick-up; termed DR operation) and trips from pick-up to drop-off points (termed service distance). Figure 4c shows the VKT of the DR fleet by time-of-day, and indicates that a significant portion of VKT is generated during the peak periods. During the peak periods, the total operating and service distance is around 95,000km and 97,000km respectively, which is around 50% of total distance during a day. On the other hand, fixed transit during the peak involves 32% (rail) and 39% (Bus) of the above daily travel distances.

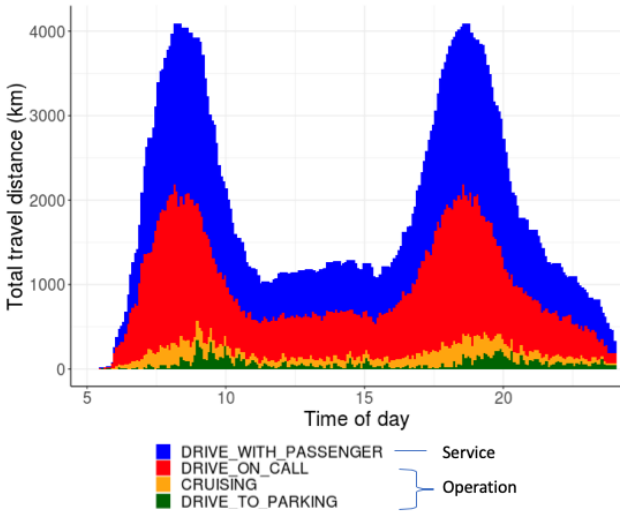
The comparison between the two scenarios suggests that introducing the DR service for FMLM can reduce the journey time from doorstep to the rail station with less waiting time, in-




(a) Number of trips in *Within-day*. Note that the MRT+feeder are counted as MRT trips as well.




(b) Veh-Km Traveled in *Supply*




(c) Detailed Veh-Km Traveled by DR service

**FIGURE 4**: *Within-day/Supply* simulation results

vehicle travel time, and no access time to bus stop, especially during the off-peak when the dispatching headway of bus is large. However, during the peak period, DR induces larger travel distances for both service and operation compared with bus and rail.

In other words, the fixed feeder is inefficient during off-peak hours and the DR feeder is inefficient during peak. Therefore, deploying always fixed feeder, as currently done in most cities, is not well adapted to the off-peak demand. On the other hand, replacing all fixed buses with DR feeder would not well adapt to peak demand. From this observation comes our idea of Flexible Transit, which deploys always the best between fixed and DR feeder in the FMLM, adapting to the demand, over the time of the day and the geographical areas.

## TRANSIT DESIGN SCHEMES

We focus on the transit system of wide urban and metropolitan areas. They generally show a transition from a central zone characterized by dense urban fabric, high population density and presence of numerous "trip attractors" (job places, commercial activities, amenities, etc.), towards sprawled suburban areas often shaped by low residential density and sparse transport demand, both temporally and spatially.

The **components** of transit systems are

- A rail system (called **Mass Rapid Transit** (**MRT**)) and
- Possibly, a **feeder** service, provided by bus, to serve the First Mile and Last Mile.

### Central and suburban areas

The MRT network is modeled as a ring-radial structure (Fig. 5), as in (*16*, *17*), which can be adopted to model several big cities around the world (e.g. Paris, Singapore, Moscow).

The entire area is composed by two parts:

- a **central area** only served by MRT with double coverage provided by radial and circular rail lines and
- a **suburban area** covered only by radial lines.

### Alternative transit schemes

We discuss three alternative transit schemes, which essentially differ in the way passengers can access MRT:

1. ***MRT-only scheme***, in which the access to MRT stations only takes place by walking.
2. ***Fixed-feeder scheme***, which includes a feeder bus lines with **fixed routes** to increase the accessibility of MRT stations in the suburban area, which can be reached either by walking or using the bus, depending on the distance from the station itself.
3. ***Flexible scheme***, in which the FMLM in the suburban area is still covered by a feeder service, but the feeder can switch between two modes, namely **fixed** and **demand-responsive** (**DR**), choosing optimally between one or the other based on the transport demand density.

We clarify that in Flexible Transit the choice of whether to deploy fixed or DR feeder is not made on-the-fly. On the contrary, we assume that, based on historical observation of the demand density, the authority would plan, for each area, the time periods when fixed or DR feeder will be operated. Such plan would be revised only on a seasonal base.

We use MRT-only as a baseline. Its poor cost-efficiency shown in the numerical results (pag.17) demonstrates that a feeder bus service in the suburban area is necessary. The fixed-feeder

scheme is what it is basically currently deployed in cities around the world. Flexible scheme is the design we propose for future generation transit.

## CONTINUOUS APPROXIMATION MODEL

Continuous Approximation (CA) is a methodology in which an urban conurbation and a transit network are represented with a parametric model, consisting of:

- A set *decision variables*, describing the layout of the transit network, i.e., line and stop spacing, headway values, etc.
- A set of *input parameters*, which are exogenous and describe the scenario, e.g., size of the area, demand density
- A set of *constraints*, which ensure basic properties like conservation of flows, not exceeding transit vehicle capacity
- A *cost function*, which we want to minimize and usually includes a weighted sum of user-centric and operator-centric terms. It summarizes the performance of transit.

CA models allow to understand the impact of the different decision variables on the performance, in an approximated, concise and computationally efficient way. Due to the high level of simplification, results obtained via CA models should not be expected to be exact and realistic and must rather be interpreted as high-level trends, which can guide transit planning considerations.

In this paper is to understand the benefits of choosing over time and geographical areas the feeder service that better suits the demand, a concept that we call *Flexible Transit*. We are not interested in exact results representative of a single specific city. For this reason, CA methodology perfectly fits our needs. Our formulation is mainly based on (*15*, *17*), which, however, do not integrate MRT and feeder. For this reason, we need to introduce some modification, which we will pinpoint in the following pages.

### Main Decision Variables

We study a circular metropolitan area of radius $R$. The transit layout, depicted in Fig. 5, is organized as described in the section of pag.10. It is described by one scalar parameter $r$, which is the radius of the central areas, and other 6 decision variables determining the transit structure, which are:

- the angular spacing $\theta_r(x)$ between radial MRT lines (based on that, we can also compute the corresponding linear spacing $S_r(x) = \theta_r(x) \cdot x$
- the spacing $S_c(x)$ between circular MRT lines;
- the spacing $s(x)$ between the MRT stations (hereinafter called just "stations") along a radial MRT line;
- the angle $\phi(x)$ between stations on a circular MRT line;
- the headway $H(x)$ on circular and radial MRT lines;
- the headway $h(x)$ of the feeder service in the suburban area.

The variation of such parameters along $x$ can be seen as an approximation of what one could observe in reality. For instance, if radial lines bifurcate with the distance $x$, we can represent this by reducing $\theta_r(x)$ with $x$. Accordingly, the headway $H(x)$ would increase with $x$ because less vehicles will travel along a single line.

### Assumptions and constraints

Along each radial MRT line, in the inward direction, a train can depart from any $x \leq R$ but always terminate in the center ($x = 0$). In the outward direction, a train always departs from the center and

can terminate at any $x \leq R$. This translates to the following constraint:

$$H(x_1) \leq H(x_2), \quad \forall x_1, x_2 \mid x_1 < x_2 < R \tag{1}$$

The radial flow $Q(x) = \frac{2\pi}{\theta_r(x) \cdot H(x)}$ is the number of trains crossing an infinitesimal annulus of radius $x$ in both inward and outward direction. We prevent this flow from increasing outward with the following constraint:

$$Q(x_1) \geq Q(x_2), \quad \forall x_1, x_2 \mid x_1 < x_2 < R \tag{2}$$

The following vehicle capacity constraint must also be respected:

$$O_j(x) \leq C_{pax,j} \quad \forall x \tag{3}$$

where $O_j(x)$ is the average vehicle occupancy at $x$ and $C_{pax,j}$ is the vehicle capacity of mode $j \in$ {MRT, fixed-feeder, DR-feeder}. $Oj$ is computed as in (17).

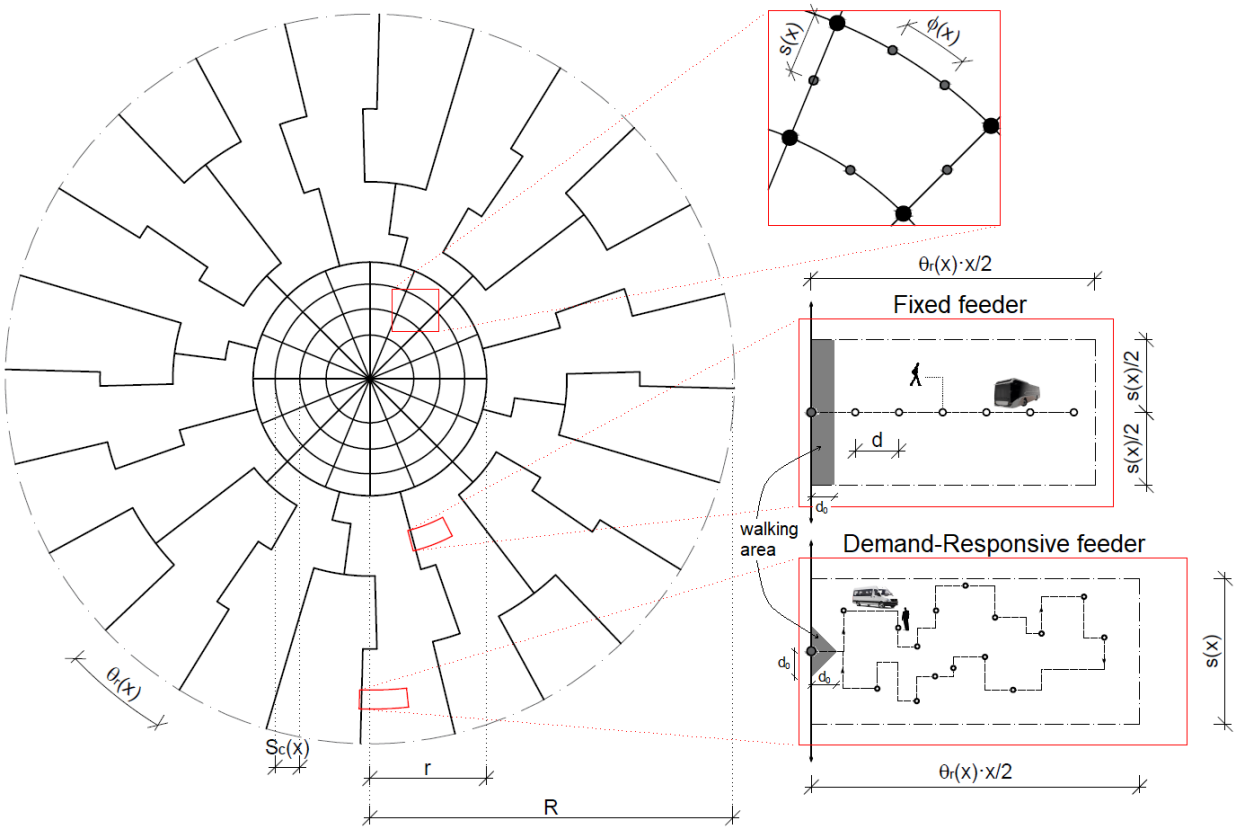



**FIGURE 5**: Transit network layout

**Feeder service**

The suburban area is divided in **FMLM sub-areas**, each determined by the spacing between the radial lines and the station's spacing along them, as in Fig. 5-right. In case of MRT-only Scheme, passengers can only walk therein. In case of Fixed or Flexible Scheme, instead, each FMLM subarea is associated to a MRT station, and is served by feeder service, i.e. a fleet of buses bringing passengers to/from that MRT station. Such service area, forming a ring sector, can be approximated

into a rectangle of length $l(x) = \theta_r(x) \cdot x/2$ and width $s$. Note that the size of an FMLM subarea depends on the MRT structure (the more the MRT lines and/or the smaller the station spacing, the smaller the FMLM areas) Such size determines the total user demand to accommodate and thus the operational strategy (fixed or DR).

*Fixed feeder*

The fixed (FX) feeder is modeled as a straight route with spacing $d$ (input parameter) between predetermined stops and the vehicle moving back and forth between MRT station and the furthest stop, as in (*2*), so that the length of a complete cycle is given by:

$$CL_{\text{FX}} = 2 \cdot \left( l(x) - \frac{d}{2} \right) = \theta_r(x)x - d \tag{4}$$

We assume that people walk on a Cartesian grid, so the average walking distance to reach the nearest bus stop is $s/4 + d/4$. If a user is closer than a certain threshold $d_{0,FX}$ to the MRT station, she directly walks to there. Such users are the ones having origin within a "walking area", represented in gray in Fig. 5. The fraction of such users is $p_{\text{walk,FX}} = d_{0,\text{FX}}/l$, where $d_{0,\text{FX}} = d/2$. The time needed to complete a cycle can be calculated as follows:

$$C_{\text{FX}} = \frac{CL_{\text{FX}}}{v_{\text{FX}}} + \tau_s \cdot \underbrace{\left( \frac{2l(x)}{d} - 1 \right)}_{\text{number of stops}} + \tau_p \cdot (1 - p_{\text{walk,FX}}) \cdot n \tag{5}$$

where $v_{\text{FX}}$ is the cruising speed of the bus, $\tau_s$ is the time lost per stop, $\tau_p$ is the time lost per passenger due to boarding/alighting operations and it is multiplied by the average number of passengers per vehicle $n = 2 \cdot \rho(x) \cdot s(x) \cdot l(x) \cdot h(x)$, being $\rho(x)$ the demand density (trips/km$^2$h per travel direction).

*Demand-Responsive feeder*

The demand-responsive (DR) feeder provides a door-to-door service, so passengers do not have to walk to any physical bus stop. It is assumed that each new request is processed in real-time via an insertion algorithm that aims at minimizing the vehicle travel distance (see (*2*)). The more passengers a single DR vehicle has to pick-up/drop-off, the longer its route, due to longer detours. Users close enough to the MRT station, i.e. at distance $d_{0,\text{DR}}$ from the station (gray area in Fig. 5) directly walk to it. Their fraction is:

$$p_{\text{walk,DR}} = \underbrace{d_{0,\text{DR}}^2}_{\substack{\text{Area of the} \\ \text{walking area}}} / \underbrace{(l(x) \cdot s(x))}_{\substack{\text{FMLM} \\ \text{subarea}}} \tag{6}$$

The cycle length ($CL_{\text{DR}}$) and time ($C_{\text{DR}}$) are computed as in (*2*) (Equations (10) and (11)).

**Cost components**

The main objective of the present work is to find the optimal transit structure able to integrate fixed and DR modes. With this aim, we formulate a generalized cost function to be minimized (*16, 17*), which combines the disutility for users due to the travel time in its different components and the costs incurred by the transport agency to provide the service (and the related externalities).

As regards the transit users, as usually done in CA work, we do not consider transit fares

(which is a reasonable assumption when most of users use monthly passes).

*User-related cost*

The user cost is related to

- Walking time $A$ to reach the bus stop or the MRT station or the destination
- Waiting time $W$ at the bus stop or the MRT station
- In-vehicle (MRT or bus feeder) time $T$ including boarding, riding, dwell and alighting time for the vehicle.
- Transfer: since any possible transfer between different transit lines is an additional disutility, we treat it as a penalty of 2 minutes of extra walking time.

Let us denote with $N(x) = 2 \cdot \rho(x) \cdot (2\pi x)$ the number of passengers in the infinitesimal annulus of radius $x$ and with $v_w$ the walking speed. Note that a percentage $p_{\text{walk}}$ of passengers does not use the feeder service and that the average walking distance for users residing in the walking area to reach the MRT stationin case of DR-feeder is $d_{0,\text{DR}}/3 + d_{0,\text{DR}}/3 = (2/3)d_{0,\text{DR}}$. Then, the total time components suffered by users at $x$ are:

$$y_{A,FX}(x) = N(x) \cdot \frac{\frac{d}{4} + \frac{s(x)}{4}}{v_w} \tag{7}$$

$$y_{A,DR}(x) = N(x) \cdot p_{\text{walk.DR}} \cdot \frac{\overbrace{2/3 \cdot d_{0,\text{DR}}}^{\substack{\text{walking distance of users} \\ \text{inwalking area}}}}{v_w} \tag{8}$$

$$y_{Wj}(x) = N(x) \cdot (1 - p_{\text{walk,j}}) \cdot \frac{h(x)}{2}, \quad j \in \{FX, DR\} \tag{9}$$

$$y_{T,FX}(x) = N(x) \cdot p_{\text{walk.DR}} \cdot \underbrace{\left(\frac{C_{FX}}{4} + \frac{d_{0,FX}}{2v_{\text{FX}}}\right)}_{\text{avg in-vehicle time}} \tag{10}$$

$$y_{T,DR}(x) = N(x) \cdot p_{\text{walk.DR}} \cdot \underbrace{\frac{C_D R}{2}}_{\text{avg in-vehicle time}} \tag{11}$$

Such formulas are obtained in a similar way than (2), but considering the headway $h(x)$ of the DR-feeder and the percentage of walking users.

*Agency-related cost*

The cost components related to the agency depend on:

- Infrastructure length $L$ (km), i.e. construction and maintenance costs
- Total distance $V$ (veh km/h) traveled by the vehicles per unit of time, i.e. operational costs
- Size $M$ (veh) of the vehicle fleet, i.e. the costs of the crew needed and the capital cost.

Denoting with $sa(x) = (2\pi)/(\theta_r(x)/2)$ the number of subareas along the ring at $x$, the agency metrics for the feeder service are computed as follows:

$$y_{Lj} = \frac{2 \cdot \left(l(x) - \frac{d}{2}\right)}{s(x)} \cdot sa(x), \quad j = \{FX, DR\} \tag{12}$$

$$y_{Vj} = \frac{CL_j}{s(x)h(x)} \cdot sa(x), \quad j = \{FX, DR\} \tag{13}$$

$$y_{Mj} = \frac{C_j}{s(x)h(x)} \cdot sa(x), \quad j = \{FX, DR\} \tag{14}$$

In the equations above, $sa(x)$ multiplies on the left the cost (per distance unit) related to a single service area. In Equation 12 we reasonably assume that the infrastructure cost related to the fixed-feeder route length does not change when switching from FX to DR operation. Equation 13 computes the vehicle-distance traveled per hour, by multiplying the cycle length $CL_j/s(x)$ by their corresponding vehicle frequency $1/h(x)$. Equation 14 computes the cost of fleet size from the number of vehicles $C_j/(s(x)h(x))$ needed to ensure the feeder service.

*Cost coefficients*

Agency's and user's metrics are converted into cost density functions by means of a set of cost coefficients, in order to compute the social cost (per unit of time) as a linear combination of those metrics. Denoting with $\mu_L$ (\$/km-h), $\mu_V$ (\$/veh-km) and $\mu_M$ (\$/veh-h) the cost coefficients related to the agency metrics, the agency's cost function is therefore $\mu_L L + \mu_V V + \mu_M M$. Similarly, the cost coefficients related to the users metrics represent the Value of Time (VoT) (\$/h) associated to each travel component, so that the user's cost function is the linear combination $\mu_A A + \mu_W W + \mu_T T$.

**Optimization problem**

To express the cost objective in a concise way, we use $i$ to indicate the type of cost component {L,V,M,A,W,T} and $j$ to specify the mode of transport {MRT, fixed-feeder, DR-feeder}, emphasizing that fixed and DR are alternative feeder services. As in (*17*), we distinguish:

- Local cost densities $\mu_{ij} y_{ij}(x)$ (\$/h-km), which vary with the distance from the center $x$ and
- Global costs $\mu_{ij} F_{ij}$ (\$/h) which are instead only related to the outermost circular line, i.e. $x = r$.

We now formulate the optimization problem to minimize it. If we denote with $Q_0 = Q(x = 0)$ the maximum value of the total radial flow of MRT vehicles (see Equation (2)) and with $H_B$ the headway at the outermost circular line, we can separate the the decision variables in two sets:

- Global decision variables $G = \{r, Q_0, H_B\}$,
- Local decision functions $D(x) = \{\theta_r(x), S_c(x), s(x), \phi(x), H(x), h(x)\}$.

The problem is therefore to minimize the total cost function in order to find the optimal values for the sets $G$ and $D(x), x \in [0, R]$:

$$Z(D(x), \underbrace{G}_{(r,H_B,Q_0)}) = \sum_{\substack{\text{cost} \\ \text{component } i}} \sum_{\text{mode } j} Z_{ij} = \sum_i \sum_j \mu_{ij} \left( \underbrace{F_{ij}(r, H_B)}_{\text{global cost}} + \int_0^R \underbrace{y_{ij}(D(x), r, x)}_{\text{local cost densities}} dx \right) \tag{15}$$

$$\min_{G, D(x)} \{Z(G, D(x)) \text{ such that Eq. (1)-(3) are satisfied}\} \tag{16}$$

For the derivation procedure of the cost components for the MRT the reader can refer to the Appendix A of (*17*), except that we consider the same headway on circular and radial lines

$H_r(x) = H_c(x) = H(x)$ and treat the spacing between radial ($s(x)$) and circular ($\phi(x)$) MRT stations as local decision variables.

**Demand pattern and travel behavior**

We assume that the transit demand density is both temporally and spatially variable and follows the Clark's law (*31*), i.e. an exponential decline from the center to the suburbs. Denoting with $x$ the distance (in kilometers) from the city center, the demand density (measured in trips per hour per square kilometer) is given by:

$$\rho(x,t) = \rho_0(t) \cdot e^{-\gamma x} \tag{17}$$

where $\rho_0$ ($pax/km^2h$) is the density of users in the center and $\gamma$ ($km^{-1}$) is the slope (also called density gradient) with which that value decreases as we move away. By changing the former $\rho_0(t)$ over time of day $t$, we can capture the temporal variation of the transport demand. In our paper we consider two "snapshots" $t^{\text{peak}}$ and $t^{\text{off-peak}}$, representing an instant during peak and off-peak hours.Note that the demand density decreases towards the outer regions. We approximately fit eq.(17) on data on Paris region [2] and we found $\gamma = 0.15$. With such model, the city center, where economic activities are more concentrated, emerges as an attractor (and thus generator) of trips from/to the periphery. A passenger first accesses the closest transit station (either by walking or via a feeder bus, rides via MRT to the station closest to her destination and finally reaches (either by walking or via a feeder), her destination. Note that, within the MRT, a passenger could change from a radial to a circular line and vice versa. The sequence of such changes obeys classic assumptions in literature (see Fig. A.1 of (*17*)).

**Optimization procedure**

The optimization of Equation 16 is non-convex. As in (*17*), we resort to bi-level optimization to solve it. The lower level subproblem consists, given any value of global variables $r, Q_0$, to solve the following *local optimization problem*, for all $x \in [0,R]$:

$$D^*(x|r,Q_0) = \left\{ \arg\min \sum_i \sum_j \mu_{ij} y_{ij}(x), \text{ such that Eq. (1)-(3) are satisfied} \right\} \tag{18}$$

We use an interior-point algorithm to solve such problem. The higher level subproblem (global optimization) is to determine the set of global variables $G^* = (r^*, Q_0^*, H_B^*)$ that minimizes the total cost $Z$, which we remember is the sum of global and local costs (Eq. 16). The following iterative procedure is implemented:

1. Initialize $r$ (sufficiently small) and repeat the global optimization procedure (described in the next steps) by increasing $r$ until the total cost found is higher than the average value from the previous 3 iterations. When this occurs, set $r^*$ equal to the value of the third to last iteration.
2. For any value of $r$, run the local optimization (Eq. 18) relaxing the constraints of Eq. 2 and 1, thus finding the vector of the different optimal (in the local sense) values of $Q(x)$ along $x$. Set $Q_0 = Q(0)$, i.e., the first element of such a vector.
3. Starting from the minimum value of the above vector
   - Run the local optimization to find $D^*(x|r,Q_0)$ for all $x \in [0,R]$ (we discretize this interval).

[2] https://www.insee.fr/fr/statistiques

- Find $H_B^* = \arg\min_{H_B} \sum_i \sum_j \mu_{uj} \cdot F_{ij}(r, H_B)$ (this problem is simple as it has only one decision variable).
- Compute the cost as in Eq. 15, i.e., $Z((r, Q_0, H_B^*), D^*(x))$
- Repeat this procedure by keeping $r$ fixed and increasing $Q_0$ until the total cost found is higher than the average value from the previous 3 iterations. When this occurs, set $Q_0^*$ equal to value of the third to last iteration.

## NUMERICAL RESULTS

We now compare the performance transit schemes of pag. 10 in a scenario representing a large urban conurbation, during peak and off-peak hours.

### Scenario Parameters

The scenario parameters (cost coefficients, vehicle speed, passenger capacities, etc.) are taken from (*15*). Here we just specify what we modify, to account for suburban areas and demand-responsive feeder, both of which are not considered there. We consider a circular area of radius $R = 25$ Km, which corresponds to the size of big metropolitan areas, as Paris region. The demand density $\rho_0(t^{\text{peak}})$ is calculated such that the average of our demand density, eq. (17), over a surface equal to the base case area of (*15*) matches the value therein, so that $\rho_0(t^{\text{peak}}) = 1500$ trips/km$^2$h. As in (*32*), we consider $\rho_0(t^{\text{off-peak}}) = (3/10)\rho_0(t^{\text{peak}})$. Also, in off-peak, we fix the value of $r$ obtained in peak hour for the fixed-feeder design scheme.

The cost coefficients of the DR feeder are the same as the fixed, except $\$_V$, which is 0.6 times than the FX bus (as in (*33*) - Table 2), time lost due to stops $\tau_s = 20$s, cruise speed $v_{\text{DR}} = 30$ km/h and $C_{\text{pax}} = 30$ passengers. The walking speed is 3 Km/h. The cost coefficient $\mu_{\text{L.MRT}}$ in the suburban area is 2.5 smaller than in the central area, as in (*34*).

### Performance of Flexible Design Scheme

We now compare the overall cost obtained with the three transit schemes, the difference in their optimal structure and the impact on user QoS, to show the superiority of our proposed Flexible transit over today transit design. The results are obtained, for each transit scheme, by applying the optimization procedure of pag. 16 on the respective continuous-approximation (CA) model. Such procedure searches for the optimal structure, i.e., the set of values of the decision variables that minimize the overall cost. We remark that such a procedure, which we implement in Matlab, is quite computational and terminates in about 10-20 minutes on an ordinary laptop for each transit scheme.

*Costs*

Fig. 6 shows that our Flexible Transit reduces the overall generalized cost, in particular during off-peak hours ( 4% of improvement). This improvement comes from an evident saving in walking cost. In order to achieve this, the operator needs to deploy more feeder vehicles, and thus kilometers traveled, which result in higher fleet and vehicle-km cost. In other words, Flexible transit tips the balance toward user-centric objectives. In any case, the overall cost of Flexible Transit always outperforms the other schemes.

Also observe that deploying a feeder, as in the Fixed or Flexible scheme, reduces the extent of needed MRT lines, since the demand in the first and last mile is served by almost infrastructure-less buses. This results in a lower infrastructure than with the MRT only scheme.

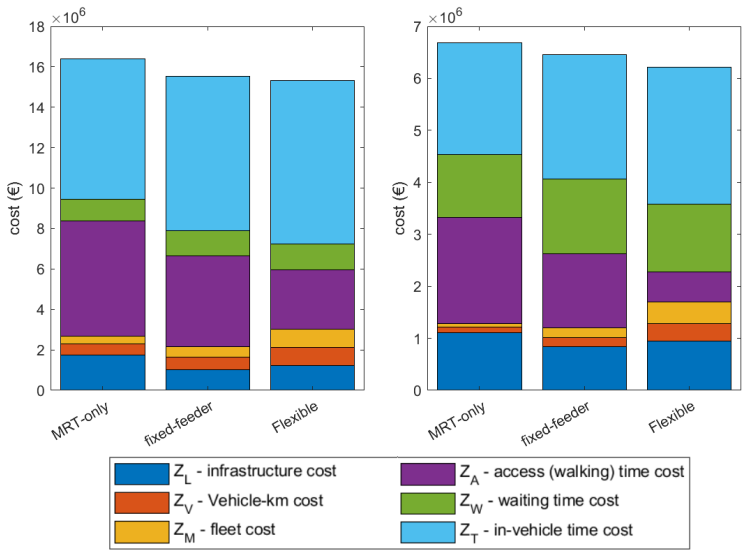



**FIGURE 6**: Cost of MRT-Only, Fixed-Feeder and Flexible, at peak and off-peak hours.

*Time and geographical adaptivity of Flexible Transit*

Fig. 7 shows the optimal structure of the three transit schemes, which explains the cost results previously discussed. Observe that the three schemes slightly differ in the the central area $x < r$, since in any case only MRT transportation is deployed there. The differences are instead visible in the suburban area.

It is important to remark how the structure of our flexible structure changes over the time and space. During off-peak hours, for $x > r$, the low demand density makes Flexible Transit prefer DR feeder. This already shows that Flexible Transit is able to vary the kind of offered feeder service spatially, to adapt to the geographic demand gradient. The adaptation to the temporal variation of the demand is instead visible when going from off-peak to peak hours, where the demand is sufficiently high up to $x = 12$ km to prefer fixed feeder, relegating demand-responsive feeder only to the further periphery.

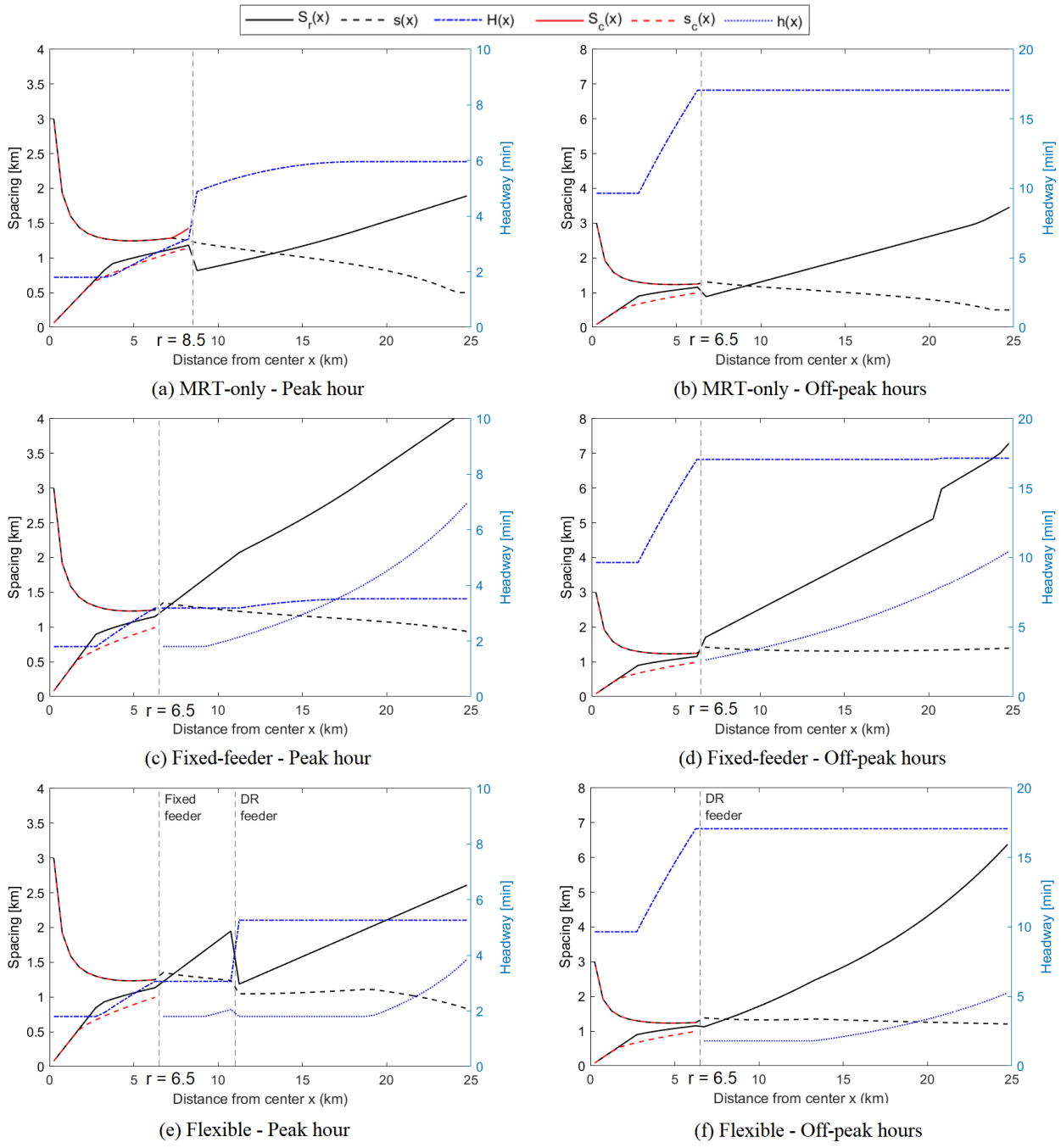
*Backbone (MRT) structure*

Observe (Fig. 7) that, for all the schemes the MRT offer is richer where the demand density is high, i.e., the headway $H(x)$ and the radial line spacing $S_r(x)$ is smaller in the suburbs closer to the center. This is also what we observe in real cities.

In the MRT Only Scheme, the line density in the entire area would be unfeasibly high. Fixed-feeder Scheme, instead, which serves the first and last mile demand via a feeder can afford sparser MRT infrastructure, whence the infrastructure savings.

*Accessibility to MRT*

Fig. 8 shows how easy it is to access MRT, depending on the distance $x$ from the center, in terms of access time and waiting time at the MRT, whose components are represented in Fig. 9. Note that the access time of MRT Only and Fixed-feeder scheme is basically the same and explodes, in particular during off-peak hours, far the from city center. The discomfort for passengers with high $x$ is further exacerbated by high waiting time for MRT. Flexible Transit, instead, compensate the




(a) MRT-only - Peak hour

(b) MRT-only - Off-peak hours

(c) Fixed-feeder - Peak hour

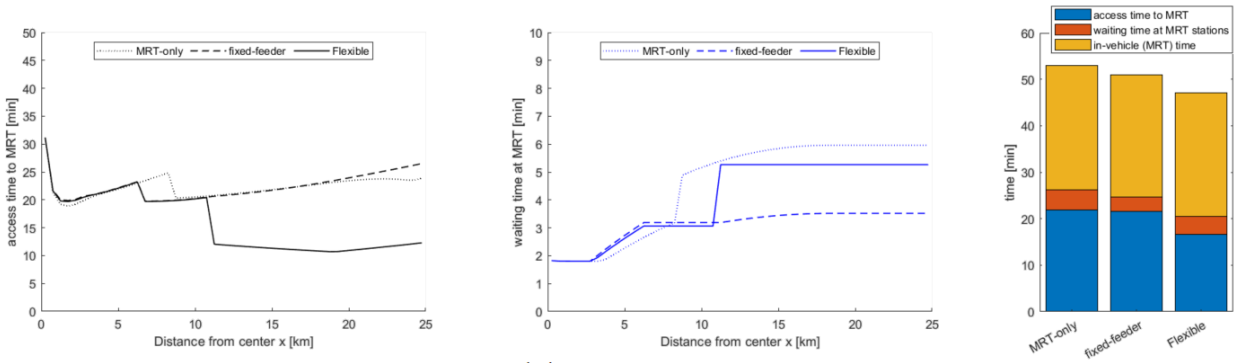
(d) Fixed-feeder - Off-peak hours

(e) Flexible - Peak hour

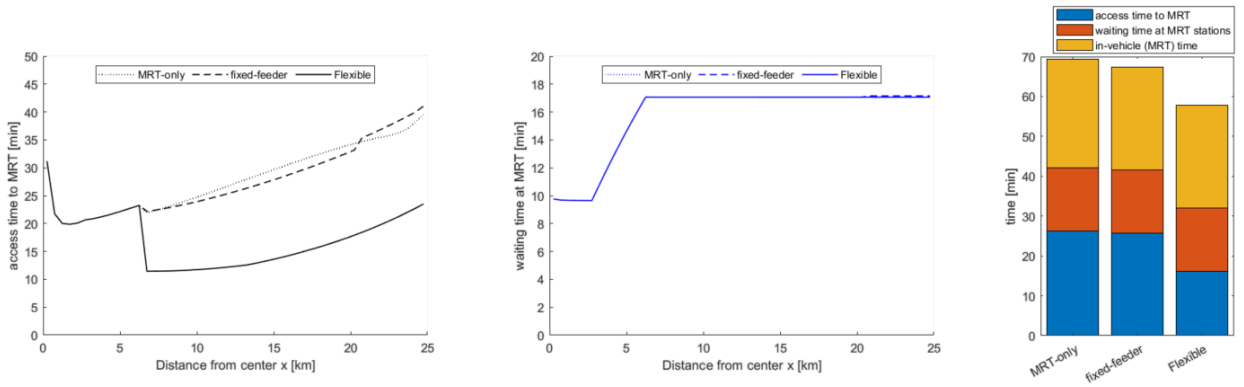
(f) Flexible - Off-peak hours

**FIGURE 7**: Optimal decision variables for the three transit schemes

increase in waiting time by guaranteed a very fast connection to MRT stations far from the center. Therefore, it prevents the accessibility to MRT from degrading in suburban areas.




(a) Peak hour




(b) Off-peak hours

**FIGURE 8**: Access to MRT

Note also that deploying a feeder (either fixed or DR) in the suburbs, instead of serving the demand with MRT Only, brings lower waiting time for MRT. Indeed, as confirmed by Fig. 7, when a feeder is deployed, less MRT lines are needed than in MRT only, which brings cost saving, which, in turn, can be used to increase the MRT frequency.

## CONCLUSION

We have presented the concept of *Flexible Transit*, which combines fixed schedule and demand-responsive transportation.

Indeed, as we show in a small microsimulation scenario, the issue with fixed-schedule transportation is poor QoS during off-peak hours in weak demand areas. On the other hand, the issue with demand-responsive is too high operational cost during peak hours. Based on this observation, Flexible Transit alternates between fixed and demand-responsive feeder to adapt to the geographical and temporal variation of the demand density.

We provide a theoretical high-level model of Flexible Transit based on Continuous Approximation. Numerical results on such a model show that Flexible Transit tilts the balance of the overall costs in favor of user-centric components, keeping the agency cost at a reasonable level, such that the overall cost (the sum of the two) is improved. An important benefit of Flexible Transit is that it limits the degradation of QoS in suburban areas.

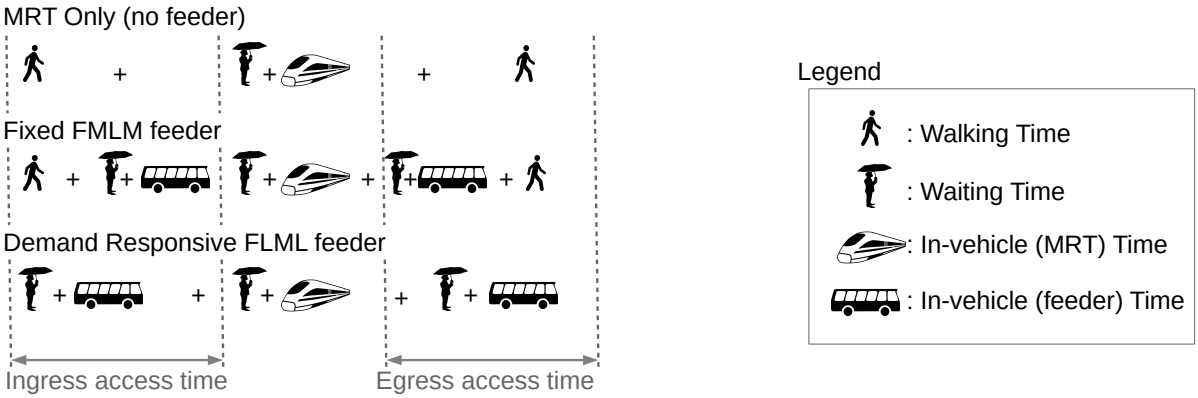



**FIGURE 9**: Components of the access time and waiting time with MRT-only, fixed feeder and demand-responsive feeder. The access time includes both ingress and egress access times.

In future work, we will verify such insights in different scenarios (different demand densities, size of the entire area). We will also consider the impact of electrification and automation, which deeply modify the agency cost components.

We believe the concept of Flexible Transit can guide planning agencies in the deployment of more efficient next-generation transit systems.

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## CONTRIBUTION OF THE AUTHORS

All authors contributed to the general concept and reviewed the paper. G.C., A.A. and G.I. proposed the idea of Flexible Transit and contributed to the sections on Continuous Approximation (CA) method. S.O.and R.S. contributed to the sections on microsimulation. G.C. and S.O. are responsible for the CA and microsimulation code and results. M.B.A. provided scientific direction.

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