Computational Advantage from Quantum Superposition of Multiple Temporal Orders of Gates

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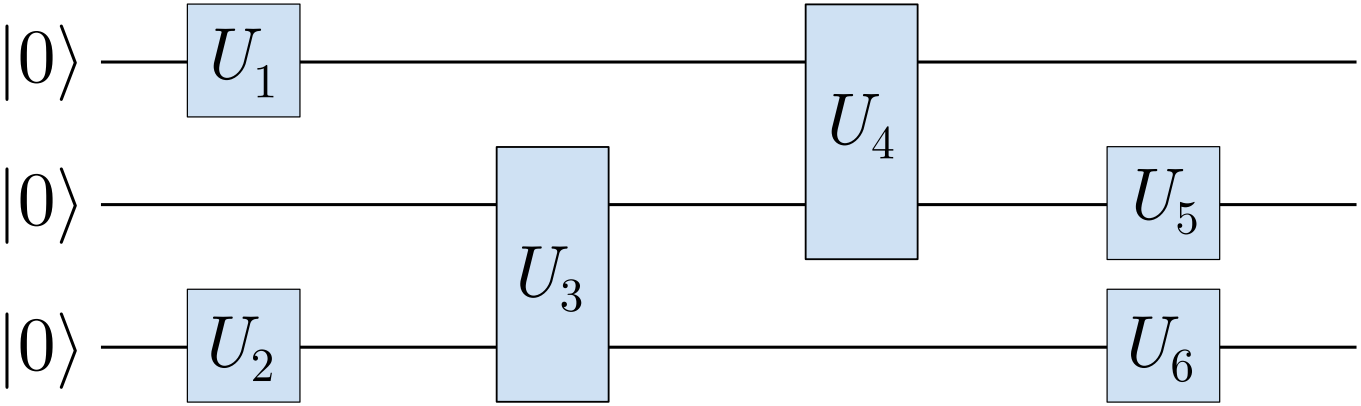
arXiv:2002.07817

with M. Taddei, J. Cariñe, D. Martínez, T. García, N. Guerrero, M. Araújo, C. Branciard, E. Gómez, S. Walborn, L. Aolita, G. Lima

Motivation

- Standard quantum circuit



➢ Can we go further?

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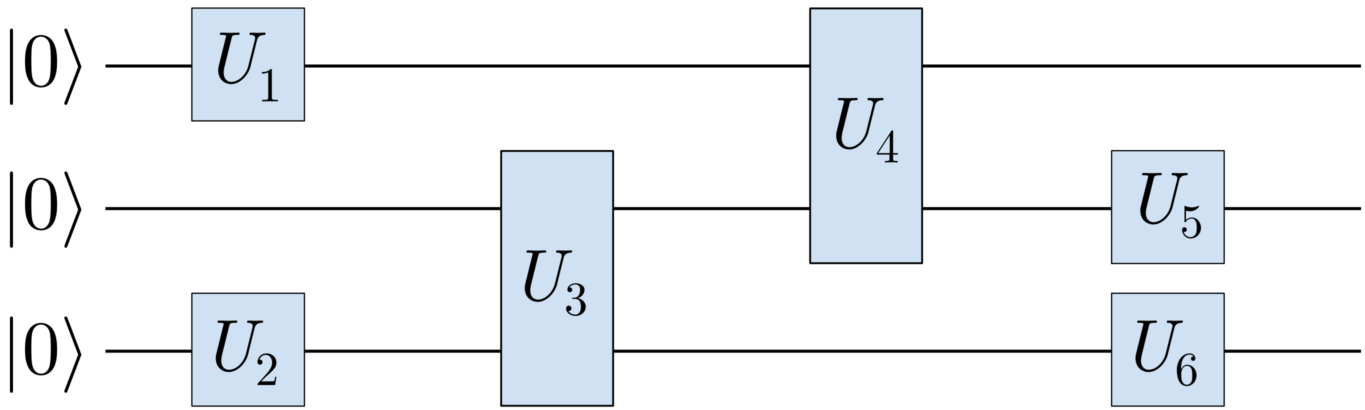

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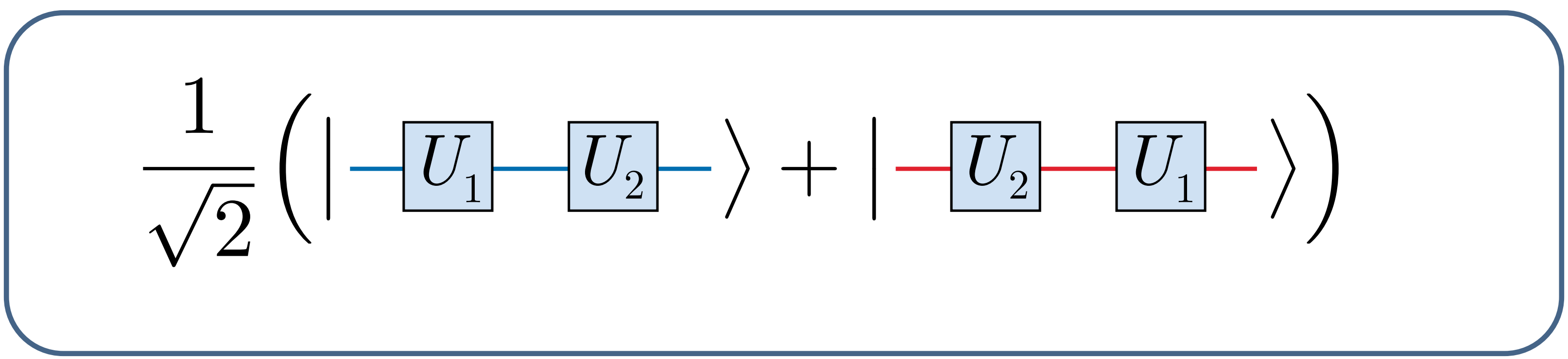


Outline

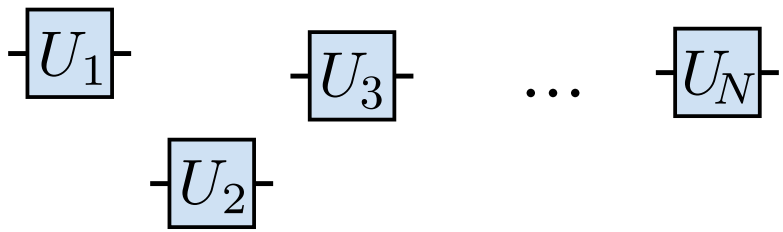
1. Higher-order quantum computation
   - The “quantum switch” – coherently controlling gate order

2. Hadamard Promise Problem

3. Experimental 4-switch

Quantum Computing with Black Boxes

- Black box access to unknown unitaries (oracles)



- Property to determine: $f(U_1, \dots, U_N)$

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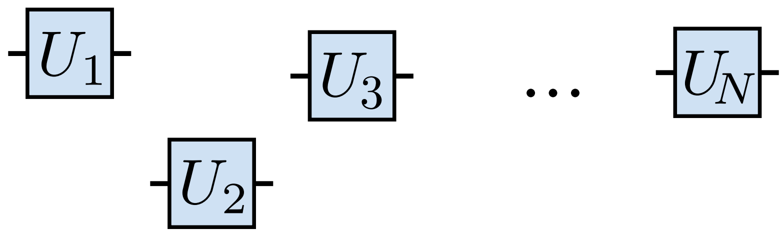
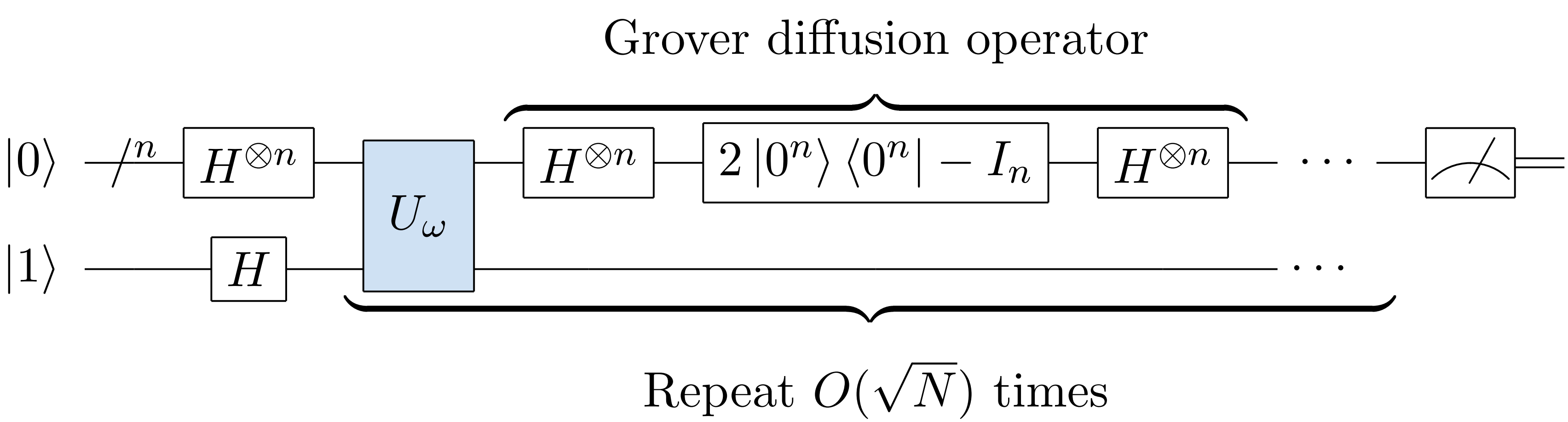


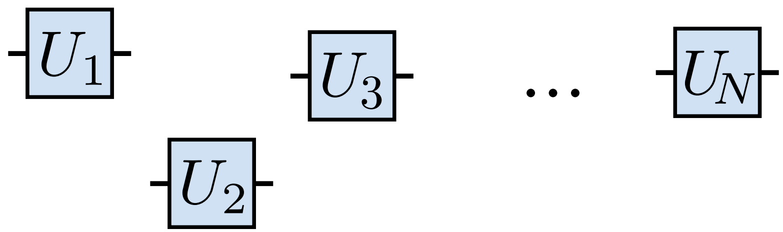
- Property to determine: $f(U_1, \dots, U_N)$

- Example: Grover's problem



Higher-Order Quantum Computation

- Black box access to unknown unitaries (oracles)



- Higher-order computation: how can the operations be composed?

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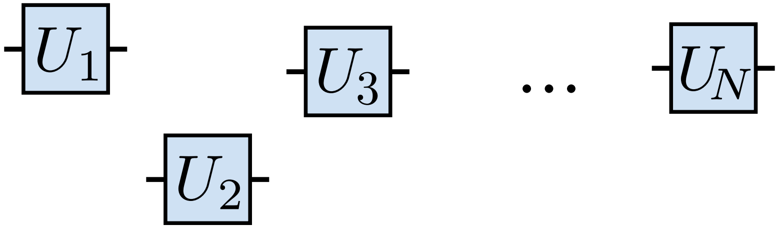

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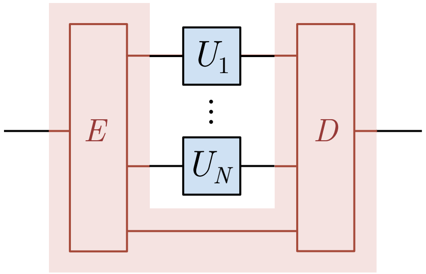

Parallel

Higher-Order Quantum Computation

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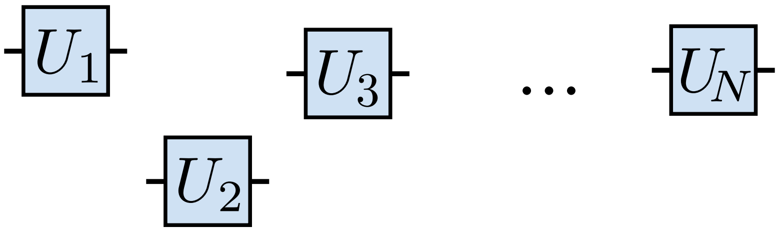

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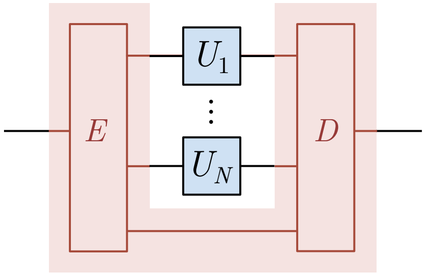

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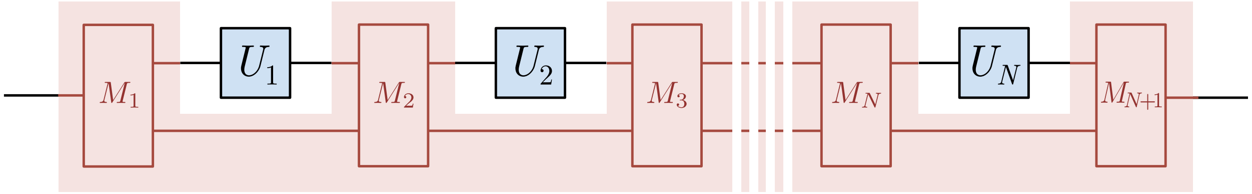

Quantum circuit (quantum comb)

[Chiribella, D'Ariano, Perinotti, PRL (2008)]

Higher-Order Quantum Computation

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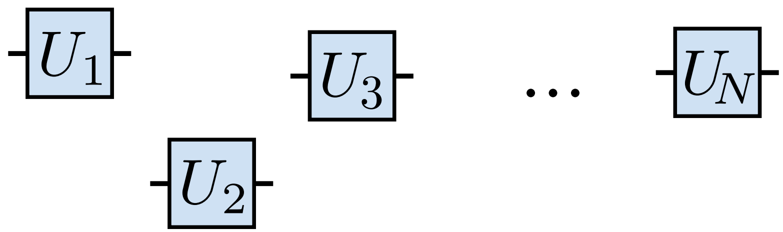

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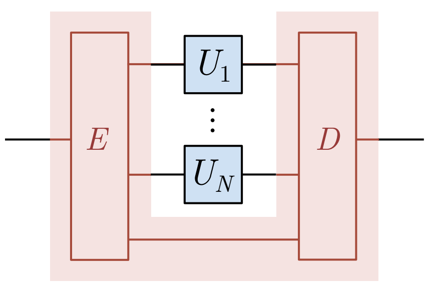

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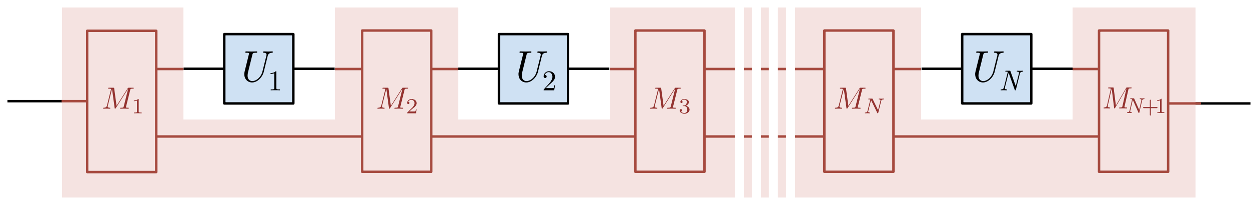

Quantum circuit (quantum comb)

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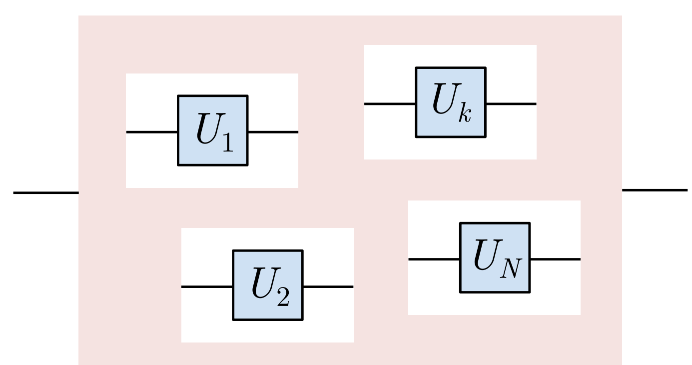
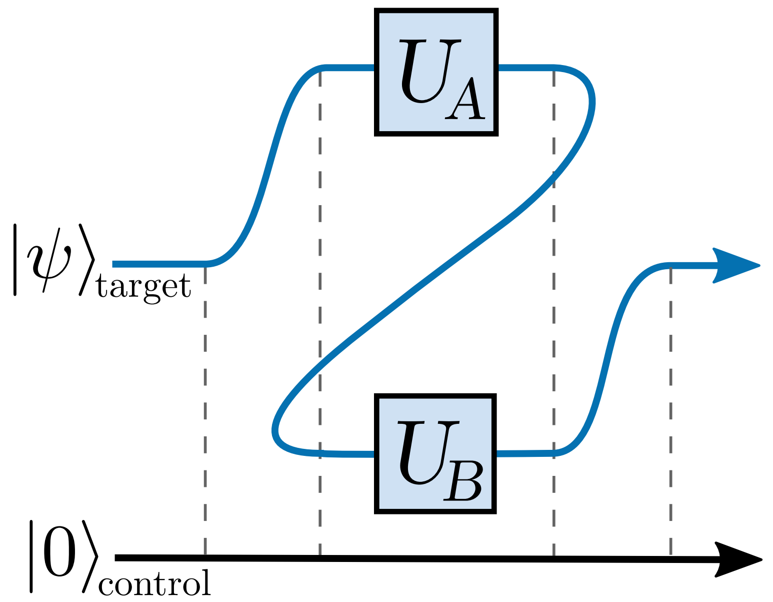
Quantum supermap
(process matrix)



[Oreshkov, Costa, Brukner, Nat. Commun (2012)]
[Wechs, **AA**, Branciard, NJP (2019)]

The Quantum Switch

- First example of a “causally indefinite” supermap
  [Chiribella, D’Ariano, Perinotti and Valiron, PRA 88 (2013); arXiv:0912.0195]



$$S_2(U_A, U_B) = |0\rangle\langle 0|^c \otimes U_B U_A + |1\rangle\langle 1|^c \otimes U_A U_B$$

- Cannot be represented in quantum circuit model
  - Would need to query one box twice

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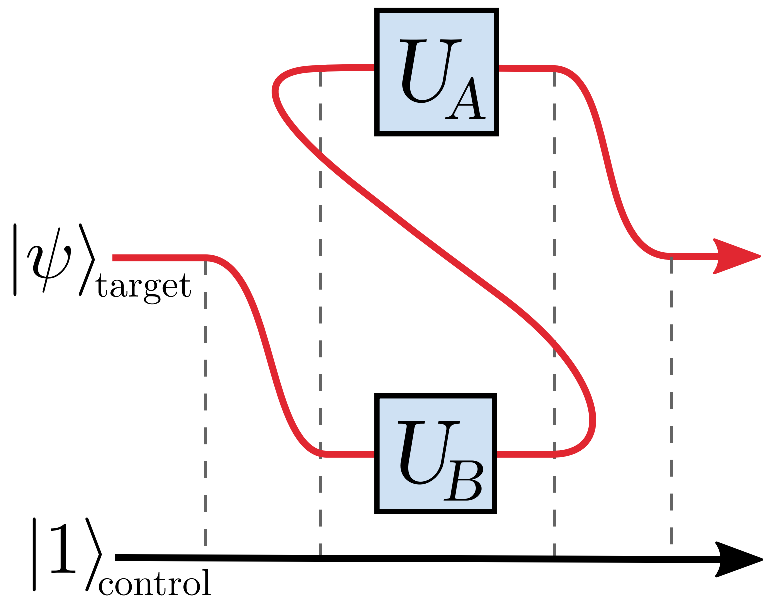

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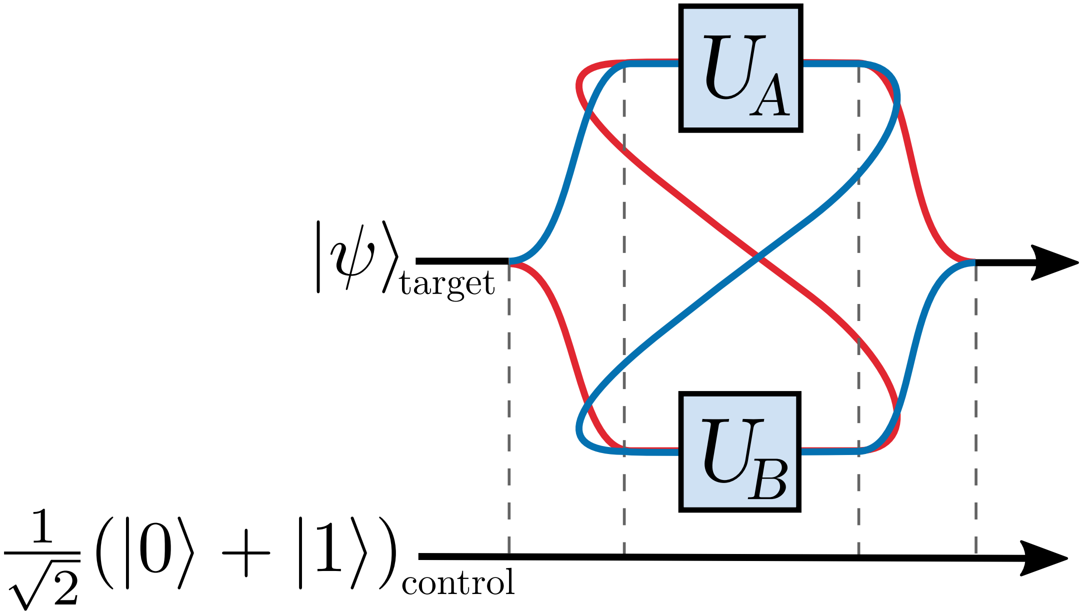

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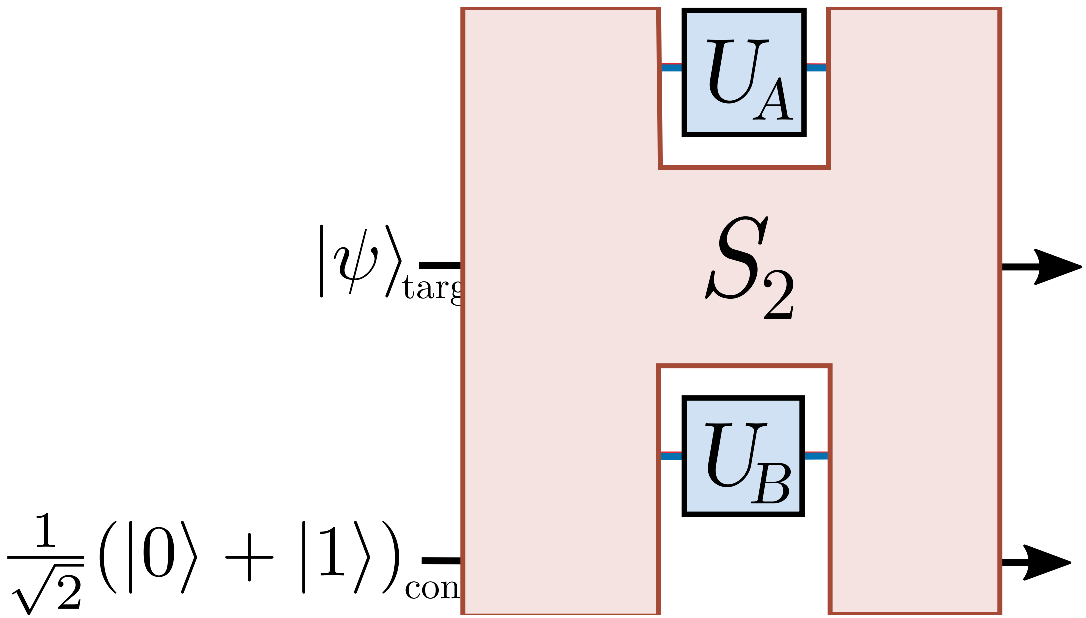

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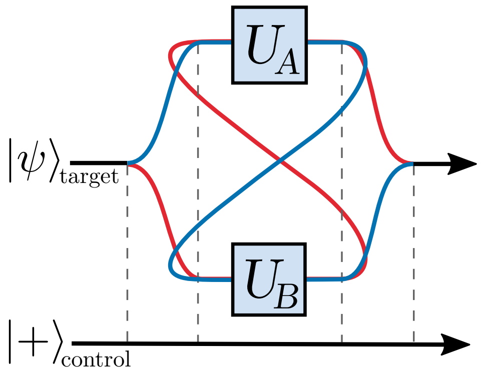
Quantum Switch: A Tool for Quantum Info?

- Quantum switch provides advantages in several settings
  - Query complexity
  - Communication complexity
  - Metrology
  - Communication through noisy channels



[Guérin *et al.*, PRL (2016); Zhao, Yang, Chiribella, PRL (2020); Ebler *et al.*, PRL (2018)]

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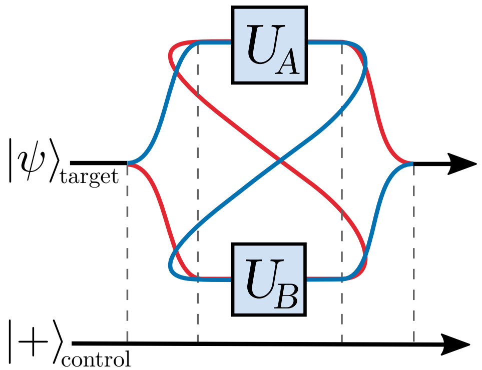


Discriminating commuting vs anticommuting U_A, U_B

[G. Chiribella, PRA (2012)]

$[U_A, U_B]=0$ or $\{U_A, U_B\}=0$

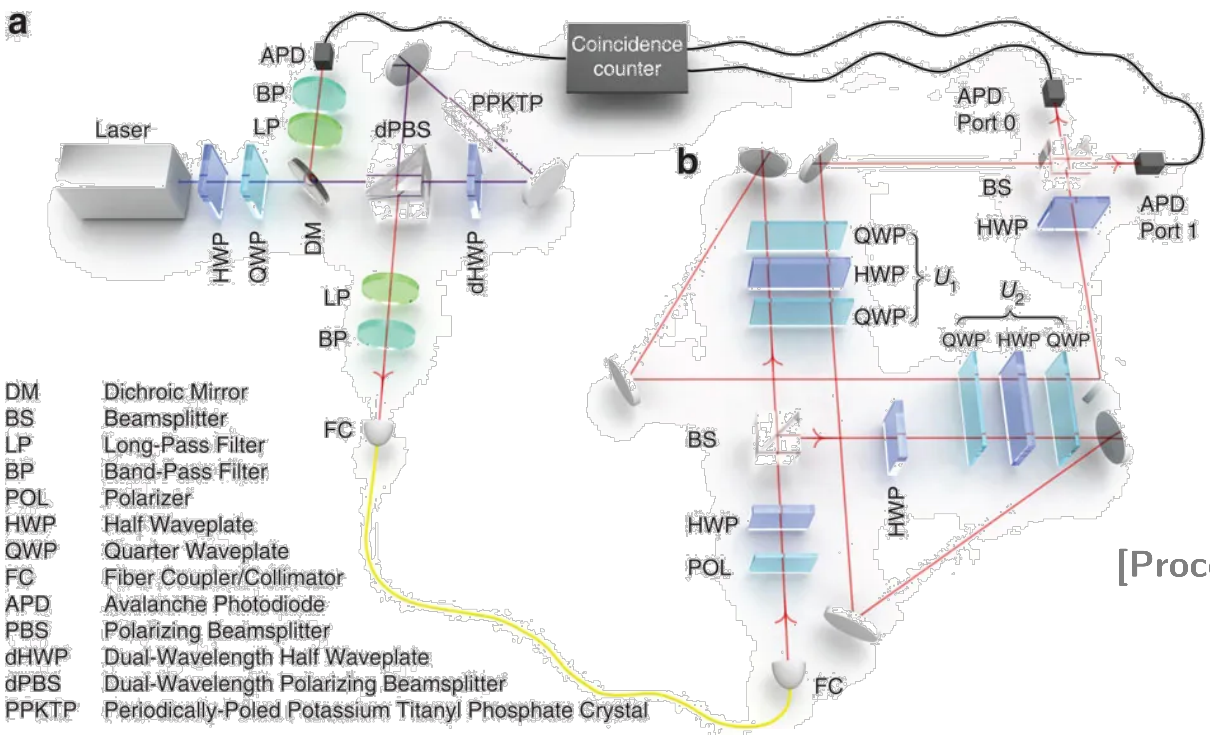
- In a quantum circuit must use U_A or U_B twice
- With the quantum switch, one query to each is enough

$$\frac{1}{\sqrt{2}}(|0\rangle^c + |1\rangle^c) \otimes |\psi\rangle^t \to \frac{1}{\sqrt{2}}(|0\rangle^c \otimes U_B U_A |\psi\rangle^t + |1\rangle^c \otimes U_A U_B |\psi\rangle^t)$$
$$= |+\rangle^c \otimes \tfrac{1}{2}\{U_A, U_B\}|\psi\rangle^t + |-\rangle^c \otimes \tfrac{1}{2}[U_A, U_B]|\psi\rangle^t$$

[Guérin *et al.*, PRL (2016); Zhao, Yang, Chiribella, PRL (2020); Ebler *et al.*, PRL (2018)]

Experimentally Achieving Causal Indefiniteness

- Optical-table experiments based on conceptually simple interferometers



One d.o.f. of photon acts as control, another as the target system

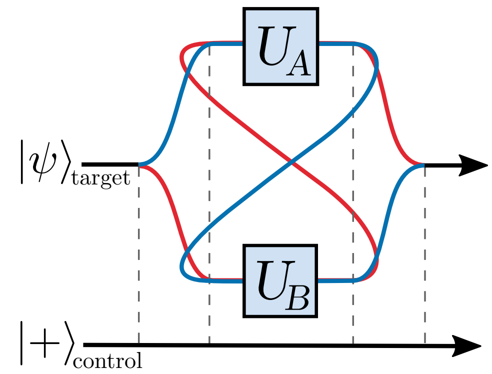
[Procopio *et al.*, Nat. Commun (2015)]
[Rubino *et al.*, Sci. Adv. (2017)]
[Goswami *et al.*, PRL (2018)]
[Wei *et al.*, PRL (2019)]
[Guo *et al.*, PRL (2020)]

How to go beyond the 2-operation quantum switch?

Quantum N-Switch

- Recall the quantum 2-switch

$$S_2(U_A, U_B) = |0\rangle\langle 0|^c \otimes U_B U_A + |1\rangle\langle 1|^c \otimes U_A U_B$$



- N-switch quantum supermap

$$S_N(U_1, \ldots, U_N) = \sum_{x \in [N!]} |x\rangle\langle x|^c \otimes \Pi_x \qquad \Pi_x := U_{\sigma_x(N)} \cdots U_{\sigma_x(1)}$$

$[N!] := \{0, 1, \ldots, N! - 1\}$

xth permutation of $\{1,\ldots,N\}$

[Colnaghi, D'Ariano, Perinotti, Facchini, PLA (2012)]

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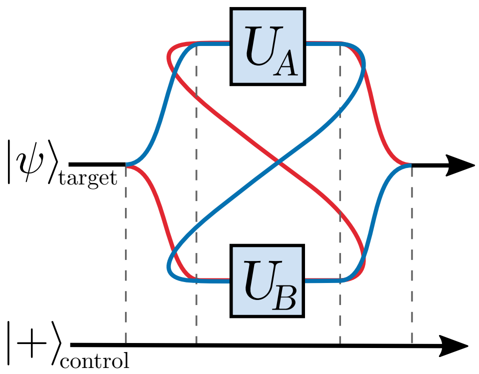

- N-switch quantum supermap

$d_c := \dim(\mathcal{H}^c) \geq N!$ $\quad$ $d_t := \dim(\mathcal{H}^t)$

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Fourier Promise Problem

Setting: Oracle access to N unitary quantum gates U_1, U_2, ..., U_N

$$\boxed{U_1} \quad \boxed{U_2} \quad \cdots \quad \boxed{U_N}$$

Promise: The unitaries obey, for some y, the property

$$\mathbb{P}_y : \forall x \ \Pi_x = \omega^{xy} \Pi_0, \text{ with } \omega = e^{i \frac{2\pi}{N!}}$$

Goal: Find y

[Araújo, Costa, Brukner, PRL (2014)]

- N-switch can solve with N queries (quadratic advantage) 😀

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[Araújo, Costa, Brukner, PRL (2014)]

- N-switch can solve with N queries (quadratic advantage) 😀
- Promise can only be satisfied when $d_t \geq N!$ 😭
  - (Take determinant of both sides of promise statement)

$$\det \Pi_x = \omega^{x d_t} \det \Pi_0 \implies e^{i\frac{2\pi}{N!} x d_t} = 1$$

Outline

1. Higher-order quantum computation
   - The “quantum switch” – coherently controlling gate order
2. Hadamard Promise Problem
3. Experimental 4-switch

Towards the Hadamard Promise Problem

- Now: $P \leq N!$ permutations $\Pi'_x, \; x = 0, \ldots, P-1$

Promise: The unitaries obey, for some y, the property

$$\mathbb{P}_y \, : \, \forall x \; \Pi'_x = m_{x,y} \, \Pi'_0,$$

with $m_{x,y} = \pm 1$ the elements of a $P \times P$ matrix M

Goal: Find y

- Determinant condition: $\det(\Pi'_x) = m_{x,y}^{d_t} \det(\Pi'_0)$
  - d_t must be even → qubits ok!

- What kind of matrix should M be?

Hadamard Promise Problem

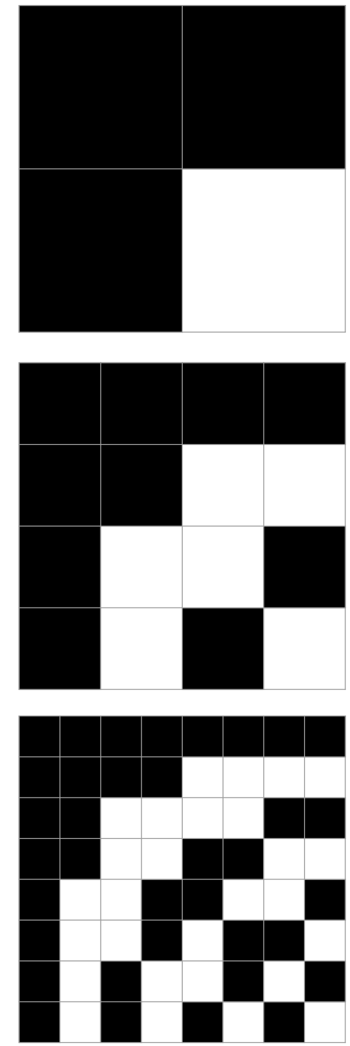
Promise: The unitaries obey, for some y, the property

$$\mathbb{P}_y : \forall x \; \Pi'_x = m_{x,y} \, \Pi'_0,$$

with $m_{x,y} = \pm 1$ the elements of a $P \times P$ Hadamard matrix M

Goal: Find y

- Answer: A Hadamard matrix!
  - ± 1-valued matrices with orthogonal rows and columns
  - Half of values in each row/column are $+1$, half -1
  - Satisfies $\frac{1}{P} M \cdot M^{\mathsf{T}} = \mathbb{1}$
  - Only possible for $P = 1$, 2 or a multiple of 4
  - Conjectured to exist for all multiples of 4



Solving the Hadamard Promise Problem

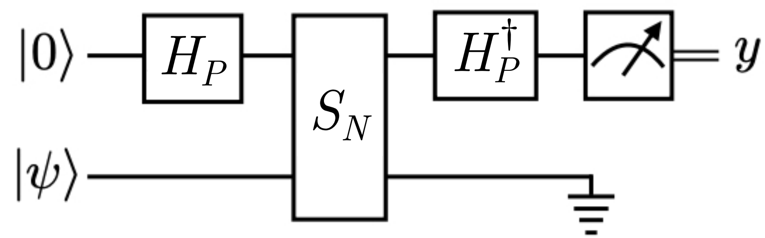
- Can solve perfectly with the N-switch
  - N-switch now coherently controls the permutations Π'_x

$$|0\rangle^c|\psi\rangle^t \xrightarrow{H_P} \frac{1}{\sqrt{P}} \sum_{x\in[P]} |x\rangle^c|\psi\rangle^t$$

$$\xrightarrow{S_N} \frac{1}{\sqrt{P}} \sum_{x\in[P]} |x\rangle^c \Pi'_x|\psi\rangle^t$$

$$= \left(\frac{1}{\sqrt{P}} \sum_{x\in[P]} m_{x,y}|x\rangle^c \right) \Pi'_0|\psi\rangle^t$$

$$\xrightarrow{H_P^\dagger} |y\rangle^c \Pi'_0|\psi\rangle^t$$



where $H_P = \frac{1}{\sqrt{P}} M$

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$$
\begin{aligned}
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&\xrightarrow{S_N} \frac{1}{\sqrt{P}}\sum_{x\in[P]}|x\rangle^c\Pi'_x|\psi\rangle^t \\
&= \left(\frac{1}{\sqrt{P}}\sum_{x\in[P]}m_{x,y}|x\rangle^c\right)\Pi'_0|\psi\rangle^t \\
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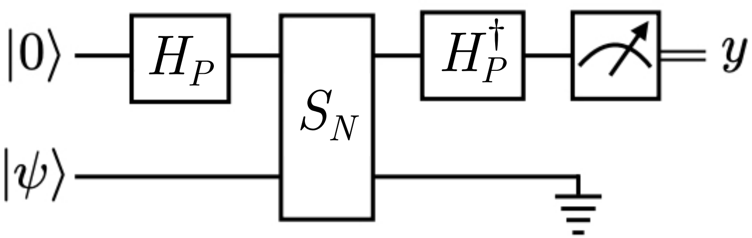

where $H_P = \frac{1}{\sqrt{P}}M$

- Solves the problem using exactly N total queries
  - Asymptotic $O(N^2)$ advantage over best known quantum circuits
- P-dimensional control, qubit target system

Outline

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3. Experimentally solving the HPP with a 4-switch

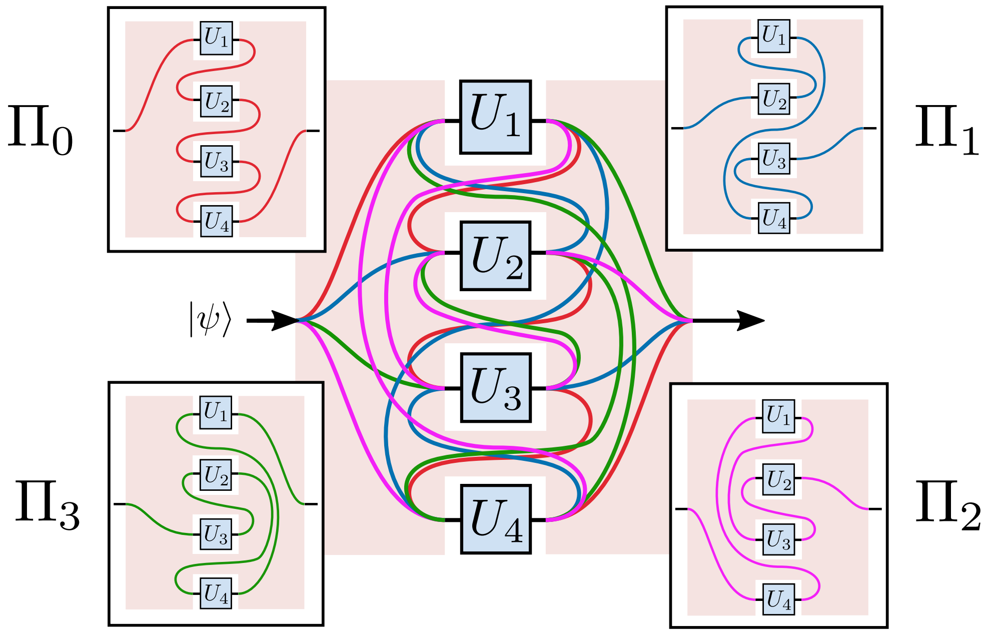
Example: $N\!=\!P\!=\!4$

- Fix 4 permutations and a Hadamard matrix M

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

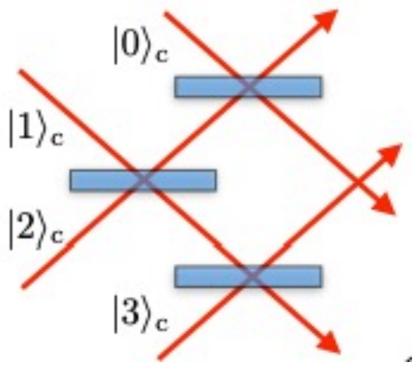
$$\Pi_0 = U_4 U_3 U_2 U_1 \quad \Pi_1 = U_3 U_4 U_1 U_2$$

$$\Pi_2 = U_1 U_4 U_2 U_3 \quad \Pi_3 = U_2 U_3 U_1 U_4$$



Experimental High-Dimensional Hadamards

- A sequence of 2^k balanced beamsplitters gives a Hadamard matrix



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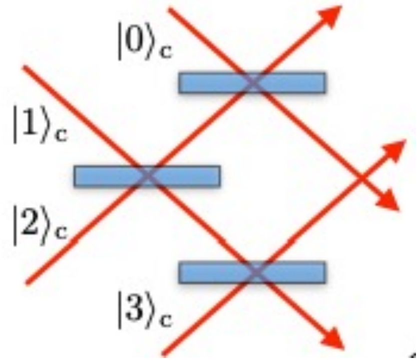
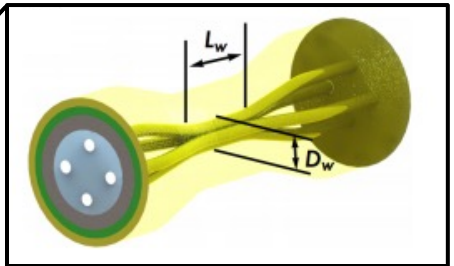
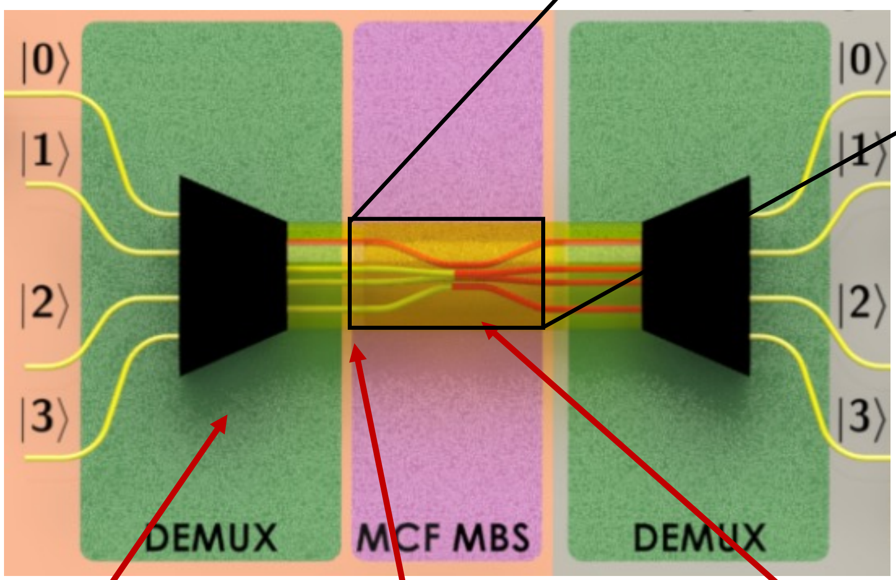


- More compactly: multicore fibre 4-port beamsplitter



[J. Cariñe *et al.*, Optica (2020)]

Single-mode fibres



Spatial demultiplexer unit

4-core fibre

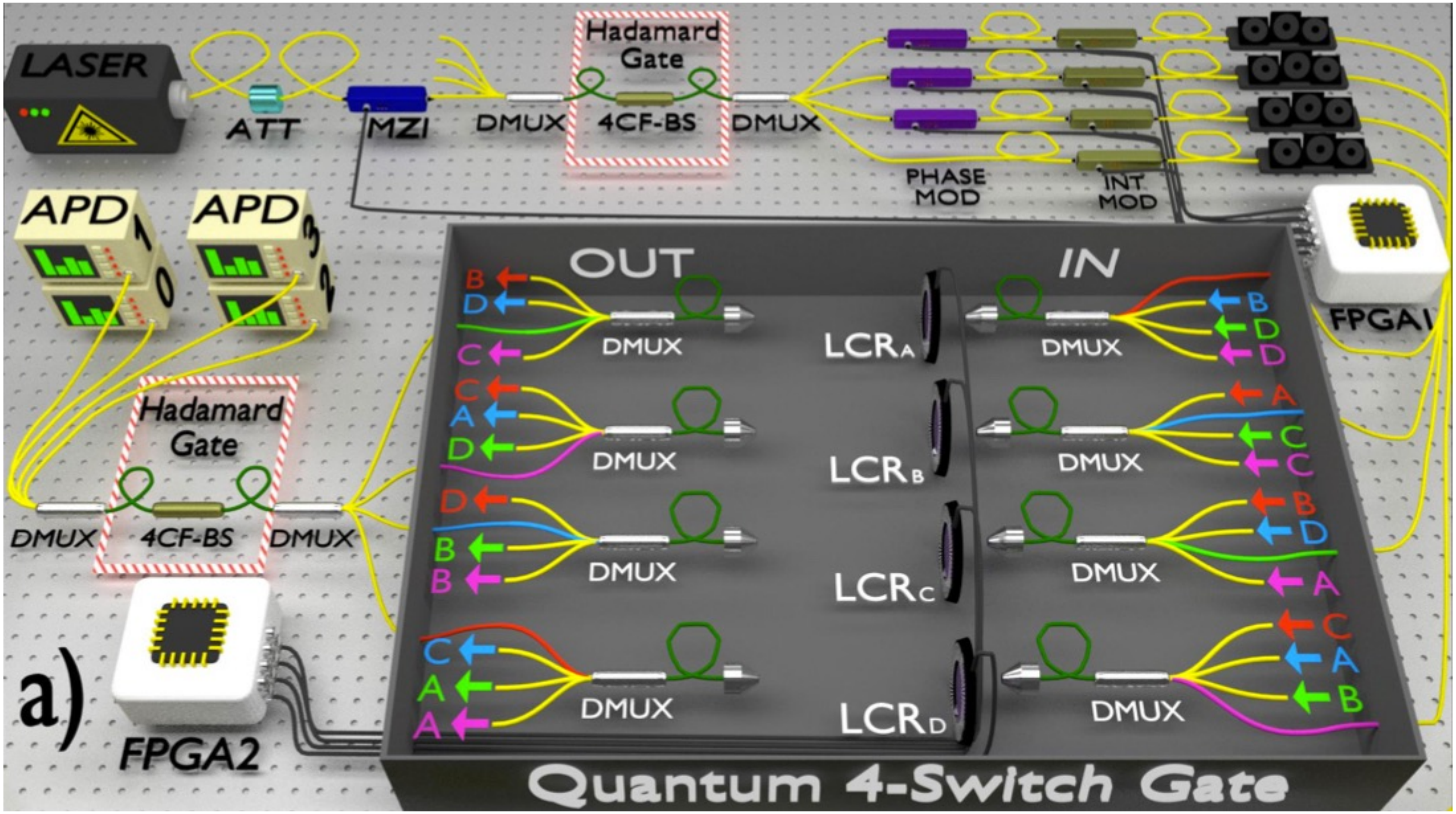
Evanescent-wave coupling

Implements

$$H_P = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

Photonic 4-switch Implementation

- Control system = path ququart;   target system = polarisation



[M. Taddei *et al.*, arXiv:2002.07817]

Experimentally Solving the HPP

- Implemented two different sets of unitaries

Table 1				
y	0	1	2	3
U_A	$\mathbb{1}$	Z	$\mathbb{1}$	Z
U_B	X	X	X	X
U_C	$\mathbb{1}$	Z	Z	$\mathbb{1}$
U_D	X	X	X	X

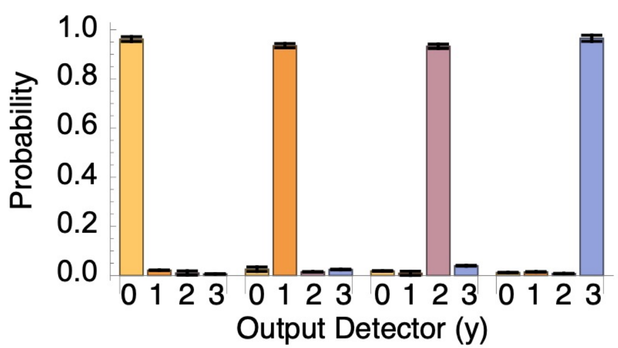
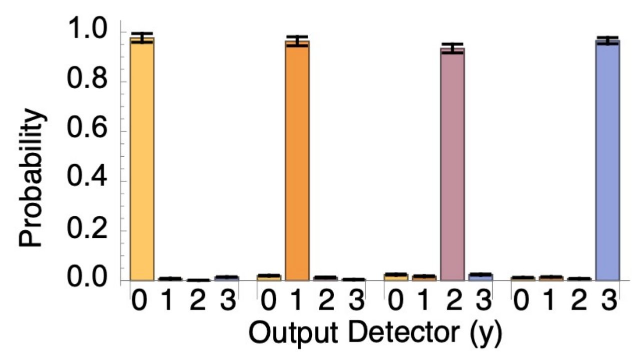


Table 2				
y	0	1	2	3
U_A	$\frac{Z+X}{\sqrt{2}}$	$\mathbb{1}$	Z	Z
U_B	$\frac{Z+X}{\sqrt{2}}$	X	X	X
U_C	$\mathbb{1}$	Z	$\mathbb{1}$	$\mathbb{1}$
U_D	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	X



- Solved the problem with 4 total queries with $P_{\text{succ}} \approx 0.95$

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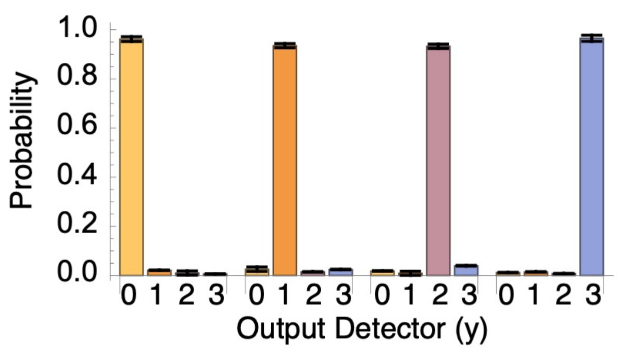
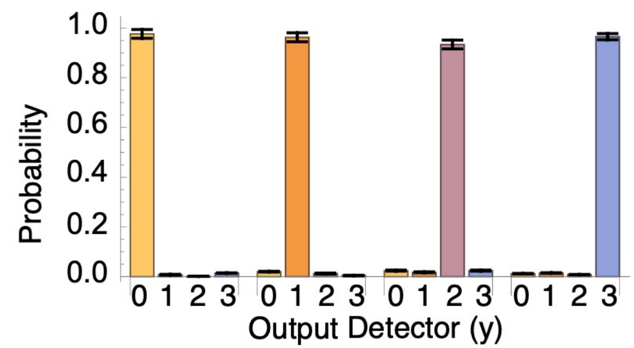
U_B	$\frac{Z+X}{\sqrt{2}}$	X	X	X
U_C	$\mathbb{1}$	Z	$\mathbb{1}$	$\mathbb{1}$
U_D	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	X





- Solved the problem with 4 total queries with $P_{\text{succ}} \approx 0.95$
- Is there an advantage over causally definite computations?

Bounding Causal Strategies

- Best P_{succ} with 4 queries in well-defined order (e.g., circuit):

Given unitaries $\{U_1^{y,i}, \ldots, U_N^{y,i}\}_i$ for each y, and prior distribution $q_{y,i}$

$$\max P_{\text{succ}} = \sum_{y,i} q_{y,i} \operatorname{Tr}[W \cdot (|y\rangle\langle y|^c \otimes \mathfrak{C}(U_1^{y,i}, \ldots, U_N^{y,i}))]$$

$$\text{s.t. } W \in \mathcal{W}^{\text{causal}}$$

Process matrix

Class of computations with well defined query order

- Finding "causal bound" is a semidefinite program!

[Araújo *et al.*, NJP (2015); Wechs, **AA**, Branciard, NJP (2019)]

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- For the sets of unitaries experimentally implemented, can have $P_{\text{succ}}=1$ with a quantum circuit!

  ➢ Untrusted regime

Table 1

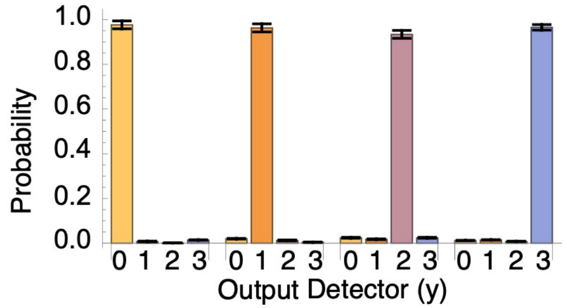
y	0	1	2	3
U_A	$\mathbb{1}$	Z	$\mathbb{1}$	Z
U_B	X	X	X	X
U_C	$\mathbb{1}$	Z	Z	$\mathbb{1}$
U_D	X	X	X	X

Bounding Causal Strategies

- 60 sets of unitaries taken from

$$\mathcal{G} = \{\mathbb{1}, Z, X, Y, \frac{Z+X}{\sqrt{2}}, \frac{Z+Y}{\sqrt{2}}, \frac{X+Y}{\sqrt{2}}, \frac{\mathbb{1}+iZ}{\sqrt{2}}, \frac{\mathbb{1}+iX}{\sqrt{2}}, \frac{\mathbb{1}+iY}{\sqrt{2}}\}$$

  - Bound: $P_{\text{succ}} \lesssim 0.92$



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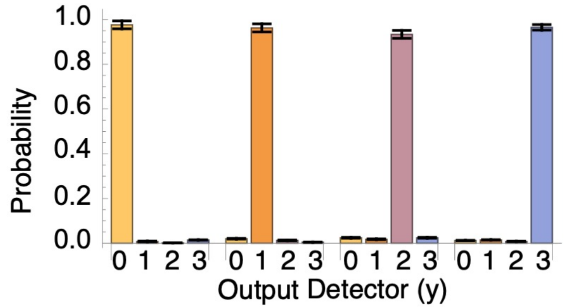

- If unitaries given in a random basis:

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➢ Violation of bounds assuming "fair" implementation

  ➢ No prior knowledge of sets of unitaries

➢ Experimentally implementing full set time-consuming, possible

Conclusions

- Query complexity advantage from superposition of query order in a problem with softer dimension constraints

- First implementation for $N>2$ gate orders
  - (modulo philosophical discussions)

- Outlook:
  - Need more choices of unitaries to properly violate causal witness
  - More applications: communication complexity, …
  - Coherent control of gate order beyond the N-switch

[J. Wechs, H. Dourdent, **AA**, C. Branciard, *in preparation*]

Conclusions

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[J. Wechs, H. Dourdent, **AA**, C. Branciard, *in preparation*]

arXiv:2002.07817

Experimental computational advantage from superposition of multiple temporal orders of quantum gates