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Geometric Categories in Cognition

Moira R. Dillon, Marianne Duyck, Stanislas Dehaene, Marie Amalric, Véronique Izard

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1 **Title** Geometric categories in cognition

2

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Abstract

At the scale in which we live, space is continuous. Nevertheless, our perception and cognition parse the world into categories, whether physical, like *scene* or *object*, or abstract, like *infinitesimal point* or *seven*. The present study focuses on two categories of special angles in planar geometry, *parallels* and *perpendiculars*, and we evaluate how these categories might be reflected in adults' basic angle discrimination. In the first experiment, participants were most precise when detecting two parallel or perpendicular lines among other pairs of lines at different relative orientations. Detection was also enhanced for two connected lines whose angle approached 90°, with precision peaking at 90°. These patterns emerged despite large variations in the scales and orientations of the angle exemplars. In the second experiment, the enhanced detection of perpendiculars persisted when stimuli were rotated in depth, indicating a capacity to discriminate shapes based on perpendicularity in three dimensions despite large variation in angles' two-dimensional projections. The results suggest that two categorical concepts which lie at the foundation of Euclidean geometry, parallelism and perpendicularity, are reflected in our discrimination of simple visual forms, and they pave the way for future studies exploring the developmental and evolutionary origins of these cognitive categories.

Keywords: categories; shape discrimination; spatial cognition; angle; geometry; parallel; perpendicular; **Significance:** In this paper, we discover that two categories of special angles in formal geometry, *parallels* and *perpendiculars*, are robustly reflected in adults' basic angle discrimination. Our results both characterize the psychophysical properties of angle discrimination, which has been debated in prior literature, and also link vision research to the conceptual, formal, and school-relevant spatial understanding that supports abstract mathematics.

47 **Introduction**

48 Categories of all kinds pervade human cognition and organize our physical and mental
 49 worlds. Some categories are present early in human development and are rooted in perception.
 50 For example, very young infants show more precise discrimination (indicated by an increase in
 51 sucking frequency) of two synthetic speech sounds separated by a fixed distance in voice onset
 52 time when that distance distinguishes voiced and unvoiced stop consonants (e.g. /b/ and /p/)
 53 compared to when it does not (Eimas et al., 1971; Liberman et al., 1961). Moreover, even early
 54 brain responses are modulated by categories of visual stimuli: Localized regions of the visual
 55 cortex of infants as young as four months respond preferentially to exemplar pictures belonging
 56 to categories like *faces* and *scenes* (Deen et al., 2017). Nevertheless, such early emerging
 57 categorical processing is reshaped by experience. For example, in the first year of life, infants’
 58 auditory discrimination becomes specialized to the phonemic categories present in their native
 59 language (Kuhl et al., 1992; Werker & Tees, 1984), and children’s explicit discrimination of
 60 phonemes develops reciprocally with culturally constructed reading and writing systems (see
 61 Anthony & Francis, 2005 for a review). Such complex developmental stories raise intriguing
 62 questions about the origins of our explicit categorical knowledge in adulthood: To what extent is
 63 such knowledge innate, rooted in our perceptual experiences, or acquired by explicit education or
 64 immersion in a specific language and culture? And what of those categories with no apparent
 65 perceptual origins or constraints, like the concept of *irrational number* or other such abstract
 66 concepts often found in formal mathematics? In the present study, we investigate whether two
 67 categories of special angles that lie at the foundation of formal, Euclidean geometry, *parallelism*
 68 and *perpendicularity* (Euclid, 1990/300 B.C.E.), are reflected in adults’ basic angle

69 discrimination. We do so by evaluating whether adults' discrimination acuity is enhanced around
70 these category boundaries in a variety of perceptual contexts.

71 At the turn of the 20th century, Wilhelm Wundt and Hermann von Helmholtz
72 independently suggested that angle discrimination may not be a wholly continuous process. They
73 observed that small angles are judged to be somewhat bigger than their actual size and that big
74 angles are judged to be somewhat smaller (von Helmholtz, 1897; Wundt, 1897). More recent
75 work on angle discrimination has aimed to explain and quantify this observation by suggesting,
76 for example, that errors in discrimination reflect orientation selectivity in the visual cortex that
77 leads to orientation distortions (Carpenter & Blakemore, 1973) or that the errors are rooted in an
78 inference about how an angle's appearance reflects its real-world size (Howe & Purves, 2005;
79 Nundy, Lotto, Coppola, Shimpi, & Purves, 2000).

80 A number of studies have focused on comparing the threshold at which we are able to
81 discriminate various angles, but the results have been mixed. In addition, such studies often
82 incompletely controlled for a variety of visual cues in the angle stimuli, such as orientation or
83 size, and so conclusions about angle discrimination specifically have been difficult to make
84 (Werkhoven & Koenderink, 1993; Wenderoth & Johnson, 1984; Snippe & Koenderink 1994;
85 Regan & Hamstra 1992). Chen & Levi (1996) suggested, for example, that there are different
86 detection thresholds for angles of specific sizes, reporting more fine-grained discrimination of
87 90° angles in a discrimination space that otherwise follows Weber's Law. While such thresholds
88 were measured for 12 different reference angles (ranging from 15° -180°), the angles were
89 presented at only two different orientations, vertical or oblique, so it remains unclear whether the
90 thresholds were generalizable to more variable orientations. Heeley & Buchanan-Smith (1996)
91 found a similar pattern of discrimination thresholds with angles presented at random orientations,

92 but they did not simultaneously vary the lengths of the lines that formed the angles, allowing for
 93 other, global shape cues to drive participants' performance. Regan, Gray, & Hamstra (1996), in
 94 contrast, found fairly constant thresholds of angle discrimination between 20° and 160° when
 95 angles were presented at a wide range of orientations. The authors suggest, however, that their
 96 discrepant findings might have been due to their unique methodology: Participants compared
 97 each stimulus to a reference angle size internalized through many practice trials, rather than to a
 98 physical display. The existing literature thus suggests that discrimination may be most precise
 99 around 90° angles, but it remains inconclusive.

100 In the present study, we measured the discrimination thresholds for many different angles
 101 using an intruder task (after Dehaene, Izard, Pica, & Spelke, 2006). We evaluated whether
 102 participants could locate an intruder that differed in angle size among five other angle exemplars
 103 of the same angle size as each other. The lines forming the angles varied considerably in their
 104 lengths and orientations to ensure that responses were made on the basis of angle alone within an
 105 individual trial. In addition, no individual lines were oriented within 10° of the horizontal or
 106 vertical to avoid any specialized angle discrimination that might occur when individual lines are
 107 orientated at the horizontal or vertical (Xu, Chen, & Kuai, 2018).

108 We hypothesized that angle discrimination may reflect categories of parallelism and
 109 perpendicularity in two ways. First, detection thresholds may be more precise when parallels or
 110 perpendiculars serve as reference angles, compared to intruder angles of other sizes. Second, the
 111 detection of angle intruders may be asymmetric such that, for reference angles near 0°/180° and
 112 90°, intruders whose angle size moves towards versus away from the parallel or perpendicular
 113 category boundaries will be easier to detect. This second prediction should hold when intruders
 114 cross the category boundary and also, perhaps, when intruders approach but do not cross the

115 boundary. In Experiment 1, we find evidence for such categories. In Experiment 2, we
116 demonstrate the robustness of these categories by imposing an additional rotation in depth in the
117 experimental displays, dissociating 3D perpendicularity from 2D angle.

118

119 **Experiment 1**

120 **Methods**

121 **Participants**

122 Eight adults (four women; $M_{age} = 25$ years; range 19-28 years) participated in this experiment.
123 The sample size was set in advance based on the maximum sample size (8 participants) used in
124 several other studies investigating angle discrimination and presenting large numbers of trials to
125 individual participants (Chen & Levi, 1996; Regan, Gray, & Hamstra, 1996; Heeley &
126 Buchanan-Smith, 1996; Snippe & Koenderink 1994; Regan & Hamstra 1992, Xu et al., 2018). In
127 addition, stimuli were piloted in advance on three of the study's authors, and effects were robust
128 enough to emerge in each pilot participant. As such, we also illustrate individual participant
129 results in the figures and **SM**. All participants had normal or corrected-to-normal vision and had
130 completed high school; most of them had also received a college or advanced degree. None were
131 informed of the purpose and hypotheses of the study until after it was completed. This study was
132 part of a research program approved by the Paris Descartes Ethics Committee (Conseil
133 d'Évaluation Éthique pour les Recherches En Santé, CERES) and each participant provided
134 informed consent prior to the experiment. Participants were paid 10€ per hour plus an additional
135 sum, which depended on their performance (see below). On average, participants earned 77.38€
136 (range 75.60€-80.07€).

137 **Design, Apparatus, Procedure, and Stimuli**

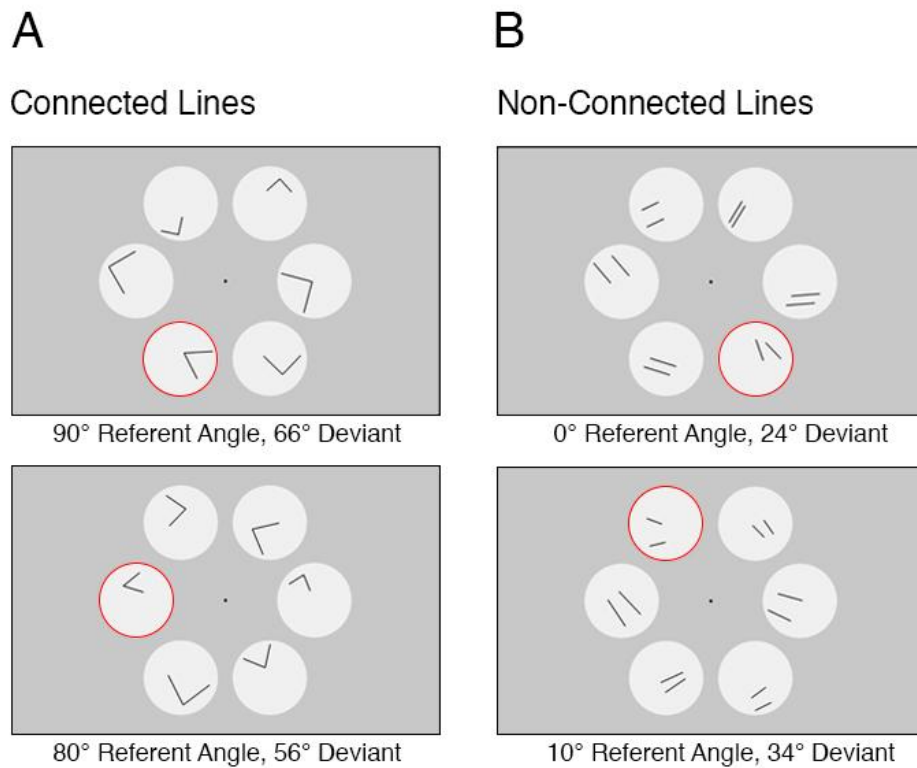
RUNNING HEAD: GEOMETRIC CATEGORIES IN COGNITION

138 For each trial, participants were presented with six angles, five of which were identical in their
139 angle size (hereafter referred to as the “reference” angle) and one of which differed in its angle
140 size (hereafter referred to as the “deviant” angle). Participants were asked to identify the deviant
141 angle. In the *connected lines* condition (**Figure 1A**), angles were composed of two lines that met
142 at one end. Participants saw trials in seven different blocks, each block with a different reference
143 angle. The angle measures of the stimuli are summarized **Table 1**. The reference angles were
144 chosen to evaluate discrimination thresholds symmetrically around 90° , and the angle differences
145 between the reference angles and each deviant were chosen to capture potentially large
146 differences in discrimination thresholds across the different references (if, e.g., discrimination
147 followed Weber’s Law). To probe the detection of 90° deviants specifically, $\pm 10^\circ$ were
148 presented in the 80° and 100° reference blocks. In the *non-connected lines* condition (**Figure**
149 **1B**), two lines were oriented relative to one another, but did not meet, and participants saw eight
150 different reference angles in separate blocks (**Table 1**). We again added 90° deviants in the 80°
151 and 100° blocks. We also added 0° (parallel) deviants in the 10° block and removed from that
152 block the -12° and -18° deviants since these values were not geometrically possible (i.e., they
153 would have resulted in -2° and -8° deviants, which would be equivalent to 2° and 8° angles).
154 Each type of deviant was presented 18 times in a random order in a block, three times at each of
155 six possible target locations. The number of trials per block thus ranged from 126 to 162. Finally,
156 despite the addition of 0° and 90° deviants in some reference blocks, those angles were not over-
157 represented in the experiment. For example, in the connected lines condition, a total of 756 90°
158 exemplars were presented, compared to 810 exemplars of 80° and 100° angles and 720
159 exemplars of the angles in the other reference blocks (**Table 1**).
160

		Reference Angles								
		0	10	25	55	80	90	100	125	155
Deviant Angle Difference	-18	18		7	37	62	72	82	107	137
	-12	12		13	43	68	78	88	113	143
	-10		0					90		
	-7	7	3	18	48	73	83	93	118	148
	-3	3	7	22	52	77	87	97	122	152
	+3	3	13	28	58	83	93	103	128	158
	+7	7	17	32	62	87	97	107	132	162
	+10					90				
	+12	12	22	37	67	92	102	112	137	167
	+18	18	28	43	73	98	108	118	143	173

161 **Table 1.** Summary of the stimuli presented in Experiments 1 & 2. The 25° and 155° reference
 162 angles were presented only in Experiment 1 and the 0° and 10° reference angles were presented
 163 only in the non-connected lines condition of Experiment 1. All angle measures are in degrees.
 164

165



166

167 **Figure 1. A.** Exemplar trials from the connected lines condition. Participants showed smaller
 168 discrimination thresholds with 90° reference angles (top) than with 80° reference angles

169 (bottom). **B.** Exemplar trials from the non-connected lines condition. Participants easily
 170 distinguished non-parallel lines among parallel lines (top) but had difficulty with larger deviants
 171 when reference angles were 10° (bottom). Here, deviants differ from the reference angles by 24°
 172 (the difference used during training trials). For illustration purposes, the correct responses are
 173 circled in red.
 174

175 Participants were seated in a lit room (62 cd.m^{-2}) at eye level and 48 cm away from the
 176 center of an LCD monitor (60 Hz) subtending 44×32 degrees of visual angle (dva). On every
 177 trial, six angles appeared at the same time in one of the six white circular placeholders
 178 (124 cd.m^{-2} , 5 dva radius). These placeholders were equally distributed around a 11 dva radius
 179 circle centered on a central black fixation dot (0.15 cd.m^{-2} , 0.4 dva radius) on a light grey screen
 180 (92 cd.m^{-2}). While maintaining their head position with a chin rest, participants were given 5 s to
 181 look at all of the angles before the angles disappeared. Participants could respond 250 ms after
 182 the presentation onset or up to 30 s after the angles disappeared. At the beginning of each trial,
 183 the cursor was positioned at the center of the screen. Participants clicked on the location of the
 184 deviant angle and received informative auditory feedback. The next trial started 500 ms after the
 185 response. After every quarter of a block, the percentage of correct responding was displayed as
 186 well as the sum earned during that block. Participants earned 0.014€ per correct response.

187 In both conditions, the lines that formed each angle were the same length, but these
 188 lengths varied across angles in the same display (chosen randomly from a uniform distribution
 189 between 2 and 4.4 dva). Lines were never within 10° of the vertical or horizontal axes of the
 190 screen. In the non-connected lines condition, one of the lines was displaced relative to the other
 191 line both along it (up to its midpoint) and orthogonally (0.6-2.5 dva) to it. Finally, to ensure that
 192 all angles in each display were presented at sufficiently different orientations, each was initially
 193 assigned to either 0° , 60° , 120° , 180° , 240° , and 300° relative to the screen and then each was

194 jittered randomly between $\pm 30^\circ$. Each angle was then translated to a random location within 0.4
195 dva of its placeholder's edge.

196 The experiment was divided into two sessions corresponding to the two conditions,
197 which took place on different days within a two-week period. The order of conditions was
198 counterbalanced across participants. Each session started with a short training phase consisting
199 of two trials per reference angle displaying $\pm 24^\circ$ angle deviants. These trials were first
200 presented with unlimited viewing time and then with the 5 s viewing time used in the actual
201 experiment. Each block started with an introductory screen instructing participants to click on the
202 shape that had a different angle size from the rest. This introductory screen also displayed one
203 example of the reference angle for that block, oriented such that a vertical line would bisect the
204 angle. The order of the blocks was random for each participant. Each session lasted
205 approximately 3 hours, with a 15-30 min break after the fourth block in each condition.

206

207 **Analyses**

208 Participants' performance for each reference angle and for both the connected and non-
209 connected line conditions was fit with down-pointing Gaussian curves constrained to chance
210 performance (chance = 0.167) at a difference of zero degrees and perfect performance at a
211 difference of infinity. The fitted Gaussians were used to estimate individual participants'
212 thresholds. These thresholds corresponded to the difference in degrees between the reference and
213 the deviant such that the participant could detect the deviant on half of the trials and was
214 guessing on the other half (i.e., performance of 0.583, halfway between chance and perfect
215 performance). To capture possible asymmetries in participants' responses, performance was fit
216 separately for smaller and larger deviants. Overall thresholds for each reference were obtained by

217 averaging these two values. Because we did not present the same number and measure of angle
 218 deviants across reference angles (e.g., we included 10° deviants in the 80° block, but not in the
 219 55° block), such thresholds were more appropriate than raw accuracies to compare performance
 220 across reference angles. This measure was decided on after pilot testing and in advance of any
 221 data collection.

222 In the connected lines condition, all eight participants performed above chance on all
 223 seven reference angle conditions with both smaller and larger deviants (binomial tests, two-
 224 tailed, all $ps < 0.05$), yielding 112 data sets with above-chance performance. In the non-
 225 connected lines condition, however, 17 of the 128 data sets did not significantly differ from
 226 chance, thus yielding unreliable detection thresholds (see **Figure 2** for the accuracy curves of a
 227 representative participant and **Figure S1** for all individual accuracy curves). To analyze the
 228 results of the non-connected lines condition, we thus used non-parametric, rank-order tests
 229 instead of parametric tests, which replaced estimated threshold values with ranks based on
 230 thresholds' relative magnitudes. As confirmation of our results for this condition using this
 231 method, we also analyzed participants' accuracy using parametric tests, and we obtained the
 232 same results (see **SM, Table S1, Figure S2**).

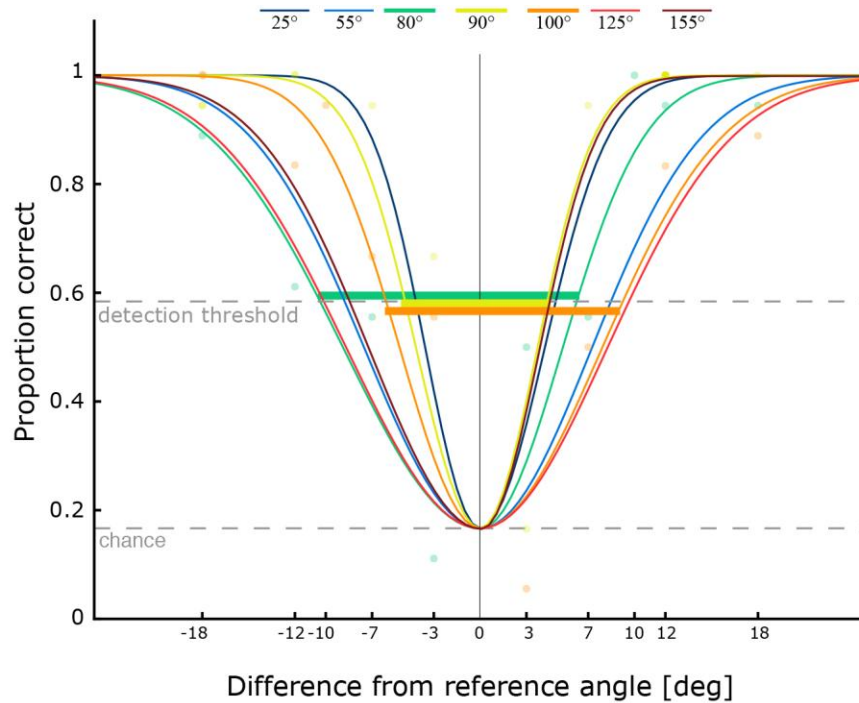
233 First, we evaluated participants' performance in the connected lines condition. Using
 234 planned one-tailed t -tests, we examined whether participants' detection thresholds were more
 235 precise for the perpendicular (90°) reference angles compared to the 80° and 100° reference
 236 angles (the closest references to 90°, a 10° difference) and to the 55° and 125° reference angles
 237 (references further from 90°, a 35° difference). In those same reference blocks, we then
 238 examined whether deviant detection was more precise when deviants approached or crossed the
 239 90° boundary compared to when they did not. For example, we evaluated whether thresholds in

240 detecting larger deviants in 80° reference blocks were more precise than detecting smaller
241 deviants in 80° reference blocks. Since all four comparisons tested whether angle discrimination
242 was influenced by the category of right angles, *p*-values were adjusted for multiple comparisons
243 using Holm's method.

244 For the non-connected lines condition, we investigated whether detection thresholds were
245 more precise for the perpendicular (90°) and parallel (0°) reference angles using planned Holm-
246 corrected, one-tailed Wilcoxon signed-ranks tests. As in the connected lines condition, we
247 compared the thresholds of detecting deviants with a 90° reference both to those with an
248 80°/100° reference and also to those with a 55°/125° reference. In addition, we also compared
249 the thresholds of detecting deviants with a 0° reference to the two closest reference blocks (10°
250 and 25°). Finally, we tested for asymmetries in detection thresholds around these categories by
251 examining whether deviant detection was more precise when deviants approached or crossed the
252 90° or 0° boundaries compared to when they did not.

253

254



255

256 **Figure 2.** Model-fit curves for one participant (S2) for all reference angles in the connected lines
 257 condition of Experiment 1. Individual data points are additionally represented for the 80°, 90°,
 258 and 100° reference blocks. These curves illustrate lower thresholds in the 90° reference block
 259 (i.e., a more narrow curve, in yellow) and lower thresholds for deviants towards versus away
 260 from 90° (i.e., an asymmetry in the curves around 0° of deviation) in the 80° (in green) and 100°
 261 (in orange) reference blocks.

262

263

264 **Results**

265 **Connected Lines Condition**

266 **Figure 3A** displays the individual and average detection thresholds for each angle
 267 reference in the connected lines condition. All participants had similarly shaped curves, in which
 268 detection thresholds were smaller as reference angles approached 0°/180°. Strikingly, all
 269 participants also showed steep drops in their detection thresholds as angle references approached
 270 90°. Group-wise analyses, summarized in **Table 2**, corroborated these results, finding
 271 significantly more precise detection thresholds for 90° reference angles compared to 80°/100°

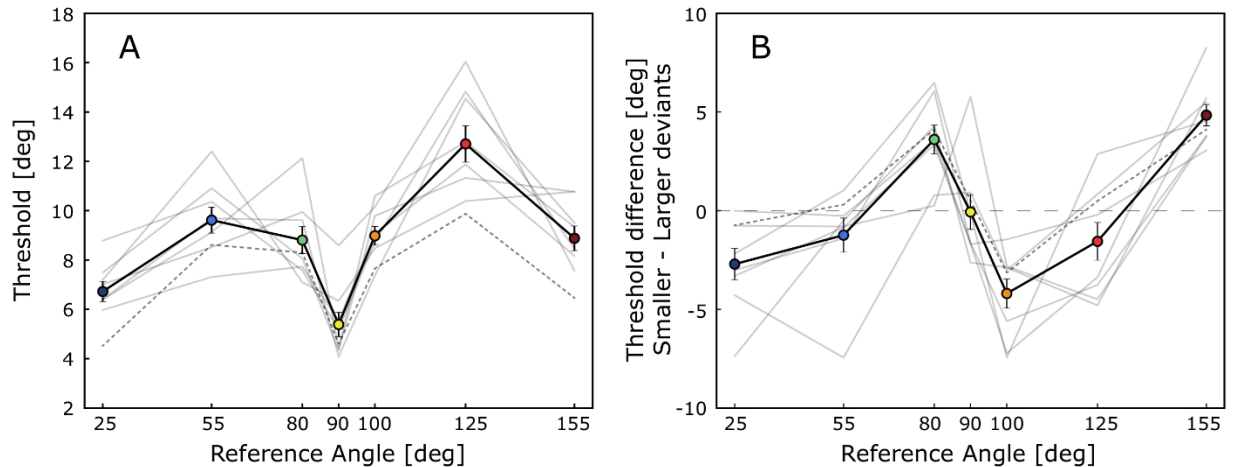
272 reference angles ($t(7) = 5.47, p < .001$, Cohen’s $d = 1.94$) and $55^\circ/125^\circ$ reference angles ($t(7) =$
 273 $10.05, p < .001$, Cohen’s $d = 3.55$). Such differences are characteristic of a categorical effect at
 274 90° .

275 Moreover, we observed significant asymmetries in the blocks where the deviants crossed
 276 the 90° boundary (80° and 100° references). In these blocks, deviant detection was more precise
 277 towards 90° compared to away from it (**Table 2**; $t(7) = -6.29, p < .001$, Cohen’s $d = 2.23$).
 278 However, no asymmetries were observed in the $55^\circ/125^\circ$ reference blocks ($t(7) = 0.21, p = .419$,
 279 Cohen’s $d = 0.07$). Thus, as deviants crossed 90° , their discrimination became more precise,
 280 again signaling an influence of the perpendicular category on performance (see **Figure 3B**).
 281
 282

Perpendicular			
	90	80/100	55/125
Connected Lines – thresholds	5	8.9***	11.2***
Connected Lines – asymmetry		toward: 6.9; away: 10.8**	toward: 11.1; away: 11.2
Non-Connected Lines – thresholds	8	13.6*	20.7*
Non-Connected Lines – asymmetry		toward: 10.1; away: 16.6*	toward: 19.5; away: 19.0
Parallel			
	0	10	25
Non-Connected Lines – thresholds	3	13.3*	20.0*
Non-Connected Lines – asymmetry		toward: 5.1; away: 21.2*	toward: 12.1; away: 28.4*

283 **Table 2.** Group-wise mean (for the connected lines condition) or median (for the non-connected
 284 lines) detection thresholds for perpendicular and parallel reference angle blocks compared to
 285 other reference blocks and detection asymmetries. Planned, Holm-corrected one-tailed t-tests (for
 286 the connected lines condition) or Wilcoxon signed-ranks tests (for the non-connected lines
 287 condition) compare the 90° or 0° reference blocks to the other reference blocks and the
 288 magnitude of the asymmetry effects. All angle measures are in degrees. * $p < .05$, ** $p < .01$,
 289 *** $p < .001$.
 290

291



292

293 **Figure 3. A.** Estimated detection thresholds for each participant (grey lines) and on average
 294 (black line) at different reference angles for the connected lines condition. All curves are
 295 characterized by threshold drops towards 0°/180° as well as a sharp drop at 90°. The dashed
 296 curve corresponds to the participant shown in **Fig. 2. B.** Asymmetries in the thresholds for
 297 detecting smaller versus larger deviants at different reference angles. Positive values on the y-
 298 axis indicate greater success in detecting larger versus smaller deviants, and negative values
 299 indicate the opposite. The dashed curve corresponds to the participant shown in **Fig. 2.** For 80°
 300 and 100° references, acuity is better (smaller thresholds) when the deviant is in the direction of
 301 90°. The asymmetry in detection thresholds crosses zero at almost exactly 90°, and there appears
 302 to be an approximately identical advantage on both sides of 90°.

303

304

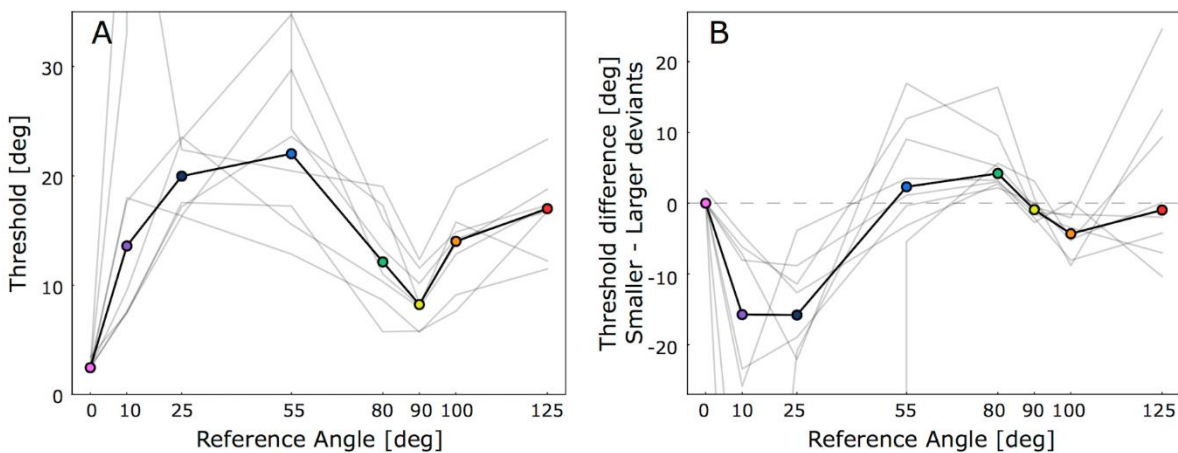
305 **Non-Connected Lines Condition**

306 **Figure 4A** displays the individual and median detection thresholds for each reference
 307 angle in the non-connected lines condition. All participants had similarly shaped curves, in
 308 which detection thresholds were smallest at 0° compared to other reference angles (see **Figure**
 309 **S1** for individual participant accuracy curves). Group-wise analyses corroborated these results
 310 (see **Table 2**), finding more precise detection thresholds for 0° reference angles compared to 10°
 311 and 25° reference angles (10°: $Z = 2.15, p = .016$, 8/8 participants showed the effect; 25°: $Z =$
 312 $2.15, p = .016$, 8/8 participants showed the effect). Moreover, participants performed better with
 313 angles deviating towards versus away from 0° with both 10° and 25° reference angles (10°: $Z =$

314 2.15, $p = .016$, 8/8 participants showed the effect; 25°: $Z = 2.15$, $p = .016$, 8/8 participants
 315 showed the effect; see **Figure 4B**) suggesting that parallel lines might serve as an anchor for
 316 judgments of discrimination.

317 Detection thresholds were quite variable across participants between 10° and 80°, though
 318 they exhibit a similar shape, first increasing at smaller reference angles and then decreasing at
 319 larger reference angles. Evidently, acute angles are difficult to differentiate with non-connected
 320 lines. Nevertheless, as in the connected lines condition, participants' precision increased around
 321 90° compared to 80°/100° reference angles ($Z = 2.15$, $p = .016$; 8/8 participants showed the
 322 effect) and to 55°/125° reference angles ($Z = 2.15$, $p = .016$; 8/8 participants showed the effect;
 323 see **Table 2**). We also observed significant asymmetries in detection thresholds, with better
 324 discrimination as deviants approached or crossed the 90° boundary in the 80°/100° reference
 325 blocks ($Z = 2.15$ $p = .016$, 8/8 participants showed the effect), but not in the 55°/125° reference
 326 blocks ($Z = 0.07$, $p = .473$, 4/8 participants showed the effect; see **Fig. 4B**).

327



328

329 **Figure 4. A.** Estimated detection thresholds for each participant (grey lines) and the group
 330 medians (black line) at different reference angles for the non-connected lines condition.
 331 Thresholds at some reference angles for some individuals exceed the graph limit. All curves are
 332 characterized by low thresholds for the 0° reference angle as well as a sharp drop at the 90°
 333 reference angle. **B.** Asymmetries in the detection thresholds for larger versus smaller deviants at

334 different reference angles. The detection threshold asymmetry crosses zero at around 90°, with
335 an approximately equal advantage on both sides.
336

337 **Discussion**

338 This experiment presents two main findings. First, acuity in angle discrimination varies
339 massively across angles. In particular, thresholds are smallest for parallels and perpendiculars.
340 When angles are acute or obtuse and far from these two categories, thresholds of discrimination
341 are high, though when angles deviate from such acute or obtuse angles and approach the parallel
342 or perpendicular categories, discrimination becomes more precise. These effects persist over
343 variations in the orientation and scale of the angle exemplars. While other studies reviewed
344 above have found similar categorical responses to parallels and perpendiculars under some
345 conditions, our results suggest that angle discrimination in general and the more precise
346 discrimination of these angle categories in particular are invariant to scale and orientation.

347 How robust, then, are the categories of parallelism and perpendicularity in the face of
348 more drastic variation in the angle stimuli, such as rotations in 3D depth, which are prevalent in
349 everyday viewing conditions? In Experiment 1, where stimuli were presented in the
350 frontoparallel plane, the higher precision for parallel and perpendicular lines could have arisen
351 from a specific sensitivity at the retinal or retinotopic level. If, however, the effect arises at a
352 more abstract representational level, i.e., that parallelism and perpendicularity are properties that
353 apply to the arrangement of lines in 3D space, then the effect should remain even if the stimuli
354 are rotated in 3D depth, off the frontoparallel plane. This manipulation applies specifically to
355 perpendicular lines: While parallel lines remain parallel even with rotation in depth (save a few,
356 “accidental” viewpoints where their projections coincide: Amir, Biederman & Hayworth, 2011;
357 2012; Biederman, Yue, & Davidoff, 2009), perpendicular lines vary greatly in their projected

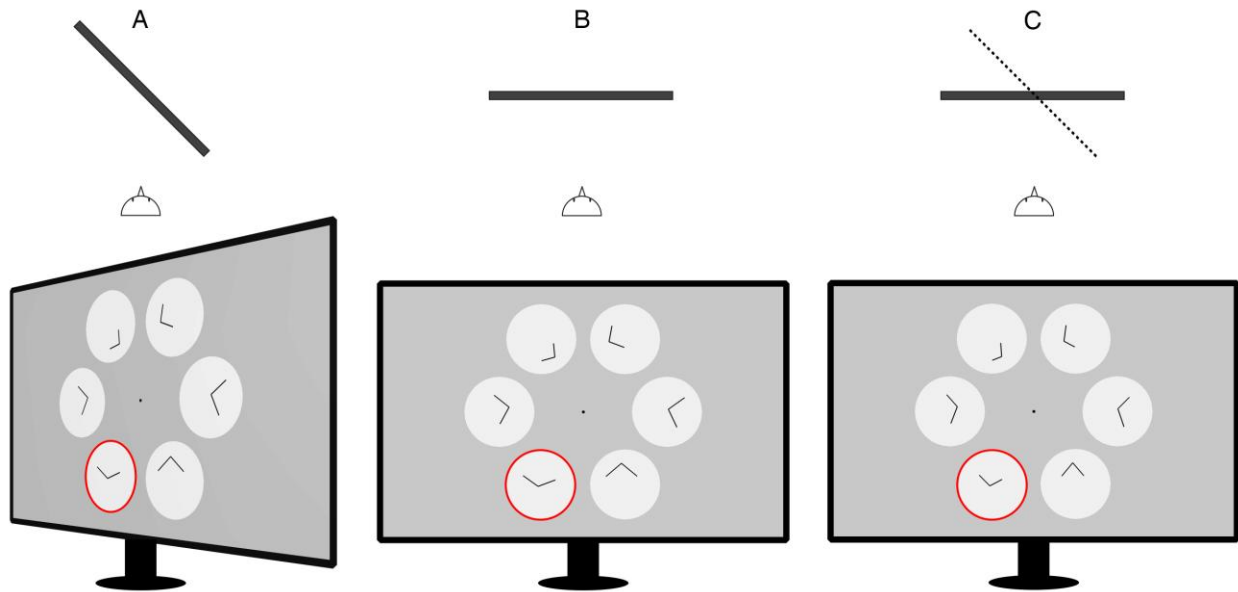
358 angle measure when the lines are rotated in depth (Nundy et al., 2000). Experiment 2 was
 359 therefore conducted both to evaluate how well the detection of perpendicular lines is preserved
 360 under viewing conditions that include additional rotations in 3D depth and to examine whether
 361 perpendicularity as a category persists under these conditions.

362

363 **Experiment 2**

364 In this experiment, we presented new participants with stimuli derived from the
 365 connected lines condition of Experiment 1. In the *slanted screen* condition (**Figure 5A**),
 366 participants viewed the same stimuli as Experiment 1, but on a screen that was rotated in depth
 367 by 45°. In the *normal viewing* condition (**Figure 5B**), participants were presented with the same
 368 stimuli as Experiment 1 with an un-rotated, frontoparallel screen (i.e., a direct replication of
 369 **Experiment 1**). By comparing participants' performance in these two conditions, we could
 370 evaluate whether both angle discrimination in general and the perpendicular category in
 371 particular persist with the rotation of our visual stimuli in depth. In the *projected stimuli*
 372 condition (**Figure 5C**), participants viewed the projection of the stimuli presented in the slanted
 373 screen condition on a frontoparallel screen. With this condition, we could evaluate whether the
 374 3D context, presenting consistent cues to the reference frame transformation (like would be
 375 present in our everyday object recognition, see Nakayama, He, & Shimojo, 1995) or some
 376 general property of the stimuli's projection (which has a less clear connection to our everyday
 377 perceptual experience) might explain participants' responses in the slanted screen condition. If
 378 participants showed the same pattern of performance on the projected stimuli condition and the
 379 slanted screen condition, then there may be some geometric information at the level of the retinal
 380 projection that is guiding responses on the slanted screen condition.

381



382

383 **Figure 5.** The top panel illustrates on overhead perspective of the setup for the screen and
 384 participant for the three conditions: (A) *slanted screen*; (B) *normal viewing*; (C) *projected*
 385 *stimuli*. The lower panel illustrates the same trial (100° referent angle and 124° deviant) as it
 386 would be seen on the screen in each condition. The angles in the slanted screen condition are
 387 identical to the angles in the projected stimuli condition when projected on to the frontoparallel
 388 plane, as illustrated here. For illustration purposes, the correct response is circled in red.
 389

390 **Methods**

391 **Participants**

392 Twelve adults (10 women; *M*_{age} = 23 years; range 19-38 years) participated in this
 393 experiment. The sample size was set in advance of data collection. With the sample size and
 394 smallest effect size from Experiment 1 (N = 8, Cohen's *d* = 1.94), our power to detect group-
 395 wise categorical effects within condition would be .999. We decided to increase our sample size
 396 by four participants compared to Experiment 1 since we also planned to compare results across
 397 conditions in this experiment and investigate effects by condition. All participants had normal or
 398 corrected-to-normal vision and had completed high school; most of them had also received a
 399 college or advanced degree. None were informed of the purpose and hypotheses of the study

400 until after it was completed. Each provided informed consent prior to the experiment and were
 401 paid 10€ per hour plus an additional sum, which depended on their performance. On average,
 402 participants earned 72.95€ (range 70.47€-75.56€).

403 **Design, Apparatus, Procedure, and Stimuli**

404 The distance of the participant to the center of the screen, the lighting conditions, the
 405 placeholder sizes, the screen luminance, the presentation duration, and the stimuli parameters
 406 were identical in all conditions of Experiment 2 to the connected lines condition of Experiment
 407 1, except that the lines that formed each angle were slightly shorter (uniform distribution from
 408 1.6 to 3.5 dva). In the projected stimuli condition, the angles were transformed by reducing all
 409 horizontal coordinates of each angle by a factor of $\cos\left(\frac{\pi}{4}\right)$, while keeping all vertical coordinates
 410 constant. This transformation was applied to the angles only so that the external reference frame
 411 in the slanted and projected stimuli conditions (placeholders' shapes and screen frame) provided
 412 cues to enforce the perception of a slanted or frontoparallel presentation of the stimuli. As in
 413 Experiment 1, trials were blocked by reference angle but fewer reference angles were presented
 414 to focus on the comparisons that were most relevant to exploring the categorical effect (see
 415 **Table 1**). Each type of deviant was presented 18 times in a random order in a block, three times
 416 at each of six possible target locations. The total number of trials per block thus ranged from 144
 417 to 162.

418 The experiment was divided into three sessions, one for each of the three experimental
 419 conditions. The sessions took place on different days, one to five days apart, and the order of the
 420 condition presented at each session was counterbalanced across participants. Sessions started
 421 with a short training phase consisting of two trials per reference angle displaying +/- 24° angle
 422 deviants. Five blocks corresponding to the five reference angles followed the training phase. The

423 order of the blocks within a session was chosen randomly for each participant. Each session
 424 lasted approximately 1.5 hours. As in Experiment 1, at the beginning of each block, participants
 425 were given a slide with instructions, showing an example of the reference angle in the vertical
 426 orientation. This slide was identical in the slanted screen, normal viewing, and projected stimuli
 427 conditions.

428 **Analyses**

429 As in Experiment 1, we fit performance with asymmetric down-pointing Gaussian
 430 curves. However, in several cases participants responded at chance (across participants: 10/120
 431 blocks in the slanted screen condition; 3/120 blocks in the normal viewing condition; and 45/120
 432 blocks in the projected condition; see **Figure S3** for individual accuracy curves). Such at-chance
 433 performance resulted in very high and unreliable estimated threshold values. We thus again used
 434 non-parametric rank tests to minimize the impact of these extreme values in our comparisons,
 435 and we also conducted parametric analyses on accuracy data, which yielded identical results
 436 except where indicated (see **SM, Table S2**).

437 As in Experiment 1, for all conditions of Experiment 2, we examined whether angle
 438 detection was more precise for the 90° reference angle compared to the other reference angles
 439 that were close to and far from 90° (i.e., 80°/100° reference angles and 55°/125° reference
 440 angles). We also evaluated whether deviant detection was more precise when deviants
 441 approached or crossed the 90° boundary compared to when they did not (all using Holm-
 442 corrected, one-tailed Wilcoxon signed-ranks tests). We only evaluated the asymmetries in the
 443 80°/100° reference angles since the asymmetries in the 55°/125° reference angles were not
 444 significant in Experiment 1. By comparing the findings across the three conditions, first we
 445 asked whether categorical effects were present in the slanted screen condition, as in the normal

446 viewing condition (we compared the thresholds and asymmetries in these two conditions using
 447 Scheirer-Ray-Hare [SRH] tests, a non-parametric equivalent of a two-way repeated measures
 448 ANOVA). Second, we asked whether the effects observed with the slanted screen were due to
 449 information present in the stimuli's projection rather than based on shape information invariant
 450 to 3D rotations (by comparing the slanted screen condition to the projected stimuli condition
 451 using a second SRH test).

452

453 **Results**

454 **Figure 6A** shows the median detection thresholds for each reference angle in all three
 455 conditions, and **Table 3** shows tests for within-condition categorical effects. First, in the slanted
 456 screen condition, we observed more precise detection thresholds for 90° reference angles
 457 compared to both the 80°/100° reference angles ($Z = 3.18, p < .001$, 12/12 participants showed
 458 the effect), and also the 55°/125° reference angles ($Z = 3.18, p < .001$, 12/12 participants showed
 459 the effect). Second, this categorical effect did not differ across the slanted screen and normal
 460 viewing conditions. An SRH test comparing these two conditions revealed a significant main
 461 effect of reference angle ($H = 8.37, p = .004$), a significant main effect of condition ($H = 5.87,$
 462 $p = .015$), but, crucially, no interaction between reference angle and condition ($H = 0.20,$
 463 $p = .653$). As such, our normal viewing condition also replicated the results of Experiment 1,
 464 finding smaller detection thresholds with the 90° reference angle compared to the other reference
 465 angles (80°/100°: $Z = 2.47, p = .007$, 11/12 participants showed the effect; 55°/125°: $Z = 2.98, p$
 466 $= .001$, 11/12 participants showed the effect).

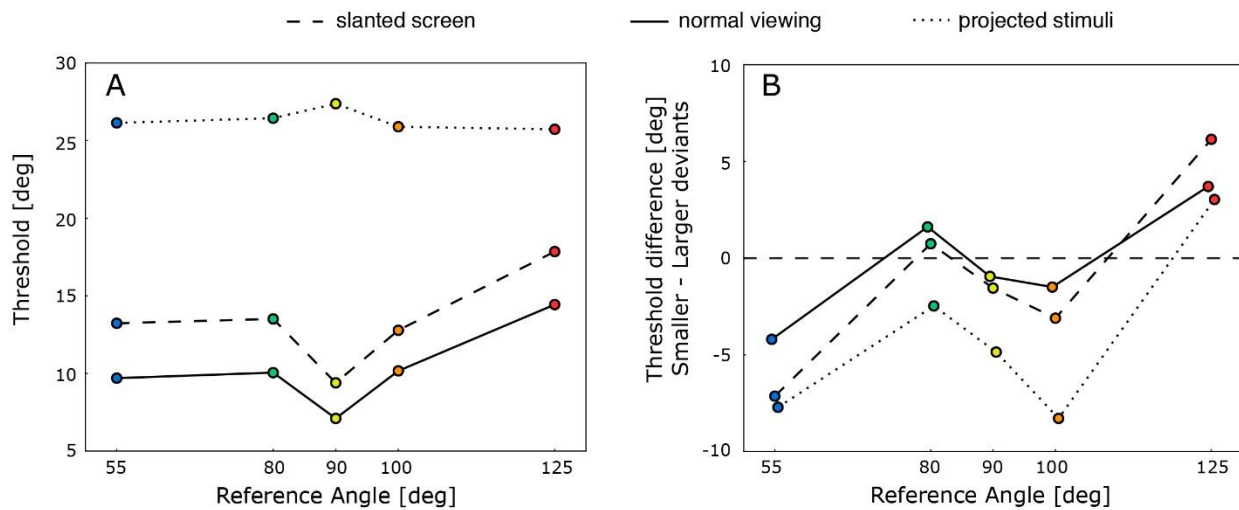
467

Perpendicular		
90	80/100	55/125

Slanted Screen - thresholds	9.4	13.2***	15.9***
Slanted Screen - asymmetry		toward: 12.8; away: 14.8*	
Normal Viewing - thresholds	7.1	10.6**	12.2**
Normal Viewing - asymmetry		toward: 9.5; away: 11.3*	
Projected Stimuli - thresholds	27.3	26.5	25.9
Projected Stimuli - asymmetry		toward: 22.1; away: 28.1	

468 **Table 3.** Group-wise median detection thresholds for perpendicular reference angle blocks
 469 compared to other reference blocks. Planned, Holm-corrected one-tailed Wilcoxon signed-ranks
 470 tests compare the 90° reference blocks to the other reference blocks. All angle measures are in
 471 degrees. ** $p < .01$ ***, $p < .001$.
 472

473



474 **Figure 6. A.** Median thresholds for different reference angles in the three conditions of
 475 Experiment 2. While the slanted screen and normal viewing conditions show a sharp drop at the
 476 90° reference angle, the projected stimuli condition does not. **B.** Asymmetries in the threshold of
 477 detection for larger versus smaller deviants at different reference angles.
 478
 479

480

481 Does the increased performance with the 90° reference angle in the slanted screen
 482 condition result from an analysis of the stimuli as projected or from an invariance of the
 483 detection of perpendicularity over rotations in depth? If the former, then the same categorical
 484 effects should be present in the projected stimuli condition. An SRH test comparing the slanted

485 screen and projected stimuli conditions revealed a significant main effect of reference angle (H
 486 = 5.89, $p = .015$), a significant main effect of condition ($H = 11.67$, $p < .001$), and an interaction
 487 between reference angle and condition ($H = 8.56$, $p = .003$). Indeed, contrary to the slanted
 488 screen condition, there was no sign of categorical effects in the projected stimuli condition, as
 489 performance with the 90° reference angle was no different from the other reference angles ($ps =$
 490 1.00).

491 Next, we assessed whether performance for reference angles $80^\circ/100^\circ$ was better when
 492 the deviant approached or crossed the 90° boundary (see **Figure 6B**). Performance was
 493 significantly asymmetric in the slanted screen condition ($Z = 1.85$, $p = 0.032$, 9/12 participants
 494 showed the effect). An SRH test comparing the thresholds towards and away from 90° in the
 495 slanted screen and normal viewing conditions revealed a significant main effect of deviant
 496 direction ($H = 5.00$, $p = .025$), condition ($H = 7.29$, $p = .007$), but no interaction between these
 497 two factors ($H = 0.02$, $p = .896$). In the normal viewing condition too, there was a significant
 498 asymmetry in detection thresholds ($Z = 1.85$, $p = .032$, 8/12 participants showed the effect).
 499 Thus, the asymmetry effects were comparable across the slanted screen and normal viewing
 500 conditions and were comparable to Experiment 1.

501 Finally, an SRH test comparing the slanted screen and projected stimuli conditions on the
 502 asymmetry of their deviant detection thresholds revealed a main effect of deviant direction
 503 ($H = 6.88$, $p = .009$), condition ($H = 11.38$, $p < .001$), and no interaction between the two factors
 504 ($H = 0.09$, $p = .770$). Indeed, asymmetries followed the same pattern in the projected stimuli
 505 condition as in the other conditions, with slightly better performance for angles deviating
 506 towards 90° versus away from 90° , though this difference was not significant ($Z = 1.20$, $p = .116$,
 507 8/12 participants showed the effect; see **SM, Table S2**). These weak asymmetry effects in the

508 projected stimuli condition may have been due to a residual property of the angle transformation,
509 rather than to participants' categorical discrimination of projected 90° angles. In particular, when
510 the stimuli were transformed, the range of angle sizes presented on the screen differed based on
511 the reference angle, with the greatest range occurring at 90°. Moreover, projected deviant angles
512 fell outside the range of projected reference angles around 90° more often for deviants towards
513 90° than for deviants away from 90°, making deviants towards 90° easier to detect.

514

515 **Discussion**

516 This experiment builds on the findings of Experiment 1 by suggesting that both angle
517 discrimination in general, and the perpendicular category in particular, are largely invariant to
518 the rotation of shape stimuli in depth. A 45° rotation of the screen in the slanted screen condition
519 degraded discrimination performance uniformly, yet left the relative discriminability of angles of
520 different sizes intact, including preserving peak discriminability at 90°. Because the 90° angles in
521 that condition were defined only in three dimensions with widely varying 2D projections, these
522 results suggest that participants detected the angle intruder by monitoring the 3D angle sizes
523 rather than their retinotopic, 2D projections. This conclusion is further supported by comparisons
524 between the slanted screen condition and the projected stimuli condition. Here, the 2D angles
525 that were projected on the retina were identical between conditions, yet accuracy varied greatly,
526 suggesting that the preserved shape discrimination in the slanted screen condition was not due to
527 properties of the 2D projection of the stimuli. Indeed, participants performed so poorly in the
528 projected stimuli condition that the data were difficult to model, and it is thus hard to make
529 specific conclusions about the characteristics of our shape discrimination with stimuli

530 transformed in this way. By contrast, rotating shapes in depth, with all of the real-world depth
 531 cues intact, led to a largely preserved ability to discriminate angles.

532

533 **General Discussion**

534 Across two experiments presenting angle-intruder detection tasks, we demonstrated that
 535 the discriminability of both connected and non-connected lines forming different angles varies
 536 greatly depending on the size of the angles being discriminated. Most notably, thresholds of
 537 discrimination were significantly smaller when lines formed parallels and perpendiculars. When
 538 angles were acute or obtuse and far from these two special angle categories, thresholds of
 539 discrimination were high, though when angles deviated from nearer acute or obtuse angles to
 540 approach the parallel or perpendicular categories, discrimination became more precise. This
 541 pattern of results persisted not only over variations in the orientations and scale of the angle
 542 exemplars, but also over their rotations in depth. In particular, when the very same angle stimuli
 543 were presented on a screen rotated 45° in depth, the relative discriminability of angles of
 544 different sizes persisted. Most remarkably, the significantly smaller threshold for detecting
 545 perpendicular angles persisted, despite the variability in the 2D projections of these angles, and
 546 this result was not due to properties of the stimuli's 2D projection. Our findings thus suggest that
 547 angle discrimination in general and the special angle categories of parallelism and
 548 perpendicularity in particular affect participants' shape judgments by their real-world angle
 549 information.

550 The prior literature had outlined different models of angle discrimination to which we can
 551 compare our results. In two cases, more precise discrimination for horizontal and vertical lines
 552 was invoked to explain better acuity for 90° and 0/180° angles: Chen & Levi (1996) suggested

553 an orientation-independent, Weber-like discrimination space except at 90°, where discrimination
 554 is heightened and rooted in the presence of vertical and horizontal lines; Xu et al.
 555 (2018) proposed that angles which include either a vertical or horizontal line are discriminated
 556 most precisely. In contrast, Heely & Buchanan-Smith (1996) proposed that, unlike
 557 the discrimination of individual line orientations, angle discrimination operates over a reference
 558 frame that is object-centered, like the discrimination of other, more complex objects. Our
 559 findings thus support and extend this last model: Not only did our stimuli specifically avoid
 560 using lines at or near the horizontal or vertical, but also more precise discrimination
 561 persisted when the angles were rotated in 3D depth, suggesting that angle discrimination unfolds
 562 at the level of the 3D angle size, as it might for the shape of complex 3D objects.

563 What then is the status of the geometric categories of *parallelism* and *perpendicularity*?
 564 Some research has suggested that sensitivity to parallelism, at least, arises early in child
 565 development. Even four-year-old children indicate that a pair of parallel lines is the “most
 566 different” from five other pairs of lines that present continuous differences in angle at varied
 567 absolute orientations and scales (Izard et al., 2011a; 2011b). Children’s knowledge of the word
 568 “parallel,” moreover, has no relation to this choice (Izard et al., 2011a; 2011b). As such,
 569 children’s judgments of parallelism may be rooted in the recognition of more basic shape
 570 properties, for example, that two lines have the same orientation or that they maintain a constant
 571 distance from each other — and perhaps these very properties contribute to the categorical
 572 effects for parallels documented here. Future research should explore whether children display
 573 the same categorical effects around the discrimination of parallels as adults and whether their
 574 specialized treatment of parallel lines changes with more visual experience or with explicit
 575 learning of formal geometry in school.

576 What about perpendicularity, whose angle measure changes greatly with rotations in 3D
 577 depth? Unlike their performance with parallel lines, not until age seven or older do children pick
 578 out a pair of perpendicular lines from other pairs of lines that vary continuously in angle, and
 579 individual children’s knowledge of the lexical terms “right angle” and “perpendicular” correlates
 580 with this choice (Izard et al., 2011a). The present task with adults did not rule out the possibility
 581 that explicitly learned conceptual representations were supporting participants’ performance. All
 582 the adults in this study had benefitted from a formal education at least through high-school,
 583 which likely included a geometry class where the concepts of parallelism and perpendicularity
 584 were taught. In particular, since participants were introduced to each block of trials with an
 585 image that depicted the reference angle, it was possible that, for the parallel and perpendicular
 586 categories, participants labeled the reference and used that label to access a stored representation
 587 of an exemplar from that category from memory, making its recognition more accurate
 588 (Firestone & Scholl, 2016). Like other studies examining advantages in discrimination for
 589 complex spatial stimuli, defined, e.g., by topological relations, it is unknown the extent to which
 590 the present findings reflect verbal coding or pure visual processing (Lovett & Franconeri, 2017).
 591 Indeed, a suite of studies using search tasks and deviant detection tasks show enhanced detection
 592 of visual stimuli including colors, simple shapes, and more complex objects when category
 593 boundaries line up with linguistic labels (see Goldstone & Hendrickson, 2010; Lupyan, 2012).
 594 Future studies using the same angle stimuli with adults from other cultures, children, and non-
 595 human animals might shed further light on whether the effects observed in this study were
 596 perceptual or cognitive.

597 Nevertheless, the detection of perpendicular angles may indeed be universal. A group of
 598 adults from the Mundurucu tribe in the Amazon, who, unlike the adults in the present sample,

599 have no specialized geometric training or vocabulary, pick out a pair of perpendicular lines as the
 600 “most different” from other pairs of lines that vary continuously in angle (Izard et al., 2011a).
 601 While categorization of perpendiculars (as in this study with the Mundurucu adults, Izard et al.,
 602 2011a) and categorical effects in discrimination (as measured with the adults in this paper) may
 603 dissociate, it is possible that a perceptually based perpendicular category arises spontaneously in
 604 development and that education in formal geometry refines it or heightens attention to it (see
 605 Piazza, Pica, Izard, Spelke, & Dehaene, 2013 for further exploration of this suggestion in the
 606 numerical domain). If this is the case, it remains a challenge to identify a perceptual learning
 607 mechanism that might operate over such a long period of development and create such a robust
 608 category.

609 Might visual experience through development bolster our recognition of 3D shape
 610 information from which a heightened sensitivity to perpendicular angles could emerge? Previous
 611 work has shown that high-level perceptual learning can occur over extended periods of time, for
 612 example, to create or refine specific object categories like letters and faces (Dehaene, Cohen,
 613 Sigman, & Vinckier, 2005; Maurer, Le Grand, & Mondloch, 2002). This kind of perceptual
 614 learning is nevertheless limited, in that categorical effects disappear when, for example, face
 615 stimuli are inverted (Maurer et al., 2002). Such limits may be adaptive to the context: While face
 616 recognition is not invariant to large 2D orientation changes, it is invariant over large changes in
 617 viewpoint (i.e., in full view or in profile), expression, and lighting (Anselmi & Poggio, 2014).

618 The perceptual learning associated with 3D object recognition may accumulate 2D and
 619 3D rotational invariance through development since objects are often seen rotated in 2D or 3D
 620 space. While newborns differentiate among 2D angle shapes given long periods of habituation
 621 (Slater, Mattock, Brown, & Bremner, 1991), seven-month-old infants fail to differentiate shapes

622 differing in angle during brief exposures to 2D forms with simultaneous variations in scale,
 623 direction, and orientation (Dillon, Izard, & Spelke, in preparation). Research with toddlers has
 624 suggested that 3D object recognition undergoes protracted development, with a spurt in the
 625 ability to recognize objects by their 3D geometric shapes between the ages of 18-24 months
 626 (Augustine, Smith, & Jones, 2011; Smith, 2009), and studies with older children have shown that
 627 the period during which we become better able to recognize unfamiliar viewpoints of 3D objects
 628 captured in 2D drawings extends to adulthood (Jüttner, Müller, & Rentschler, 2006; Jüttner,
 629 Wakui, Petters, Kaur, & Davidoff, 2013; Landau, Hoffman, & Kurz, 2006). Further
 630 developmental work, investigating invariances in angle detection may begin to shed light on how
 631 such shape representations become more robust to every-day 3D viewing conditions.

632 Even if angle detection in general becomes more robust to rotations in 3D depth through
 633 development, such development alone does not explain the emergence of a perpendicular
 634 category. A rotationally invariant representation of perpendicularity could be singled out because
 635 a perpendicular line is the most symmetrical position for a line relative to another line (i.e.,
 636 splitting a line in half) or to a plane, and both infants and young children show some sensitivity
 637 to symmetry (Bornstein, Ferdinandsen, & Gross, 1981; Huang, Xue, Spelke, Huang, Zheng,
 638 Peng, 2018). A rotationally invariant representation of perpendicularity could also derive from
 639 representations of the absolute vertical and horizontal. These orientations are robustly
 640 represented early in development and across animal species (Appelle, 1972). For example, 5-
 641 month-old infants take longer to look towards a deviant oblique line among other oblique lines
 642 versus among vertically oriented lines the same angle measure away (Franklin, Catherwood,
 643 Alvarez, & Axelsson, 2010).

644 Finally, the present study raises questions about the relations between our perception of
645 angles and our conception of formal Euclidean geometry. Euclid famously did not explain what
646 he means by “equal” angles when, at the beginning of his *Elements* (Book 1, Definition 10), he
647 defines a *perpendicular* as a line with equal angles to either side as it stands up from a baseline.
648 This definition is strikingly perceptual: We see what he means. Do we need to see the
649 abstractions of geometry to conceive of them? Are they driven by mental imagery of specific
650 spatial exemplars, abstract idealized concepts, or linguistic representations? Indeed, what are the
651 mental representations guiding our geometric reasoning? Individuals differ in their ability to
652 discriminate angles: Might individual differences in our angle discrimination predict our ability
653 to reason about the properties of points, lines, and figures on the Euclidean plane? Could training
654 in angle discrimination cause short- or long-term benefits to geometric reasoning or judgment?
655 We are only just beginning to probe how the human mind navigates the perceptual and
656 conceptual worlds of geometry.

657

658 **Author Contributions**

659 All authors conceived of and designed the study. M. Duyck programmed the stimuli. M.
660 R. Dillon, M. Duyck, and V. Izard collected the data. M. R. Dillon, M. Duyck, and V. Izard
661 performed the data analysis. M. R. Dillon, M. Duyck, and V. Izard drafted the manuscript, and
662 all authors provided critical revisions. All authors approved the final version of the manuscript.

663

664

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671 **Authors' Note**

672 The full data set and analysis code are available on the Open Science Framework at:

673 <https://osf.io/wa64q/>. Code for the generation of the experimental stimuli are available upon

674 request.

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