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M. SGHIAR. Prime numbers, Goldbach's conjecture, De Polignac's conjecture, Legendre's conjecture, Landau's conjecture, Mersenne's conjecture, and the Fermat number conjecture


Prime numbers, Goldbach's conjecture, De Polignac's conjecture, Legendre's conjecture, Landau's conjecture, Mersenne's conjecture, and the Fermat number conjecture

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Abstract : Inspired by the article [3], the introduction of the function $\hat{\mathbb{S}}$ whose integer zeros are the prime numbers, will allow me to demonstrate analytically the Goldbach's conjecture [6], the De Polignac's Conjecture [7], the Legendre's conjecture [9], Landau's conjecture [10], Mersenne's conjecture [11] , and the Fermat number conjecture [12] .

Résumé : Inspiré de l'article [3], l'introduction de la fonction $\hat{\mathbb{S}}$ dont les zéros entiers sont les nombres premiers, va me permettre de démontrer analytiquement la conjecture de Goldbach [6], la conjecture de De Polignac [7], la conjecture de Legendre [9] et la conjecture de Landau [10], la conjecture de Mersenne [11] et la conjecture des nombres de Fermat (12).

Keywords : Prime Number, the Goldbach conjectures, the Gamma function, the De Polignac's Conjecture, the Legendre's conjecture, the Landau's conjecture, Mersenne's conjecture, Fermat numbe.

Code MSC : 11P32, 11N05, 11N36, 11A41

M. SGHIAR. Prime numbers, Goldbach's conjecture, De Polignac's conjecture, Legendre's conjecture, Landau's conjecture, Mersenne's conjecture, and the Fermat number conjecture

In this version I apply the same techniques to prove the Mersenne's conjecture [11] , and the Fermat number conjecture [12] .

I- INTRODUCTION

In 1742, in a letter to Leonhard Euler, Christian Goldbach conjectured [6] : Any even integer greater than 3 can be written as the sum of two prime numbers.

De Polignac's conjecture, stated by Alphonse De Polignac in 1849 [7], is as follows : Any non-zero even number $2m$ is equal to the difference of two consecutive prime numbers in an infinite number of ways.

Inspired by the function \mathbb{S} which I introduced in the article [3], I introduce the function $\hat{\mathbb{S}}$ which, also like \mathbb{S} , allows to find the prime numbers .

The study of the function $S_{2m}(z) = \hat{\mathbb{S}}^2(z) + \hat{\mathbb{S}}^2(2m - z)$ will allow me to demonstrate the Goldbach conjecture. And the study of the function : $S_{2m}(z) = \hat{\mathbb{S}}^2(z) + \hat{\mathbb{S}}^2(2m + z)$ will allow me to demonstrate the De Polignac's conjecture.

By y the same techniques I will prove the Legendre's conjecture [9], and Landau's conjecture [10] .

Recall that the four conjectures studied in this article, Edmund Landau, has characterized them, at the 1912 International Congress of Mathematicians, as "unattackable at the present state of mathematics" and are now known as Landau's problems [10].

Finally I give answers on the Mersenne's conjecture [11] , and the Fermat number conjecture [12]

In this article I assume known the functions zeta ζ , Gamma $\Gamma : z \mapsto \int_0^{+\infty} t^{z-1} e^{-t} dt$ and their properties (see [4] and [5]).

II- THE PROOF OF THE GOLDBACH CONJECTURE :

Theorem 1 : [The Goldbach conjecture]

Any even integer greater than 3 can be written as the sum of two prime numbers.

Proposition 1. [3] :

Let $\mathbb{S}(z) = \zeta(-\frac{\Gamma(z)+1}{z/2})$.

If $z \in \mathbb{N}^*$ then $\mathbb{S}(z) = 0 \iff z$ is a prime number

Proof :

It follows from Wilson's theorem [1] - which shows that p is prime if and only $(p-1)! \equiv -1 \pmod{p}$ - and the fact that the trivial zeros of ζ are $-2\mathbb{N}^*$.

Proposition 2 :

Let $\hat{\mathbb{S}}(z) = \sin(\frac{\Gamma(z)+1}{z}\pi)$.

If $z \in \mathbb{N}^*$ then $\hat{\mathbb{S}} = 0 \iff z$ is a prime number

Proof : Inspired by **proposition 1**

Proposition 3 : If $m \in \mathbb{N}^* \setminus \{1\}$, then the equation :

$$\sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right) = 0$$

admits at least one solution z in \mathbb{N} with z and $2m - z$ are primes.

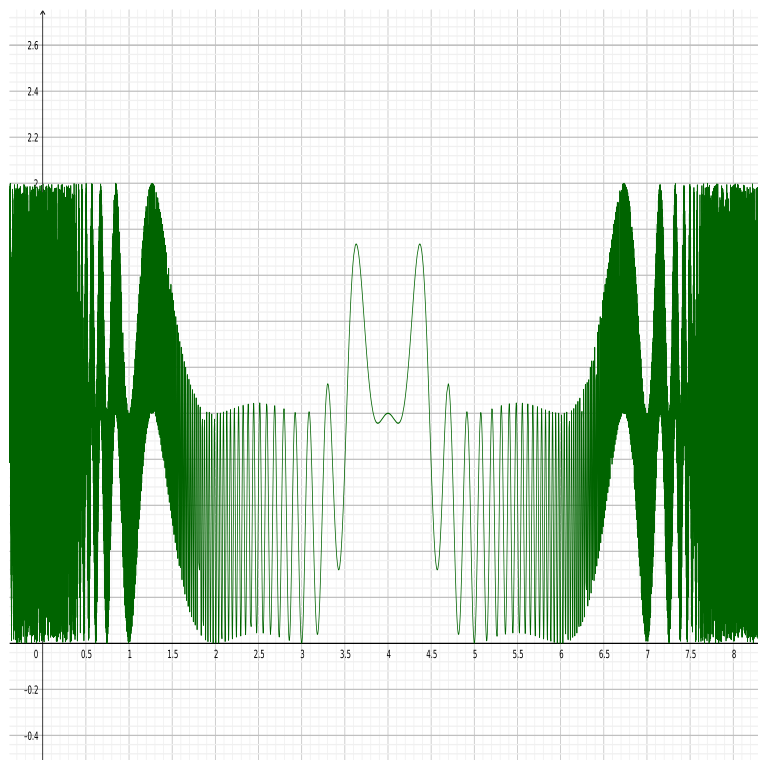


FIGURE 1 – $S(z)$ for $m=8$

Proof of Proposition 3 :

Consider the restriction to \mathbb{R}^{+*} of the function :

$$S(z) = \sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right)$$

1 step : If the function $S(z) = \sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right)$ has a local extremum at the point $p \neq 2$ with p is prime :

In this case $\frac{dS}{dz}(p) = 0$ and according to proposition 2 we have $\hat{S}(p) = \sin\left(\frac{\Gamma(p)+1}{p}\pi\right) = 0$

We have :

$$\frac{dS}{dz}(z) = 2\pi \sin\left(\frac{\Gamma(z)+1}{z}\pi\right) \cos\left(\frac{\Gamma(z)+1}{z}\pi\right) \frac{z\Gamma'(z) - (\Gamma(z)+1)}{z^2} + 2\pi \sin\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right) \cos\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right) \frac{-(2m-z)\Gamma'(2m-z) + (\Gamma(2m-z)+1)}{(2m-z)^2}$$

But as $\frac{-(2m-p)\Gamma'(2m-p)+(\Gamma(2m-p)+1)}{(2m-p)^2} \neq 0$ (Because $-z\Gamma'(z) + \Gamma(z) + 1$) vanishes in $z \sim 2.5 \notin \mathbb{N}$), then $2\sin\left(\frac{\Gamma(2m-p)+1}{2m-p}\pi\right) \cos\left(\frac{\Gamma(2m-p)+1}{2m-p}\pi\right) = 0$

And since we cannot have $\cos\left(\frac{\Gamma(2m-p)+1}{2m-p}\pi\right) = 0$, because otherwise $\frac{\Gamma(2m-p)+1}{2m-p} \in \mathbb{N} + \frac{1}{2}$, and therefore $p \in 2\mathbb{N}$, which is not the case, then $\sin\left(\frac{\Gamma(2m-p)+1}{2m-p}\pi\right) = 0$, and from **proposition 2**, we conclude that $2m - p$ is prime.

2 step : Let us show the existence of local minimums z in $]1, m[$ as soon as $m \geq 5$, with $S(z) = 0$, $\frac{\Gamma(z)+1}{z} \in \mathbb{N}$ and $\frac{\Gamma(2m-z)+1}{2m-z} \in \mathbb{N}$:

First S admits a local extremum at a point $z \in]1, m[$ if $m \geq 5$: Indeed :

$$S(z) = 1 - \frac{1}{2}(\cos(2\frac{\Gamma(z)+1}{z}\pi) + \cos(2\frac{\Gamma(2m-z)+1}{2m-z}\pi))$$

If S does not admit a local extremum at a point $z \in]1, m[$, then $T(z) = (\cos(2\frac{\Gamma(z)+1}{z}\pi) + \cos(2\frac{\Gamma(2m-z)+1}{2m-z}\pi))$ does not admit a local extremum in $z \in]1, m[$, and in this case, by symmetry of the curve of S with respect to the line $x = m$, we have $\frac{dT}{dz}(z) \leq 0, \forall z \in]1, m[$ or $\frac{dT}{dz}(z) \leq 0, \forall z \in]m, 2m-1[$. By symmetry suppose that $\frac{dT}{dz}(z) \leq 0, \forall z \in]1, m[$:

$$\frac{dT}{dz}(z) = -2\pi \sin(2\frac{\Gamma(z)+1}{z}\pi) \frac{z\Gamma'(z) - (\Gamma(z)+1)}{z^2} - 2\pi \sin(2\frac{\Gamma(2m-z)+1}{2m-z}\pi) \frac{-(2m-z)\Gamma'(2m-z) + (\Gamma(2m-z)+1)}{(2m-z)^2}$$

By growth of the function $\frac{z\Gamma'(z)-(\Gamma(z)+1)}{z^2}$ in $\in [3, +\infty[$ we have : $\frac{(2m-z)\Gamma'(2m-z)-(\Gamma(2m-z)+1)}{(2m-z)^2} > \frac{z\Gamma'(z)-(\Gamma(z)+1)}{z^2}$ $> 1, \forall z \in [3, m[$, so :

$$\sin(2\frac{\Gamma(z)+1}{z}\pi) \geq \sin(2\frac{\Gamma(2m-z)+1}{2m-z}\pi) \frac{(2m-z)\Gamma'(2m-z)-(\Gamma(2m-z)+1)}{(2m-z)^2} \frac{z^2}{z\Gamma'(z)-(\Gamma(z)+1)}; \forall z \in [3, m[$$

And for z such that $\sin(2\frac{\Gamma(2m-z)+1}{2m-z}\pi) = 1$ (z exists because the function $L(x) = \frac{\Gamma(x+1)+1}{x+1} - \frac{\Gamma(x)+1}{x}$ check $L(x) \geq 1, \forall x \geq 3.2$), we will have $\sin(2\frac{\Gamma(z)+1}{z}\pi) > 1$, which is absurd. And therefore $S(z) = \sin^2(\frac{\Gamma(z)+1}{z}\pi) + \sin^2(\frac{\Gamma(2m-z)+1}{2m-z}\pi)$

admits local extremum z_1 in $\in]3, m[$

If z_1 is not a local minimum, by taking the same reasoning by replacing 1 by z_1 , we deduce the existence of a local extremum z_2 in $[3, z_1[$ or in $]z_1, m[$ and hence there is a local minimum z between z_1 et z_2 .

Let z be a minimum of S in $\in]1, m[$.

If $S(z) \neq 0$, then $\frac{\Gamma(z)+1}{z}\pi \neq 0, \text{ mod}(\pi)$ ou $\frac{\Gamma(2m-z)+1}{2m-z}\pi \neq 0, \text{ mod}(\pi)$

By symmetry of the curve of S with respect to the line $x = m$, we can assume that $\frac{\Gamma(2m-z)+1}{2m-z}\pi \neq 0, \text{ mod}(\pi)$ and that $\sin^2(\frac{\Gamma(z)+1}{z}\pi) \leq \sin^2(\frac{\Gamma(2m-z)+1}{2m-z}\pi)$.

If $\sin^2(\frac{\Gamma(z)+1}{z}\pi) < \sin^2(\frac{\Gamma(2m-z)+1}{2m-z}\pi)$: If z' is too close to z with $\sin^2(\frac{\Gamma(2m-z')+1}{2m-z'}\pi) < \sin^2(\frac{\Gamma(2m-z)+1}{2m-z}\pi)$, then $S(z') < S(z)$ which contradicts the minimality. It follows that $\sin^2(\frac{\Gamma(z)+1}{z}\pi) = \sin^2(\frac{\Gamma(2m-z)+1}{2m-z}\pi)$

But from the equation :

$$0 = \frac{dS}{dz}(z) = 2\pi \sin\left(\frac{\Gamma(z)+1}{z}\pi\right) \cos\left(\frac{\Gamma(z)+1}{z}\pi\right) \frac{z\Gamma'(z) - (\Gamma(z)+1)}{z^2} + 2\pi \sin\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right) \cos\left(\frac{\Gamma(2m-z)+1}{2m-z}\pi\right) \frac{-(2m-z)\Gamma'(2m-z) + (\Gamma(2m-z)+1)}{(2m-z)^2}$$

We deduce that : $0 = \sin\left(\frac{\Gamma(z)+1}{z}\pi\right) \cos\left(\frac{\Gamma(z)+1}{z}\pi\right)$, and therefore $0 = \sin\left(\frac{\Gamma(z)+1}{z}\pi\right)$ by minimality of $S(z)$. Thus $S(z) = 0$ with $\frac{\Gamma(z)+1}{z} \in \mathbb{N}$ and $\frac{\Gamma(2m-z)+1}{2m-z} \in \mathbb{N}$

3 step : Let us show the existence of the roots $z \in \mathbb{N} \cap]1, m[$:

The case where $m \in \{2, 3, 4\}$ is easy to see, since the Goldbach conjecture holds in these cases.

If $m \geq 5$, from step 2, we have $S(z) = 0$ with : $z \in]1, m[$, $\frac{\Gamma(z)+1}{z} \in \mathbb{N}$ and $\frac{\Gamma(2m-z)+1}{2m-z} \in \mathbb{N}$

If $z \notin \mathbb{N}$:

- If $z = n + r$ with $r \geq 2$, $r \in \mathbb{R}^+ \setminus \mathbb{N}$ and $n \geq 1$

Let $\frac{\Gamma(z)+1}{z} = k$.

we deduce : $\prod_{i=1}^n (n-i+r)\Gamma(r) + 1 = kn + kr$

So $\Gamma(r) = \frac{-1+kn+kr}{\prod_{i=1}^n (n-i+r)}$

As $r \geq 2$, then $\Gamma'(r) > 0$, and therefore by deriving we will have :

$$k \prod_{i=1}^n (n-i+r) > (-1+kn+kr) \sum_{j=1}^n \prod_{i \neq j; i=1}^n (n-i+r)$$

So $\frac{1}{r} \geq \frac{1}{-\frac{1}{k}+n+r} > \sum_{i=1}^n \frac{1}{n-i+r}$, which is not the case, and $z \in \mathbb{N}$.

- If $z = n + r$ with $r < 2$, $r \in \mathbb{R}^+ \setminus \mathbb{N}$ and $n \geq 1$

By setting $z = (n + 3) + (r - 3)$ we can suppose that $z = n + r$ with $r < -1$, $r \in \mathbb{R}^- \setminus \mathbb{N}$ and $n \geq 1$

Let $\frac{\Gamma(2m-z)+1}{2m-z} = l$, we deduce :

$$\prod_{i=1}^{2m-n-1} (2m - n - i - r)\Gamma(1 - r) + 1 = l(2m - n - 1) + l(1 - r)$$

So $\Gamma(1 - r) = \frac{-1+l(2m-n-1)+l(1-r)}{\prod_{i=1}^{2m-n-1}(2m-n-i-r)}$

But $-\Gamma'(1 - r) < 0$ if $r < -1$.

So :

$$-l \prod_{i=1}^{2m-n-1} (2m - n - i - r) < -(-1+l(2m-n-1)+l(1-r)) \sum_{j=1}^{2m-n-1} \prod_{i \neq j; i=1}^{2m-n-1} (2m - n - i - r)$$

Thus : $\frac{1}{1-r} \geq \frac{1}{-\frac{1}{l}+2m-n-1+(1-r)} > \sum_{i=1}^{2m-n-1} \frac{1}{2m-n-i-r}$, which is not the case, and $z \in \mathbb{N}$.

Note : *This result of step 3 can be noticed in the graphs of S_{2m} : If the curve of S_{2m} touches the segment $]1, 2m[$ at a point p , then p is a prime number !!*

4 step : Proof of proposition 3

The case where $m \in \{2, 3, 4\}$ is easy to see, since the Goldbach conjecture holds in these cases.

If $m \geq 5$, in step 3 we have shown the existence of the roots $z \in \mathbb{N} \cap]1, m[$ such that $\frac{\Gamma(z)+1}{z} \in \mathbb{N}$ and $\frac{\Gamma(2m-z)+1}{2m-z} \in \mathbb{N}$. And from proposition 2 we deduce that z and $2m - z$ are primes with $2m = z + (2m - z)$

Proof of Theorem 1 :

It is deduced directly from **proposition 3**

III - The proof of Alphonse de Polignac's conjecture [7]

Lemma 1 :

Let f and g be two continuous functions from $[N, +\infty[$ over \mathbb{R}^+ with $\lim_{z \rightarrow +\infty} f(z) = +\infty$ and $\lim_{z \rightarrow +\infty} g(z) = +\infty$

If $M(z) = \sin^2(f(z)\pi) + \sin^2(g(z)\pi)$, then there exists $z \in [N, +\infty[$ such that $M(z)$ is a minimum of M .

Proof : Let :

$$\overline{M(f(z)\pi, g(z)\pi)} = \sin^2(\overline{f(z)\pi}) + \sin^2(\overline{g(z)\pi})$$

Where $\overline{Z} = Z \bmod(2\pi)$

From hypotheses on f and g we deduce that $\{(\overline{f(z)\pi}, \overline{g(z)\pi}), z \in [N, +\infty[\} = [0, 2\pi] \times F$ where F is a part of $[0, 2\pi]$. If \tilde{F} is the closure of F , then $[0, 2\pi] \times \tilde{F}$

is a compact of $[0, 2\pi] \times [0, 2\pi]$ therefore (see [2]) \overline{M} has minima in points $(\overline{f(z)\pi}, b)$ of $[0, 2\pi] \times \tilde{F}$.

But if z_n is a sequence such that $z_n \rightarrow z$, then $\overline{g(z_n)\pi} \rightarrow b$. But as $\overline{g(z_n)\pi} \rightarrow \overline{g(z)\pi}$, then $b = \overline{g(z)\pi}$, and \overline{M} has minima in points $(\overline{f(z)\pi}, \overline{g(z)\pi})$. And therefore M has minima at points z .

The Polignac conjecture is a conjecture about the theory of numbers. It was stated by Alphonse de Polignac in 1849 [7].

Theorem 2 : The conjecture of Alphonse de Polignac [7]

Any non zero even number $2m$ is equal to the difference of two consecutive prime numbers in an infinite number of ways.

Proof :

It is demonstrated by the same technique used above to prove the Goldbach conjecture using the function S^+ instead of S :

$$S^+(z) = \sin^2\left(\frac{\Gamma(z) + 1}{z}\pi\right) + \sin^2\left(\frac{\Gamma(2m + z) + 1}{2m + z}\pi\right)$$

- Using Lemma 1 above, we show the existence of an infinite number of minima z of S^+ .

And by showing as in steps 2, 3, above :

- The existence of an infinite number of local minimums z of S^+ in $]1, \infty[$, with $S(z) = 0$, $\frac{\Gamma(z)+1}{z} \in \mathbb{N}$ and $\frac{\Gamma(2m+z)+1}{2m+z} \in \mathbb{N}$

- The existence of an infinity of integer z roots of S^+ ($z \in \mathbb{N}$).

Then using proposition 2 - as in step 4 above - we deduce that z and $2m + z$ are prime with $2m = (2m + z) - z$

Corollary : [The conjecture of twin prime numbers] [8]

There exists an infinity of prime numbers p such that $p + 2$ is also prime.

IV - The proof of Legendre's conjecture [9]

The Legendre conjecture, proposed by Adrien-Marie Legendre, is stated as follows :

Theorem 3 : [Legendre's conjecture] : For all $n \geq 1$, there is a prime number between n^2 et $(n + 1)^2$

Proof :

We consider the function defined on $]1, 2n]$:

$$S_{le,n}(z) = \sin^2\left(\frac{\Gamma(z) + 1}{z}\pi\right) + \sin^2\left(\frac{\Gamma(n^2 + z) + 1}{n^2 + z}\pi\right)$$

The result is true if $n < 2$.

- By the same techniques used above, we see that there are integer z roots of $S_{le,n}(z)$ ($z \in \mathbb{N} \cap]1, 2n]$ if $n \geq 2$). Then by using proposition 2 -as in step 4 above- we deduce that z and $n^2 + z$ are primes with $n^2 + z \in [n^2, (n+1)^2]$

V - The proof of the Landau's conjecture [10]

Theorem 4 [Landau's conjecture] : There exists an infinity of primes p such that $p-1$ is a perfect square. (or in other words : there is an infinity of prime numbers of the form $n^2 + 1$).

Proof :

We consider the function defined on $[1, +\infty[$:

$$S_{la}(z) = \sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2(\pi\sqrt{z-1})$$

- Using Lemma 1 above, we show the existence of an infinity of minima z of S_{la}

- By the same techniques used above, we see that there exists an infinity of integer z roots of $S_{la}(z)$ ($z \in \mathbb{N} \cap [3, +\infty[$) with $z = n^2 + 1$. Then by using proposition 2 -as in step 4 above- we deduce that z is a prime with $z = n^2 + 1$

Note :

One may wonder why the proof of the Landau's conjecture does not work by replacing $p-1$ by $p+1$ the reason is as follows : First we must place ourselves in the interval $]3.2, +\infty[$ to be able to use step 2(because in $]3.2, +\infty[$ we have $L(x) = \frac{\Gamma(x+1)+1}{x+1} - \frac{\Gamma(x)+1}{x} \geq 1, \forall x \geq 3.2$).

Then using the function as in Lemma 1 (passing to classes) :

$$\overline{M(f(z), g(z))} = \sin^2(\overline{f(z)\pi}) + \sin^2(\overline{g(z)\pi})$$

$S_{la}(z) = \sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2(\pi\sqrt{z-1})$ admits root **5** in $[4, 6]$, while $S_{la}(z) = \sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2(\pi\sqrt{z+1})$ admits no root in $[4, 6]$.

VI - The proof of Mersenne's conjecture [11]

A Mersenne number is a number of the form $2^n - 1$ (where n is a non-zero natural number), a prime Mersenne number, is therefore a prime number of this form.

Theorem 5 : There is an infinite number of Mersenne prime numbers.

Proof : It suffices to apply the previous study to the function defined on $]0, +\infty[$:

$$S_{me}(z) = \sin^2\left(\frac{\Gamma(z)+1}{z}\pi\right) + \sin^2\left(\frac{\Gamma(2^z-1)+1}{2^z-1}\pi\right)$$

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VII - The proof of the Fermat number conjecture [12]

A Fermat number is a number that can be written as $2^{2^n} + 1$, with n natural integer. The n -th Fermat number, $2^{2^n} + 1$, is denoted by F^n .

Theorem 6 : If $n \geq 5$, then $F^n = 2^{2^n} + 1$ is not a prime number.

Proof : We consider the function :

$$S_{fe}(z) = \sin^2(z\pi) + \sin^2\left(\frac{\Gamma(2^{2^z} + 1) + 1}{2^{2^z} + 1}\pi\right)$$

If $z \geq 5$ is a root integer of S_{fe} , then as in the note above, $S_{fe}(z) = \sin^2(z\pi) + \sin^2\left(\frac{\Gamma(2^{2^z} + 1) + 1}{2^{2^z} + 1}\pi\right)$ will have the root **5** in the interval $[4, 6]$, which is not the case. Hence the result.

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