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# Contact forces pre-integration for the whole body estimation of legged robots

Mederic Fourmy<sup>†</sup>, Thomas Flayols<sup>†</sup>, Nicolas Mansard<sup>†</sup> and Joan Solà<sup>†\*</sup>

**Abstract**—State estimation, in particular estimation of the base position, orientation and velocity, plays a big role in the efficiency of legged robot stabilization. The estimation of the base state is particularly important because of its strong correlation with the underactuated dynamics, i.e. the evolution of center of mass and angular momentum. Yet this estimation is typically done in two phases, first estimating the base state, then reconstructing the center of mass from the robot model. The underactuated dynamics is indeed not properly observed, and any bias in the model would not be corrected from the sensors. While it has already been observed that force measurements make such a bias observable, these are often only used for a binary estimation of the contact state. In this paper, we propose to simultaneously estimate the base and the underactuation state by using all measurements simultaneously. To this end, we propose several contributions to implement a complete state estimator using factor graphs. Contact forces altering the underactuated dynamics are pre-integrated using a novel adaptation of the IMU pre-integration method, which constitutes the principal contribution. IMU pre-integration is also used to measure the positional motion of the base. Encoder measurements are then participating to the estimation in two ways: by providing leg odometry displacements, contributing to the observability of IMU biases; and by relating the positional and centroidal states, thus connecting the whole graph and producing a tightly-coupled whole-body estimator. The validity of the approach is demonstrated on real data captured by the Solo12 quadruped robot.

## I. INTRODUCTION

Legged robot estimation has from the start been focused on the integration of odometry derived from the kinematic chain [1], later fused with Inertial Measurement Units (IMU) to capture higher movement dynamics [2]. This approach led to the use of Bayesian filters introduced for quadruped robots in [3] and adapted for humanoid robots in [4]. Focusing on proprioceptive odometry, they naturally produced drifting estimation of the robot position and yaw and have been complemented by loosely coupled corrections from exteroceptive sensors [5].

Another line of research is the estimation of centroidal quantities such as the center of mass, centroidal momentum and their derivative which is especially important for the balance and predictive control of humanoid robots. They all have in common a two-step approach where a first estimator outputs the base state and this is used by a second



Fig. 1: LAAS-Gepetto robot Solo12 from the Open Dynamic Robot Initiative

estimator to obtain the centroidal quantities. While [6] uses simplified models such as the LIPM, [7] applies the full dynamical model to develop Kalman Filters using kinematic measurements fused with sensor wrenches while permitting the estimation of an external force or kinematic model bias. [8], [9] propose a fusion based on a careful spectral analysis of the measurement sources from which a cascade of complementary filters is derived.

Coming from the drone community in the context of Visual Inertial Odometry [10], more recent works start to introduce nonlinear optimization to solve real time estimation problems using the factor graph paradigm [11]. The seminal work [12] showed that IMU measurements could be efficiently integrated in such a framework using the so called pre-integration, which was improved later with proper estimation on the  $SO(3)$  manifold [13]. Ref. [14] was the first work applying this framework to a biped robot (Cassie). It derived factors to tightly fuse IMU measurements with pre-integrated leg odometry and off-the-shelf visual odometry. Shortly after in [15], another team proposed similar methods on a quadruped robot (ANYmal) replacing leg odometry by velocity measurements coming from a lower level IMU kinematic filter. They then built on these results by introducing a new pre-integrated factor taking explicitly into account the bias from low level filter velocity, made observable by vision [16]. The leg odometry factors used in these works rely implicitly on gyroscope measurement integration for point feet robots.

However, none of these factor graph based works included the robot dynamics into the estimation yet. The most closely related work is that proposed by [17] in the context of aerial vehicles. They incorporated the linear dynamics of the rotors as a pre-integrated factor within a visual inertial framework which permits them to observe an external force being applied to the drone.

In this paper we describe a method to perform whole-

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body state estimation of legged robots. The estimated state contains positional states of the base (position, velocity and orientation), and centroidal states (CoM position and velocity, and angular momentum). The method fuses information from IMU, joint encoders and contact forces. A unique factor graph is employed to represent the estimation problem, yielding a tightly coupled fusion which is solved by nonlinear least squares optimization. One key contribution is the pre-integration of the force measurements into a unique measurement, in what constitutes an adaptation of the IMU pre-integration technique widely used in visual-inertial odometry and SLAM. We present results on a real dataset from Solo12 quadruped with 12 DoF, see Fig. 1.

## II. PROBLEM STATEMENT

### A. Centroidal dynamics

The robot dynamics is described by the well-known:

$$\mathbf{M}(q)\dot{v}_q + h(q, v_q) = \boldsymbol{\tau}_q + \sum_k \mathbf{J}_k^\top \mathbf{f}_k \quad (1)$$

where  $q, v_q, \dot{v}_q, \boldsymbol{\tau}_q$  are the position, velocity, acceleration and torques of the robot in configuration space,  $\mathbf{f}_k$  are the contact forces (written as 3D forces in this paper),  $\mathbf{M}$  is the generalized inertia matrix,  $h$  the sum of gravity, Coriolis and centrifugal forces, and  $\mathbf{J}_k$  the jacobians of the contact points. Because of the underactuated nature of legged robots, the configuration is often separated into  $q = (\mathbf{p}_b, \mathbf{R}_b, q_a)$  where  $\mathbf{p}$  is the position in world frame of the robot basis (typically, the torso),  $\mathbf{R}_b$  the rotation of the basis body with respect to the world and  $q_a$  are the joint configuration of the actuated joints.

While (1) represents the whole dynamics, a subpart of it is of particular importance for legged robots. The centroidal dynamics is written by the two equations:

$$m\ddot{\mathbf{c}} = m\mathbf{g} + \sum_k \mathbf{f}_k \quad , \quad \dot{\mathcal{L}} = \sum_k (\mathbf{p}_k - \mathbf{c}) \times \mathbf{f}_k \quad (2)$$

where  $\mathbf{c}, \mathcal{L}$  are the position of the center of mass (CoM) and angular momentum (AM) around the CoM (both expressed in world frame),  $m$  is the robot total mass, and the  $\mathbf{p}_k$  are the positions of the contact points in world frame. The centroidal dynamics is an exact part of (1), but more deeply reveals the underactuation: the robot can move only if applying the proper forces to the environment, as the joint torques alone cannot lead to any modification of the Center of Mass (CoM) or angular momentum (AM).

The classical approach in estimation of legged-robot state is to first estimate the basis state, and then reconstruct the centroidal state ( $\mathbf{c}, \dot{\mathbf{c}}, \mathcal{L}$ ) using the joint position and velocity measurements, and the robot model. This assumes that there is no direct measurement of the centroidal state. Consequently, we would not be able to recover the exact centroidal state if there is any bias in the robot model.

Yet, we can see from the centroidal dynamics that the force measurements are connected to the variation of the centroidal state. As observed in [18], a proper fusion of the force measurements with an estimation of the state

of the basis makes the centroidal state observable. As the basis state is also observable from inertial and kinematic measurements [3], a direct consequence is that the fusion of force, inertial and kinematic measurements makes the basis and centroidal states observable.

In the following, we propose a practical method to implement a basis and centroidal estimator based on IMU, joint coders and force sensors. Based on factor graph, this method can also be added into a SLAM estimator to also estimate the absolute positions of the basis and the CoM.

### B. Estimation problem MAP formulation

Our 12 degrees of freedom (DoF) quadruped robot Solo12 is equipped with an IMU, joint encoders monitoring the joint angles  $\mathbf{q}_a \in \mathbb{R}^{12}$  and angle velocities  $\dot{\mathbf{q}}_a \in \mathbb{R}^{12}$ . We will often drop the dependency of certain measurements on  $\mathbf{q}_a$  and  $\dot{\mathbf{q}}_a$  for readability once stated. Joint torques can also be estimated from motor current. These quantities are available at 1kHz. The goal is to reconstruct the full state of the robot, namely the base position  $\mathbf{p} \in \mathbb{R}^3$ , base velocity  $\mathbf{v} \in \mathbb{R}^3$  and base orientation  $\mathbf{R} \in SO(3)$ , as well as the robot's CoM position  $\mathbf{c} \in \mathbb{R}^3$ , the CoM velocity  $\dot{\mathbf{c}} \in \mathbb{R}^3$  and the centroidal AM  $\mathcal{L} \in \mathbb{R}^3$  at this same frequency. All quantities are expressed with respect to the world frame  $w$ :

$$\mathbf{x} = [\mathbf{p}, \mathbf{v}, \mathbf{R}, \mathbf{c}, \dot{\mathbf{c}}, \mathcal{L}] \triangleq [{}^w\mathbf{p}_b, {}^w\mathbf{v}_b, {}^w\mathbf{R}_b, {}^w\mathbf{p}_c, {}^w\mathbf{v}_c, {}^w\mathcal{L}]$$

Some measurements are affected by time varying biases  $\mathbf{b} = [\mathbf{b}_a, \mathbf{b}_\omega, \mathbf{b}_p]$  that can be estimated, where  $\mathbf{b}_a/\mathbf{b}_\omega$  refer respectively to the accelerometer/gyro biases and  $\mathbf{b}_p$  is the bias on the CoM centroidal measurement.

We pose the estimation as an optimization problem where variables  $\chi$  are composed of a trajectory of states  $\mathbf{x}_i$  at times  $t_i$ , also called Key Frames (KF), to which the biases  $\mathbf{b}_i$  are associated. This problem can be represented as a factor graph (Fig. 2) which is a bipartite graph where *nodes* correspond to the variables and *factors* to nonlinear constraints coming from the noisy sensor measurements. Then, under the assumption that  $\chi$  has a multivariate Gaussian distribution and that measurement noises are Gaussian and independent, the maximum a-posteriori (MAP) problem is equivalent to solving the weighted least squares problem,

$$\chi^* = \arg \min_{\chi} \sum_k \|\mathbf{r}_k(\chi)\|_{\boldsymbol{\Sigma}_k}^2 \quad (3)$$

Each factor  $k$  corresponds to a residual function  $\mathbf{r}(\cdot)$  of any number of variables from  $\chi$  and a covariance matrix  $\boldsymbol{\Sigma}$  associated to sensor noises. In the remaining of the paper, we will refer to measurement quantities with a tilde, *e.g.*  $\tilde{a}$ , and Gaussian noises affecting them as  $\mathbf{n}_a$ . The following section describes how these residuals are obtained from measurements.

## III. FACTOR GRAPH FOR BASE-CENTROIDAL ESTIMATION

In the context of graph optimization estimation, factors can apply to a single KF (unary factors) or distant KFs. KFs are created at relatively low rate to avoid saturating the estimation problem with too many variables. We incorporate

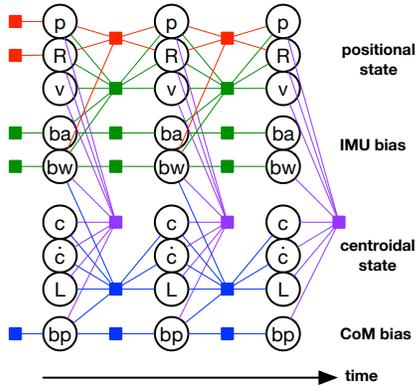


Fig. 2: Factor graph including factors of IMU (green), leg odometry (red), centroidal kinematics (purple) and pre-integrated force-torque (blue). Columns are keyframes. State block nodes are  $p$ : position;  $R$ : orientation;  $v$ : velocity;  $ba$ : acc. bias;  $bw$ : gyro bias;  $c$ : CoM position;  $\dot{c}$ : CoM velocity;  $L$ : angular momentum;  $bp$ : CoM measurement bias. Single-ended factors are priors.

several kinds of factor to (3): state prior (0), leg odometry (LO), pre-integrated IMU deltas (I), pre-integrated force measurements (F), centroidal kinematics (CK), and bias drifts (B), yielding the optimization problem (Fig. 2),

$$\begin{aligned} \chi^* = \arg \min_{\chi} & \|\mathbf{r}_i^0\|_{\Sigma_i^0}^2 + \sum_{i,j} \|\mathbf{r}_{i,j}^I\|_{\Sigma_{i,j}^I}^2 + \sum_{i,j} \|\mathbf{r}_{i,j}^{LO}\|_{\Sigma_{i,j}^{LO}}^2 \\ & + \sum_{i,j} \|\mathbf{r}_{i,j}^F\|_{\Sigma_{i,j}^F}^2 + \sum_i \|\mathbf{r}_i^{CK}\|_{\Sigma_i^{CK}}^2 + \sum_{i,j} \|\mathbf{r}_{i,j}^B\|_{\Sigma_{i,j}^B}^2 \end{aligned} \quad (4)$$

These factors are described in the following paragraphs.

#### A. State priors $\mathbf{r}^0$

In the posed problem, the base position and yaw angle are unobservable since we do not have access to absolute measurements (coming from GPS for instance). In order to ground the estimates to a reference and not have a floating problem, we impose a prior on the initial position and orientation of the base.

$$\mathbf{r}^0(\mathbf{x}_0) = \begin{bmatrix} \mathbf{p}^0 - \tilde{\mathbf{p}}^0 \\ \text{Log}(\mathbf{R}^0 \top \mathbf{R}^0) \end{bmatrix} \quad (5)$$

where  $\text{Log}$  denotes the logarithm application mapping elements of  $\text{SO}(3)$  to the  $\mathbb{R}^3$  representation of its Lie algebra  $\mathfrak{so}(3)$ .

#### B. Leg odometry factor $\mathbf{r}^{LO}$

Assuming that we have access to an accurate contact detection, for each foot in contact between KFi and KFj, we can write that  ${}^w\mathbf{p}_{wl}^i = {}^w\mathbf{p}_{wl}^j + \mathbf{n}_{LO}$  where  $\mathbf{n}_{LO}$  denotes a Gaussian noise accounting for potential slip of the foot and kinematic inaccuracies. This pseudo measurement can be used to derive a 3D factor for each foot  $l$  in contact:

$$\mathbf{r}_l^{LO}(\mathbf{p}^i, \mathbf{R}^i, \mathbf{p}^k, \mathbf{R}^k) = \mathbf{p}^i + \mathbf{R}^i {}^b\mathbf{p}_{bl}^i - (\mathbf{p}^k + \mathbf{R}^k {}^b\mathbf{p}_{bl}^k) \quad (6)$$

Where  ${}^b\mathbf{p}_{bl}^i$  and  ${}^b\mathbf{p}_{bl}^k$  are the contact position in base frame at times  $i$  and  $j$  acquired from  $\mathbf{q}_a$  via forward kinematics.

Assuming good contact detection, leg odometry fused with the IMU is enough to properly observe 6DoF motion with as few as 2 contact feet at a time. Note that contrary to previous works [14], [16], this leg odometry factor does not require gyroscope integration

#### C. Generalized pre-integration theory

Pre-integration refers to the integration of high rate proprioceptive sensory data in an efficient way in the context of factor graphs. It was initially developed to deal with IMU measurements [12], [13]. In fact, if a standard integration is conducted naively to derive a factor measurement between two KFs, then the IMU data needs to be reintegrated at each solver iteration because the integral depends on the states we are to estimate. The IMU pre-integration theory solves this issue and can be generalized to any proprioceptive sensor as shown in [19]–[21]. Delta quantities  $\Delta_{ik}$  are defined between KFs  $i$  and  $k$  so that  $\mathbf{x}_k = \mathbf{x}_i \boxplus \Delta_{ik}$ . A suitable composition  $\boxplus$  is chosen so that  $\Delta_{ik}$  does not depend on the initial states  $\mathbf{x}_i$ . Dependence on other, small-varying parameters  $\mathbf{b}$  such as sensor bias is linearized as  $\Delta_{ik}(\mathbf{b}) = \Delta_{ik}(\bar{\mathbf{b}}) \oplus \mathbf{J}_{\mathbf{b}}^{\Delta}(\mathbf{b} - \bar{\mathbf{b}})$ . Then we can pre-integrate  $\Delta_{ik} \triangleq \Delta_{ik}(\bar{\mathbf{b}})$  once during data gathering, and use it to define residuals that are later evaluated many times by the optimizer.

1) *Delta pre-integration:* We perform pre-integration incrementally as follows. First,  $\bar{\Delta}_{ii}$  is initialized to the null motion. Its covariance  $\Sigma_{ik}^{\Delta}$  and the Jacobian  $\mathbf{J}_{\mathbf{b}_i}^{\delta_k}$  are set to zero. At each reception of sensor data  $\tilde{\mathbf{z}}_k$  at  $t_k$ , we integrate during  $\delta t$  to obtain the delta corresponding to a single data sample,

$$\delta_k = f(\tilde{\mathbf{z}}_k, \bar{\mathbf{b}}_i, \delta t), \quad (7)$$

using the bias  $\bar{\mathbf{b}}_i$  available in KF  $i$ . This single delta is integrated onto the delta pre-integrated so far using the delta composition law,

$$\bar{\Delta}_{ik} = \bar{\Delta}_{ij} \circ \delta_k. \quad (8)$$

The Jacobian of the pre-integrated delta with respect to biases, as well as the delta covariance, are also pre-integrated,

$$\mathbf{J}_{\mathbf{b}_i}^{\Delta_{ik}} = \mathbf{J}_{\Delta_{ij}}^{\Delta_{ik}} \mathbf{J}_{\mathbf{b}_i}^{\Delta_{ij}} + \mathbf{J}_{\delta_k}^{\Delta_{ik}} \mathbf{J}_{\mathbf{b}_i}^{\delta_k} \quad (9)$$

$$\Sigma_{ik}^{\Delta} = \mathbf{J}_{\Delta_{ij}}^{\Delta_{ik}} \Sigma_{ij}^{\Delta} \mathbf{J}_{\Delta_{ij}}^{\Delta_{ik} \top} + \mathbf{J}_{\delta_k}^{\Delta_{ik}} \mathbf{J}_{\mathbf{z}_k}^{\delta_k} \Sigma_{\mathbf{z}_k} \mathbf{J}_{\mathbf{z}_k}^{\delta_k \top} \mathbf{J}_{\delta_k}^{\Delta_{ik} \top} \quad (10)$$

where  $\mathbf{J}_{\mathbf{b}_i}^{\delta_k}$ ,  $\mathbf{J}_{\mathbf{z}_k}^{\delta_k}$ ,  $\mathbf{J}_{\Delta_{ij}}^{\Delta_{ik}}$  and  $\mathbf{J}_{\delta_k}^{\Delta_{ik}}$  are the Jacobians  $\mathbf{J}_{\mathbf{x}}^y = \partial y / \partial \mathbf{x}$  of (7,8), computed according to Lie theory [22].

2) *Residual definition:* The pre-integrated  $\bar{\Delta}_{ik}$  is used at the end of the pre-integration to define the residual,

$$\mathbf{r}_{ik} = (\bar{\Delta}_{ik} \oplus \mathbf{J}_{\mathbf{b}_i}^{\Delta_{ik}}(\mathbf{b}_i - \bar{\mathbf{b}}_i)) \ominus \hat{\Delta}_{ik} \quad (11)$$

where  $\mathbf{b}_i$  is the current bias value,  $\hat{\Delta}_{ik} = \mathbf{x}_k \boxminus \mathbf{x}_i$  is the expected delta between keyframes, and  $\{\oplus, \ominus\}$  are the composite plus and minus operators described in [22]. That is,  $\{\oplus, \ominus\}$  are  $\{+, -\}$  for vectors, and for rotations we have  $\mathbf{R} \oplus \theta \triangleq \mathbf{R} \text{Exp}(\theta)$  and  $\mathbf{R}_2 \ominus \mathbf{R}_1 \triangleq \text{Log}(\mathbf{R}_1 \top \mathbf{R}_2)$ . The residual clearly depends on the keyframe states  $\mathbf{x}_i, \mathbf{x}_k$  and the bias  $\mathbf{b}_i$ . It has an associated covariance  $\Sigma_{ik}^{\Delta}$ .

#### D. IMU pre-integration factor $\mathbf{r}^I$

The states involved in this integration are the base states  $\mathbf{x}_b = [\mathbf{p}, \mathbf{v}, \mathbf{R}]$  with deltas  $\Delta = [\Delta\mathbf{p}, \Delta\mathbf{v}, \Delta\mathbf{R}]$ . The IMU produces biased and noisy measurements  $\tilde{\mathbf{z}} = [\tilde{\mathbf{a}}, \tilde{\boldsymbol{\omega}}]$  of the base proper acceleration and angular velocity, with bias  $\mathbf{b} = [\mathbf{b}_a, \mathbf{b}_\omega]$  and noise  $\mathbf{n} = [\mathbf{n}_a, \mathbf{n}_\omega]$ . The pre-integration method in [13] can be put in the formalism above by realizing that defining  $\delta = f(\tilde{\mathbf{z}}, \mathbf{b}, \delta t)$  in (7) as

$$\delta_k = \begin{bmatrix} \delta\mathbf{p} \\ \delta\mathbf{v} \\ \delta\mathbf{R} \end{bmatrix}_k = \begin{bmatrix} \frac{1}{2}(\tilde{\mathbf{a}} - \mathbf{b}_a - \mathbf{n}_a)\delta t^2 \\ (\tilde{\mathbf{a}} - \mathbf{b}_a - \mathbf{n}_a)\delta t \\ \text{Exp}((\tilde{\boldsymbol{\omega}} - \mathbf{b}_\omega - \mathbf{n}_\omega)\delta t) \end{bmatrix}_k \quad (12)$$

and the composition law  $\Delta_{ik} = \Delta_{ij} \circ \delta_k$  in (8) as

$$\begin{aligned} \Delta\mathbf{p}_{ik} &= \Delta\mathbf{p}_{ij} + \Delta\mathbf{v}_{ij}\delta t + \Delta\mathbf{R}_{ij}\delta\mathbf{p}_k \\ \Delta\mathbf{v}_{ik} &= \Delta\mathbf{v}_{ij} + \Delta\mathbf{R}_{ij}\delta\mathbf{v}_k \\ \Delta\mathbf{R}_{ik} &= \Delta\mathbf{R}_{ij}\delta\mathbf{R}_k \end{aligned} \quad (13)$$

then  $\mathbf{x}_k = \mathbf{x}_i \boxplus \Delta_{ik}$  is [13, eq. 32] and  $\Delta_{ik} = \mathbf{x}_k \boxminus \mathbf{x}_i$  is [13, eq. 33]. Full details can be found in [19, Section 3.4].

#### E. Force pre-integration factor $\mathbf{r}^F$

The Newton-Euler equations (2) relate the evolution of a system's CoM and AM due to gravity, external forces and torques in a chosen inertial frame. In the case of a legged robot with punctual contact feet, only forces  $\mathbf{f}_l$  are applied at each limb contact  $l$ , with no torque. We assume that at each limb contact we have access to noisy local force measurements  $\tilde{\mathbf{f}}_l = {}^l\mathbf{f}_l + \mathbf{n}_f$ . To transform them to the body frame  $b$ , we compute  ${}^b\mathbf{R}_l(\mathbf{q}_a) \in SO(3)$  and  ${}^b\mathbf{p}_{bl}(\mathbf{q}_a) \in \mathbb{R}^3$  from the articulations configuration  $\mathbf{q}_a \in \mathbb{R}^{12}$ . The lever arm  $(\mathbf{p}_l - \mathbf{c})$  in (2) uses a measurement of the CoM position in base frame  ${}^b\mathbf{p}_{bc}(\mathbf{q}_a) \in \mathbb{R}^3$ . This measure is biased due to inaccuracies in the robot model and therefore we add a bias variable to estimate,  $\mathbf{b}_{bc} \in \mathbb{R}^3$  so that  ${}^b\tilde{\mathbf{p}}_{bc} = {}^b\mathbf{p}_{bc} + \mathbf{b}_{bc} + \mathbf{n}_{bc}$ . Assuming constant forces during the interval  $\delta t$  the integral of (2) yields the discrete dynamic model for the centroidal states:

$$\begin{aligned} \mathbf{c}^k &= \mathbf{c}^{k-1} + \dot{\mathbf{c}}^{k-1}\delta t + \frac{1}{2}\mathbf{g}\delta t^2 + \frac{1}{2m}\mathbf{R}^{k-1}\sum_l {}^b\mathbf{R}_l^k(\tilde{\mathbf{f}}_l^k - \mathbf{n}_f^k)\delta t^2 \\ \dot{\mathbf{c}}^k &= \dot{\mathbf{c}}^{k-1} + \mathbf{g}\delta t + \frac{1}{m}\mathbf{R}^k\sum_l {}^b\mathbf{R}_L^k(\tilde{\mathbf{f}}_l^k - \mathbf{n}_f^k)\delta t \\ \mathcal{L}^k &= \mathcal{L}^{k-1} + \mathbf{R}^k\sum_l ({}^b\mathbf{p}_{bl}^k - {}^b\tilde{\mathbf{p}}_{bc}^k + \mathbf{b}_{bc}^k + \mathbf{n}_{bc}^k) \times {}^b\mathbf{R}_l^k(\tilde{\mathbf{f}}_l^k - \mathbf{n}_f^k)\delta t \end{aligned} \quad (14)$$

Analogously to the IMU case, it is possible to pre-integrate force measurements to derive a factor on the states  $\mathbf{x}_c = [\mathbf{c}, \dot{\mathbf{c}}, \mathcal{L}, \mathbf{R}]$  using deltas  $\Delta_c = [\Delta\mathbf{c}, \Delta\dot{\mathbf{c}}, \Delta\mathcal{L}, \Delta\mathbf{R}]$ . The rotation measured by the gyroscope has to be included too for the pre-integration to work. Then the bias vector is  $\mathbf{b} = [\mathbf{b}_{bc}, \mathbf{b}_\omega]$ . We define measurements  $\mathbf{z}_k$  to be:

$$\mathbf{z}_k = \left[ {}^b\tilde{\mathbf{p}}_{bc}^k, \tilde{\boldsymbol{\omega}}^k, \left[ \tilde{\mathbf{f}}_l^k, {}^b\tilde{\mathbf{p}}_{bl}^k, {}^b\tilde{\mathbf{R}}_l^k \right]_{l=1\dots 4} \right] \quad (15)$$

The following operators are enough to particularize the method in Section III-C to the contact forces case. Integrating

$\mathbf{z}_k$  during  $\delta t$  yields  $\delta_k$  in (7) as,

$$\delta_k(\mathbf{z}_k, \mathbf{b}_i, \delta t) = \begin{bmatrix} \frac{1}{2m}\sum_l {}^b\tilde{\mathbf{R}}_l^k \tilde{\mathbf{f}}_l^k \delta t^2 \\ \frac{1}{m}\sum_l {}^b\tilde{\mathbf{R}}_l^k \tilde{\mathbf{f}}_l^k \delta t \\ (\sum_l {}^b\tilde{\mathbf{p}}_{bl}^k - ({}^b\tilde{\mathbf{p}}_{bc}^k - \mathbf{b}_{bc}) \times ({}^b\tilde{\mathbf{R}}_l^k \tilde{\mathbf{f}}_l^k))\delta t \\ \text{Exp}((\tilde{\boldsymbol{\omega}}^k - \mathbf{b}_\omega)\delta t) \end{bmatrix} \quad (16)$$

and the delta composition law in (8) as

$$\Delta \circ \delta = \begin{bmatrix} \Delta\mathbf{c} + \Delta\dot{\mathbf{c}}\delta t + \Delta\mathbf{R}\delta\mathbf{c} \\ \Delta\dot{\mathbf{c}} + \Delta\mathbf{R}\delta\dot{\mathbf{c}} \\ \Delta\mathcal{L} + \Delta\mathbf{R}\delta\mathcal{L} \\ \Delta\mathbf{R}\delta\mathbf{R} \end{bmatrix}. \quad (17)$$

The expected delta  $\Delta_{ik} = \mathbf{x}_k \boxminus \mathbf{x}_i$  between KFs  $i, k$  reads

$$\Delta_{ik} = \begin{bmatrix} \Delta\mathbf{c}_{ik} \\ \Delta\dot{\mathbf{c}}_{ik} \\ \Delta\mathcal{L}_{ik} \\ \Delta\mathbf{R}_{ik} \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{i\top}(\mathbf{c}^k - \mathbf{c}^i - \dot{\mathbf{c}}^i\Delta t_{ik}) \\ \mathbf{R}^{i\top}(\dot{\mathbf{c}}^k - \dot{\mathbf{c}}^i - \mathbf{g}\Delta t_{ik}) \\ \mathbf{R}^{i\top}(\mathcal{L}^k - \mathcal{L}^i) \\ \mathbf{R}^{i\top}\mathbf{R}^k \end{bmatrix}. \quad (18)$$

Finally, the propagation of the state  $\mathbf{x}_i$  to  $\mathbf{x}_k$  using  $\Delta_{ik}$ , which can be used to retrieve a state at any time  $k$  between keyframes in the trajectory, is

$$\mathbf{x}^k = \mathbf{x}^i \boxplus \Delta_{ik} = \begin{bmatrix} \mathbf{c}^i + \dot{\mathbf{c}}^i\Delta t_{ik} + \mathbf{R}^i\Delta\mathbf{c}_{ik} + \frac{1}{2}\mathbf{g}\Delta t_{ik}^2 \\ \dot{\mathbf{c}}^i + \mathbf{R}^i\Delta\dot{\mathbf{c}}_{ik} + \mathbf{g}\Delta t_{ik} \\ \mathcal{L}^i + \mathbf{R}^i\Delta\mathcal{L}_{ik} \\ \mathbf{R}^i\Delta\mathbf{R}_{ik} \end{bmatrix} \quad (19)$$

#### F. Centroidal kinematics factor $\mathbf{r}^{CK}$

We need to relate base states to centroidal quantities to ground their estimate. For that, we can rely on the inertial-kinematic model for the computation of the CoM position wrt. base frame  ${}^b\mathbf{p}_{bc}(\mathbf{q}_a) \in \mathbb{R}^3$ , the CoM velocity wrt. base frame  ${}^b\mathbf{v}_{bc}(\mathbf{q}_a, \dot{\mathbf{q}}_a) \in \mathbb{R}^3$ , the inertial matrix  $\mathcal{I}(\mathbf{q}_a) \in \mathbb{R}^{3 \times 3}$ , and the kinematic momentum due to gesticulation of the robot limbs only  $\mathcal{L}_a(\mathbf{q}_a, \dot{\mathbf{q}}_a) \in \mathbb{R}^3$ . In particular, the computed CoM position is here considered an independent measurement, affected by Gaussian noise and a slowly varying bias  ${}^b\tilde{\mathbf{p}}_{bc} = {}^b\mathbf{p}_{bc} + \mathbf{b}_{bc} + \mathbf{n}_{bc}$ . The angular velocity from the IMU is used and its bias has to be incorporated in the factor  ${}^b\tilde{\boldsymbol{\omega}}_b = {}^b\boldsymbol{\omega}_b + \mathbf{b}_\omega + \mathbf{n}_\omega$ . In the end the equations used to derive the factor are:

$$\begin{aligned} \mathbf{c} &= \mathbf{R}({}^b\tilde{\mathbf{p}}_{bc} - \mathbf{b}_{bc} - \mathbf{n}_{bc}) + \mathbf{p} \\ \dot{\mathbf{c}} &= \mathbf{v} + \mathbf{R}(({}^b\tilde{\boldsymbol{\omega}}_b - \mathbf{b}_\omega - \mathbf{n}_\omega) \times ({}^b\tilde{\mathbf{p}}_{bc} - \mathbf{b}_{bc} - \mathbf{n}_{bc}) \\ &\quad + ({}^b\tilde{\mathbf{v}}_{bc} - \mathbf{n}_{vc})) \\ \mathcal{L} &= \mathbf{R}(\mathcal{I}({}^b\tilde{\boldsymbol{\omega}}_b - \mathbf{b}_\omega - \mathbf{n}_\omega) + \mathcal{L}_a) \end{aligned} \quad (20)$$

Then, the residual  $\mathbf{r}^{CK} \in \mathbb{R}^9$  is simply expressed as:

$$\mathbf{r}^{CK} = \begin{bmatrix} {}^b\tilde{\mathbf{p}}_{bc} - (\mathbf{R}^T(\mathbf{c} - \mathbf{p}) + \mathbf{b}_p) \\ ({}^b\tilde{\boldsymbol{\omega}}_b - \mathbf{b}_\omega) \times ({}^b\tilde{\mathbf{p}}_{bc} - \mathbf{b}_p) + {}^b\tilde{\mathbf{v}}_{bc} - \mathbf{R}^T(\dot{\mathbf{c}} - \mathbf{v}) \\ \mathcal{I}({}^b\tilde{\boldsymbol{\omega}}_b - \mathbf{b}_\omega) + \mathcal{L}_a - \mathbf{R}^T\mathcal{L} \end{bmatrix} \quad (21)$$

#### G. Bias prior and drift factors

Bias prior and drift factors have trivial residuals  $\mathbf{r}^{\text{prior}} = \mathbf{b} - \mathbf{b}^{\text{ref}}$  and  $\mathbf{r}^{\text{drift}} = \mathbf{b}_k - \mathbf{b}_i$  respectively, each with its own associated covariance  $\Sigma$ .

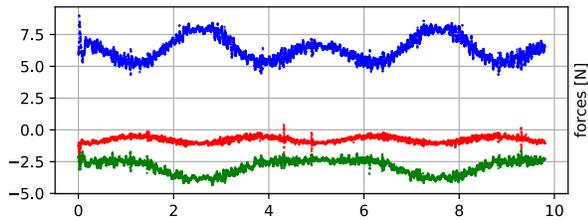


Fig. 3: Force estimation on X-Y-Z axis (r-g-b) of one Solo12 leg expressed in world frame using proprioceptive sensors during the *sinXYZ* trajectory

#### IV. CONTACT FORCES RECONSTRUCTION

We tested our solution on the open source robot Solo12 [23] built recently at LAAS. Solo12 is not equipped with contact nor force sensors. Yet, it is possible to invert the robot dynamic equation (1) in order to reconstruct the contact forces knowing the robot configuration and derivatives  $\dot{v}_q, v_q, q$  and joint torques given by the motor current. Some of these quantities are hard to obtain directly since they depend on the state being estimated (base orientation) or on numerical differentiation ( $\ddot{q}_a$ ). For these reasons, we pre-calculated these forces by benefiting from an internal filter of the IMU for the base orientation, and centered window differentiation for the articulation acceleration  $\ddot{q}_a$ . Fig. 3 shows an example of the force reconstruction of one leg.

#### V. RESULTS

In this section we present results of the application of the estimator on datasets taken at LAAS-Toulouse on our platform Solo12: *sinXYZ* corresponds to a trajectory where the robot moves by following a sine wave reference with the feet fixed in place. We use the state estimation framework WOLF, developed at IRI-Barcelona, which relies on Google Ceres as the graph solver. For all runs, Key Frames were created at 4Hz for fair comparison.

##### A. Base estimation through inertial kinematic fusion

First, to validate the use of our kinematic factor, we include uniquely the IMU and LO factors to obtain an Inertial Kinematics (IK) estimator which conceptually includes the same information as estimators such as [3]. In Fig. 4, we compare our state estimation at 1kHz with Motion Capture (Mo-Cap) up-sampled from 200Hz to 1kHz. Velocity in base frame is also shown in Fig. 5.

Artificially removing contact factors (considering only 1, 2 or 3 feet in contact) can help us gain confidence in the use of this kinematic factor in situations where we rarely have all feet in contact, like for example with trotting gaits. In Fig. 6, we can see that only considering 1 foot in contact during the whole trajectory results in a drifting position, but as soon as 2 or more feet are in contact, the system is constrained enough for the drift to remain below around 5mm on all axes.

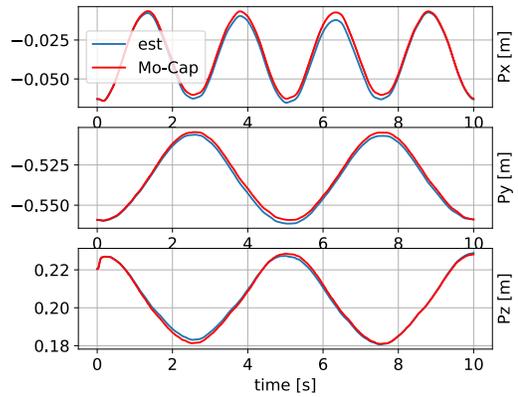


Fig. 4: *sinXYZ* trajectory base position from the IMU+kinematics estimator (blue) vs Mo-Cap (red)

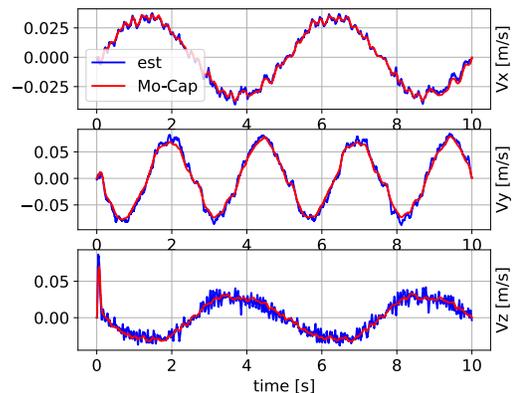


Fig. 5: *sinXYZ* trajectory base velocity in base frame from the IMU+kinematics estimator (blue) vs Mo-Cap (red)

##### B. Centroidal estimation

Now, on the same trajectory, we deploy the full estimator with all factors described in Section III to jointly estimate the base and centroidal quantities. A ground truth on the centroidal quantities is difficult to obtain since no direct sensor can provide us with this information contrary to the base state. We can however validate our method by comparing it a two step procedure: first, estimate the base state with a state Kalman Filter as implemented in [24], then compute the centroidal quantities directly from the robot kinematic model. The full estimator should be able to infer a bias on the  ${}^b\tilde{p}_{bc}$  measure so we artificially add a constant disturbance in the robot dynamic model on the lever of the base link of [0.03, 0.06, 0.04] cm, which then corresponds to a CoM bias of [-0.0197, -0.0394, 0.0263]. Fig. 8 shows that the bias estimated with our method closely matches the introduced bias. Fig. 9 shows a comparison between the base and CoM reconstruction with our method and with the two-step base KF with geometric CoM. Note that the base-CoM difference on the z axis reflects the fact that limbs of the robot lower naturally its CoM. The estimated CoM velocity follows closely the velocity of the base as shown in Fig. 7.

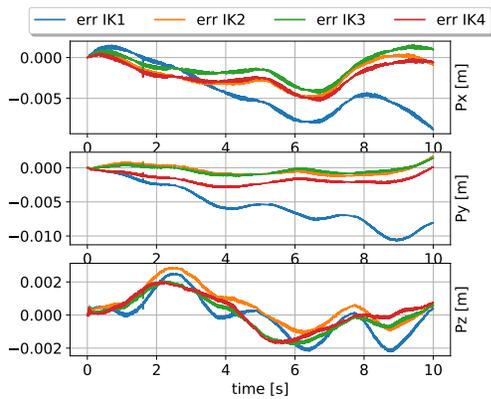


Fig. 6: *sinXYZ* trajectory base position error with different numbers of feet used for the leg odometry factors: 1 (blue), 2 (orange), 3 (green), 4 (red) from the IMU+kinematics estimator

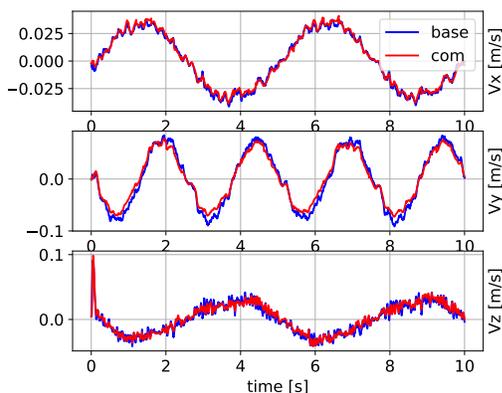


Fig. 7: Base (blue) vs CoM (red) velocities from the full estimator

## VI. CONCLUSION

In this paper, we introduced the first realization of the tightly coupled estimation of base and centroidal quantities of a legged robot using factor graph optimization. This was enabled by the introduction of a new pre-integrated factor on the contact forces of the robot, making the CoM geometric measurement bias observable. We also designed a simple factor for point leg sensors that does not rely on gyro measurement. We demonstrated the viability of the approach on a dataset taken on the quadruped robot Solo 12. Within this framework, we plan to integrate exteroceptive measurement in a SLAM context to obtain a drift free estimation of base and centroidal states useful for planning and feedback control.

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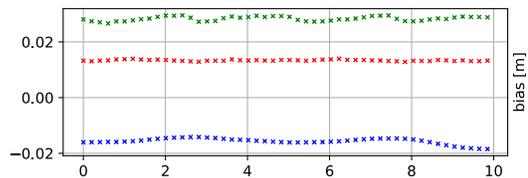


Fig. 8: Estimation of bias on CoM measurement from the full estimator along x-y-z axis ( red-green-blue) in base frame

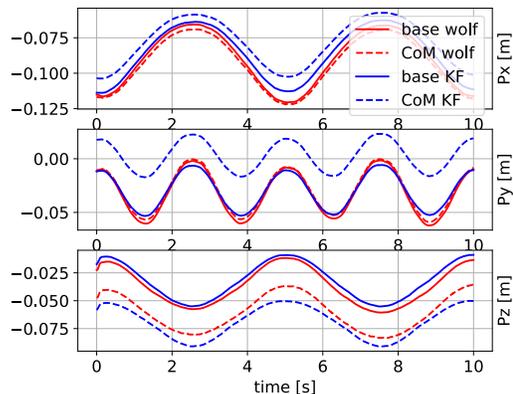


Fig. 9: Comparison of the estimates between decoupled the KF base estimator and geometric CoM reconstruction (blue) and the tightly coupled estimator presented in this paper (red) on the *sinXYZ* trajectory with artificial base link CoM bias.

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