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# Higher-order Automation in TLAPS

(Work in progress)

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$\text{TLA}^+$ , as a logical framework, is often presented as set theory in *first-order* logic [Lam02]. This is not strictly true, as some features of  $\text{TLA}^+$  can reasonably be called “second-order”. The dedicated prover of  $\text{TLA}^+$ , TLAPS [CDL<sup>+</sup>12], relies on backend solvers, most of which only support first-order logic. Isabelle is the only exception. Thus there is a class of problems that are currently very difficult for TLAPS to handle. This work aims at bridging this gap, by extending TLAPS with a higher-order backend solver, namely Zipperposition.

Zipperposition is a superposition-based automatic theorem prover envisioned as a vehicle for prototyping various extensions to superposition calculi. Its support for higher-order is based on a complete calculus for extensional polymorphic clausal higher-order logic [BBT<sup>+</sup>19]. Recent pragmatic extension to full higher-order logic [Vuk20] and an improved higher-order unification procedure [VBN20] were the main factors that contributed to Zipperposition winning the higher-order division of CASC-J10 theorem proving competition [Sut20].

$\text{TLA}^+$  has several second-order features. To illustrate a few, we turn to a simple formalization. The goal is to define the sum over a series:

$$\sum_{i=1}^n s_i$$

That expression has two parameters: the natural number  $n$ , and the term  $s$ . The variable  $i$  is bound in  $s_i$ ; thus, there is a lambda-abstraction “ $\lambda i. s_i$ ” hidden behind the notation.  $\text{TLA}^+$  allows the declaration of second-order operators, that is, operators that take first-order operators as arguments. We can then represent the parameter  $s$  by an operator  $S(\_)$  given as argument to `sum`.

We want to define the sum by recursion on  $n$ . Module `NaturalsInduction` provides utilities to define recursive *functions* on `Nat`, but not recursive operators. This implies that we cannot represent  $n$  as an argument of the sum operator. Instead, we must define `sum(S)` as a  $\text{TLA}^+$  function on `Nat`, so that the full expression will be `sum(S)[n]` (assuming  $n \in \text{Nat}$ ).

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```
1 EXTENDS TLAPS, Naturals, NaturalsInduction
2
3 (** For all operators S(_) and n ∈ Nat, sum(S)[n] is
4     the sum of all S(i) for 0 ≤ i < n *)
5 sum(S(_)) ==
6   LET sumRec[m ∈ Nat] ==
7     IF m = 0 THEN 0 ELSE S(m) + sumRec[m - 1]
8   IN
9     sumRec
10
11 THEOREM SumDefConclusion ==
12   ASSUME NEW S(_)
13   PROVE NatInductiveDefConclusion(sum(S), 0, LAMBDA v,n : S(n) + v)
14   OMITTED
15
```

```

16 THEOREM SumDef ==
17   ASSUME NEW S(_), NEW n ∈ Nat
18   PROVE sum(S)[n] = IF n = 0 THEN 0 ELSE S(n) + sum(S)[n - 1]
19   (* Isabelle fails. SumDefConclusion is a lemma that needs to be
20     instantiated with the operator S(_) *)
21   BY SumDefConclusion DEF NatInductiveDefConclusion

```

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The definition of `sum` in lines 5-9 is not enough by itself, because  $\text{TLA}^+$  does not guarantee the recursive function exists. The existence of a function that matches the definition must be proven manually. Module `NaturalsInduction` provides generic theorems for this. By following a simple pattern of theorems (given at the end of the module’s source code), we can recover the basic facts we need about `sum`.

Theorem `SumDefConclusion` essentially states that a function matching the recursive definition exists. This is expressed by the predicate `NatInductiveDefConclusion`—its precise definition does not matter to us. Next, theorem `SumDef` also states that `sum` matches the intended definition, but in a form that is more practical to us. Unfortunately, TLAPS fails to prove that theorem.

To prove the goal, one has to first instantiate `SumDefConclusion` with the operator `S(_)`, and then finish the proof with the definition of `NatInductiveDefConclusion`. This instantiation step is problematic, because the instance is a first-order operator. It is thus *second-order* reasoning. Currently Isabelle is the only backend that is able to perform this kind of reasoning, which makes the proof script very fragile.

So far, our work has lead us to extend TLAPS with an export to the TPTP language, a standard input format for automatic provers, and use this export to make Zipperposition solve  $\text{TLA}^+$  proof obligations. Despite this being a work-in-progress, we managed to prove theorem `SumDef` with Zipperposition.

## References

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