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# Generalization of second order quasi-phase matching in whispering gallery mode resonators using Berry Phase

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► **To cite this version:**

Alejandro Lorenzo-Ruiz, Yoan Léger. Generalization of second order quasi-phase matching in whispering gallery mode resonators using Berry Phase. OSA Advanced Photonic Congress, Jul 2020, Washington, United States. hal-02990227

**HAL Id: hal-02990227**

**<https://hal.science/hal-02990227v1>**

Submitted on 5 Nov 2020

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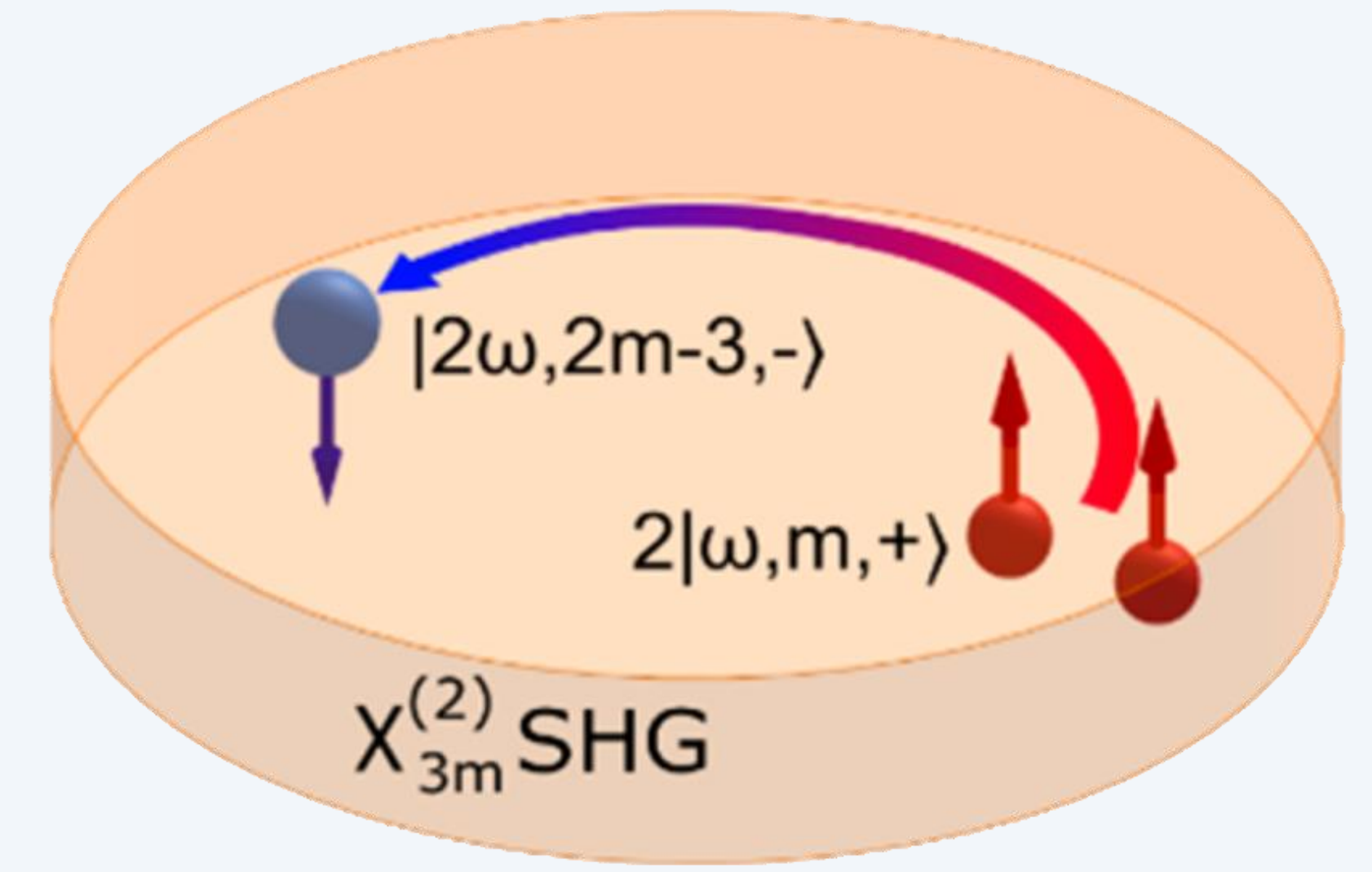
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## Introduction

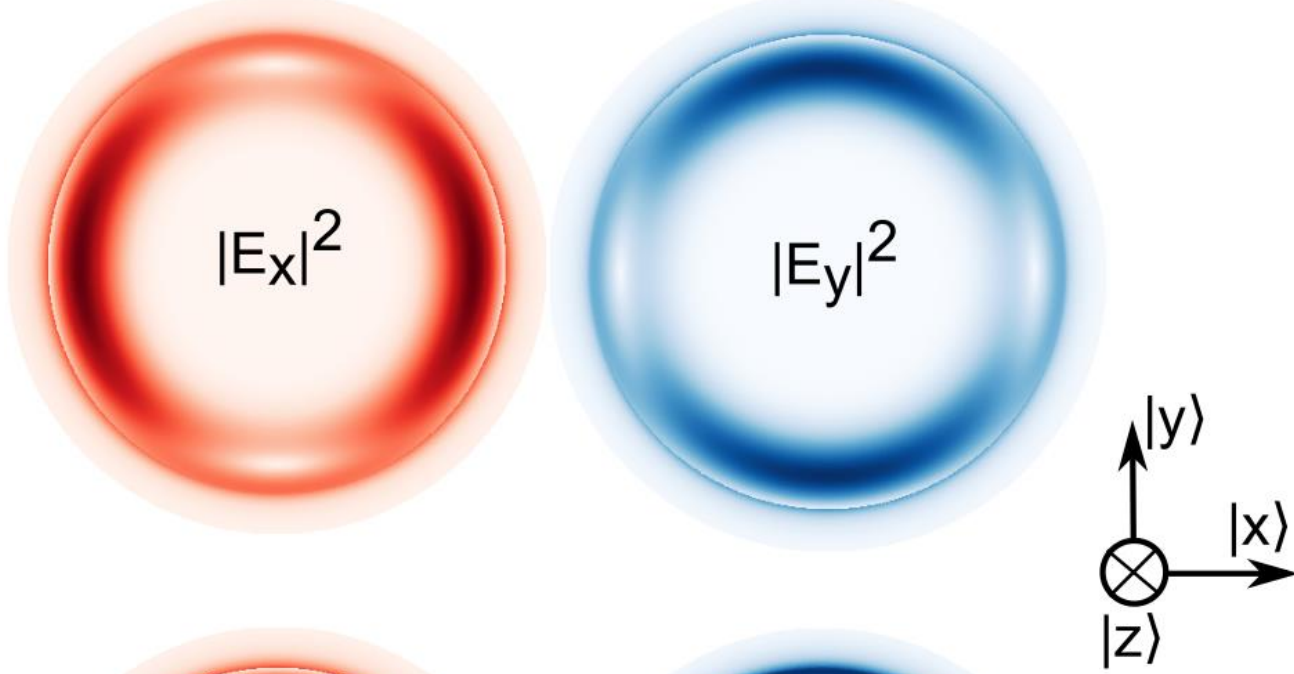
Second order processes in circular microresonators are highly investigated to design and fabricate high efficiency integrated devices:

- Need to conserve momentum conservation → Quasi-phase matching (QPM) → Azimuthal number selection rules ( $\Delta m$ )
- Natural QPM in WGM strongly depend on crystal symmetry
- Different additional quanta appear for different materials and crystallographic orientations
- No simple explanation of the physical source of these quanta in literature

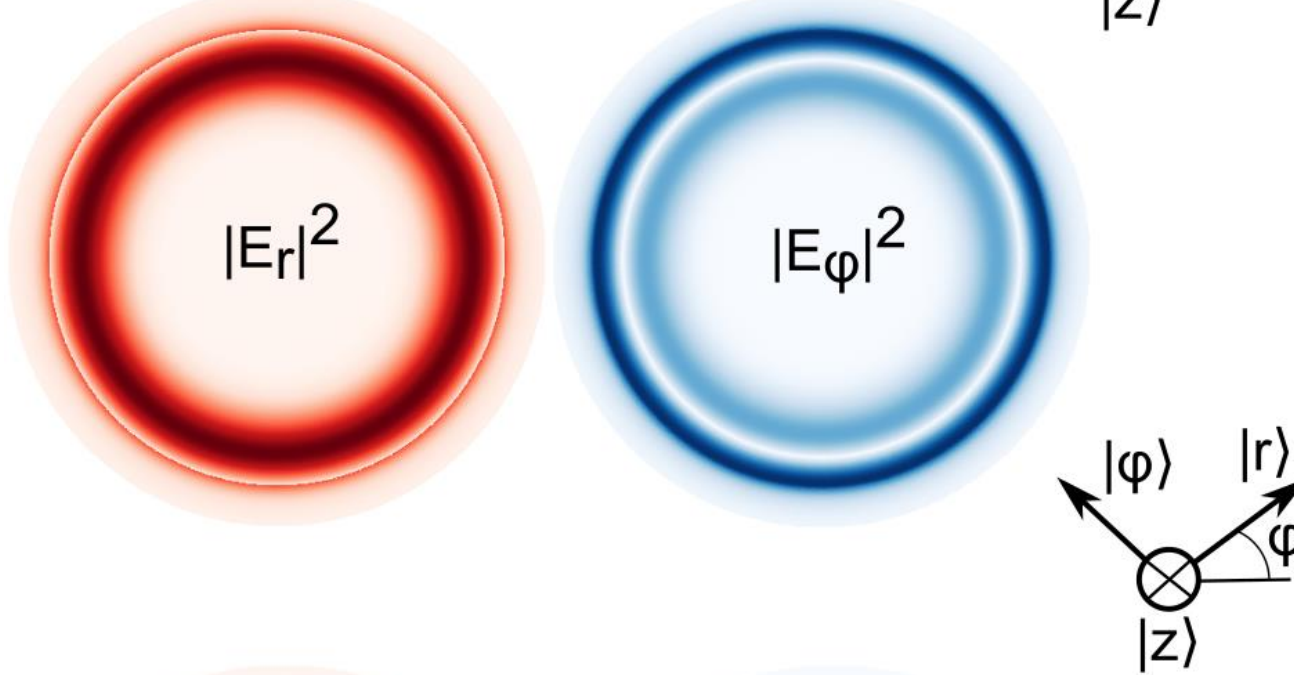


## Transverse Spin Angular Momentum (TSAM) of photons: circular polarization basis

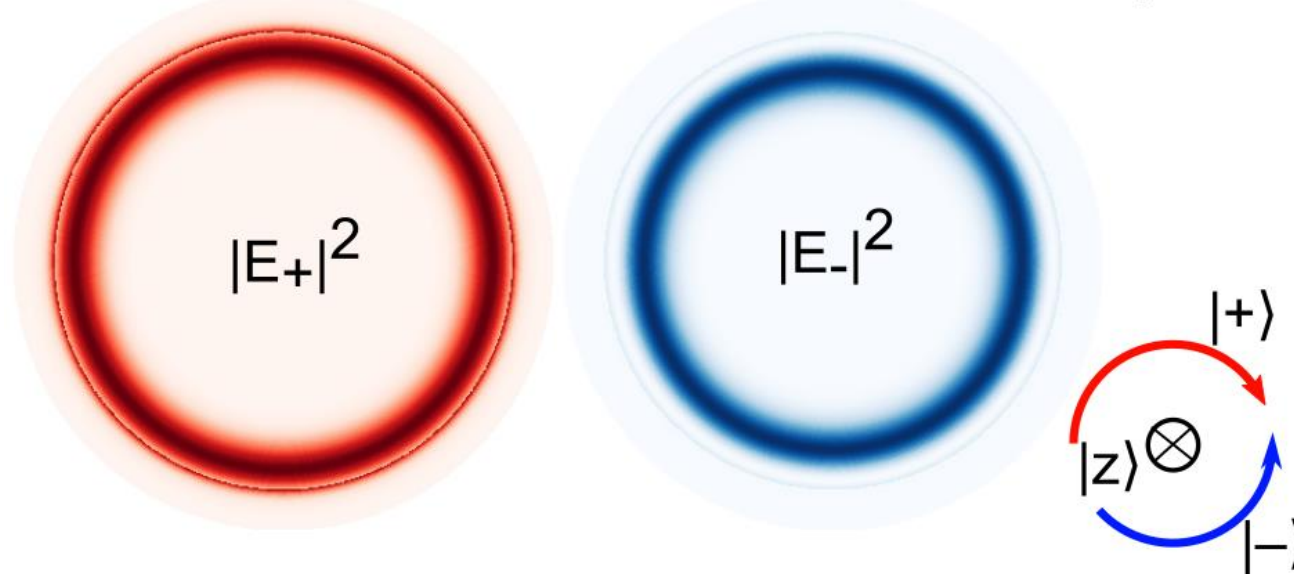
- Cartesian fixed basis



- Rotating polar basis



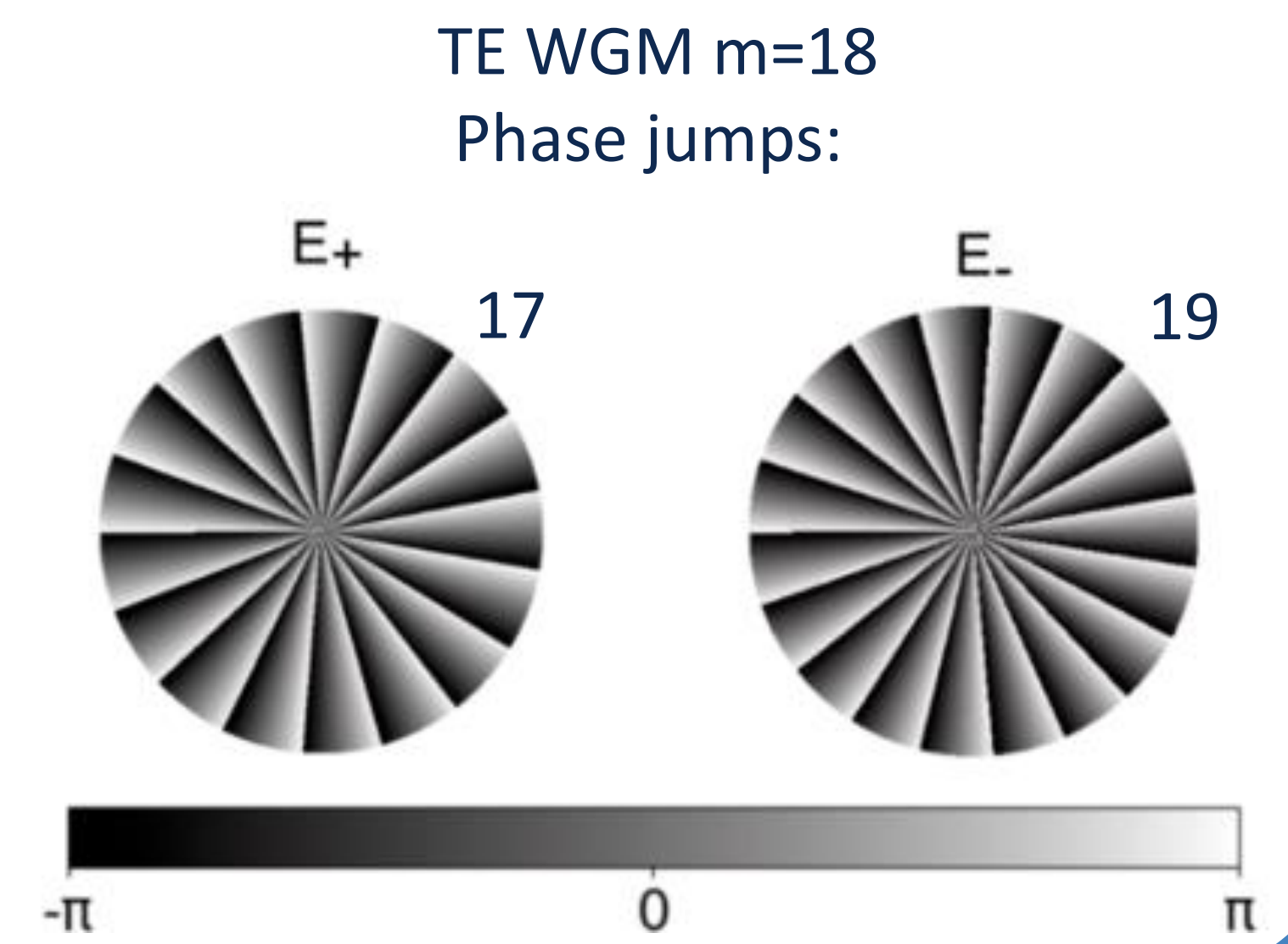
- Circular polarization basis (CP basis) along resonator axis → TSAM components<sup>(1)</sup>



## Berry phase in TSAM components: spin up & down polarization of photons

$$\begin{cases} E_r = \frac{m}{r} S_m(r) e^{im\phi} \\ E_\phi = \frac{i\tilde{k}_1}{2} T_m(r) e^{im\phi} \end{cases} \begin{cases} |+\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle) \\ |-\rangle = \frac{1}{\sqrt{2}}(|x\rangle - i|y\rangle) \end{cases} \rightarrow \begin{cases} E_+ = \frac{1}{\sqrt{2}}(S_m(r) + T_m(r)) e^{i(m-1)\phi} \\ E_- = \frac{1}{\sqrt{2}}(S_m(r) - T_m(r)) e^{i(m+1)\phi} \end{cases}$$

- Opposite additional quanta on azimuthal dependence appears on both CP components
- This phenomenon is a direct consequence of the Berry phase experienced by each CP component during the revolution within the resonator<sup>(2)</sup>



## Generalization of nonlinear susceptibility tensor and azimuthal selection rules

General form of the nonlinear tensor can be written in the circular polarization basis keeping the  $d_{ij}$  parameters known from the usual cartesian representation

$$\begin{pmatrix} P_+ \\ P_- \\ P_z \end{pmatrix} = \epsilon_0 \begin{pmatrix} \frac{\sqrt{2}(d_{11} + d_{12} + id_{16} + id_{22})}{4} & \frac{\sqrt{2}(d_{11} - 3d_{12} - 3id_{16} + id_{22})}{4} & \frac{\sqrt{2}(d_{13} - id_{23})}{2} & -id_{14} + \frac{d_{15}}{2} - \frac{d_{24}}{2} & \frac{d_{15}}{2} + \frac{d_{24}}{2} & \frac{\sqrt{2}(d_{11} + d_{12} - id_{16} - id_{22})}{4} \\ \frac{\sqrt{2}(d_{11} - 3d_{12} + 3id_{16} - id_{22})}{4} & \frac{\sqrt{2}(d_{11} + d_{12} - id_{16} - id_{22})}{4} & \frac{\sqrt{2}(d_{13} + id_{23})}{2} & \frac{d_{15}}{2} + \frac{d_{24}}{2} & id_{14} + \frac{d_{15}}{2} - \frac{d_{24}}{2} & \frac{\sqrt{2}(d_{11} + d_{12} + id_{16} + id_{22})}{4} \\ id_{14} + \frac{d_{15}}{2} - \frac{d_{24}}{2} & -id_{14} + \frac{d_{15}}{2} - \frac{d_{24}}{2} & d_{33} & \frac{\sqrt{2}(d_{13} - id_{23})}{2} & \frac{\sqrt{2}(d_{13} + id_{23})}{2} & \frac{d_{15}}{2} + \frac{d_{24}}{2} \end{pmatrix} \begin{pmatrix} E_+ E_+ \\ E_- E_- \\ E_z E_z \\ 2E_- E_z \\ 2E_+ E_z \\ 2E_+ E_- \end{pmatrix}$$

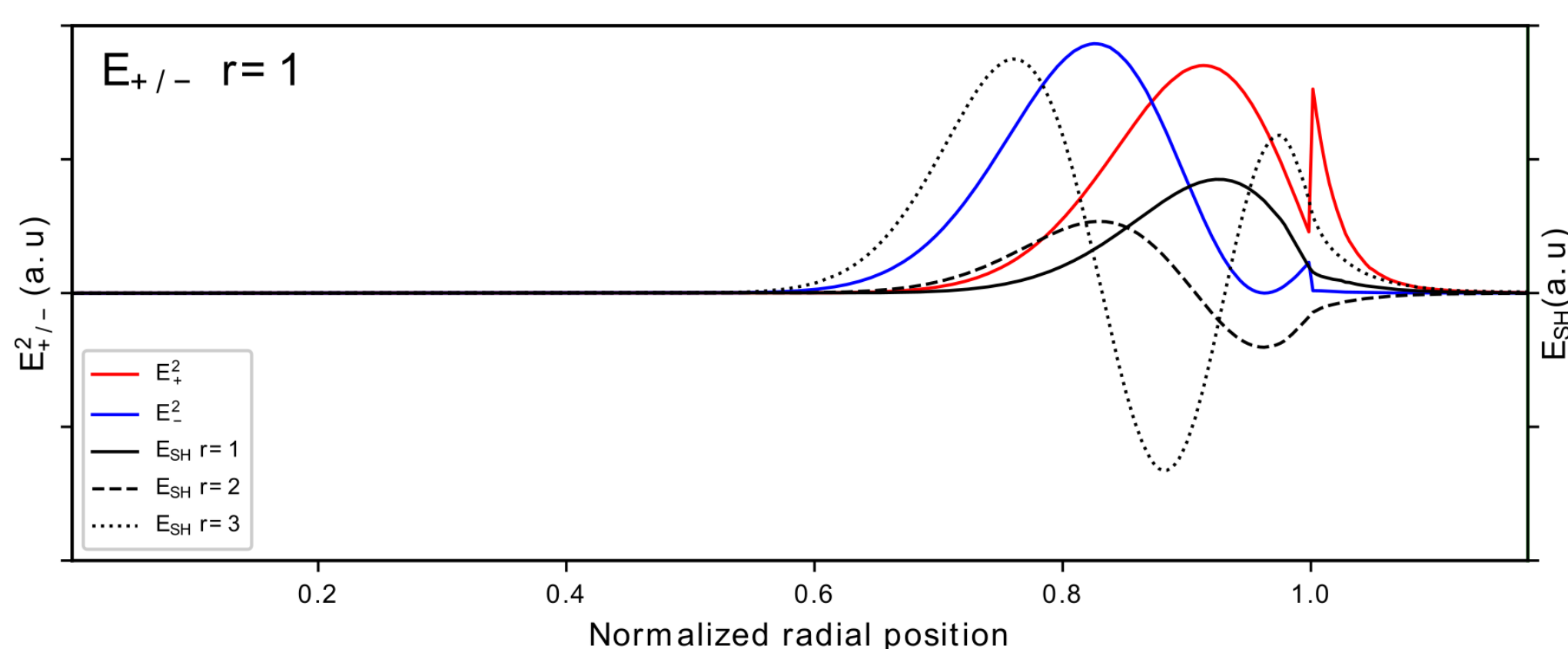
Each tensor element is associated to a single creation/annihilation process in CP basis with a unique azimuthal number mismatch

$$\begin{bmatrix} +1 & -3 & -1 & -2 & 0 & -1 \\ +3 & -1 & +1 & 0 & +2 & +1 \\ +2 & -2 & 0 & -1 & +1 & 0 \end{bmatrix}$$

Examples:

- Zinc-blende: TE-TE → TM  $\Delta m = \pm 2$
- LnNbO<sub>3</sub> z-cut: TE+TE → TM  $\Delta m = \pm 3$
- LnNbO<sub>3</sub> x-cut: TE+TE / TM → TM  $\Delta m = \pm 1$
- Wurtzite: TE+TE → TM  $\Delta m = 0$

## Choosing the right WGM: towards better design of resonators for nonlinear processes



$$\beta_2 = \frac{i d_{14}}{4} \frac{\int d^3x \epsilon E_{2z}^* (E_{1+}^2 - E_{1-}^2)}{[\int d^3x \epsilon |E_1|^2][\int d^3x \epsilon |E_2|^2]^{\frac{1}{2}}}$$

- Azimuthal dependence can be inserted in the overlap integral<sup>(3)</sup>.
- A precise optimization can be made using higher radial orders
- Knowing exactly the  $\Delta m$  condition allows a precise tuning of integrated coupling systems

## Conclusion

- TSAM description of WGMs to explicit the Berry phase dependence
- General expression to obtain the nonlinear susceptibility tensor in circular polarization basis
- Origin of the different azimuthal selection rules presents in second order processes in circular resonators
- Useful description to optimize and design highly efficient devices.
- Detailed work available at Lorenzo-Ruiz, Alejandro, and Yoan Léger. "Generalization of second order quasi-phase matching in whispering gallery mode resonators using Berry Phase." *ACS Photonics* (2020).

This work is funded by the French national research agency through the project ORPHEUS ANR-17-CE24-0019

(1) Aiello, A. et al. From transverse angular momentum to photonic wheels. *Nature Photon* 9, (2015): 789–795.

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(3) Rodriguez, A. et al. " $\chi$ (2) and  $\chi$ (3) harmonic generation at a critical power in inhomogeneous doubly resonant cavities." *Optics express* 15.12 (2007): 7303-7318.