Generalization of second order quasi-phase matching in whispering gallery mode resonators using Berry Phase

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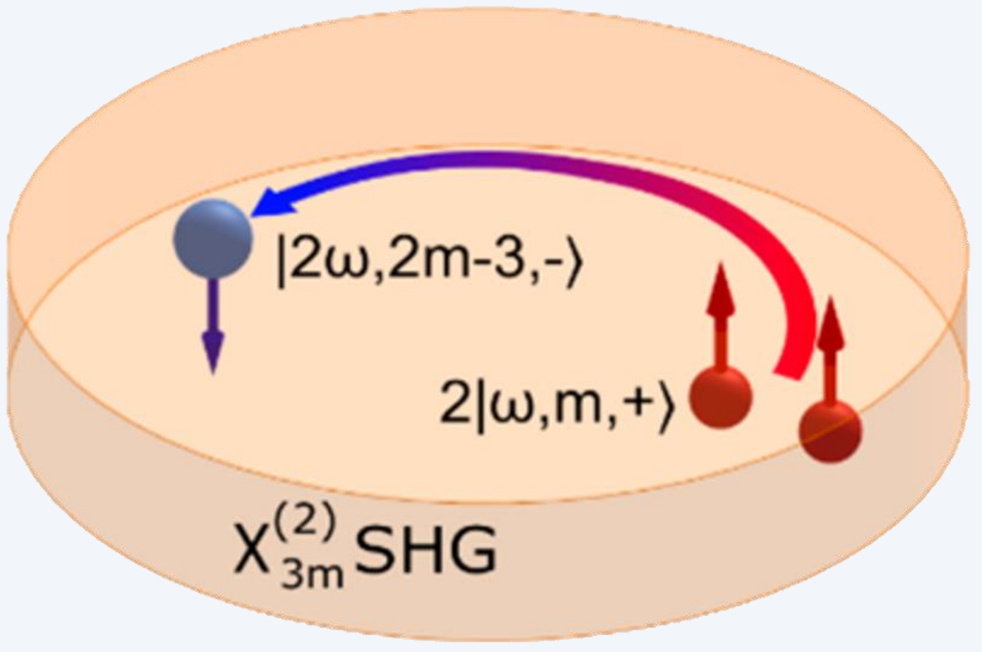
Introduction

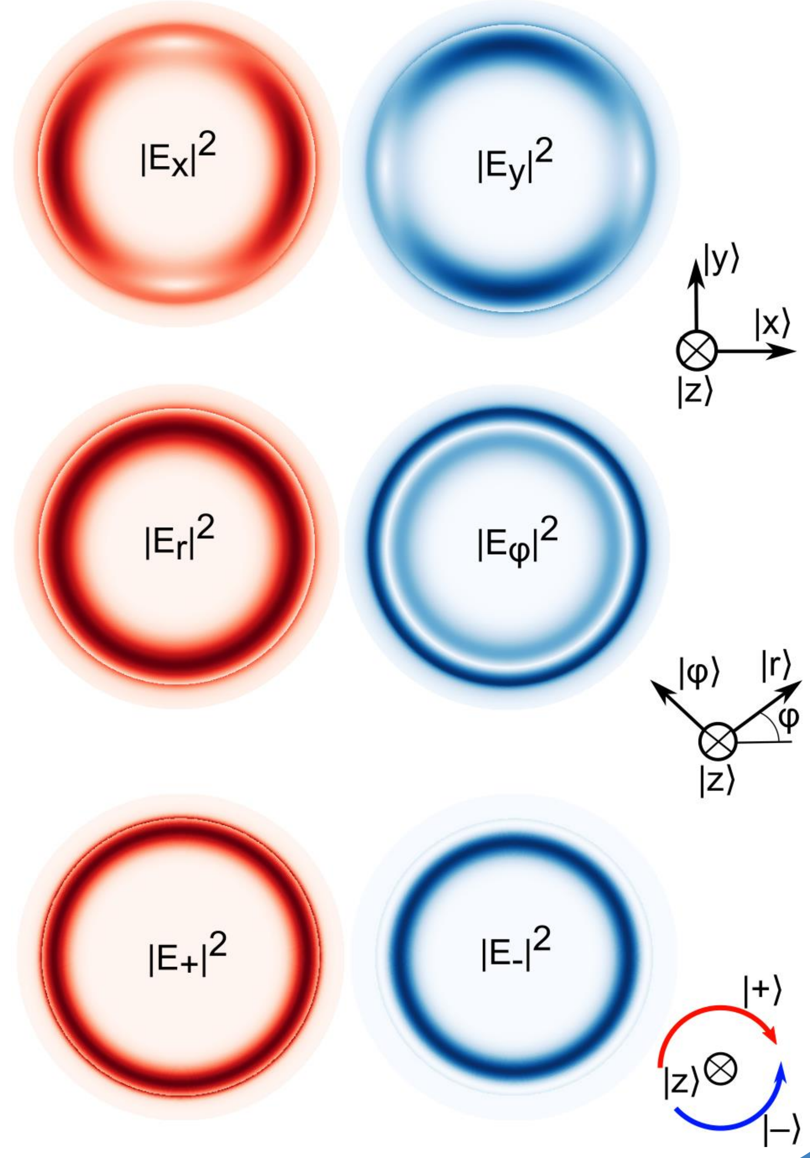
Second order processes in circular microresonators are highly investigated to design and fabricate high efficiency integrated devices:

- Need to conserve momentum conservation → Quasi-phase matching (QPM) → Azimuthal number selection rules (Δm)
- Natural QPM in WGM strongly depend on crystal symmetry
- Different additional quanta appear for different materials and crystallographic orientations
- No simple explanation of the physical source of these quanta in literature



Transverse Spin Angular Momentum (TSAM) of photons: circular polarization basis

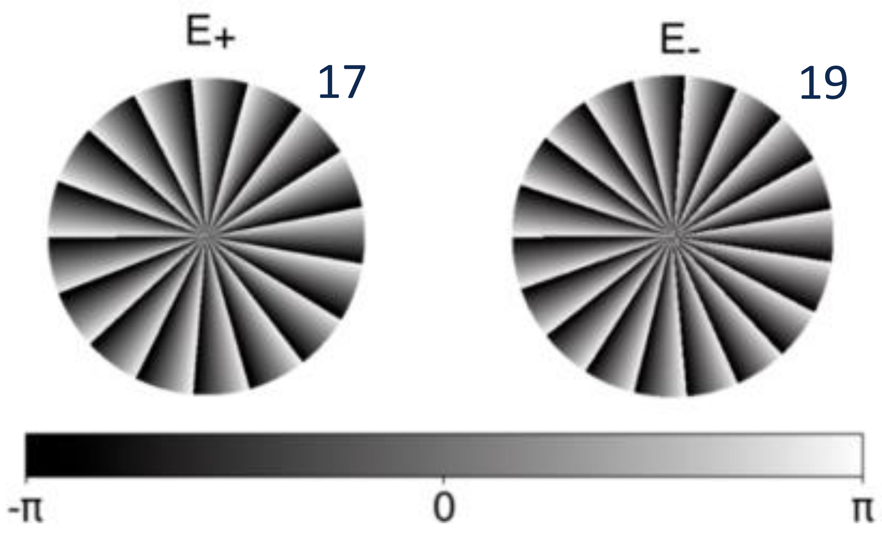
- Cartesian fixed basis
- Rotating polar basis
- Circular polarization basis (CP basis) along resonator axis → TSAM components[1]



Berry phase in TSAM components: spin up & down polarization of photons

$$\begin{cases} E_r = \dfrac{m}{r} S_m(r) e^{im\varphi} \\ E_\varphi = \dfrac{i\tilde{k}_1}{2}\, i\, T_m(r) e^{im\varphi} \end{cases} \quad \xrightarrow{\begin{cases} |+\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle) \\ |-\rangle = \frac{1}{\sqrt{2}}(|x\rangle - i|y\rangle) \end{cases}} \quad \begin{cases} E_+ = \dfrac{1}{\sqrt{2}}\left(S_m(r) + T_m(r)\right) e^{i(m-1)\varphi} \\ E_- = \dfrac{1}{\sqrt{2}}\left(S_m(r) - T_m(r)\right) e^{i(m+1)\varphi} \end{cases}$$

- Opposite additional quanta on azimuthal dependence appears on both CP components
- This phenomenon is a direct consequence of the Berry phase experienced by each CP component during the revolution within the resonator[2]



Generalization of nonlinear susceptibility tensor and azimuthal selection rules

General form of the nonlinear tensor can be written in the circular polarization basis keeping the d_{ij} parameters known from the usual cartesian representation

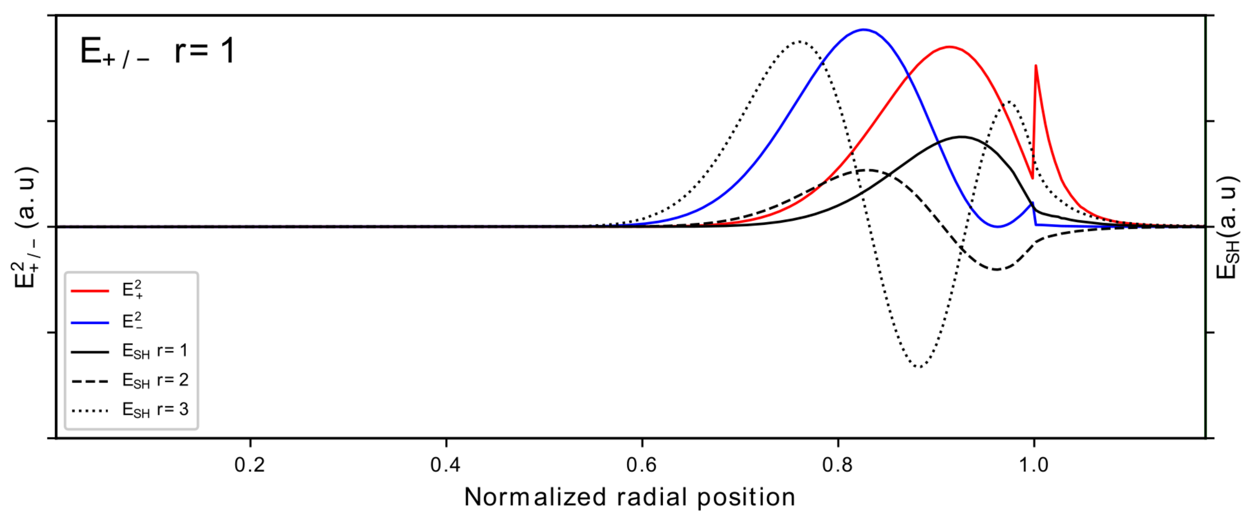
$$\begin{pmatrix} P_+ \\ P_- \\ P_z \end{pmatrix} = \epsilon_0 \begin{bmatrix} \frac{\sqrt{2}\,(d_{11}+d_{12}+id_{16}+id_{22})}{4} & \frac{\sqrt{2}\,(d_{11}-3d_{12}-3id_{16}+id_{22})}{4} & \frac{\sqrt{2}\,(d_{13}-id_{23})}{2} & -id_{14}+\frac{d_{15}}{2}-\frac{d_{24}}{2} & \frac{d_{15}}{2}+\frac{d_{24}}{2} & \frac{\sqrt{2}\,(d_{11}+d_{12}-id_{16}-id_{22})}{4} \\ \frac{\sqrt{2}\,(d_{11}-3d_{12}+3id_{16}-id_{22})}{4} & \frac{\sqrt{2}\,(d_{11}+d_{12}-id_{16}-id_{22})}{4} & \frac{\sqrt{2}\,(d_{13}+id_{23})}{2} & \frac{d_{15}}{2}+\frac{d_{24}}{2} & id_{14}+\frac{d_{15}}{2}-\frac{d_{24}}{2} & \frac{\sqrt{2}\,(d_{11}+d_{12}+id_{16}+id_{22})}{4} \\ id_{14}+\frac{d_{15}}{2}-\frac{d_{24}}{2} & -id_{14}+\frac{d_{15}}{2}-\frac{d_{24}}{2} & d_{33} & \frac{\sqrt{2}\,(d_{13}-id_{23})}{2} & \frac{\sqrt{2}\,(d_{13}+id_{23})}{2} & \frac{d_{15}}{2}+\frac{d_{24}}{2} \end{bmatrix} \begin{pmatrix} E_+E_+ \\ E_-E_- \\ E_zE_z \\ 2E_-E_z \\ 2E_+E_z \\ 2E_+E_- \end{pmatrix}$$

Each tensor element is associated to a single creation/annihilation process in CP basis with a unique azimuthal number mismatch

$$\begin{bmatrix} +1 & -3 & -1 & -2 & 0 & -1 \\ +3 & -1 & +1 & 0 & +2 & +1 \\ +2 & -2 & 0 & -1 & +1 & 0 \end{bmatrix}$$

Examples:
- Zinc-blende: TE-TE → TM Δm = ±2
- LnNbO3 z-cut: TE+TE → TM Δm = ±3
- LnNbO3 x-cut: TE+TE /TM→ TM Δm = ±1
- Wurtzite: TE+TE → TM Δm = 0

Choosing the right WGM: towards better design of resonators for nonlinear processes



$$\beta_2 = \frac{i\, d_{14}}{4} \frac{\int d^3x\, \varepsilon\, E^*_{2z}\left({E_{1+}}^2 - {E_{1-}}^2\right)}{\left[\int d^3x\, \varepsilon |E_1|^2\right]\left[\int d^3x\, \varepsilon |E_2|^2\right]^{\frac{1}{2}}}$$

- Azimuthal dependence can be inserted in the overlap integral[3].
- A precise optimization can be made using higher radial orders
- Knowing exactly the Δm condition allows a precise tuning of integrated coupling systems

Conclusion

- TSAM description of WGMs to explicit the Berry phase dependence
- General expression to obtain the nonlinear susceptibility tensor in circular polarization basis
- Origin of the different azimuthal selection rules presents in second order processes in circular resonators
- Useful description to optimize and design highly efficient devices.
- Detailed work available at Lorenzo-Ruiz, Alejandro, and Yoan Léger. "Generalization of second order quasi-phase matching in whispering gallery mode resonators using Berry Phase." *ACS Photonics* (2020).

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