

Equation of State of hcp Fe-C-Si Alloys and the Effect of C Incorporation Mechanism on the Density of hcp Fe alloys at 300 K 2

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1	Equation of State of <i>hcp</i> Fe-C-Si Alloys and the Effect of C Incorporation					
2	Mechanism on the Density of <i>hcp</i> Fe alloys at 300 K					
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14						
15	Key Points:					
16 17	• Synchrotron X-ray diffraction and density functional theory calculations were performed on <i>hcp</i> Fe-C-Si alloy with 4 at% C and 3 at% Si					
18 19	• Different incorporation mechanisms were examined and the <i>hcp</i> Fe-C-Si alloy sample takes the interstitial form					
20	• Assuming a wrong incorporation mechanism leads to incorrect density determination and					

- a more enhanced density contrast between the alloy and pure Fe.
- 21 22

23 Abstract

Si and C are cosmochemically abundant elements soluble in hcp Fe under pressure and 24 temperature, and could therefore be present in the Earth's inner core. While recent ab initio 25 26 calculations suggest that the observed inner core density and velocities could be matched by an Fe-C-Si alloy, the combined effect of these two elements has only recently started to be 27 investigated experimentally. We therefore carried out synchrotron X-ray diffraction 28 measurements of an hcp Fe-C-Si alloy with 4 at% C and 3 at% Si, up to ~150 GPa. Density 29 functional theory calculations were also performed to examine different incorporation 30 31 mechanisms. These calculations suggest interstitial C to be more stable than substitutional C below ~350 GPa. In our calculations, we also find that the lowest energy incorporation 32 mechanism in the investigated pressure range (60-400 GPa) is one where two C atoms occupy 33 one atomic site; however this is unlikely to be stable at high temperatures. Notably, substitutional 34 C is observed to decrease the volume of the *hcp* Fe, while interstitial C increases it. This allows 35 us to use experimental and theoretical equations-of-state to show unambiguously that C in the 36 experimental hcp Fe-C-Si alloys is not substitutional, as is often assumed. This is crucial since 37 assuming an incorrect incorporation mechanism in experiments, leads to incorrect density 38 determinations of ~4%, undermining attempts to estimate the concentration of C in the inner 39 core. In addition, the agreement between our experiments and calculations, support Si and C as 40 being light elements in the inner core. 41

42 **1 Introduction**

The only direct investigation of the inaccessible Earth's deep interior is provided by seismological studies of sound wave velocities through the Earth. The comparison between seismological observations and elastic properties of candidate materials at the relevant conditions present in the deep Earth, allows to relate geophysical data with the chemistry and temperature of

the Earth's interior. In the Earth's mantle, mineralogical and compositional models can match 47 the observed velocities to approximately 1% (e.g. Irifune et al., 2008; Murakami et al., 2012; 48 Bass & Zhang, 2015; Pamato et al., 2016; Kurnosov et al., 2017). Conversely, for the inner core, 49 sound wave velocities of pure Fe or Fe-Ni alloys in the *hcp* structure usually obtained from 50 mineral physics are much higher (10-30 %) than those from seismic observations (Vočadlo, 51 2007; Belonoshko et al., 2007; Martorell, Brodholt, et al., 2013). Numerous arguments were 52 proposed over the years to explain these discrepancies, including, for example, large 53 compositional effects, pervasive partial melting, anelasticity or the stabilization of bcc Fe 54 (Vočadlo, 2007; Belonoshko et al., 2003, 2007, 2017, 2019; Antonangeli et al., 2004). 55

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Theoretical calculations have shown that the sound velocities observed in the inner core could be 57 matched by hcp Fe due to a strong non linear softening of the elastic properties at 360 GPa above 58 7000 K (Martorell, Vočadlo, et al., 2013). However, even though the velocities of the inner core 59 were matched by pure Fe, the density remained too high (by 2-3%) to agree with geophysical 60 observations, and so lighter elements are still needed to explain this density mismatch. 61 Furthermore, the suggested pre-melting softening is generally suppressed in iron alloys (e.g. 62 63 Martorell et al., 2016). With the exception of Fe_7C_3 (Chen et al., 2014; Prescher et al., 2015), all the iron compounds examined so far exhibit higher velocities than those found in the core. For 64 the case of Fe₇C₃ a strong non linear softening has been also observed (Li et al., 2016), but, 65 66 while the calculated velocities agree with seismic data, the density is far too low (by $\sim 8\%$). Many recent computational and experimental studies addressing core compositions point to the 67 need of two or more distinct light elements (e.g. Badro et al. 2104; Morard, Nakajima, et al., 68 69 2017; Li et al. 2018; Edmund et al., 2019). However, the identity of this light element mixture is

70 still largely debated. Very recently Li et al. (2018), reported ab initio molecular dynamics (AIMD) calculations which reveal the observed inner-core density and sound velocities can be 71 simultaneously matched by a range of ternary and quaternary Fe alloys, all containing carbon. 72 This constrains possible solid solution's space in a multiple light elements model explaining all 73 of the major seismic constraints. However, even when limiting the discussion to mineral physics 74 properties, several aspects still remain uncertain, including the different incorporation 75 mechanisms of C in Fe and its effect on seismic properties. From density functional theory 76 (DFT) calculations, Huang et al. (2005) suggested that C takes the substitutional form in hcp Fe 77 at 0 K above approximately 300 GPa, as confirmed by Li et al. (2019) who, using ab initio 78 molecular dynamics, also found that C takes the substitutional form at 360 GPa and 6500 K. 79 However, at lower pressures, interstitial C and other complex defect structures may be stabilized 80 (Huang et al., 2005; Caracas, 2017; Li et al., 2019). In fact, the thermodynamic model of Fei & 81 Brosh (2014), based on experiments at pressures <20 GPa, and more recently the work by Yang 82 et al. (2019) indicate that the interstitial form is preferred up to inner-core pressures. The change 83 of incorporation mechanism may be related to the variance of solubility. Lord et al. (2009), for 84 example, found a negligible C solubility in *hcp* Fe approaching 50 GPa. However, more recently, 85 Mashino et al. (2019) performed melting experiments up to 255 GPa and predicted a solubility of 86 1 wt.% in hcp Fe at inner-core boundary conditions. A question remains as to whether different 87 incorporation mechanisms of C in iron have an effect on the seismic properties of an Fe-C alloy; 88 89 does it matter if the carbon is on an interstitial, substitutional or other site within the *hcp* crystal structure? 90

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92 Multiple lines of evidence, from geochemical and cosmochemical arguments (e.g. Allègre et al., 1995), to core formation models based on metal-silicate partitioning (e.g. Siebert et al., 2013; 93 Fischer et al., 2015), to isotopic considerations (e.g. Fitoussi et al., 2009; Shahar et al., 2009), 94 support silicon to be present in the core. Due to the almost equal Si partitioning between liquid 95 and *hcp* iron (Alfè et al., 2002), the quantity of Si present in the inner core is not significantly 96 different from that expected for the liquid core. However, many recent studies point out that Si is 97 very unlikely to be the only light element in the core (Badro et al., 2014; Morard, Andrault, et al., 98 2017) and specifically in the inner core (Martorell et al., 2016; Antonangeli et al., 2018; Edmund 99 et al., 2019). An alloy in the ternary Fe-C-Si system thus seems to provide the simplest 100 composition possibly accounting for the bulk of geophysical and geochemical evidences of the 101 inner core. In particular, very recent AIMD calculations (Li et al., 2018) indicate that an hcp 102 Fe₆₀C₂Si₂ alloy is expected to have density, compressional and shear sound velocities matching 103 those of PREM at 360 GPa for simulation temperatures between 6000 and 7000 K. 104

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To confirm this prediction, both high-pressure experiments and further simulations need to be 106 performed to clarify the effects of C on the seismic properties of Fe and Fe-Si alloys over a wider 107 range of pressures and temperatures. Thus, here we address the effect of the C incorporation 108 109 mechanism on density by combining synchrotron X-ray diffraction (XRD) measurements on an hcp Fe-C-Si alloy (with 4 at % C and 3 at % Si at 300 K and up to ~150 GPa), with DFT 110 111 calculations. In particular, we examined the different incorporation mechanisms of carbon in the structure of the alloy as a function of pressure. Our results are then used to discuss the density of 112 a Fe-C-Si alloy at inner core conditions. 113

114 **2 Materials and Methods**

115 **2.1 Experiments**

The starting material, with a nominal composition of Fe 93 at%, C 4 at%, Si 3 at%, was synthesised by an ultra-rapid quench method at the ICMPE (Institut de Chimie et des Matériaux de Paris-Est, Paris, France) using nominally pure Fe (Neyco F-12734), Si (Wacker) and Fe₃C (Neyco FC-76857/1). Homogeneous Fe-alloys, in the form of ribbons, approximately 25 μ m thick and of 20-30 mm width, were synthesized following the procedure described in Morard et al. (2011).

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The chemical composition of the synthesized alloys was measured by means of a Cameca SX100 electron microprobe analyser equipped with a cold finger and an O₂ flux at Camparis center-Sorbonne Université. The resulting composition for the alloy is 3 (\pm 0.2) at% Si, 4 (\pm 0.4) at% C and 93 (\pm 0.06) at% Fe (Fe₉₃C₄Si₃). In addition, scanning electron microscopy (SEM) analyses were performed on the sample using a Zeiss Ultra 55 scanning electron microscope installed at the IMPMC (Institut de Minéralogie, de Physique des Matériaux et de Cosmochimie), confirming homogeneity at the micron scale (Miozzi, Morard, et al., 2020).

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Samples were prepared for the high-pressure experiments by crushing parts of a ribbon specimen and compressing it between two diamonds (~ 600 μ m culet), in order to obtain the desired thickness (usually between 5 and 10 microns). The sample was then loaded in a rhenium gasket of 20 μ m thickness, with a hole of 60 μ m, with 100/300 μ m bevelled anvils. High pressures were generated using a Le Toullec-type (membrane driven) diamond anvil cell. Tungsten carbide seats designed for diamonds with conical support were used (Boehler & De Hantsetters, 2004). The sample was loaded alongside Mo as a pressure calibrant, with Ne acting as the pressure transmitting medium; - the equation of state parameters for the Mo pressure calibrant were taken to be: $V_0 = 31.14 \text{ Å}^3$ (Litasov et al., 2013; Ross & Hume-Rothery, 1963), $K_0 = 260(1)$ GPa and K' = 4.19(5).

141

Angle-dispersive XRD measurements were carried out at the ID27 beamline of the European Synchrotron Radiation Facility in Grenoble (Mezouar et al., 2005). In the experiments, a monochromatic beam of 33 keV (λ = 0.3738 Å) was focused on an area < 3 × 3 µm² (full width at half maximum). Data collection times were typically between 10 and 30 s; diffraction patterns were collected on a MAR CCD detector at pressures between 48 and 150 GPa at 300 K.

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2-D diffraction images were integrated using the software DIOPTAS (Prescher & Prakapenka, 148 2015), using calibration parameters (sample detector distance and orientation) derived from the 149 diffraction pattern of a cerium dioxide (CeO₂) standard. Diffraction patterns were indexed with 150 PD Indexer (http://pmsl.planet.sci.kobe148u.ac.jp/~seto/?page id=20). The sample started to 151 transform from the *bcc* structure stable at ambient pressure to the *hcp* structure at ~19 GPa. All 152 *bcc* reflections were absent by ~ 22 GPa. At least five sample diffraction lines are clearly 153 observed for the *hcp* phase over the entire pressure range. However, the diffraction patterns are, 154 to some extent, non-ideal in terms of randomness of the distribution (both 101 and 102 155 reflections showed intensity variation around the powder ring). Also, the 002 reflection, while 156 157 present over the entire pressure range, is very weak, due to a small preferential orientation, likely induced by sample thinning before loading. Therefore, it was not possible to perform full profile 158 data refinements. The unit-cell parameters were thus calculated from the d-spacings of the 159 160 reflections 100, 101, 102, and 110 using the software Unit Cell by Holland & Redfern (1997).

161 **2.2 DFT Calculations**

DFT calculations were performed by using a $4 \times 4 \times 2$ hcp supercell, the VASP code (Kresse & 162 Joubert, 1999) and PAW potential (Blöchl, 1994). The valence states $3p^63d^64s^2$, $3s^23p^2$ and 163 $2s^22p^2$ were adopted for Fe, Si and C, respectively. We employed the generalised gradient 164 approximation (GGA); the Perdew-Wang (Perdew & Wang, 1992) exchange-correlation 165 functional was used. We adopted the ordered symmetrical structure for the alloy by placing solid 166 solution atoms with the maximum sum of distances between them to achieve homogeneous 167 dispersion. The octahedral interstitial site was adopted for the interstitial carbon atoms as the 168 tetrahedral interstitial site was reported to be less favourable (Caracas, 2017). A plane-wave 169 cutoff energy of 400 eV and 12 irreducible k-points were used to converge the results. 170

171

172 **3 Results and Discussion**

173

3.1 Equations of state

The diffraction pattern of the investigated alloy collected at 140 GPa (the upper end of the pressure range investigated) and indexed as hexagonal closed packed (*hcp*) iron, is shown in Figure 1.

Measured unit-cell volumes of hcp Fe₉₃C₄Si₃ alloy are presented in Figure 2 and show a smooth, continuous trend with increasing pressure, indicating the absence of any phase transition within the pressure range investigated.

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The *P*–*V* data, reported in Table S1, were fitted using a third-order Birch–Murnaghan equation of state (BM3 EoS; Birch, 1947) with the software EosFit7 (Angel et al., 2014). All the parameters were refined resulting in the following EoS parameters: $V_0 = 22.7(2)$ Å³, $K_0 = 148(18)$ GPa, and $K_0' = 5.8(4)$ (Table 1). The EoS parameters are usually defined at 0 GPa (e.g., V_0 , K_0 and K_0'). However, as iron alloys in the *hcp*-structure are not stable and cannot be quenched at room pressure and temperature, the EoS parameters can be defined at any other pressure in order to compare different EoS in the pressure stability field of *hcp*-phases. The parameters refined relative to a pressure scale from 50 GPa are: $V_{P=50} = 18.80(1)$ Å³, $K_{P=50} = 394(6)$ GPa, and *K'* $P_{P=50} = 4.8(2)$.

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The measured unit-cell volumes of Fe₉₃C₄Si₃ are larger than those of pure *hcp* Fe, Fe-Si, and Fe-191 C alloys reported in the literature (Figure 2a). If compared to the pure hcp Fe EoS as determined 192 by Dewaele et al. (2006), the P-V trend established here shows a different curvature, with the 193 difference in the volumes increasing at high pressure, thus highlighting a difference in 194 compressibility. On the other hand, differences are less pronounced when compared to recent 195 EoS of pure *hcp* Fe reported by Sakai et al. (2014), Yang et al. (2019) and Miozzi, Matas, et al. 196 (2020). Specifically, the new data on the $Fe_{93}C_4Si_3$ alloy determined here, show a compressibility 197 curve comparable to the more recent results for pure *hcp* Fe, but with higher volumes. 198

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3.2 Comparison of Fe-C-Si alloys

A comprehensive comparison of the equations of state parameters K_0 and K_0' of the different 200 alloys considered here is illustrated in the inset in Figure 2a. The data published by Dewaele et 201 al. (2006), Sakai et al. (2014), Yang et al. (2019) and Miozzi, Matas, et al. (2020) for pure Fe and 202 by Edmund et al. (2019) and Yang et al. (2019) for Fe-Si and Fe-C alloys, respectively, were 203 reevaluated in order to obtain the variance-covariance matrixes (Angel, 2000) and to construct 204 the confidence ellipses (Table 1, Inset Figure 2a). The confidence ellipses of our dataset are 205 larger than those reported in the literature. This might be due to the different pressure and 206 volume uncertainties, how these are taken into account in the analysis, and to the number and 207

208 range of measured pressure points in the different studies. Here, both pressure and volume uncertainties were weighted, whereas most of the studies from the literature report an equally-209 weighted fit. When performing an equally-weighted fit, errors are smaller and thus also the 210 correlation between K_0 and K_0' is smaller. In addition, given the trade-off between V_0 , K_0 and K_0' , 211 it is important to measure the volume of the *hcp* structure at the lowest pressure in order to 212 213 constrain the refinement of the room temperature EoS (see e.g., Miozzi, Matas, et al. (2020)). In our study, due to the lack of low-pressure points (we report *P-V* data from 48 GPa, see Table S1) 214 the EoS refinement provides a less robust constraint of the reference volume at P = 0 (i.e. V_0). 215

216 The ellipse representing our data and the one of Yang et al. (2019) for pure Fe intersect at the 68.3% confidence level. Our data are also in good agreement with the recent EoS of Fe reported 217 by Miozzi, Matas, et al. (2020), which extends the data set to lower pressures, and also, within 218 experimental error, with that of Sakai et al. (2014). The confidence ellipses are in fact parallel 219 and the error bars intersect. All this indicates that the value of the isothermal room pressure bulk 220 modulus obtained in this study for Fe93C4Si3 is very similar to the values obtained for pure Fe 221 from Sakai et al (2014), Yang et al (2019) and Miozzi, Matas, et al. (2020). A critical review of 222 the EoS parameters of pure Fe is beyond the aim of this paper. It is important to note, however, 223 that the confidence ellipse built with the K_0 and K_0' values of Miozzi, Matas, et al. (2020) 224 intersects only with that of Sakai et al. (2014) for pure Fe, while, that of Dewaele et al. (2006) 225 does not intersect with our results and also does not intersect with other results in the literature 226 227 for pure Fe, as a direct consequence of the different compressibility already mentioned.

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Edmund et al. (2019), recently suggested that *hcp* Fe can contain up to 5 wt% Si (approximately 8 at%) without any significant change of the unit cell volume (Figure 2a). As shown by both

231 compression curve and the confidence ellipses, in fact, the data by Edmund et al. (2019) are in good agreement with the data from Dewaele et al. (2006) for hcp Fe. Yang et al. (2019) reported 232 that the incorporation of C has a direct effect on the volume of hcp Fe. In particular, their fitting 233 results showed that addition of C to Fe leads to larger unit cell volumes at the same pressure, 234 with the magnitude of the increase remaining constant up to ~ 135 GPa, the highest investigated 235 pressure, with compression curves almost parallel. The authors fitted a BM3 EoS with K_0' fixed 236 to 4.79 (value taken from Fei et al., 2016 for pure Fe) to systematically compare the EoS 237 parameters for pure Fe, Fe-0.31 wt% C (~1.4 at% C), and Fe-1.37 wt% C (~6.1 at% C). 238 However, when the data of Yang et al. (2019) are fitted without constraining K_0' , the resulting 239 magnitude of the volume increase is not the same. Furthermore, it can be seen that extrapolation 240 of the EoS established for Fe-6.1 at% C and Fe-1.4 at% C crosses above 120 GPa, with the 241 volumes of the first becoming smaller than the second and approaching the volume of pure Fe 242 (Figure 2b). Even when limiting comparison to the directly investigated pressure range, we note 243 that Fe-6.1 at% C has smaller volumes than the sample studied here, which contains less C (4 244 at% C), and volumes similar to those of Fe-Si alloy with 8 at% Si (Edmund et al. 2019) and pure 245 Fe by Dewaele et al. (2006). Moreover, as shown in the inset in Figure 2a, the confidence 246 ellipses representing our data and those of Yang et al. (2019) on pure Fe and Fe-1.4 at % C 247 intersect, indicating a shift only in V_0 without changing the compressibility, while that of Fe-6.1 248 at. %C is closer to those of Fe-8 at% Si (Edmund et al. 2019) and pure Fe according to Dewaele 249 250 et al. (2006).

3.3 Effect of the incorporation mechanism of carbon on the compressibility of *hcp*Fe While Si is incorporated via substitution in iron without introducing major changes in the volume (Edmund et al., 2019), the strong effect of small amounts of C on the volume of *hcp* Fe may well

255 be related to its interstitial position in the *hcp* structure (Caracas, 2017; Yang et al., 2019). Impurities, such as C, in a solid can be present as substitutional or interstitial atoms. In addition, 256 impurities can develop defect clusters, possibly modifing the physical and mechanical properties 257 of the material (Smidt & Sprague, 1972; Li et al., 2017; Li et al., 2019) including its volume and 258 its compressibility. Therefore, it is essential to address the different incorporation mechanisms of 259 carbon in the structure of the alloy at different pressures, to evaluate their relevance in defining 260 the properties of the Earth's core. To this end, in our *ab initio* calculations, we considered both 261 binary Fe-C and ternary Fe-C-Si alloys. At first we relaxed the structures and calculated 262 enthalpies for the binary substitutional $Fe_{62}C_2$, interstitial $Fe_{64}C_2$ and dimer $Fe_{63}C_2$ (two C at one 263 atomic site) alloys (see the structural illustration in Figure S1). Then, we also calculated volumes 264 and fitted the EoS of pure Fe, Fe₆₀C₂Si₂ via substitution, Fe₆₂C₂Si₂ via interstitial carbon and 265 $Fe_{61}C_2Si_2$ with a dimer (Table S2 and Table 1, respectively). The composition of the calculated 266 alloys corresponds to Fe-~3 at% C and ~3 at% Si. 267

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Calculations using a 16 Fe atoms supercell (Huang et al. 2005) support a transition from 269 interstitial to substitutional C at pressure of ~ 300 GPa at 0 K. At high temperature, calculations 270 based on the quasiharmonic approximation (QHA) (Li et al. 2019) showed that the substitutional 271 alloy is more stable than the interstitial one at 360 GPa and 6500 K, confirming that the 272 temperature effect does not significantly vary the relative stability (at least at inner core 273 274 pressures). They also showed that the inclusion of C as a dimer is more stable than both substitutional and interstitial alloying at 360 GPa and 0 K, but entropy does not favour the dimer 275 and it becomes less stable than substitutional C at 6500 K (Li et al., 2019). The enthalpies of C in 276 277 different forms calculated at 0 K in this study are shown as a function of pressure in Figure 3.

278 The enthalpy calculations show that interstitial C is more stable than substitutional C below \sim 350 GPa. The difference in transition pressure with Huang et al. (2005) may be due to the size 279 effect and concentration dependence. The dimer is shown to be more stable than both the 280 substitutional and interstitial forms at all examined pressures from 60 to 400 GPa. However, on 281 qualitative grounds, considering the temperature effect and entropy cost in forming the dimer, 282 this could become unfavourable at high temperatures. Quantitative high-temperature calculations 283 will be necessary to determine at what temperature the dimer structure becomes unstable, but, in 284 any case, it definitely has its own stability field over a large pressure range extending to a certain 285 286 temperature. Overall, these theoretical studies show that the interstitial alloy is more stable at low pressures, while the substitutional defect is more stable at 360 GPa. This also implies that a 287 change of C incorporation mechanism is likely under inner-core conditions. 288

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290 The calculated volumes of the alloys considered, together with those that we measured 291 experimentally, at different pressures are displayed in Figure 4.

Substitutional C decreases the volume of the *hcp* unit cell relative to that of pure Fe, while interstitial C increases it. The dimer also increases the volume of *hcp* Fe but to a much lesser degree than the interstitial defect. Calculated and experimentally determined volumes are in excellent agreement at pressures above \sim 60 GPa, suggesting that C in the experimental sample is interstitial.

At lower pressures (below 60 GPa) the calculated volumes are smaller than those determined experimentally. This difference is attributed to the approximations in the theoretical approach. DFT may fail to capture the correct electron correlation at low pressures, becoming more reliable at high pressures as a consequence of the increased delocalization of the electrons This mismatch between calculations and experiments at low pressures has also been observed for *hcp* Fe (Figure
5).

Actually, in the case of pure Fe the difference in vlumes is much more pronounced, with the experimental and computational results starting to agree closely only at ~200 GPa.

To better understand the effect of C and the incorporation mechanism, Figure 6 shows the 305 pressure dependence of the volume difference $\Delta V = V - V_{Fe}$ (where V is the unit-cell volume of the 306 alloy and $V_{\rm Fe}$ is the reference unit-cell volume for hcp Fe) for hcp Fe and Fe-Si-C alloys. The 307 experimental volumes for $Fe_{93}C_4Si_3$ are compared to the different experimental V_{Fe} available in 308 the literature, whereas the calculated volumes are compared to *hcp* Fe from our DFT simulations. 309 Regardless of the reference $V_{\rm Fe}$ used, we can unambiguously confirm that C in the experimental 310 311 sample is not substitutional as the ΔV for substitutional C is negative. In the pressure range of the 312 experiments (50-150 GPa), the experimental volume differences are close, although slightly 313 higher, to those reported for the dimer configuration, whereas at pressures above 200 GPa, all 314 (except when using the EoS of Miozzi, Matas, et al., 2020) approach the ΔV expected for interstitial C. Once considering the different EoS of hcp Fe and the differences among the 315 316 experiments (pressure scales, hydrostaticity, etc) and possible problems with the calculations at 317 low pressures, this analysis seems to suggest that C in the experimental sample is interstitial. The more stable dimer might be present in some domains at lower pressures. Possibly the sample 318 synthesis process at ambient pressure and high temperature favored interstitial carbon. Dimer 319 320 formation in the quenched samples might then be entropically unfavorable or prevented by kinetic barriers and slow diffusion of vacancies. 321

Previous computational studies (Huang et al., 2005; Caracas, 2017; Li et al., 2019) suggest that C
 preferentially occupies the octahedral interstitial site at low pressure. The presence of C slightly

expands the octahedron containing the impurity, resulting in an increase in unit cell volume 324 (Figure 2 and 4). According to Caracas (2017), the volume increase is almost linear with the 325 amount of C and is approximately independent of pressure within the stability field of the *hcp* 326 phase (see their Figure 2). In agreement with Caracas (2017), the volume increase is almost 327 linear with C content, from 1.4 at% (Yang et al. 2019) to 4 at% C (this study) and almost 328 independent of pressure. Contrarily to the other datasets, Fe-6.1 at% C by Yang et al. (2019) 329 seems not to follow the same trend; above ~80 GPa the volume ratio decreases as a function of 330 pressure and is much lower than that expected on the basis of the other data for lower C contents, 331 332 and below that calculated by Caracas (2017) for a similar composition (Figure S2). These data by Yang et al (2019) also disagree with the EoS predicted for a similar composition (Fe-6 at% C) by 333 Huang et al. (2005) and, therefore, it appears that Fe-6.1at% C by Yang et al. (2019) may not be 334 an interstitial solid solution. 335

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To summarise, even a small amount of C in the alloy has a strong effect on the volume of *hcp* Fe. Calculated and experimentally determined volumes are in excellent agreement at pressures above ~ 60 GPa, suggesting that C in the experimental sample is interstitial. While substitutional C decreases the volume of the *hcp* unit cell, interstitial C increases it. The inclusion of C as a dimer, predicted by calculations to be the most stable, also increases the volume of *hcp* Fe, but to a lesser degree than the interstitial defect.

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3.4 Effect of the incorporation mechanism of carbon on the density

The effect of C on the density of *hcp* Fe is critical to estimation of the effect of C on the physical properties of the Earth's inner core. A common practice in mineral physics is to calculate the density of the alloy assuming a substitutional solid solution, which is valid, for example, for an Fe-Si alloys as Si only enters in the *hcp* structure as a substitutional impurity. In this way, the density of a substitutional alloy $A_{1-x}B_x^s$ (s denotes substitutional) is calculated as follows:

349
$$\boldsymbol{\rho} = \frac{\boldsymbol{N} \cdot [(\mathbf{1} - \boldsymbol{x}) \cdot \mathbf{M}_{\mathbf{A}} + \boldsymbol{x} \cdot \mathbf{M}_{\mathbf{B}}]}{\boldsymbol{V}} \qquad (\text{eq. 1})$$

where *N* is the number of atoms in unit cell, *V* is the volume of unit cell. M_A , M_B are the molar mass for A, B and x is the molar fraction.

However, this is not directly applicable for other alloys like Fe-C, as C can have other solid solution mechanisms, such as incorporation interstitially. Following the established approach for *fcc* Fe-C alloys reported for example in Waseda et al. (2011), the density of an interstitial solid solution $A_{1-x}B_x^i$, (i denotes interstitial) in this case, is calculated as follows:

356
$$\boldsymbol{\rho} = \frac{\boldsymbol{N} \cdot \left[\mathbf{M}_{\mathbf{A}} + \frac{x}{1-x} \mathbf{M}_{\mathbf{B}} \right] / \mathbf{V}} \qquad (\text{eq. 2})$$

357 where the number N of atoms in unit cell in *hcp* Fe is two while in *fcc* is four.

The density in the case of the dimer defect, with two C atoms sitting at one lattice site, can be computed as a combination of one substitutional and one interstitial C.

In this study, the density of Fe-C-Si alloy considering Si as substitutional and C as interstitial $A_{1-x-y}B_x^SC_y^I$, is calculated as follow:

362
$$\boldsymbol{\rho} = \frac{\boldsymbol{N} \cdot \left[\left(\frac{1-x-y}{1-y} \right) \cdot \mathbf{M}_{\mathrm{A}} + \frac{x}{1-y} \cdot \mathbf{M}_{\mathrm{B}} + \frac{y}{1-y} \mathbf{M}_{\mathrm{C}} \right] / V \qquad (\text{eq. 3})$$

363 while if both Si and C are substitutional, the density for $A_{1-x-y}B_x^SC_y^S$ is simply:

364
$$\boldsymbol{\rho} = \frac{\boldsymbol{N} \cdot \left[(1 - \boldsymbol{x} - \boldsymbol{y}) \cdot \mathbf{M}_{\mathrm{A}} + \boldsymbol{x} \cdot \mathbf{M}_{\mathrm{B}} + \boldsymbol{y} \cdot \mathbf{M}_{\mathrm{C}} \right]}{\boldsymbol{V}} \qquad (\mathrm{eq.}\ 4)$$

Further details of density calculations of the Fe-C-Si alloy considering Si as substitutional and C both as interstitial and as substitutional are reported in the supplementary material. The derived density evolution for *hcp* Fe-C-Si with pressure is shown in Figure 7.

368

The density of the alloy experimentally determined in this study, with 4 at% C, is ~6.5 % smaller than that of pure Fe when C is assumed to be substitutional, whereas it is only is ~2.5 % smaller when C is assumed to be interstitial, roughly independent of pressure. Density calculations assuming the interstitial incorporation mechanism are in remarkable agreement with DFT calculations also showing a density reduction with added interstitial C in the Fe structure (Caracas, 2017; Li et al., 2018). Caracas (2017) indicated that 1 wt.% (~4.5 at%) C would decrease *hcp* Fe density by approximately 2.8%.

376

Yang et al. (2019) reported the density of Fe-1.37 wt % C (~6.1 at% C) to be 5.6% smaller than 377 the of pure *hcp* Fe. These authors state that interstitial C in the *hcp* structure enhanced the density 378 difference between pure Fe and the alloy. However, although the authors clearly assume C as 379 interstitial, the densities they report (see their Figure 2) were calculated assuming substitutional 380 solid solution (see our Figure 7). When the densities are calculated starting from the reported 381 volumes using the equation for interstitial carbon, C, the resulting values are anomalously large, 382 even larger than those of pure Fe (see Figure 7). As already discussed, it appears that the Fe-1.37 383 wt. % C studied by Yang et al. (2019) might not be an interstitial solid solution. 384

385

The correct interpretation of the incorporation mechanism is essential for evaluating the identity and amount of light element(s) in the Earth's core. As reported above, the incorporation 388 mechanism affects both the volume and the mass of the unit cell. Substitutional C lowers the volume of the *hcp* unit cell while the interstitial and, to a lesser degree, the dimer mechanisms 389 increase it (Figure 4 and 6). At the same time, the interstitial mechanism increases the mass 390 while the substitutional mechanism reduces it. The two effects act to cancel each other out when 391 calculating density. In fact, as shown by our DFT simulations, in which both the changes in unit-392 cell volume and unit-cell contents are taken into account in an internally consistent way, the 393 density difference between a substitutional, interstitial and dimer alloy is almost negligible 394 (Figure 8). 395

396

This is different in the case of the experiments, in which the volume is directly measured, 397 without any prior assumption of C distribution. However, assumptions of the effective mass of 398 the unit-cell have to be made to derive density from the measured volumes. As illustrated in 399 Figure 8, the way this is done might lead to very different densities with very dissimilar 400 conclusions concerning the amount of light elements required to account for PREM values. In 401 particular, if the volume measured in the experiments is due to an interstitial mechanism but is 402 interpreted as being substitutional, the resulting density will be significantly lower, and a much 403 smaller concentration of light element would be required to match PREM. These results indicate 404 that it is vital to know the exact incorporation mechanism of the alloy and its effect on the elastic 405 properties with changing pressure and temperature, to correctly interpret the properties of the 406 407 inner core. To this end, the combination of experiments and theoretical studies provide unique capabilities. 408

409

410 4 Conclusions

We have carried out volume and density measurements on an *hcp* Fe-C-Si alloy with 4 at% C and 3 at% Si up to ~150 GPa, using a combination of synchrotron powder diffraction and DFT simulations, carried out at the same pressure conditions, to facilitate interpretation of the experiments and to validate the theoretical approach. In particular *hcp* Fe-C-Si alloys with a different incorporation mechanism - substitutional, interstitial, and dimer - were examined to precisely address the effect of C on the physical properties of the alloy.

417

Enthalpy calculations suggest that interstitial C is more stable than substitutional C below ~350 418 419 GPa. While the dimer, with two C atoms replacing an Fe atom, is more stable than both the interstitial and substitutional C, it is destabilized by temperature and, at ambient temperature, it 420 can be kinetically inhibited by a diffusion barrier. Indeed, we find that C in the experimentally 421 422 investigated hcp Fe-C-Si alloy sample takes the interstitial form, as supported by the very good agreement at pressure above ~ 60 GPa between the experimentally determined compression curve 423 424 and that obtained from calculations for the interstitial alloy. Calculated values for density in the 425 literature are often determined assuming a substitutional solid solution. However, this is not applicable for an interstitial-type solid solution, such as that reported here. Assuming the wrong 426 incorporation mechanism when estimating density from measured volumes can lead to incorrect 427 density determination, resulting in a more enhanced density contrast between the alloy and pure 428 Fe. In turn, this can affect density-velocities systematics (Birch's law) resulting in erroneous 429 interpretation of inner core composition. Finally, the agreement between our experiments and 430 calculations, supports previous work (Li et al., 2018) suggesting Si and C combined can account 431 fore core densities and velocities. 432

433

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- 442

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V_0 (Å ³)	K_0 (GPa)	K	composition	Reference		
experiments						
22.7(2)	148(18)	5.8(4)	Fe-4 at% C-3 at% Si	This study		
22.524(62)	172.4(6.0)	4.64(14)	Fe-8 at% Si	Edmund et al. (2019)		
22.5(1)	170(10)	4.7(2)	Fe-8 at% Si	Edmund et al. (2019) (our fit)		
22.37(4)	168.9(4.8)	5.18(14)	Fe-1.4 at% C	Yang et al. (2019)		
22.40(6)	164(8)	5.4(3)	Fe-1.4 at% C	Yang et al. (2019) (our fit)		
22.37(6)	182.2(4.2)	4.68(17)	Fe-6.1 at% C	Yang et al. (2019)		
22.44(5)	174(6)	4.88(18)	Fe-6.1 at% C	Yang et al. (2019) (our fit)		
22.26(6)	169.7(5.2)	5.19(16)	Fe	Yang et al. (2019)		
22.32(5)	163(5)	5.36(5)	Fe	Yang et al. (2019) (our fit)		
22.468(24)	165(fixed)	4.97(4)	Fe	Dewaele et al. (2006)		
22.31(8)	178(7)	4.75(12)	Fe	Dewaele et al. (2006) (our fit)		
22.80(2)	129(6)	6.2(2)	Fe	Miozzi, Matas, et al. (2020)		
22.80(7)	130(6)	6.2(2)	Fe	Miozzi, Matas, et al. (2020) (our fit)		
22.468 (fixed)	155.3(2.2)	5.37(14)	Fe	Sakai et al. (2014)		
22.5(2)	151(15)	5.5(4)	Fe	Sakai et al. (2014) (our fit)		
calculations						
20.40	281	4.4	$Fe_{60}C_2$ ^s Si ₂	This study		
20.85	305	4.3	$Fe_{62}C_2^iSi_2$	This study		
20.39	313	4.3	$\operatorname{Fe}_{61}\operatorname{C_2}^{d}\operatorname{Si}_2$	This study		
21.46	291	4.34	Fe-6 at% C ⁱ	Huang et al. (2005)		
20.29	312	4.3	Fe	This study		

^s substitutional C, ⁱ interstitial C, ^d dimer C

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639 Figure Captions

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Figure 1. Angle-dispersive x-ray powder diffraction pattern of hcp Fe-C-Si alloy (+ Ne) 641 collected at ~140 GPa and 300 K. The reflection positions of Neon (orange) and Fe-C-Si alloy 642 (grey) are shown by the tick markers. 643 Figure 2. a) Unit-cell volume of Fe-C-Si alloy as a function of pressure. Error bars are smaller 644 than symbols. Lines correspond to the fit of the data to a third-order Birch-Murnaghan equation 645 of state. Literature data for Fe-alloys (Edmund et al., 2019; Yang et al., 2019) and reference EoS 646 for pure Fe (Dewaele et al., 2006; Sakai et al., 2014; Yang et al., 2019; Miozzi, Matas, et al., 647 2020) are shown for comparison. Inset: confidence ellipses centred on each data point, 648 representing the 68.3% confidence level. The reported 1σ error bars for K_0 and K_0' were derived 649 from least-squares refinements. b) Unit-cell volumes of Fe-C alloys showing effect of C in the 650 unit cell volume of *hcp* Fe. 651 Figure 3. Calculated enthalpy of carbon (referenced to the isolated atom) in the form of 652 substitutional, interstitial and dimer defects in the *hcp* Fe-C alloy at 0 K. 653 Figure 4. Calculated and measured unit-cell volumes of Fe-C-Si alloys with pressure showing 654 the effect of different incorporation mechanisms of C in the alloy structure. The solid lines are 655 the fit of the data to a third-order Birch-Murnaghan EoS. The red solid line corresponds to the 656 EoS of Fe calculated in this study. 657 Figure 5. Comparison between calculations and experiments for *hcp* Fe and Fe-C-Si alloys as a 658 function of pressure. 659 **Figure 6.** Volume variation, $\Delta V = V - V_{\rm Fe}$ (where V is the unit-cell volume of the alloy and $V_{\rm Fe}$ is 660

the reference unit-cell volume for *hcp* Fe) for *hcp* Fe alloys as a function of pressure. For the

experimental results from the present work different reference EoS for pure Fe (Dewaele et al., 2006; Sakai et al., 2014; Yang et al., 2019; Miozzi, Matas, et al., 2020) were taken from the literature. The calculated volumes of $Fe_{62}C_2Si_2$ (C interstitial), $Fe_{61}C_2Si_2$ (dimer) and $Fe_{60}C_2Si_2$ (substitutional) are normalized to *hcp* Fe from our DFT simulations.

Figure 7. Density evolution with pressure for different *hcp* Fe-C-Si alloys. Solid lines represent densities calculated considering C as interstitial, whereas dashed lines are densities calculated assuming C as substitutional, starting from the same volumes.

Figure 8. Density variation with pressure for different hcp Fe-C-Si alloys calculated at 300 K 669 (experiments) and 0 K (DFT calculations). Dashed lines represent density calculated considering 670 C as substitutional, solid lines are densities calculated assuming C as interstitial and dotted lines 671 are densities calculated with C as a dimer. For simplicity, only recent data for pure Fe from Yang 672 et al. (2019) are shown as reference. The DFT calculations, in which both the changes in unit-673 cell volume and unit-cell contents are taken into account in an internally consistent way, show 674 that the density difference between a substitutional, interstitial and dimer alloy is almost 675 negligible. In the case of the experiments, the volume is directly measured, without any *a priori* 676 assumption of C distribution and assumptions of effective mass of the unit-cell have to be made 677 to derive density from the measured volumes. If the experimental results are interpreted 678 incorrectly as substitutional, then it would erroneously predict much lower densities. 679

- 680
- 681
- 682

683

Figure 1.

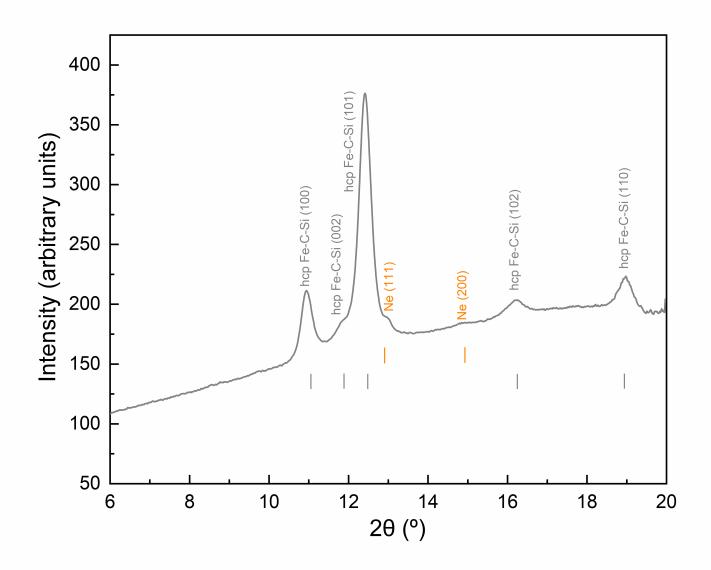


Figure 2.

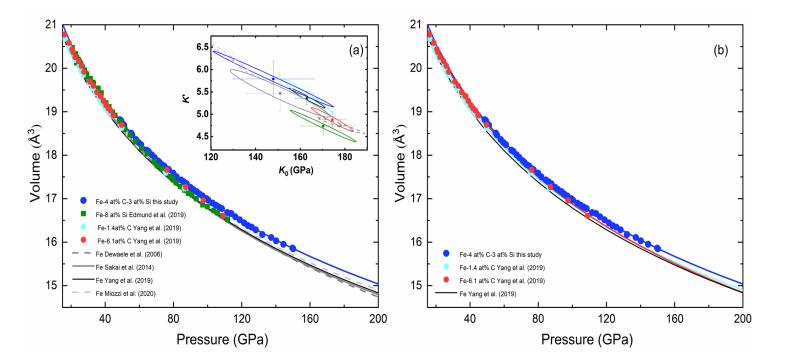


Figure 3.

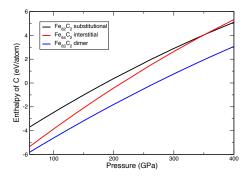


Figure 4.

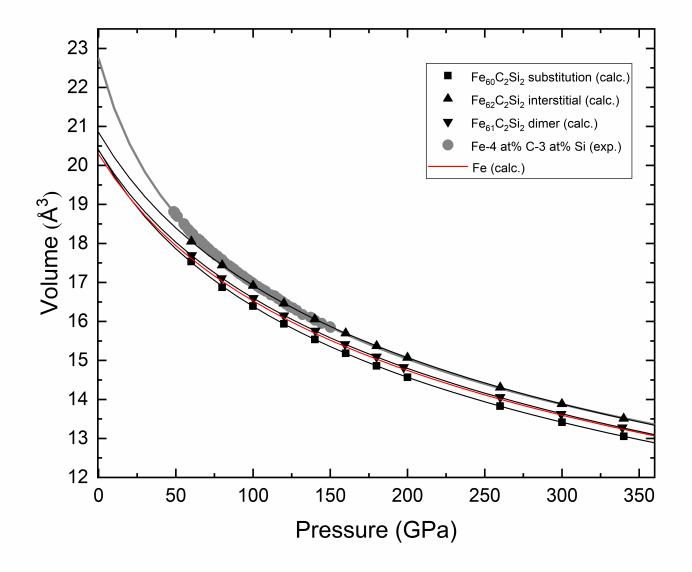


Figure 5.

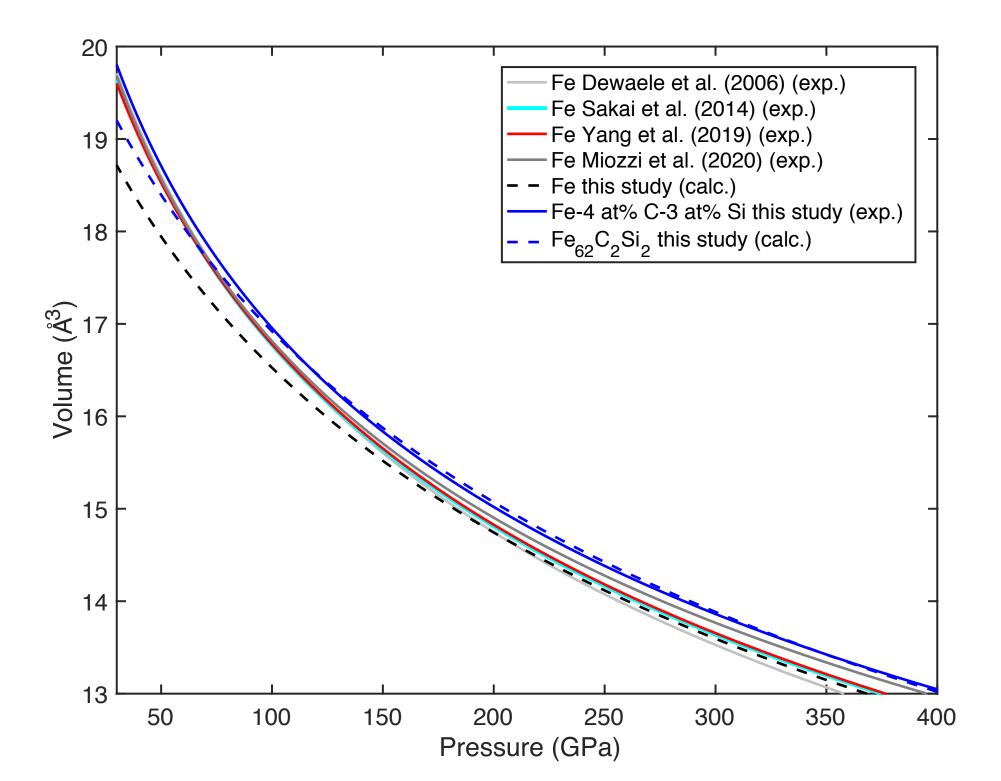


Figure 6.

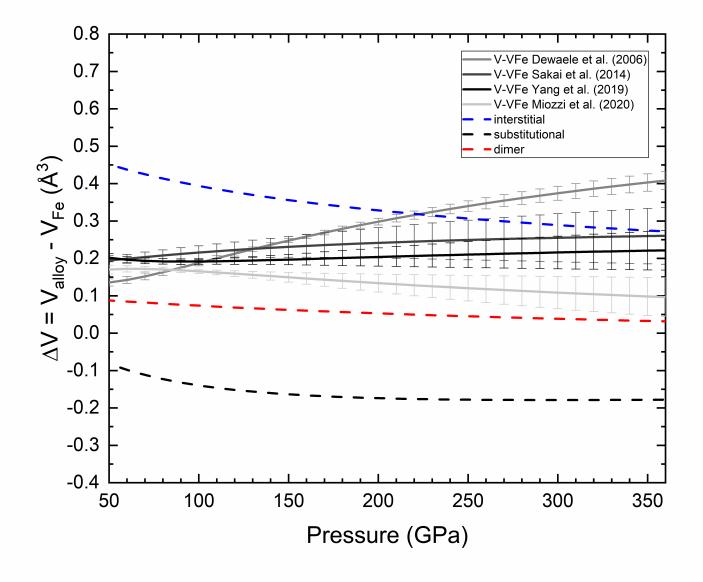


Figure 7.

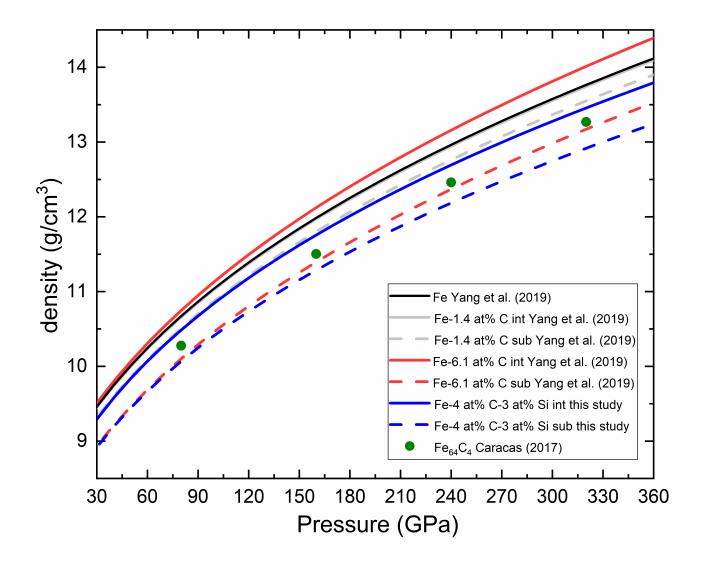
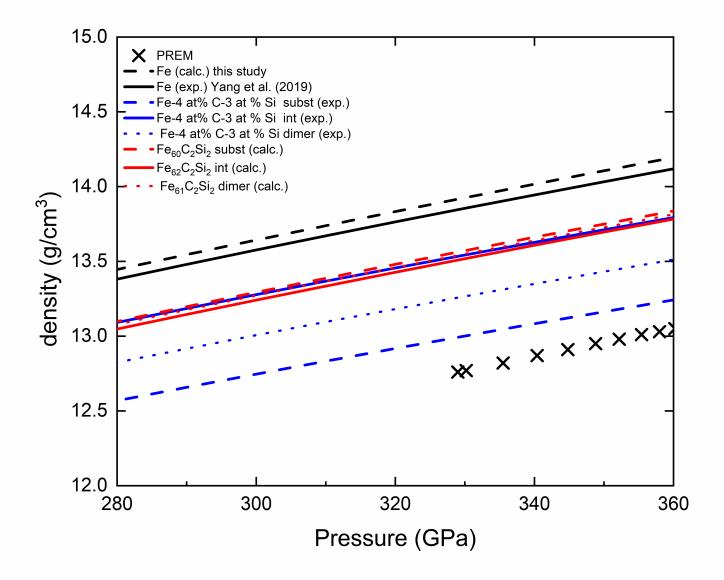


Figure 8.



Supporting Information for

Equation of State of *hcp* Fe-C-Si Alloys and the Effect of C Incorporation Mechanism on the Density of *hcp* Fe alloys at 300 K

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Text S1 Figure S1 to S2 Tables S1 to S3 Tables S4 (File uploaded separately)

Introduction

These supplemental materials include: Figure S1, showing details of the different configurations of C in *hcp* Fe; Figure S2, showing the effect of interstitial C on the volume of *hcp* Fe; Text S1, describing the density calculations for different alloys: substitutional, interstitial and dimer; Tables S1 and S2, reporting unit cell parameters and density of *hcp* Fe-Si-C from experiments and calculations, respectively; Table S3, summarizing the results from enthalpy calculations; Table S4 (uploaded separately) reporting the resulting stress matrices.

Text S1.

As reported in the main text, we denote the alloy with both substitutional and interstitial solute elements as $A_{1-x-y}B_x^S C_y^I$, where A is the solid solvent, B and C are the substitutional and interstitial solutes, respectively. *x* and *y* are the molar fraction. The density of the alloy can be calculated by

$$\rho = \frac{N \cdot \left[\left(\frac{1 - x - y}{1 - y} \right) \cdot M_{A} + \frac{x}{1 - y} \cdot M_{B} + \frac{y}{1 - y} M_{C} \right]}{V}$$

where *N* is the number of atoms in unit cell, and it is two for the *hcp* structure. *V* is the volume of unit cell. M_A , M_B and M_C are the molar mass for A, B and C respectively.

In the case of Fe- 4 at% C and 3 at% Si investigated in this study, the density at P= 48.8 GPa, V = 18.816 Å³ (see Table S1) is calculated:

ρ

$$=\frac{2 \cdot \left[\left(\frac{1-0.03-0.04}{1-0.04}\right) \cdot 55.845 + \frac{0.03}{1-0.04} \cdot 28.085 + \frac{0.04}{1-0.04} \cdot 12.011\right]}{11.331}$$

 $\rho = 9.792 \ g/cm^3$

Where $V = 18.816 \times 10^{-24} \times 6.022 \times 10^{23} = 11.331 \ cm^3$

The density calculations for other alloys can be derived from the equation above. If both B and C are substitutional, the density for $A_{1-x-y}B_x^S C_y^S$ is then:

$$\rho = \frac{N \cdot [(1 - x - y) \cdot M_{A} + x \cdot M_{B} + y \cdot M_{C}]}{V}$$

In the case of Fe- 4 at% C and 3 at% Si investigated in this study, the density at P= 48.8 GPa, V = 18.816 Å³ (see Table S1) is calculated:

$$\rho = \frac{2 \cdot \left[(1 - 0.03 - 0.04) \cdot 55.845 + 0.03 \cdot 28.085 + 0.04 \cdot 12.011 \right]}{11.331}$$

$$\rho = 9.400 \ g/cm^3$$

If both B and C are interstitial, the density for $A_{1-x-y}B^I_xC^I_y$ is then

$$\rho = \frac{N \cdot \left[\mathsf{M}_{\mathsf{A}} + \frac{x}{1 - x - y} \mathsf{M}_{\mathsf{B}} + \frac{y}{1 - x - y} \mathsf{M}_{\mathsf{C}} \right]}{V}$$

The dimer defect with two C atoms sitting at one lattice site is actually also a combination of one substitutional and one interstitial C.

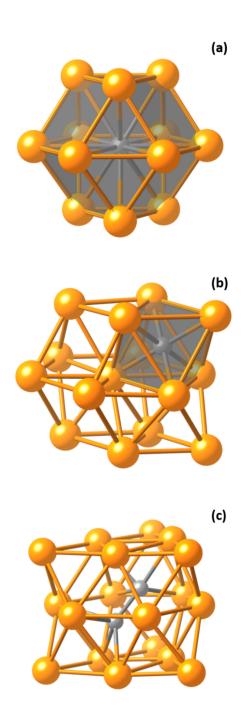


Figure S1. Illustration of the different configurations of C in *hcp* Fe. (a) substitutional; (b) octahedral interstitial site; (c) 2 C atoms occupying one atomic site (dimer defect)

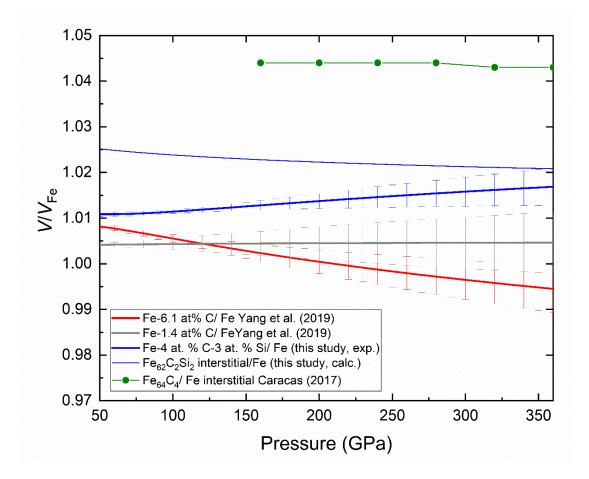


Figure S2. Pressure dependence of the volume ratio V/V_{Fe} (where V is the unit-cell volume of the alloy and V_{Fe} is the reference unit-cell volume for *hcp* Fe) for *hcp* Fe and Fe-Si alloys with different amounts of C. For all of the experimental results, i.e. those from the present work and those from Yang et al. (2019), the reference V_{Fe} was taken from Yang et al. (2019). In addition, the calculated volumes of Fe₆₂C₂Si₂ (C interstitial) normalized to *hcp* Fe from our DFT simulations, are also plotted, as are the results for Fe₆₄C₄ (~6 at% C) by Caracas (2017).

P(GPa)	$V(\text{\AA}^3)$	a (Å)	$c(\text{\AA})$	$\rho(g/cm^3)^{int}$	$\rho(g/cm^3)^{sub.}$	$\rho(g/cm^3)^{dimer}$
48.6(4)) 18.816(11)	2.388(3)	3.812(15)	9.792	9.400	9.592

49.3(4)	18.799(11)	2.387(3)	3.811(14)	9.801	9.409	9.601
49.9(5)	18.742(11)	2.385(3)	3.805(15)	9.831	9.437	9.630
51.1(4)	18.691(11)	2.383(3)	3.802(14)	9.858	9.463	9.656
55.2(5)	18.507(11)	2.375(3)	3.788(14)	9.956	9.557	9.753
55.8(5)	18.464(11)	2.373(3)	3.785(14)	9.979	9.580	9.775
58.0(5)	18.359(11)	2.369(3)	3.777(14)	10.036	9.634	9.831
59.6(6)	18.291(11)	2.366(3)	3.772(14)	10.073	9.670	9.868
61.0(6)	18.244(11)	2.364(3)	3.769(13)	10.099	9.695	9.893
64.3(4)	18.115(11)	2.360(4)	3.756(16)	10.171	9.764	9.963
65.4(4)	18.088(11)	2.358(4)	3.757(15)	10.171	9.779	9.903 9.978
67.4(5)	18.009(11)	2.355(4)	3.749(16)	10.231	9.822	10.022
69.0(5)	17.944(11)	2.353(4) 2.352(4)	3.749(10)	10.251	9.857	10.022
71.0(4)	17.868(11)	2.332(4) 2.349(4)	3.744(17) 3.738(17)	10.208	9.899	10.038
71.9(5)	17.836(11)	2.349(4)	3.738(17) 3.737(17)	10.312	9.899 9.917	10.101
					9.917 9.948	10.119
73.3(5)	17.780(11)	2.346(4)	3.730(17)	10.363		
74.6(5)	17.750(11)	2.344(4)	3.730(16)	10.380	9.965	10.168
76.0(5)	17.691(11)	2.342(4)	3.725(16)	10.415	9.998	10.202
77.2(5)	17.670(11)	2.340(4)	3.725(15)	10.427	10.010	10.214
79.9(5)	17.587(10)	2.336(4)	3.720(16)	10.476	10.057	10.262
81.8(5)	17.505(10)	2.334(4)	3.709(17)	10.525	10.104	10.310
84.6(6)	17.429(10)	2.330(4)	3.707(16)	10.571	10.148	10.355
86.1(6)	17.387(10)	2.328(3)	3.703(15)	10.597	10.173	10.381
87.7(7)	17.343(10)	2.326(3)	3.702(14)	10.624	10.199	10.407
90.0(7)	17.262(10)	2.322(3)	3.696(15)	10.674	10.247	10.455
91.6(8)	17.211(10)	2.320(3)	3.691(15)	10.705	10.277	10.487
93.7(8)	17.162(10)	2.317(3)	3.690(14)	10.736	10.306	10.517
95.2(8)	17.107(10)	2.316(3)	3.683(14)	10.770	10.339	10.551
97.0(7)	17.064(10)	2.313(3)	3.683(14)	10.797	10.365	10.577
99.4(7)	17.000(10)	2.310(3)	3.679(13)	10.838	10.405	10.617
100.3(7)	16.969(10)	2.309(3)	3.675(13)	10.858	10.424	10.636
103.0(7)	16.898(10)	2.305(3)	3.672(13)	10.903	10.467	10.681
105.0(6)	16.848(10)	2.303(3)	3.668(14)	10.936	10.498	10.713
106.7(6)	16.808(10)	2.302(3)	3.664(15)	10.962	10.523	10.738
108.3(6)	16.772(10)	2.299(3)	3.663(14)	10.985	10.546	10.761
111.6(6)	16.680(10)	2.295(3)	3.656(13)	11.046	10.604	10.821
113.9(7)	16.659(10)	2.294(3)	3.656(12)	11.061	10.618	10.834
116.3(8)	16.583(10)	2.291(3)	3.649(13)	11.111	10.666	10.884
118.9(8)	16.515(10)	2.289(3)	3.641(12)	11.156	10.710	10.929
121.4(9)	16.443(10)	2.286(3)	3.633(13)	11.205	10.757	10.976
123.7(10)	16.380(9)	2.283(3)	3.628(13)	11.248	10.798	11.019
125.8(10)	16.345(9)	2.281(3)	3.626(12)	11.272	10.821	11.043
128.3(9)	16.283(9)	2.279(3)	3.619(12)	11.315	10.863	11.085
132.0(12)	16.176(9)	2.275(3)	3.609(12)	11.390	10.935	11.157
137.2(10)	16.108(9)	2.271(3)	3.606(12)	11.438	10.981	11.205
140.2(10)	16.030(9)	2.268(3)	3.598(12)	11.494	11.034	11.260
144.4(10)	15.952(9)	2.266(3)	3.586(13)	11.550	11.088	11.315
149.8(10)	15.863(9)	2.263(3)	3.576(15)	11.615	11.150	11.378
150.1(9)	15.848(9)	2.262(3)	3.575(14)	11.626	11.161	11.389

 Table S1. Experimental lattice parameters and resulting densities of Fe-C-Si alloy

	Fe		Fe ₆₀ C ₂ Si ₂ substitutional			Fe ₆₂ C ₂ Si ₂ interstitial			Fe ₆₁ C ₂ Si ₂ dimer		
Р	V	ρ	Р	V	ρ	Р	$V(\lambda^3)$	ρ	Р	$V(\lambda^3)$	ρ
(GPa	$(Å^3)$	(g/cm ³	(GPa)	(Å ³)	(g/cm ³	(GPa)	$V(A^3)$	(g/cm^3)	(GP	$V(\mathbf{A}^{2})$	(g/c

)))				a)		m ³)
	17.61	10.52	59.98	17.53		60.01	18.0	10.18		17.69	
60	7	8	3	5	10.153	1	53	3	60.33	7	10.224
	17.02	10.89	80.00	16.87		80.00	17.4	10.53		17.10	
80	9	1	3	7	10.549	1	42	9	79.702	4	10.578
	16.52	11.22	99.98	16.39		100.0	16.9	10.86		16.59	
100	2	5	7	5	10.859	1	19	5	100.254	5	10.903
	16.08			15.93		119.9	16.4	11.16		16.15	
120	6	11.53	120	7	11.171	9	63	6	119.975	1	11.481
	15.69	11.81		15.53		140.0	16.0	11.44		15.76	
140	8	5	140	7	11.459	1	60	7	140.389	0	11.737
	15.34	12.08	159.9	15.18			15.6	11.71		15.41	
160	9	4	8	3	11.726	160	98	1	159.567	5	11.987
	15.03	12.33	179.9	14.86			15.3			15.09	
180	2	8	9	2	11.98	180	71	11.96	179.979	5	12.205
	14.74			14.57			15.0	12.19		14.82	
200	3	12.58	200	0	12.22	200	72	7	197.562	5	12.87
	14.00	13.24	259.9	13.82		260.0	14.3	12.84		14.05	
260	8	1	9	8	12.875	1	09	7	260.014	8	11.203
	13.59	13.64	300.0	13.41		299.9	13.8			13.62	
300	6	1	1	6	13.271	9	85	13.24	299.51	6	13.279
	13.23	14.01		13.05		339.9	13.5	13.60		13.27	
340	1	8	340	3	13.639	9	09	8	339.157	8	13.627

Table S2. Volumes and densities of *hcp* Fe and Fe-C-Si alloys from DFT calculations.

$\mathbf{D}(\mathbf{C}\mathbf{D}_{2})$	E	nthalpy (eV/ator	n)
P (GPa)	$Fe_{62}C_2$	$Fe_{64}C_2$	Fe ₆₃ C ₂
60	-3.68	-5.39	-5.85
80	-3.05	-4.59	-5.22
100	-2.48	-3.77	-4.53
120	-1.92	-3.14	-4.10
140	-1.20	-2.39	-3.40
160	-0.79	-1.77	-2.93
180	-0.27	-1.14	-2.26
200	0.37	-0.47	-1.85
260	2.00	1.36	-0.35
300	3.01	2.77	0.93
340	3.81	3.90	1.66
360	4.18	4.24	2.13
400	5.11	5.27	3.11

Table S3. Enthalpy calculations for the different alloys