

Baryon-Interacting Dark Matter: heating dark matter and the emergence of galaxy scaling relations

Benoit Famaey, Justin Khoury, Riccardo Penco, Anushrut Sharma

▶ To cite this version:

Benoit Famaey, Justin Khoury, Riccardo Penco, Anushrut Sharma. Baryon-Interacting Dark Matter: heating dark matter and the emergence of galaxy scaling relations. Journal of Cosmology and Astroparticle Physics, 2020, 2020 (06), pp.025-025. 10.1088/1475-7516/2020/06/025. hal-02984987

HAL Id: hal-02984987

https://hal.science/hal-02984987

Submitted on 1 Nov 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Baryon-Interacting Dark Matter: heating dark matter and the emergence of galaxy scaling relations

Benoit Famaey, a,* Justin Khoury, b,† Riccardo Penco, c,‡ and Anushrut Sharmab,§

- ^a Université de Strasbourg, CNRS, Observatoire astronomique de Strasbourg, UMR 7550, F-67000 Strasbourg, France
 - ^b Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104
 - ^c McWilliams Center for Cosmology, Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213

Abstract

The empirical scaling relations observed in disk galaxies remain challenging for models of galaxy formation. The most striking among these is the Mass Discrepancy-Acceleration Relation (MDAR), which encodes both a tight baryonic Tully-Fisher relation (BTFR) and the observed diversity of galaxy rotation curves through the central surface density relation (CSDR). Building on our earlier work [1], we propose here that the MDAR is the result of interactions between baryons and 'Baryon-Interacting Dark Matter' (BIDM), which heat up the dark matter. Following a bottom-up, hydrodynamical approach, we find that the MDAR follows if: i) the BIDM equation of state approximates that of an ideal gas; ii) the BIDM relaxation time is order the Jeans time; iii) the heating rate is inversely proportional to the BIDM density. Remarkably, under these assumptions the set of hydrodynamical equations together with Poisson's equation enjoy an anisotropic scaling symmetry. In the BIDM-dominated regime, this gives rise to an enhanced symmetry which fully captures the low-acceleration limit of the MDAR. We then show that, assuming a cored pseudo-isothermal profile at equilibrium, this set of equations gives rise to parameters reproducing the MDAR. Specifically, in the flat part of the rotation curve the asymptotic rotational velocity matches the parametric dependence of the BTFR. Moreover, in the central region of high-surface brightness galaxies, the profile reproduces the CSDR. Finally, by studying the time-dependent approach to equilibrium, we derive a global combination of the BTFR and CSDR, which matches the expectations in low surface-brightness galaxies. The form of the heating rate also makes model-independent predictions for various cosmological observables. We argue that our scenario satisfies existing observational constraints, and, intriguingly, offers a possible explanation to the EDGES anomaly.

1 Introduction

The nature of the dark sector of the Universe is certainly one of the most important questions of modern physics. Over the years, a picture has emerged in which the Universe is composed of $\sim 5\%$

^{*}benoit.famaey@astro.unistra.fr

[†]jkhoury@sas.upenn.edu

[†]rpenco@andrew.cmu.edu

[§]anushrut@sas.upenn.edu

baryonic matter, $\sim 25\%$ cold dark matter (CDM)—which for all practical purposes does not interact with itself or with baryons—and the rest by a cosmological constant Λ . While this Λ CDM model is very successful on large scales, a few tensions remain.

On cosmological scales, one notable tension is the value of the Hubble constant as inferred from the Cosmic Microwave Background (CMB)—which has drifted towards smaller values together with a larger matter density $\Omega_{\rm m}$ with better successive data from the WMAP mission [2], and even more so after Planck [3]—to be contrasted with the higher value obtained from measurements of Type Ia supernovae and lensing time-delays [4, 5]. Whether this tension might be resolved through understanding systematics or whether it is a sign of new physics is still under debate (see [6] and references therein). Meanwhile, an interesting anomaly has surfaced around redshift $z \sim 20$, where the EDGES experiment has reported an anomalously strong absorption in the measured 21 cm signal [7]. If not due to foreground contamination, this signal might indicate an over-cooling of the HI gas with respect to standard expectations, or a modification of the soft photon background beyond the CMB contribution.

On galactic scales, a number of observational challenges to the standard Λ CDM model have also been actively debated in recent years, as galactic observations and numerical simulations of galaxies have improved in tandem [8]. Galaxy formation and evolution are processes that happen on \sim kpc scales, where the physics of baryons can play a major role through gravitational feedback in modifying the quasi-equilibrium configuration of CDM on secular timescales.

The most interesting challenge is that baryons and dark matter (DM) in galaxies seem to conspire in ways that were a priori unexpected, giving rise to tight scaling relations. The most famous such scaling relation is the baryonic Tully-Fisher relation (BTFR) [9–12], relating the fourth power of the asymptotic circular velocity of disk galaxies to their baryonic mass, $V_{\rm flat}^4 \sim M_{\rm b}$. Interestingly, when matching the mass-function of DM halos to the luminosity function of galaxies—a procedure known as abundance matching (AM)—one gets a stellar-to-halo mass relation that nicely reproduces the normalization of the BTFR [13, 14], especially at baryonic masses around $10^{10} \,{\rm M}_{\odot}$.

However, as shown by [14] using Navarro-Frenk-White (NFW) profiles [15] for the assigned DM halos, the AM-predicted curvature of the BTFR is at odds with the data. This might be attributable to large uncertainties in AM at low masses, but is definitely problematic at high masses (above stellar masses of $\sim 10^{11} M_{\odot}$) where AM systematically overpredicts the halo mass of disk galaxies [16]. Furthermore, the observed small intrinsic scatter (only ~ 0.025 dex for the orthogonal scatter) of the BTFR is in 3.6σ disagreement with AM expectations [14]. While some outliers to the BTFR at the low and high-mass ends have been recently pointed out [17, 18] and still need to be confirmed by more observations due to possible systematics (e.g., on the inclination at the low mass end) or unknowns (e.g. on the asymptotic flat velocity and on the total gas mass at the high mass end), the tightness of the BTFR for the bulk of low-z high-quality galaxy rotation curves remains challenging in the Λ CDM context.

Another aspect of the baryon-DM conspiracy is the *diversity* of rotation curves. Galaxies with the same asymptotic circular velocity—hence "twins" of identical total baryonic mass on the BTFR—can display a broad range of rotation curve *shapes*, consistent with central DM densities ranging from cuspy NFW-like central profiles as predicted in DM-only simulations, to very large, constant-density cores of DM [19]. There is in fact a positive correlation between the average DM density

within 2 kpc and the baryon-induced rotational velocity at that radius [20]. The circular velocity slope close to the center is thus directly correlated to the surface density of baryons. In other words, the rotation curve shapes of late-type spiral galaxies are all similar when expressed in units of disk scale-length [21], and the DM core size correlates with scale-length [22].

Another way to express this correlation is the *central surface density relation* [CSDR, 23] between the central surface density of stars and the central dynamical surface density, related to the slope of the rotation curve. For small disk galaxies dominated by DM, the expectation *a priori* would have been instead that galaxies at a given maximum velocity scale display *similar* rotation curves because they should be embedded in similar DM halos. Thus this can be considered as a strong version of the old "core-cusp" problem [24].

The diversity of galaxy rotation curves at a given velocity scale, their uniformity at a given baryonic surface density scale, together with the BTFR, can be summarized through what is nowadays known in disk galaxies as the *Radial Acceleration Relation* (RAR), or more generally as the *Mass Discrepancy-Acceleration Relation* (MDAR). This encodes a unique observational relation between the total gravitational field and the Newtonian acceleration generated by baryons at every radius [25–29].

While the general shape of the MDAR might be a natural outcome of Λ CDM [13, 30–33], and despite debates on its universality [34–36], its normalization and very small scatter, the latter which could be entirely accounted for by observational errors on the inclination and distance of galaxies, remain puzzling [37]. For instance, it has recently been argued that feedback becomes efficient at a characteristic acceleration scale similar to the one present in the MDAR, thereby explaining the transition from baryon-dominated to DM-dominated regimes in the MDAR [38]. While interesting, this does not per se explain the details of the diversity of rotation curves encoded in the tightness of the MDAR, which should be related to the subtleties of the core-cusp transformation process.

While the MDAR reduces to the BTFR in the flat part of rotation curves, the fact that galaxies obey the BTFR does not a priori imply that they will obey the MDAR in the rising parts of rotation curves. The fact that they do observationally is at the root of the diversity problem, as shown in [20]. As reported by [19], feedback in cosmological hydrodynamical simulations from the EAGLE and APOSTLE projects is unable to produce large constant-density cores of DM as required by the data in a significant fraction of low-mass disk galaxies. On the other hand, the recent NIHAO simulations [39, 40] are much more efficient at forming cores and predict a tight MDAR, but in turn have problems at reproducing the most cuspy, steeply rising rotation curves [20]. This illustrates that the effect of feedback on the central DM distribution in various cosmological hydrodynamical simulations is still far from settled, and that reproducing in detail the observed diversity of rotation curve shapes together with a tight MDAR still raises an interesting challenge for simulations of galaxy formation.

1.1 Approaches to the MDAR

In this context, it is natural to explore whether the above challenges find their root in a modification of the fundamental nature of DM. Alternatives to Λ CDM exploring different properties of the DM sector are usually concerned with changing the DM particle mass [41] or self-interactions [42].

Interactions with photons [43] or neutrinos [44] in the early Universe have also been considered, affecting the linear regime. In self-interacting DM some recent encouraging results have shown how underdense halos can indeed be associated with extended baryonic disks [45–47], in line with the trend of the MDAR. While very encouraging, rotation curve fits are still made with two parameters [47] and do not fully explain *ab initio* the tightness of the BTFR, as well as, *e.g.*, its tension with AM at high masses.

Given the tight correlation between the Newtonian gravitational field generated by baryons and the total gravitational field, the most direct and also most radical alternative explanation is that the gravitational law is, at least effectively, modified in galaxies [48–50]. This paradigm, known as Modified Newtonian Dynamics (MOND) [51, 52] was proposed almost 40 years ago by Milgrom [53, 54]. Within this framework, the MDAR was actually predicted well before it was precisely assessed by observations. The challenge with this approach is to reproduce the large-scale successes of Λ CDM, in particular the exquisitely-measured CMB temperature anisotropies. There are additional challenges with the mass discrepancy in galaxy clusters [52, 55], subgalactic scales [e.g., 56], as well as solar system constraints [57, 58] (though see [59]).

Less radical is the idea that DM acts as CDM cosmologically but generates an effective modification of gravity on galactic scales through long-range interactions [60–64]. A recent prototypical example in this category is based on DM superfluidity [65–79]. More radical approaches include Modified DM [80–83] and Verlinde's emergent gravity [84, 85], both inspired by gravitational thermodynamics. All such approaches boil down to some version of MOND on galaxy scales.

Another route, as yet very much unexplored, is that the tight conspiracy between the distribution of baryons and the gravitational field in galaxies is the outcome of relatively short-range interactions between baryons and DM, which reorganize the DM distribution in the desired way without effectively modifying gravity.

In [1] we proposed a novel mechanism along these lines. The idea put forward was that the desired DM profile may naturally emerge as the equilibrium configuration resulting from DM-baryon short-range (collisional) interactions. This required replacing the traditional collisionless Boltzmann equation describing the DM fluid by a collisional Boltzmann transport equation with two fluids. The first and second order moments of this equation yield respectively the traditional Jeans' equation (akin to hydrostatic equilibrium) and a heat transport equation describing the exchange of energy between baryons and DM. For static and isotropic configurations, the heat equation implies an actual equilibrium between the divergence of the heat flux within the DM fluid and the heating rate due to baryons. By retro-engineering the observationally-inferred knowledge of the MDAR in rotationally-supported disk galaxies, it was shown that an equilibrium configuration reproducing the MDAR can be attained if: i) the heating rate is inversely proportional to the DM density; and ii) if the relaxation time of DM particles is comparable to the dynamical time.

Specifically, in [1] we concentrated on collisional interactions between heavy DM particles and baryons, in which baryons effectively cooled the DM medium. We could then demonstrate that, as long as the BTFR was obeyed at large radii, the MDAR would be satisfied at all radii. While setting the stage for follow-up studies, our original model suffered from a few important caveats. Firstly, the BTFR had to be assumed at equilibrium, and it was unclear how it might be achieved in the time-dependent case. Secondly, since the mechanism relied on cooling the DM fluid to reach

equilibrium, one would need to start from relatively hot initial conditions, in contradiction with the successes of Λ CDM on large scales, or, alternatively, the center of DM halos would need to be strongly up-scattered by very efficient feedback before being allowed to cool again. An additional concern is that the cooling mechanism could lead, in self-consistent simulations, to flattened DM halos or prominent dark disks, once halos have an initial spin. Finally, we assumed that we could coarse-grain the baryonic and DM distribution functions over a typical scale of a few pc, which cannot be the case for purely collisional interactions between DM particles and stars without strongly enhancing the DM density around stars.

1.2 Baryon-interacting DM

In this paper we build on and further develop the original scenario of [1] in several crucial ways. Most importantly, instead of baryon-DM interactions cooling the DM medium, we now focus exclusively on the case where the DM fluid is *heated* by baryons. This is a priori more desirable from the point of view of galaxy formation, since DM heating can transform cusps into cores in central regions of galaxy halos. It also avoids the concern of forming flattened halos or dark disks. A second key difference pertains to the form of DM-baryon interactions. Whereas our original analysis [1] focused exclusively on short-range particle-particle collisions between DM and baryons, in the present analysis we remain general about the form of such interactions, which could happen on a pc-range.

The basic framework is otherwise similar to [1]. After reviewing the MDAR in Sec. 2, we set up in Sec. 3 a bottom-up approach to identify phenomenologically the kind of DM-baryon interactions necessary to reproduce the MDAR. By taking the first few velocity moments of a collisional Boltzmann transport equation, we obtain a hydrodynamical description of DM governed by a continuity equation, a Jeans' or momentum equation, and, crucially, a heat equation describing energy exchange between DM and baryon components. These are supplemented by the standard Poisson equation determining the gravitational field.

The microphysics of DM is encoded in three physical quantities. The first quantity is the DM equation of state, $P = P(\rho, v)$, specifying the pressure as a function of density ρ and velocity dispersion v (equivalently, temperature). The second quantity is the relaxation time, t_{relax} , which fixes the thermal conductivity. The relaxation time is the characteristic time for DM to reach equilibrium either through self-interactions or interactions with other sectors, such as baryons. The third quantity is the heating rate, $\dot{\mathcal{E}}$, which is determined by the microphysics of DM-baryon interactions.

Remarkably, the set of hydrodynamical equations is invariant under a one-parameter anisotropic space-time scaling transformation, $\vec{x} \to \lambda \vec{x}$, $t \to \lambda^z t$, for any z, provided that the DM pressure, relaxation time and heating rate transform suitably. We take this as a powerful hint to fix the parametric dependence of each quantity. Starting with the equation of state, it turns out that the ideal gas form

$$P = \rho v^2 \tag{1}$$

is invariant for any z. What makes the ideal gas equation of state particularly appealing is its universality. It is valid as long as DM is sufficiently dilute, in the sense that the average inter-

particle separation is large compared to the mean free path.

The scaling symmetry requires that the relaxation time transform as $t_{\rm relax} \to \lambda^z t_{\rm relax}$. A natural choice in galactic dynamics which satisfies the desired scaling is the Jeans time,

$$t_{\rm relax} \sim \frac{1}{\sqrt{G\rho}}$$
 (2)

We will show that this choice allows us to reproduce the MDAR.

The final ingredient is the heating rate. To fix its form, we assume that the heating rate explicitly breaks scaling invariance for any z except z=1/2. This choice is empirically motivated by the BTFR, since the relevant ratio $V_{\rm flat}^4/M_{\rm b}$ is invariant under the z=1/2 transformation. We will argue in Sec. 3.4 that this scaling, together with physically-plausible assumptions, fixes the dependence of the heating rate to

$$\frac{\dot{\mathcal{E}}}{m} \sim a_0 v \frac{\rho_{\rm b}}{\rho} \,. \tag{3}$$

The proportionality constant, which has units of acceleration, has been fixed empirically to match the MDAR characteristic acceleration scale a_0 . This scale must somehow emerge from the microphysics of DM-baryon interactions.

Once the equation of state, relaxation time and heating rate are fixed, we will show that in the DM-dominated regime our equations enjoy a larger, approximate symmetry. Namely, the circular velocity curves $V_1(R)$ and $V_2(R)$ of two DM-dominated exponential disks with different scale lengths L_1 and L_2 and different total baryonic masses $M_{\rm b,1}$ and $M_{\rm b,2}$ must be related by:

$$V_2(R) = \left(\frac{M_{\rm b,2}}{M_{\rm b,1}}\right)^{1/4} V_1\left(\frac{L_1}{L_2}R\right) . \tag{4}$$

This encodes both the BTFR and the CSDR, at the root of the diversity of rotation curves.

We will then explore in more details in Sec. 4 how Eqs. (1)–(3) are sufficient ingredients to reproduce the MDAR. Specifically, we begin in Sec. 4.1 by recalling how a cored pseudo-isothermal profile can, for suitable choice of its central density and core radius, reproduce the MDAR. Our working assumption, therefore, is that DM halos, through DM self-interactions and baryon-DM energy exchange, reach a cored pseudo-isothermal profile in the region enclosing the galactic disk.

By focusing on static, equilibrium configurations, we proceed in Sec. 4.2 to show that the cored pseudo-isothermal profile, with suitable parameters to reproduce the MDAR, is a solution to our hydrodynamical equations. Specifically, in the flat part of the rotation curve the rotational velocity asymptotes to

$$V_{\text{flat}}^4 \sim a_0 G M_{\text{b}} \log \frac{R_0}{r} \,. \tag{5}$$

The prefactor matches the parametric dependence of the BTFR. Unfortunately within the static analysis we are unable to determine the arbitrary radius R_0 (which must be larger than the galaxy) or its scatter. Meanwhile, in the central region of galaxies, we show in Sec. 4.3 that, for high-surface brightness (HSB) galaxies which are baryon-dominated near the center, the DM profile reproduces the CSDR with the behavior of the 'simple' interpolating function of MOND [86]. In Sec. 4.4 we go beyond the equilibrium treatment and study the time-dependent approach to equilibrium,

considering only average quantities suitable for the DM-dominated regime. This allows us to derive a particular combination of the DM velocity dispersion and surface density, which matches the combination of BTFR and CSDR. Therefore, if one takes the BTFR as a given (per the equilibrium analysis), this constraint yields the central density relation naturally for DM-dominated galaxies.

We move on in Sec. 5 to analyze the astrophysical and cosmological implications of our model. The form of the heating rate (3) allows us to derive very general results, irrespective of the underlying microphysical model. The only assumption is that whatever DM-baryon interactions are at the root of this heat exchange still apply in the astrophysical/cosmological context of interest. For this purpose, the inverse-density dependence of $\dot{\mathcal{E}}/m$ is a welcome feature phenomenologically. It implies a suppressed heat exchange in the early universe, allowing us to comfortably satisfy constraints from the CMB and the large scale structure. Intriguingly, as shown in Sec. 5.3 the heat exchange between DM and baryons, which acts to cool the neutral gas prior to the Cosmic Dawn, provides a possible explanation to the anomalous EDGES signal at $z \simeq 17$. This is unlike other DM-baryon explanations of the EDGES excess, such as millicharged DM, which typically run afoul of CMB constraints [87, 88].

It remains to construct a full-fledged model of particle physics that realizes the desired interactions. In the Conclusions section (Sec. 6) we will discuss various promising avenues for model building to be pursued elsewhere.

2 The MDAR and galactic scaling relations

Since the MDAR (or MOND-like phenomenology) is an empirical fact about rotationally-supported galaxies, the scaling relations it implies must emerge in any phenomenologically-viable DM model. To set the stage, we begin with a brief review of the galactic scaling relations of interest.

The MDAR is a relation between the total gravitational field g and the Newtonian acceleration g_b generated by the observed distribution of baryons [28]:

$$g = \begin{cases} g_{\rm b} & g_{\rm b} \gg a_0\\ \sqrt{a_0 g_{\rm b}} & g_{\rm b} \ll a_0 \,, \end{cases}$$
 (6)

where $a_0 \simeq 10^{-10} \,\mathrm{m/s^2}$. Numerically, this characteristic acceleration coincides with the Hubble scale $a_0 \simeq \frac{1}{6} c H_0$. The DM interpretation of the MDAR is that DM should only dominate when the baryonic acceleration drops below a_0 , and furthermore the effect of DM in this regime should be such that $g \simeq \sqrt{a_0 g_{\rm b}}$.

An immediate corollary of the MDAR is the BTFR [11]. At large distances outside the baryon distribution, the baryonic acceleration can be approximated by $g_{\rm b} \simeq G M_{\rm b}/r^2$, where $M_{\rm b}$ is the total baryonic mass. Furthermore, in this regime the DM-dominated relation $g \simeq \sqrt{a_0 g_{\rm b}}$ applies. Substituting $g = V_{\rm flat}^2/r$, where $V_{\rm flat}$ is the rotational velocity, we obtain

$$V_{\text{flat}}^4 = a_0 G M_{\text{b}} \,. \tag{7}$$

Thus the MDAR implies the BTFR in the flat part of rotation curves, but the fact that galaxies obey the BTFR does not imply that they will obey the MDAR in the rising parts of rotation

curves. The fact that they observationally do is at the root of the diversity of rotation curve shapes problem [19, 20].

The diversity of shapes is related to the central surface density relation [CSDR, 23], which is another consequence of the MDAR:

$$\Sigma(0) = \begin{cases} \Sigma_{\rm b}(0) & \Sigma_{\rm b}(0) \gg \frac{a_0}{G} \\ \sqrt{\frac{2}{\pi}} \frac{a_0}{G} \Sigma_{\rm b}(0) & \Sigma_{\rm b}(0) \ll \frac{a_0}{G} \end{cases}, \tag{8}$$

where the central dynamical surface density $\Sigma(0) = \int_{-\infty}^{\infty} dz \, \rho(\vec{x})$, with z denoting the coordinate transverse to the disk, can be evaluated from the rotation curve. Similarly, the baryonic surface density is $\Sigma_b = \int_{-\infty}^{\infty} dz \, \rho_b(\vec{x})$. The dynamical surface density Σ is the sum of Σ_b and the DM central surface density, $\Sigma_{\rm DM}$. For a spherically-symmetric DM profile, the latter is defined by

$$\Sigma_{\rm DM} = 2 \int_0^\infty \mathrm{d}r \, \rho(r) \,. \tag{9}$$

High-surface brightness (HSB) galaxies correspond to $\Sigma_{\rm b} \gg a_0/G$ and are baryon-dominated in the central region. Low-surface brightness (LSB) galaxies have $\Sigma_{\rm b} \ll a_0/G$ and are DM-dominated everywhere.

LSB galaxies are particularly interesting because they imply a scaling symmetry, which is at the root of the MOND paradigm [53, 54, 89]. Indeed the idea of MOND is that below the acceleration scale a_0 , corresponding to the DM-dominated regime, dynamics are invariant under the space-time scaling

$$\vec{x} \to \lambda \vec{x}$$
; $t \to \lambda t$. (10)

This implies, in particular, that, two LSB exponential disks of same total mass M_b but different scale-lengths L_1 and L_2 , will have identical rotation curves expressed in scale-length units. More generally, combining this with the BTFR, the circular velocities V_1 and V_2 of two LSB disks should be related by

$$V_2(R) = \left(\frac{M_{\rm b,2}}{M_{\rm b,1}}\right)^{1/4} V_1\left(\frac{L_1}{L_2}R\right) , \qquad (11)$$

where R is the axisymmetric radius within the galactic plane of each galaxy.

One can think of the above scaling relations as follows. The BTFR (7) is a global constraint, relating the asymptotic rotational velocity to the total baryonic mass at large R. The CSDR (8) constrains the total and baryonic central surface densities as $R \to 0$. For DM-dominated LSB galaxies, these two scaling relations can be summarized by the scale invariant equation (11). More generally, all these scaling relations can be summarized by the MDAR (6), which is a local relation between the baryonic and DM gravitational accelerations valid at every point in the galaxy.

3 Baryon-Interacting Dark Matter

We begin with a brief review of the general framework laid out in [1]. The starting point is a generalization of the usual collisionless Boltzmann equation for DM to a *Boltzmann transport* equation, which includes a collisional integral encoding interactions between DM particles and

baryons. For simplicity, we will restrict our attention to the zeroth, first and second velocity moments of this equation, which respectively enforce mass, momentum and energy conservation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0; \tag{12a}$$

$$\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right) u^i + \frac{1}{\rho} \partial_j P^{ij} = g^i;$$
(12b)

$$\frac{3}{2} \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \right) \frac{T}{m} + \frac{1}{\rho} P^{ij} \partial_i u_j + \frac{1}{\rho} \vec{\nabla} \cdot \vec{q} = \frac{\dot{\mathcal{E}}}{m} \,. \tag{12c}$$

Here, $\vec{u} \equiv \langle \vec{v} \rangle$ is the bulk DM velocity, $P^{ij} \equiv \rho \left\langle \left(v^i - u^i \right) \left(v^j - u^j \right) \right\rangle$ is the pressure tensor, $T \equiv \frac{m}{3} \langle |\vec{v} - \vec{u}|^2 \rangle$ is the local DM temperature, and $\vec{q} \equiv \frac{1}{2} \rho \langle (\vec{v} - \vec{u}) | \vec{v} - \vec{u} |^2 \rangle$ is the heat flux. The local heating rate $\dot{\mathcal{E}}$ is due to interactions with baryons. The (total) gravitational acceleration \vec{g} is determined as usual by the Poisson equation

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \left(\rho + \rho_{\rm b} \right) \,. \tag{13}$$

The baryon mass density $\rho_b(\vec{x})$ will be treated as an input specified by observations. Moreover, in what follows we will be interested in velocity distributions that are approximately isotropic, in which case

$$P_{ij} \simeq P\delta_{ij}$$
 valid for $|\vec{u}| \ll v$, (14)

where we have introduced the one-dimensional velocity dispersion $v = \sqrt{T/m}$.

3.1 General scaling symmetry

Having reviewed the framework of [1], let us discuss the scaling properties of the above equations. Setting $\dot{\mathcal{E}} = 0$ temporarily, notice that (12) and (13) are invariant under the anisotropic space-time scaling transformation

$$\vec{x} \to \lambda \vec{x}$$
; $t \to \lambda^z t$, (15)

valid for arbitrary z, with the various quantities transforming as 1

$$v \rightarrow \lambda^{1-z}v;$$

$$\vec{u} \rightarrow \lambda^{1-z}\vec{u};$$

$$\vec{g} \rightarrow \lambda^{1-2z}\vec{g};$$

$$\rho \rightarrow \lambda^{-2z}\rho;$$

$$\rho_{b} \rightarrow \lambda^{-2z}\rho_{b};$$

$$P^{ij} \rightarrow \lambda^{2-4z}P^{ij};$$

$$\vec{q} \rightarrow \lambda^{3-5z}\vec{q}.$$

$$(16)$$

Notice that the transformation laws for P^{ij} and \vec{q} are compatible with their definition in terms of ρ , \vec{v} and \vec{u} . The above is a symmetry of the collisionless equations. In order for it to survive as

Note that this scaling symmetry is different than the one considered in [1] because ρ_b transforms differently. They agree only for z = 1/2.

a symmetry of the collisional equations (i.e., with non-zero $\dot{\mathcal{E}}$), the heating rate must transform as

$$\frac{\dot{\mathcal{E}}}{m} \to \lambda^{2-3z} \frac{\dot{\mathcal{E}}}{m} \,. \tag{17}$$

The transformation rules (16) and (17) could at first glance be dismissed as a trivial consequence of dimensional analysis, with units of length and time kept separate due to the non-relativistic nature of our system. This becomes more manifest by rescaling ρ , ρ_b , P, \vec{q} and $\dot{\mathcal{E}}$ in Eqs. (16) and (17) by a factor of G—a procedure that does not affect Eqs. (12). Nevertheless, in what follows we will demand that this scaling is actually an emergent symmetry of the DM sector and its interactions with baryons, at least for a specific value of z. This requirement, together with some physically-motivated assumptions, will place stringent constraints on the DM equation of state, the heat flux, and the heating rate.

3.2 DM equation of state

In order to solve Eqs. (12) one must specify, among other things, an equation of state for DM, which for our purposes will be a relation of the form $P = P(\rho, v)$. The explicit form of such a relation depends on the microscopic details of the DM sector. The requirement that the equation of state be scale invariant for some particular value of z places a nontrivial constraint on its functional form.

Remarkably, there is a very general assumption one can make to obtain an equation of state that is scale invariant for any z. Namely, we assume that DM is sufficiently dilute, in the sense that $n\lambda^3 \ll 1$, where $n = \frac{\rho}{m}$ is the number density of DM particles, and $\lambda = \frac{1}{mv}$ their mean thermal wavelength. In this regime one can perform a virial expansion of the DM equation of state, which at lowest order generically reduces to that for an ideal gas:

$$P = \rho v^2 \,. \tag{18}$$

It is easy to check that this relation is the *only* equation of state that is invariant under the symmetry transformations (16) for arbitrary z.

3.3 Heat flux and relaxation time

In the limit where deviations from thermal equilibrium are small,² Fourier's law provides us with an approximate yet explicit expression for the heat flux \vec{q} :

$$\vec{q} \simeq -\kappa m \vec{\nabla} v^2 \,, \tag{19}$$

where κ is the thermal conductivity,

$$\kappa = \mathcal{O}(1) \frac{\rho \, v^2 t_{\text{relax}}}{m} \,, \tag{20}$$

²To be more precise, in the spherically symmetric case we will consider later on, Fourier's law is valid provided $\left|\frac{d \log v^2}{d \log r}\right| \ll 1$.

and t_{relax} denotes the relaxation time. This parameter can be thought of as the characteristic time for DM to reach equilibrium due to self-interactions or interactions with other sectors, e.g. with baryons.

The scaling transformations (16) immediately imply that t_{relax} must transform as a time scale:

$$t_{\rm relax} \to \lambda^z t_{\rm relax}$$
. (21)

Once again one might be tempted to attribute this scaling to dimensional analysis and therefore conclude that it is devoid of any physical significance. However, a generic relaxation mechanism will emphatically not give rise to a $t_{\rm relax}$ with this scaling property for arbitrary values of z. Imagine for instance that DM reaches thermal equilibrium due to self-interactions. The cross section for such processes will generically have a velocity dependence of the form $\sigma = \sigma_0(c/v)^{\alpha}$ for a fixed α , and with σ_0 a constant built out of microscopic scales and couplings. The relaxation time is in turn the inverse of the self-interaction rate σnv , i.e., $t_{\rm relax} = \frac{m(v/c)^{\alpha}}{\sigma_0 \rho v}$. We conclude therefore that in this scenario $t_{\rm relax} \to \lambda^{(3-\alpha)z-1+\alpha} t_{\rm relax}$, which agrees with (21) only for one particular value of z, namely $z = \frac{1-\alpha}{2-\alpha}$.

More broadly, one should keep in mind that multiple relaxation mechanisms might be at play over different characteristic time scales, in which case the relaxation time should be the shortest of such scales. Given that there is currently no direct evidence for sizable DM self-interactions, it is plausible that the associated time scale could be longer than the dynamical time in galaxies. It is then important to consider the possibility of other relaxation mechanisms. This naturally suggests another time scale, which interestingly scales like (21) for any z—the Jeans time $\frac{1}{\sqrt{G\rho}}$. A possible mechanism giving rise to such a relaxation time was discussed for instance in [1].

Indeed, we will see below that, in order to reproduce the MDAR, the relaxation time must indeed be proportional to the Jeans time, *i.e.*,

$$t_{\rm relax} = \frac{\mathcal{O}(1)}{\sqrt{G\rho}}.$$
 (22)

In the flat part of the rotation curve, where $\rho(r) \simeq \frac{v^2}{2\pi G r^2}$, this reduces to $t_{\rm relax} \sim \frac{r}{v}$. Combining this expression with the one for the thermal conductivity in Eq. (20), we obtain

$$\kappa m = \mathcal{N}\sqrt{\frac{\rho}{G}} v^2, \qquad (23)$$

where \mathcal{N} is some $\mathcal{O}(1)$ constant.

3.4 Heating rate

By working in the dilute limit and assuming that t_{relax} is determined by the Jeans time, we have been able to "kick the can down the road" and preserve scale invariance without committing to any particular value of z. In order to write down an explicit expression for the heating rate, we will now have to fix z.

To this end we will use the BTFR as an observational guiding principle. The fact that the ratio $V_{\rm flat}^4/M_{\rm b}$ appears to be a universal constant in rotationally-supported galaxies suggests that

this quantity should not transform under our scaling symmetry. This will be the case only if the scaling exponent takes the value

$$z = 1/2. (24)$$

We henceforth assume that our heating rate explicitly breaks scale invariance for any z down to scale invariance for z = 1/2 only.

We will now show, based on plausible physical assumptions, that the z = 1/2 scaling symmetry

$$\frac{\dot{\mathcal{E}}}{m} \to \lambda^{1/2} \frac{\dot{\mathcal{E}}}{m},\tag{25}$$

fixes the parametric dependence of the heating rate $\dot{\mathcal{E}}/m$ due to DM-baryon interactions. On physical grounds, we expect $\dot{\mathcal{E}}/m$ to depend on $\rho, \rho_{\rm b}$, both of which transform as $\rho_{\rm b}, \rho \to \lambda^{-1}\rho_{\rm b}, \rho$, as well as the velocity of DM and baryon components. In rotationally-supported galaxies it is reasonable to neglect the DM bulk velocity relative to its velocity dispersion, $|\vec{u}| \ll v$. Indeed, in most of our analysis we will focus on equilibrium situations and ignore the spin of the halo. We will assume the opposite for baryons, $v_{\rm b} \ll |\vec{V}_{\rm b}|$, which is also justified in disk galaxies. This leaves us with two velocity variables, v and $V_{\rm b}$. These two are comparable in the flat part of rotation curves, whereas $V_{\rm b} \ll v$ in the central region of galaxies. To simplify the discussion, we shall only keep track of the dependence on v, keeping in mind that $\dot{\mathcal{E}}/m$ more generally will depend on both v and $V_{\rm b}$.

Given the transformation law $v \to \lambda^{1/2}v$, the most general form for the heating rate compatible with (25) is

$$\frac{\dot{\mathcal{E}}}{m} = v F\left(\frac{\rho_{\rm b}}{\rho}, \frac{v^2}{\rho}\right). \tag{26}$$

In order to fix completely the form of \mathcal{E} , we will make two additional assumptions. First, since in our scenario DM heats up due to interactions with baryons, it is natural to assume that it is an extensive quantity as a function of the number of baryons. In other words, the heating rate should be linear in ρ_b :

$$\frac{\dot{\mathcal{E}}}{m} = v \frac{\rho_{\rm b}}{\rho} f\left(\frac{v^2}{\rho}\right) \,. \tag{27}$$

From a model-building perspective, this is certainly the simplest possibility. This is arguably also the most reasonable behavior one can have in the DM dominate regime $\rho_b/\rho \ll 1$. We will assume however that Eq. (27) holds more generally.

Notice that f has dimensions of acceleration. Therefore, the second assumption we will make is that the f is approximately constant, and of order the characteristic acceleration scale a_0 appearing in the MDAR. Thus the heating rate is fixed to be

$$\frac{\dot{\mathcal{E}}}{m} = Ca_0 v \frac{\rho_{\rm b}}{\rho} \,, \tag{28}$$

where C is another constant. For concreteness we will assume $C \sim \mathcal{O}(10^{-1})$, which offered a good fit to rotation curves in the cooling case [1]. The assumption that f is of order a_0 is also quite natural from a phenomenological viewpoint, given that we are trying to reproduce a result such

as the MDAR which features a characteristic acceleration scale. At the same time, the obvious downside of treating a_0 as a fundamental scale is that it is unclear why it should numerically coincide with a cosmological acceleration scale. We will assume that this "coincidence" is resolved by a different mechanism that operates over much longer, cosmological time scales, such that a_0 can be treated as a constant parameter for our purposes. This appears to be well supported by current observations [90]. It is also worth noting that the inverse density dependence in (28) is helpful for the phenomenological viability of the mechanism. As we will see in Sec. 5, it suppresses the heating rate in high-density environments, such as the early universe.

Finally, a brief word about the sign of C, which determines whether DM is cooled ($\dot{\mathcal{E}} < 0$) or heated ($\dot{\mathcal{E}} > 0$) by baryons. Whereas [1] primarily studied the cooling case for concreteness, here we focus exclusively on the heating case. This is a priori more desirable, since DM heating can transform the cusps into cores in the central regions of galaxy halos. Moreover, the opposite case of DM cooling can lead to flattened halos, or too prominent dark disks, once the halos have an initial spin. These unwanted features are absent with DM heating. Finally, we will argue in Sec. 4.4 that with heating it is possible to derive a combination of the BTFR and CSDR by studying the dynamical approach to equilibrium.

3.5 Deep-MOND scaling as an approximate enhanced symmetry

To summarize, given our expressions for the equations of state, the heat flux and the heating rate, Eqs. (12) reduce to:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \, \vec{u}) = 0; \tag{29a}$$

$$\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right) \vec{u} + \frac{1}{\rho} \vec{\nabla} \left(\rho v^2\right) = \vec{g};$$
(29b)

$$\frac{3}{2} \left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \right) v^2 + v^2 \vec{\nabla} \cdot \vec{u} - \frac{1}{\rho} \vec{\nabla} \cdot \left(\mathcal{N} \sqrt{\frac{\rho}{G}} v^2 \vec{\nabla} v^2 \right) = C a_0 v \frac{\rho_b}{\rho}; \tag{29c}$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \left(\rho + \rho_{\rm b}\right) \,. \tag{29d}$$

As discussed previously, these equations are invariant under the scaling transformations (15) and (16) with z = 1/2.

In fact, in the DM-dominated regime, where $\rho_{\rm b}$ can be neglected compared to ρ in the Poisson

equation (29d),³ our equations enjoy a larger, approximate symmetry under the rescaling

$$\vec{x} \rightarrow \lambda \vec{x};$$

$$t \rightarrow \lambda^{y} t;$$

$$v \rightarrow \lambda^{1-y} v;$$

$$\vec{u} \rightarrow \lambda^{1-y} \vec{u};$$

$$\vec{g} \rightarrow \lambda^{1-2y} \vec{g};$$

$$\rho \rightarrow \lambda^{-2y} \rho;$$

$$\rho_{b} \rightarrow \lambda^{1-4y} \rho_{b},$$

$$(30)$$

for an arbitrary y [1]. These transformations reduce to our original z = 1/2 scale symmetry for y = 1/2, but for other values of y they represent a new type of symmetry that is only approximately valid in DM-dominated regions.

Despite its approximate validity, this enhanced symmetry has interesting observational consequences. Imagine that a galaxy with scale length L_1 , total baryonic mass $M_{\rm b,1}$ and rotation curve \vec{V}_1 is a solution to our equations. It immediately follows that our equations must also admit a solution with L_2 , $M_{\rm b,2}$ and \vec{V}_2 given by

$$L_2 = \lambda L_1;$$
 $M_{b,2} = \lambda^{4-4y} M_{b,1};$ $\vec{V}_2(\lambda \vec{x}) = \lambda^{1-y} \vec{V}_1(\vec{x}).$ (31)

This is equivalent to the statement that the rotation curves of two galaxies with different scale lengths and different total baryonic masses must be related as follows:

$$\vec{V}_2(\vec{x}) = \left(\frac{M_{\rm b,2}}{M_{\rm b,1}}\right)^{1/4} \vec{V}_1 \left(\frac{L_1}{L_2} \vec{x}\right) , \qquad (32)$$

which precisely matches (11).

In the particular case of y=1, the scaling transformations (30) reduce to the "relativistic" deep-MOND scaling law [89], and the result (31) becomes particularly simple: two galaxies with the same total baryonic mass but different scale lengths L_1 and L_2 have rotation curves related by $\vec{V}_2(\vec{x}) = \vec{V}_1\left(\frac{L_1}{L_2}\vec{x}\right)$. This behavior appears to be supported by observations [21].

4 MDAR as Spontaneous Breaking of Scale Invariance

As shown above, the scaling of our equations implies that, in the DM-dominated regime, the baryonic mass-asymptotic velocity scaling should follow the BTFR scaling, $M_{\rm b} \propto V_{\rm flat}^4$. Regarding the normalization of the BTFR, it is known that if one starts from abundance matching with NFW halos, one typically reproduces the correct zero-point of the relation in the baryonic mass range $\sim 10^{10} {\rm M}_{\odot}$ to $\sim 10^{11} {\rm M}_{\odot}$, albeit with too large scatter [14]. The curvature of the predicted BTFR then implies too large $V_{\rm flat}$ (or too large enclosed DM mass) at the low-mass end, still with

³Notice that in this limit one cannot necessarily neglect the righthand side of Eq. (29c). For instance, for equilibrium solutions the right-hand side is exactly equal to the last term on the left-hand side, and is therefore not negligible.

too large scatter. Given that we are starting from the right normalization in the intermediate-mass regime, one would expect that our heating mechanism expels DM out of the baryonic disk region of low-mass disk galaxies, thereby bringing $V_{\rm flat}$ down to follow the $M_{\rm b} \propto V_{\rm flat}^4$ scaling with the zero-point set by intermediate-mass galaxies.

In order to make more concrete analytic predictions hereafter, we will now assume that, through their own self-interactions together with the baryon-DM energy exchange mechanism, DM halos reach a cored pseudo-isothermal profile in the region where the baryonic disk is sitting. In this Section we will demonstrate that the set of equations (29) is fully consistent with such a cored pseudo-isothermal profile, with parameters that reproduce the MDAR.

4.1 Cored pseudo-isothermal profile

Let us now first show how the cored pseudo-isothermal profile parameters should be arranged to reproduce the MDAR. The profile has the following form:

$$\rho(r) = \frac{\rho_0}{1 + \left(\frac{r}{r_c}\right)^2} \,. \tag{33}$$

Thus it is specified by two parameters: the central density, ρ_0 , and the core radius, r_c . Equivalently, the core radius can be traded for the (asymptotic) velocity dispersion, denoted by v_{∞} , using

$$r_{\rm c} = \frac{v_{\infty}}{\sqrt{2\pi G\rho_0}} \,. \tag{34}$$

Note that v_{∞} is defined at infinity because the velocity dispersion profile we are considering is *not* strictly isothermal.

The ability of such cored pseudo-isothermal profile to fit galactic rotation curves has been well-studied, e.g., [91]. Consider first the large distance $r \gg r_{\rm c}$ regime:

$$\rho(r \gg r_{\rm c}) \simeq \frac{\rho_0 r_{\rm c}^2}{r^2} = \frac{v_{\infty}^2}{2\pi G r^2}.$$
(35)

This implies a flat rotation curve with $V_{\text{flat}} = \sqrt{2}v_{\infty}$. Hence DM dominates in this regime, and the assumption of spherical symmetry is justified. To match the BTFR (7), the velocity dispersion must be related to the total baryonic mass via

$$v_{\infty}^{4} = \frac{1}{4} a_0 G M_{\rm b} \,. \tag{36}$$

This fixes one parameter of the cored pseudo-isothermal profile (33), which thus simplifies to

$$\rho(r) = \frac{1}{4\pi G} \frac{\sqrt{a_0 G M_b}}{r_c^2 + r^2} \,. \tag{37}$$

The second parameter can be fixed by the CSDR (8). For the cored pseudo-isothermal profile, (9) gives

$$\Sigma_{\rm DM} = \pi \rho_0 r_c \,. \tag{38}$$

To proceed, we must distinguish between LSB galaxies, which are DM-dominated everywhere, and HSB galaxies, where baryons dominate in the central region. For LSB galaxies ($\Sigma_{\rm b} \ll a_0/G$), (8) implies

$$\rho_0 r_{\rm c} = \frac{2}{\pi} \sqrt{\frac{a_0}{2\pi G} \Sigma_{\rm b}(0)} \,. \tag{39}$$

Combined with (34) and the first constraint (36), we can solve for the core radius of LSB galaxies:

$$r_{\rm c} = \frac{1}{4} \sqrt{\frac{\pi M_{\rm b}}{2\Sigma_{\rm b}(0)}}$$
 (LSB galaxies). (40)

For HSB galaxies ($\Sigma_b \gg a_0/G$), on the other hand, the CSDR (8) does not directly constrain $\Sigma_{\rm DM}$. The answer depends on the assumed functional form for the MDAR. (In the MOND parlance, this reflects the freedom in choosing the interpolating function.)

From a symmetry perspective, the cored pseudo-isothermal profile spontaneously breaks the z=1/2 scaling symmetry by introducing an explicit scale, r_c (or equivalently, ρ_0). Notice, however, that the scaling symmetry is restored in the flat part of the rotation curve (i.e., $r \gg r_c$). Indeed, in this region $\rho(r)$ approximates a singular isothermal profile (35), which transforms covariantly for any z:

$$\rho(r) \simeq \frac{v_{\infty}^2}{2\pi G r^2} \to \lambda^{-2z} \rho(r) \,. \tag{41}$$

The spontaneous symmetry breaking scale r_c (as well as v_{∞}) will be fixed through other sources of spontaneous breaking, namely baryons.

4.2 Flat part of the rotation curve and the BTFR

We now show that a cored pseudo-isothermal profile, with suitable parameters to reproduce the MDAR, is a solution to the set of equations (29). We will primarily be interested in equilibrium solutions to these equations with negligible DM halo spin. In this case, the DM bulk velocity can be set to zero, *i.e.*, $\vec{u} = 0$, and the continuity equation (29a) is trivially satisfied. Equations (29b)–(29d) then reduce to

$$\vec{\nabla} \left(\rho v^2 \right) = \rho \vec{g} \,; \tag{42a}$$

$$\vec{\nabla} \cdot \left(\sqrt{\frac{\rho}{G}} v^2 \vec{\nabla} v^2 \right) = -\frac{C}{\mathcal{N}} v a_0 \rho_b; \tag{42b}$$

$$\vec{\nabla} \cdot \vec{g} = -4\pi G \left(\rho + \rho_{\rm b} \right) \,. \tag{42c}$$

In the flat part of the rotation curve $(r \gg r_c)$, the gravitational field is dominated by DM $(\rho \gg \rho_b)$, and spherical symmetry is a good approximation. The Jeans equation (42a) and Poisson equation (42c) are approximately solved by

$$\rho(r) \simeq \frac{v^2(r)}{2\pi G r^2},\tag{43}$$

where, as we will verify a posteriori, v(r) is a slowly-varying function. Meanwhile, the velocity profile v(r) is determined by the heat equation (42b), which, upon assuming spherical symmetry and using (43), simplifies to

$$\frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r} \left(v^4 r \frac{\mathrm{d}v}{\mathrm{d}r} \right) = -\sqrt{\frac{\pi}{2}} \frac{C}{\mathcal{N}} v a_0 G \rho_{\mathrm{b}}. \tag{44}$$

Approximating v as nearly constant on the right-hand side, this can be readily integrated once:

$$r\frac{\mathrm{d}v^4}{\mathrm{d}r} = -\frac{1}{\sqrt{2\pi}}\frac{C}{\mathcal{N}}a_0GM_{\mathrm{b}}.$$
 (45)

In turn this implies

$$v^4(r) = \frac{1}{\sqrt{2\pi}} \frac{C}{N} a_0 G M_b \log \frac{R_0}{r},$$
 (46)

where R_0 is an arbitrary scale. Thus v only varies logarithmically, which justifies our assumption.

Some remarks are in order. First, the logarithmic dependence of v(r) implies that scale invariance is not quite restored for $r \gg r_{\rm c}$. Rather it is spontaneously broken, analogously to the breaking of scale invariance by radiative corrections (as in Coleman-Weinberg [92]), with R_0 playing the role of a dimensional transmutation scale. Second, using the approximate relation $V \simeq \sqrt{2}v$, the rotation curve is nearly flat with

$$V_{\text{flat}}^4 \sim a_0 G M_{\text{b}} \log \frac{R_0}{r} \,. \tag{47}$$

It is encouraging that the prefactor matches the parametric dependence of the BTFR (7). Unfortunately within our static equilibrium analysis we are not able to fix the scale R_0 , nor determine its scatter. To do so, we will need to go beyond the equilibrium treatment and analyze the dynamical evolution towards equilibrium. This will be the focus of Sec. 4.4.

4.3 Cored region and the central density relation in HSB galaxies

Consider the central region of galaxies ($r \ll r_c$). In this region the DM density can be approximated as nearly constant, $\rho \simeq \rho_0$, hence (42a) reduces to

$$\vec{\nabla}v^2 \simeq \vec{g} \,. \tag{48}$$

The solution is $v^2 = -\Phi + \alpha v_{\infty}^2$, where α is an $\mathcal{O}(1)$ constant. The precise value of this constant is irrelevant for us. The important point is that v^2 approaches $\sim v_{\infty}^2$ near the origin, while its gradient is fixed by the gravitational field.

To make headway analytically, we imagine working sufficiently close to the center that the baryon distribution looks like an infinite disk but sufficiently far that the disk appears infinitely thin. In other words, we work in the regime $L_z \ll r \ll L$, where L_z is the scale height and L the disk length of the baryon distribution. As a result, the baryon distribution is approximated by a surface density Σ_b :

$$\rho_{\rm b} \simeq \Sigma_{\rm b} \delta(z) \,.$$
(49)

For distances $\ll L$, the surface density is nearly homogeneous and given by the central value, $\Sigma_{\rm b}(0)$.

With this approximation, the heat equation (42b) implies a discontinuity in the normal component of the heat flux, which by symmetry fixes its magnitude:

$$\sqrt{\frac{\rho_0}{G}}v_{\infty}|\nabla_{\perp}v^2| = \frac{C}{2\mathcal{N}}a_0\Sigma_{\mathbf{b}}(0). \tag{50}$$

Using (34), (38), and (48), this implies

$$\Sigma_{\rm DM} g_{\perp} = \sqrt{\frac{\pi}{2}} \frac{C}{2\mathcal{N}} a_0 \Sigma_{\rm b}(0) \,. \tag{51}$$

The transverse component of the gravitational field is solved similarly by integrating Poisson's equation (42c). For HSB galaxies, which are baryon-dominated near the center, this gives

$$g_{\perp}^{\rm HSB} \simeq 2\pi G \Sigma_{\rm b}(0)$$
. (52)

It then follows from (51) that

$$\Sigma_{\rm DM} = \sqrt{\frac{\pi}{2}} \frac{C}{2N} \frac{a_0}{2\pi G}$$
 (HSB galaxies). (53)

Thus our heat equation implies $\Sigma_{\rm DM} \sim a_0/G$. This matches behavior of the 'simple' interpolating function [86], and is consistent with observations [93].

4.4 Approach to equilibrium and central density relation in LSB galaxies

Up to now our analysis has focused on static, equilibrium configurations. Within this framework, we were able to reproduce the parametric dependence of the BTFR, up to the logarithm of a scale R_0 whose magnitude and scatter remain undetermined. We were also able to derive the CSDR for HSB galaxies.

By going beyond the equilibrium treatment and considering the approach to equilibrium, we will now show how the central density relation, which is at the root of the problem of diversity of rotation curves, can be naturally reached by our DM-fluid interacting with baryons. Specifically, we will derive a constraint on a particular combination of the DM temperature and surface density, which matches the combination of BTFR and CSDR. Therefore, if one takes the BTFR as a given (per the equilibrium analysis), then this constraint yields the central density relation naturally.

We begin with a few general comments. In the standard Λ CDM model, halo virialization is achieved through violent relaxation, a manifestly non-equilibrium process that drives the DM distribution towards the attractor NFW profile within a few dynamical times. Our proposed DM-baryon interactions offer another relaxation channel. These interactions have a characteristic time on the order of a dynamical time and thus "compete" with violent relaxation [1]. Therefore we do not expect our halos to necessarily reach a NFW profile early on. Crucially, since the interactions considered here tend to heat up DM, they can plausibly prevent the formation of cold central cusps and instead generate constant density cores, as needed in most LSB galaxy halos.

A rigorous dynamical analysis to back this intuition would require numerical simulations, which is beyond the scope of this work. In what follows we offer a simple, back-of-the-envelope analysis

of the time-dependent problem. Because the derivation ignores density and velocity gradients, and relies instead on average quantities, it can only reproduce the CSDR in the DM-dominated regime (valid for LSB galaxies). This is sufficient for our purposes, since we have already established the central density relation in HSB galaxies within the equilibrium treatment.

The starting point is our set of DM fluid equations (29). It is convenient to translate these equations in terms of the entropy density per DM particle, given by the Sackur-Tetrode equation:

$$s = \ln\left((2\pi)^{3/2} \frac{m^4 v^3}{\rho}\right) + \frac{5}{2}.$$
 (54)

This allows us to eliminate v and express our equations (29) in terms of ρ , \vec{u} and s. In what follows we will keep v around for simplicity, but it should be understood via (54) as an implicit function of ρ and s. It is straightforward to combine the continuity (29a) and heat equation (29c) to obtain an equation for the entropy density:

$$\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right) s + \frac{1}{\rho v^2} \vec{\nabla} \cdot \vec{q} = \frac{\dot{\mathcal{E}}}{m}, \tag{55}$$

with the heat flux expressed as

$$\vec{q} = -\frac{2}{3}\mathcal{N}\sqrt{\frac{\rho}{G}}v^4\vec{\nabla}\left(s + \ln\rho\right) \,. \tag{56}$$

This equation is supplemented by the continuity (29a), momentum (29b) and Poisson (29d) equations.

To simplify the analysis, at this point we approximate mass and entropy densities as nearly uniform, thereby neglecting their gradients: ∇s , $\nabla \rho \simeq 0$. In other words, we treat ρ and s as average quantities. It follows from (56) that the heat flux can also be neglected, $\vec{q} \simeq 0$. Hence (55) simplifies to

$$\frac{\partial s}{\partial t} = \frac{\dot{\mathcal{E}}}{mv^2} \,. \tag{57}$$

Not surprisingly, the entropy of DM particles increases as they are heated by baryons.

Assuming that the initial DM entropy (at virialization) is negligible compared to its final value (at equilibrium), (57) can be schematically integrated over a relaxation time to give

$$\frac{\dot{\mathcal{E}}}{mv^2} t_{\text{relax}} \sim 1. \tag{58}$$

This expresses the condition for equilibrium. Substituting (28) and (22), we obtain

$$\frac{\Sigma_{\rm DM}^3}{v^2} \sim \frac{a_0 \rho_{\rm b}}{G^2} \,,\tag{59}$$

where we have used (34) and (38) to estimate the DM surface density as $\Sigma_{\rm DM} \sim \sqrt{\frac{\rho v^2}{G}}$.

Meanwhile, we know that the central baryonic surface density of an exponential disk of scale-length L is $\Sigma_{\rm b}(0)=\frac{M_{\rm b}}{2\pi L^2}$. Assuming an approximate linear relation $L_{\rm z}\approx L/8$ between disk

scale-length and scale-height, we can approximate the mean baryon density by $\rho_{\rm b} \sim \frac{M_{\rm b}}{L^3} \sim \frac{\Sigma_{\rm b}^{3/2}(0)}{\sqrt{M_{\rm b}}}$. Substituting into (59), we obtain

$$\frac{\Sigma_{\rm DM}^3}{v^2} \sim \frac{\left(\frac{a_0 \Sigma_{\rm b}(0)}{G}\right)^{3/2}}{\sqrt{a_0 G M_{\rm b}}} \,. \tag{60}$$

Hence, taking the BTFR $v^2 \sim \sqrt{a_0 G M_b}$ as a given, we get

$$\Sigma_{\rm DM} \propto \sqrt{\frac{a_0 \Sigma_{\rm b}(0)}{G}}$$
 (61)

This is the desired CSDR, valid for DM-dominated (LSB) galaxies. Because the analysis relied on the *average* density, it is not surprising that the result matches the DM-dominated CSDR. On the other hand, we have already seen within the equilibrium treatment that such a relation holds for HSB galaxies.

It will be important to quantify the numerical coefficient in (60), as well as its scatter. This will require numerical simulations of galaxy formation within our scenario, which is beyond the scope of the present analysis. It is nevertheless encouraging that the correct parametric dependence of the scaling relations derives from a back-of-the-envelope analysis.

5 Cosmological Implications and Constraints

In this Section we consider a few astrophysical and cosmological implications of our model. We will be able to derive very general results, using only the form of the heating rate (28), without specifying an explicit microphysical model. The analysis does rely, however, on the assumption that the physics underlying our DM-baryon interactions still apply in the various environments studied below, such as in the early universe. For instance, if heat transport is due to collective excitations of a DM medium (e.g., fluid or solid), our working assumption is that this DM condensed state is a valid description in these environments.

For comparison with the constraints below, we will set $C = \frac{1}{10}$ for concreteness and assume $a_0 = 10^{-8}$ cm/s². Our heating rate (28) then becomes

$$\frac{\dot{\mathcal{E}}}{m} = 10^{-9} \frac{\rho_{\rm b}}{\rho} v \frac{\rm cm}{\rm s^2} \,. \tag{62}$$

Thus the predicted heating rate is determined simply by the DM-to-baryon fraction and velocity dispersion in the relevant environments.

5.1 Early universe

DM-baryon interactions can affect the evolution in the early universe. In the case of interest where baryons heat up DM, the dominant constraint comes spectral distortions of the CMB taking place in the redshift range $10^4 \lesssim z \lesssim 10^6$ [94]. In the standard cosmological model, baryons are kept in thermal equilibrium with photons by Compton scattering until $z \simeq 200$. This process effectively

cools photons, causing small spectral distortions. This cooling will be enhanced if baryons shed part of their thermal energy to DM, resulting in larger and potentially observable spectral distortions.

This effect was studied in detail in the case of light DM ($m \ll m_b$) scattering elastically with baryons and/or photons [94]. It is straightforward to translate their result to a constraint on the energy exchange rate $\dot{\mathcal{E}}$. Consider the energy exchange rate per baryon, $\frac{\dot{\mathcal{E}}n}{n_b}$, relative to the thermal energy $\sim m_b v_b^2$ per baryon, where n_b and v_b are respectively the baryon number density⁴ and velocity dispersion. Let us compare this to the Hubble rate by defining

$$\epsilon \equiv \frac{\dot{\mathcal{E}}n/n_{\rm b}}{Hm_{\rm b}v_{\rm b}^2} = \frac{C}{6} \frac{a_0 v}{Hv_{\rm b}^2},\tag{63}$$

where the last step follows from (28).

The effect on spectral distortions will be negligible if $\epsilon \ll 1$ in the redshift range $10^4 \lesssim z \lesssim 10^6$. It is easy to check that ϵ increases in time in this range, hence the constraint is most stringent at $z \simeq 10^4$. Since baryons are in thermal equilibrium with radiation, we have $v_{\rm b}^2 = T_{\gamma}/m_{\rm b}$, with T_{γ} denoting the CMB temperature. Substituting $T_{\gamma} \simeq 2 {\rm eV}$ and $H \simeq 10^{-27} {\rm eV}$ at $z \simeq 10^4$, together with our fiducial values $C = \frac{1}{10}$ and $a_0 = 10^{-8} {\rm cm/s^2}$, we obtain

$$\epsilon|_{z=10^4} \simeq 10 \, \frac{v}{c} \,. \tag{64}$$

Since our DM particles are assumed non-relativistic at that time, $v \ll c$, the resulting spectral distortions are indeed negligible.

5.2 Merging clusters

Merging galaxy clusters constrain the DM self-interaction cross section per unit mass [95–98],

$$\frac{\sigma}{m} \lesssim \frac{\text{cm}^2}{\text{g}}$$
 (65)

The precise numerical value of the coefficient depends on the assumptions, but is $\mathcal{O}(1)$ or less [97, 98]. This can be translated to a constraint on the heating rate of DM per unit mass, $\frac{\dot{\mathcal{E}}}{m} \simeq \rho \frac{\sigma}{m} v^3$, where we have used a characteristic energy exchanged per collision of mv^2 for DM-DM scattering. Substituting the characteristic density $\rho \simeq 10^{-24} \mathrm{g/cm^3}$ and velocity $v \simeq 10^3$ km/s for merging clusters, the bound (65) translates to

$$\frac{\dot{\mathcal{E}}}{m} \lesssim \frac{\mathrm{cm}^2}{\mathrm{s}^3} \,. \tag{66}$$

Although (65) was derived assuming DM self-interactions, the end result applies equally well to our heating rate obtained from DM-baryon scattering. Substituting into (62) the DM-baryon ratio $\rho \sim 10 \, \rho_{\rm b}$ in clusters and relative velocity $v \simeq 10^3$ km/s, we obtain

$$\frac{\dot{\mathcal{E}}}{m}\Big|_{\text{clusters}} \simeq 10^{-2} \, \frac{\text{cm}^2}{\text{s}^3} \,.$$
 (67)

⁴For the purpose of this simple estimate, we ignore the distinction between nuclei and free electrons.

This comfortably satisfies (66). On the flip side, a couple order of magnitude improvement in the observational bound (65) would probe our predicted heating rate, thereby highlighting the power of merging clusters for detecting DM-baryon interactions.

5.3 Cosmic Dawn and the EDGES anomaly

The recent measurement of the 21-cm absorption spectrum from the Cosmic Dawn epoch by the EDGES collaboration revealed an excess signal [7]. If real, the excess could indicate that the hydrogen gas at $z \simeq 17$ was cooler than predicted by the standard Λ CDM model. A possible explanation is that interactions between DM and baryons acted to cool the neutral gas prior to the Cosmic Dawn [99].

For instance, sub-GeV DM particles scattering elastically with baryons with velocity-dependent cross section,

$$\sigma_{\rm int}(v) = \sigma_1 \left(\frac{v}{1 \text{ km/s}}\right)^{-4}, \tag{68}$$

would explain the signal if

$$\sigma_1 \gtrsim 10^{-20} \text{ cm}^2$$
. (69)

The strong velocity dependence of (68) is necessary to evade cosmological and astrophysical bounds [99–101]. Detailed model-building analyses, however, show that it is difficult to construct explicit particle physics models that are compatible with other constraints [87, 102–104].

Equations (68) and (69) can be translated to a heating rate per unit mass using $\frac{\dot{\mathcal{E}}}{m} \simeq n_{\rm b}\sigma_{\rm int}(v)v^3$. Substituting the cosmological baryon number density $n_{\rm b} = 2 \times 10^{-7}(1+z)^3$ cm⁻³ evaluated at $z \simeq 17$, together with the characteristic velocity v = 1 km/s, the bound (69) translates to

$$\frac{\dot{\mathcal{E}}}{m} \gtrsim 10^{-8} \frac{\text{cm}^2}{\text{s}^3} \,. \tag{70}$$

This is how large the heating rate ought to be to explain the EDGES excess. In our case, substituting into (62) the cosmological ratio $\rho \simeq 6 \rho_b$, together with v = 1 km/s, our predicted heating rate is

$$\frac{\dot{\mathcal{E}}}{m}\Big|_{z=17} \simeq 2 \times 10^{-5} \, \frac{\text{cm}^2}{\text{s}^3} \,.$$
 (71)

Thus our heating mechanism can explain the EDGES excess.

6 Conclusions

Among the small-scale challenges of Λ CDM [8], the conspiracy between DM and baryon distributions in disk galaxies, embodied in the MDAR, is arguably one of the most tantalizing. The MDAR is a unique relation between the total gravitational field and the Newtonian acceleration generated by baryons alone at every radius in disk galaxies. In particular, both the tightness of the BTFR and the diversity of galaxy rotation curves that it implies [20] remain challenging within the Λ CDM framework, where this conspiracy must arise through feedback processes. While semi-empirical arguments based on abundance matching can reproduce the general shape of the MDAR,

its normalization, and especially its very small scatter, remain challenging [37]. Relatedly, it has recently been pointed out that stellar feedback is related to a characteristic acceleration of order a_0 . While promising, this is not sufficient yet to explain the details of the diversity of rotation curves encoded in the tightness of the MDAR, which should be related to the subtleties of the core-cusp transformation process. On the numerical front, much progress has been made in obtaining the MDAR from hydrodynamical simulations of galaxy formation, as reviewed in the Introduction, though challenges – related to the extreme tightness of the BTFR and diversity of rotation curves – still remain.

Given these challenges, it is worthwhile to entertain the alternative possibility that the baryon-DM conspiracy embodied by the MDAR is due to new, non-gravitational interactions between the two sectors. Traditionally, work in this direction has focused on postulating a new long-range force acting on baryons, thereby effectively modifying gravity. This force could be either fundamental or, as in superfluid DM, emergent from the DM medium.

The idea pursued in this paper, building on our earlier work [1], is that the MDAR is the result of *direct* (non-gravitational) interactions between DM and baryons, instead of an effective modification of gravity or feedback processes. The main difference with our earlier work is to consider that this interaction *heats* the DM-fluid. The approach followed has been completely "bottom-up". Using a hydrodynamical description of DM, our goal has been to identify which such DM-baryon interactions are necessary to reproduce the MDAR.

In this framework, the microphysics of DM is encoded in three physical quantities: the DM equation of state, $P = P(\rho, v)$; the relaxation time, t_{relax} , which enters in the heat conductivity; and the energy exchange rate $\dot{\mathcal{E}}$, which is determined by DM-baryon interactions. A key result of this work is that the MDAR is obtained if the following conditions are satisfied:

- 1. The equation of state is approximately that of an ideal gas, $P = \rho v^2$. This will generically be realized in the dilute limit, where the average inter-particle separation is large compared to the mean free path.
- 2. The relaxation time is set by the Jeans time, $t_{\rm relax} \sim \frac{1}{\sqrt{G\rho}}$. This can be achieved naturally, for instance, if DM is in a Knudsen regime [1].
- 3. The heating rate satisfies the master relation $\frac{\dot{\mathcal{E}}}{m} \sim Ca_0v\frac{\rho_b}{\rho}$. This is the most important relation as it informs us about the necessary DM-baryon particle interactions.

To be clear, we do not claim that these are unique nor necessary, but they are *sufficient* to obtain the MDAR. Remarkably, with these assumptions the set of hydrodynamical equations, together with Poisson's equation, enjoy an anisotropic scaling symmetry, which offers yet another guide for model building. Moreover, in DM-dominated regions this scaling symmetry is enhanced to a one-parameter family of scalings, implying the scaling relation (11), which fully captures the low-acceleration limit of the MDAR.

In this paper, we built on and further developed the original scenario of [1] in several crucial ways. Most importantly, as stated above, instead of baryon-DM interactions cooling the DM medium, we focused exclusively on the case where the DM fluid is *heated* by baryons. This is indeed a priori more desirable from the point of view of galaxy formation, since DM heating can transform cusps into cores in the central regions of galaxy halos. It also avoids the concern of forming flattened halos

or dark disks. A second key difference pertains to the form of DM-baryon interactions. Whereas our original analysis [1] focused exclusively on short-range particle-particle collisions between DM and baryons, in the present analysis we remained general about the form of such interactions. This opens up a wider range of possibilities for particle physics model-building.

We then showed how, assuming a cored pseudo-isothermal profile, the above hydrodynamical ingredients give rise at equilibrium to suitable parameters reproducing the MDAR. Specifically, in the flat part of the rotation curve the asymptotic rotational velocity matches the parametric dependence of the BTFR, up to a logarithm in r. Meanwhile, in the central region of HSB galaxies, where baryons dominate, the DM profile reproduces the CSDR with the behaviour of the 'simple' interpolating function of MOND. Finally, by studying the time-dependent approach to equilibrium, we derived a constraint on a combination of the DM velocity dispersion and surface density, which matches the combination of BTFR and CSDR. Therefore, if one takes the BTFR as a given (per the equilibrium analysis), this constraint yields the CSDR naturally.

Remarkably, the form of the heating rate makes definite, model-independent predictions for various cosmological and astrophysical observables. The only assumption of course is that the underlying DM-baryon effective theory responsible for the heating rate is still valid in these different environments. Assuming this is the case, we argued that our model satisfies various observational constraints, and, intriguingly, offers a possible explanation to the EDGES excess. Of course, there will be many more phenomenological loops to go through once we have an explicit particle physics realization, but it is reassuring that our heating rate so far appears to be observationally viable.

Our framework offers a number of avenues for further development. Three particularly important directions are:

- Including the dynamics of baryons: In our framework we focused our attention on the dynamics of the DM sector, treating baryons as an external source. This is a reasonable approximation provided that the typical energy lost by a baryon is not significant enough to affect its dynamics over the time scales of interest. Using the expression (28) for our heating rate $\dot{\mathcal{E}}$, one can estimate the energy lost by a baryon per unit length to be $\frac{dE_b}{d\ell} \gtrsim \frac{Cm_b a_0 v}{V_b}$. Even keeping in mind that $C \sim \mathcal{O}(10^{-1})$, this quantity could become large enough in some LSBs, and a more accurate treatment would require including the dynamics of baryons.
- Numerical simulations of galaxy formation: Our scenario is ripe for a fully dynamical study of galaxy formation. Because our equations are cast in simple hydrodynamical terms, it should be straightforward to modify existing hydrodynamical codes to include our heating rate. For this purpose, the formulation in terms of entropy density presented in Sec. 4.4 may be most convenient. Such numerical studies would inform us, among other things, on the stability of the equilibrium solution, in particular whether the outskirts of galaxy disks are not too severely perturbed by interactions with DM. It would allow us to check whether the equilibrium configuration is reached dynamically on the predicted time scale. Furthermore, such an analysis would also allows us to quantify the expected scatter for the BTFR, in particular for the characteristic scale R_0 appearing in the logarithm.
- Building a particle physics model: In this paper we have adopted a purely bottom-up approach based on an effective hydrodynamical description of the DM sector. It would be very interesting to deduce what type of constraints the heating rate (28) poses on the underlying

microscopic interactions between baryons and DM. One promising way of ensuring that our scenario is compatible with small-scale (e.g., solar system) constraints would be to consider interactions that involve collective excitations emerging at scales of $\mathcal{O}(pc)$. We leave the exploration of this interesting possibility for future work.

Acknowledgements: We thank Lasha Berezhiani and Scott Dodelson for helpful discussions. B.F. acknowledges support from the Agence Nationale de la Recherche (ANR) project ANR-18-CE31-0006, and from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 834148). J.K. is supported in part by the US Department of Energy (HEP) Award DE-SC0013528, NASA ATP grant 80NSSC18K0694, and a W. M. Keck Foundation Science and Engineering Grant. R.P. is supported in part by the National Science Foundation under Grant No. PHY-1915611.

References

- [1] B. Famaey, J. Khoury, and R. Penco, "Emergence of the mass discrepancy-acceleration relation from dark matter-baryon interactions," JCAP **1803** (2018) no. 03, 038, arXiv:1712.01316 [astro-ph.CO].
- [2] WMAP, G. Hinshaw et al., "Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results," Astrophys. J. Suppl. 208 (2013) 19, arXiv:1212.5226 [astro-ph.CO].
- [3] **Planck**, N. Aghanim et al., "Planck 2018 results. VI. Cosmological parameters," arXiv:1807.06209 [astro-ph.CO].
- [4] A. G. Riess et al., "Milky Way Cepheid Standards for Measuring Cosmic Distances and Application to Gaia DR2: Implications for the Hubble Constant," Astrophys. J. **861** (2018) no. 2, 126, arXiv:1804.10655 [astro-ph.CO].
- [5] S. Birrer et al., "H0LiCOW IX. Cosmographic analysis of the doubly imaged quasar SDSS 1206+4332 and a new measurement of the Hubble constant,"

 Mon. Not. Roy. Astron. Soc. 484 (2019) 4726, arXiv:1809.01274 [astro-ph.CO].
- [6] L. Knox and M. Millea, "The Hubble Hunter's Guide," arXiv:1908.03663 [astro-ph.CO].
- [7] J. D. Bowman, A. E. E. Rogers, R. A. Monsalve, T. J. Mozdzen, and N. Mahesh, "An absorption profile centred at 78 megahertz in the sky-averaged spectrum," Nature **555** (2018) no. 7694, 67–70.
- [8] J. S. Bullock and M. Boylan-Kolchin, "Small-Scale Challenges to the ΛCDM Paradigm," Ann. Rev. Astron. Astrophys. **55** (2017) 343–387, arXiv:1707.04256 [astro-ph.CO].
- [9] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, "The Baryonic Tully-Fisher relation," Astrophys. J. **533** (2000) L99–L102, arXiv:astro-ph/0003001 [astro-ph].

- [10] E. Papastergis, E. A. K. Adams, and J. M. van der Hulst, "An accurate measurement of the baryonic Tully-Fisher relation with heavily gas-dominated ALFALFA galaxies," Astron. Astrophys. 593 (2016) A39, arXiv:1602.09087 [astro-ph.GA].
- [11] F. Lelli, S. McGaugh, and J. Schombert, "The Small Scatter of the Baryonic Tully-Fisher Relation," Astron. J. Lett. **816** (2016) no. 1, L14, arXiv:1512.04543 [astro-ph.GA].
- [12] F. Lelli, S. S. McGaugh, J. M. Schombert, H. Desmond, and H. Katz, "The baryonic TullyFisher relation for different velocity definitions and implications for galaxy angular momentum," Mon. Not. Roy. Astron. Soc. 484 (2019) no. 3, 3267–3278, arXiv:1901.05966 [astro-ph.GA].
- [13] A. Di Cintio and F. Lelli, "The mass discrepancy acceleration relation in a ΛCDM context," Mon. Not. Roy. Astron. Soc. 456 (2016) no. 1, L127–L131, arXiv:1511.06616 [astro-ph.GA].
- [14] H. Desmond, "The scatter, residual correlations and curvature of the SPARC baryonic Tully-Fisher relation," Mon. Not. Roy. Astron. Soc. Letters 472 (1) (2017) L35-L39, arXiv:1706.01017 [astro-ph.C0].
- [15] J. F. Navarro, C. S. Frenk, and S. D. M. White, "A Universal density profile from hierarchical clustering," Astrophys. J. 490 (1997) 493-508, arXiv:astro-ph/9611107 [astro-ph].
- [16] L. Posti, A. Marasco, F. Fraternali, and B. Famaey, "Galaxy disc scaling relations: A tight linear galaxyhalo connection challenges abundance matching," Astron. Astrophys. 629 (2019) A59, arXiv:1909.01344 [astro-ph.GA].
- [17] P. E. Mancera Pia et al., "Off the Baryonic TullyFisher Relation: A Population of Baryon-dominated Ultra-diffuse Galaxies," Astrophys. J. 883 (2019) no. 2, L33, arXiv:1909.01363 [astro-ph.GA].
- [18] P. M. Ogle, T. Jarrett, L. Lanz, M. Cluver, K. Alatalo, P. N. Appleton, and J. M. Mazzarella, "A Break in Spiral Galaxy Scaling Relations at the Upper Limit of Galaxy Mass," Astrophys. J. 884 (2019) no. 1, L11, arXiv:1909.09080 [astro-ph.GA].
- [19] K. A. Oman et al., "The unexpected diversity of dwarf galaxy rotation curves," Mon. Not. Roy. Astron. Soc. 452 (2015) no. 4, 3650-3665, arXiv:1504.01437 [astro-ph.GA].
- [20] A. Ghari, B. Famaey, C. Laporte, and H. Haghi, "Dark matterbaryon scaling relations from Einasto halo fits to SPARC galaxy rotation curves," Astron. Astrophys. 623 (2019) A123, arXiv:1811.06554 [astro-ph.GA].
- [21] R. A. Swaters, R. Sancisi, T. S. van Albada, and J. M. van der Hulst, "The rotation curves shapes of late-type dwarf galaxies," Astron. Astrophys. **493** (2009) 871, arXiv:0901.4222 [astro-ph.CO].
- [22] F. Donato, G. Gentile, and P. Salucci, "Cores of dark matter haloes correlate with stellar

- scalelengths," Mon. Not. Roy. Astron. Soc. **353** (2004) no. 2, L17-L22, arXiv:astro-ph/0403206 [astro-ph].
- [23] F. Lelli, S. S. McGaugh, J. M. Schombert, and M. S. Pawlowski, "The Relation between Stellar and Dynamical Surface Densities in the Central Regions of Disk Galaxies," Astrophys. J. 827 (2016) no. 1, L19, arXiv:1607.02145 [astro-ph.GA].
- [24] W. J. G. de Blok, "The Core-Cusp Problem," Adv. Astron. 2010 (2010) 789293, arXiv:0910.3538 [astro-ph.CO].
- [25] R. H. Sanders, "Mass discrepancies in galaxies Dark matter and alternatives," Astron. & Astrophys. 2 (1990) 1.
- [26] S. S. McGaugh, "The Mass discrepancy acceleration relation: Disk mass and the dark matter distribution," Astrophys. J. 609 (2004) 652-666, arXiv:astro-ph/0403610 [astro-ph].
- [27] G. Gentile, B. Famaey, and W. J. G. de Blok, "THINGS about MOND," Astron. Astrophys. **527** (2011) A76, arXiv:1011.4148 [astro-ph.CO].
- [28] S. McGaugh, F. Lelli, and J. Schombert, "Radial Acceleration Relation in Rotationally Supported Galaxies," Phys. Rev. Lett. 117 (2016) no. 20, 201101, arXiv:1609.05917 [astro-ph.GA].
- [29] F. Lelli, S. S. McGaugh, J. M. Schombert, and M. S. Pawlowski, "One Law to Rule Them All: The Radial Acceleration Relation of Galaxies," Astrophys. J. 836 (2017) no. 2, 152, arXiv:1610.08981 [astro-ph.GA].
- [30] B. W. Keller and J. W. Wadsley, "ΛCDM is Consistent with SPARC Radial Acceleration Relation," Astrophys. J. Lett. 835 (2017) no. L17, , arXiv:1610.06183 [astro-ph.GA].
- [31] J. F. Navarro, A. Bentez-Llambay, A. Fattahi, C. S. Frenk, A. D. Ludlow, K. A. Oman, M. Schaller, and T. Theuns, "The origin of the mass discrepancy-acceleration relation in ΛCDM," Mon. Not. Roy. Astron. Soc. 471 (2017) 1841, arXiv:1612.06329 [astro-ph.GA].
- [32] A. D. Ludlow et al., "Mass-Discrepancy Acceleration Relation: A Natural Outcome of Galaxy Formation in Cold Dark Matter Halos,"

 Phys. Rev. Lett. 118 (2017) no. 16, 161103, arXiv:1610.07663 [astro-ph.GA].
- [33] J. I. Read, G. Iorio, O. Agertz, and F. Fraternali, "Understanding the shape and diversity of dwarf galaxy rotation curves in ΛCDM," Mon. Not. Roy. Astron. Soc. 462 (2016) 3628, arXiv:1601.05821 [astro-ph.GA].
- [34] D. C. Rodrigues, V. Marra, A. del Popolo, and Z. Davari, "Absence of a fundamental acceleration scale in galaxies," Nature Astronomy 2 (Jun, 2018) 668-672, arXiv:1806.06803 [astro-ph.GA].
- [35] S. S. McGaugh, P. Li, F. Lelli, and J. M. Schombert, "Presence of a fundamental acceleration scale in galaxies," Nature Astronomy 2 (Nov. 2018) 924–924.

- [36] P. Kroupa, I. Banik, H. Haghi, A. H. Zonoozi, J. Dabringhausen, B. Javanmardi, O. Müller, X. Wu, and H. Zhao, "A common Milgromian acceleration scale in nature," Nature Astronomy 2 (Nov, 2018) 925–926, arXiv:1811.11754 [astro-ph.GA].
- [37] P. Li, F. Lelli, S. McGaugh, and J. Schombert, "Fitting the radial acceleration relation to individual SPARC galaxies," Astron. Astrophys. **615** (2018) A3, arXiv:1803.00022 [astro-ph.GA].
- [38] M. Y. Grudi, M. Boylan-Kolchin, C.-A. Faucher-Gigure, and P. F. Hopkins, "Stellar feedback sets the universal acceleration scale in galaxies," arXiv:1910.06345 [astro-ph.GA].
- [39] I. M. Santos-Santos, A. Di Cintio, C. B. Brook, A. Macci, A. Dutton, and R. Domnguez-Tenreiro, "NIHAO XIV. Reproducing the observed diversity of dwarf galaxy rotation curve shapes in LambdaCDM," Monthly Notices of the Royal Astronomical Society 473 (2017) no. 4, 43924403, arXiv:1706.04202 [astro-ph.GA].
- [40] A. A. Dutton, A. V. Macci, A. Obreja, and T. Buck, "NIHAO XVIII. Origin of the MOND phenomenology of galactic rotation curves in a LambdaCDM universe," Mon. Not. Roy. Astron. Soc. 485 (2019) no. 2, 1886–1899, arXiv:1902.06751 [astro-ph.GA].
- [41] P. Bode, J. P. Ostriker, and N. Turok, "Halo formation in warm dark matter models," Astrophys. J. **556** (2001) 93–107, arXiv:astro-ph/0010389 [astro-ph].
- [42] D. N. Spergel and P. J. Steinhardt, "Observational evidence for selfinteracting cold dark matter," Phys. Rev. Lett. 84 (2000) 3760–3763, arXiv:astro-ph/9909386 [astro-ph].
- [43] J. A. Schewtschenko, R. J. Wilkinson, C. M. Baugh, C. Boehm, and S. Pascoli, "Dark matter?radiation interactions: the impact on dark matter haloes," Mon. Not. Roy. Astron. Soc. 449 (2015) no. 4, 3587–3596, arXiv:1412.4905 [astro-ph.CO].
- [44] C. Boehm, A. Olivares-Del Campo, S. Palomares-Ruiz, and S. Pascoli, "Phenomenology of a Neutrino-DM Coupling: The Scalar Case," in Proceedings, Prospects in Neutrino Physics (NuPhys2016): London, UK, December 12-14, 2016. 2017. arXiv:1705.03692 [hep-ph]. https://inspirehep.net/record/1598775/files/arXiv:1705.03692.pdf.
- [45] A. Kamada, M. Kaplinghat, A. B. Pace, and H.-B. Yu, "How the Self-Interacting Dark Matter Model Explains the Diverse Galactic Rotation Curves," Phys. Rev. Lett. 119 (2017) no. 11, 111102, arXiv:1611.02716 [astro-ph.GA].
- [46] P. Creasey, O. Sameie, L. V. Sales, H.-B. Yu, M. Vogelsberger, and J. Zavala, "Spreading out and staying sharp creating diverse rotation curves via baryonic and self-interaction effects," Mon. Not. Roy. Astron. Soc. 468 (2017) no. 2, 2283–2295, arXiv:1612.03903 [astro-ph.GA].

- [47] T. Ren, A. Kwa, M. Kaplinghat, and H.-B. Yu, "Reconciling the Diversity and Uniformity of Galactic Rotation Curves with Self-Interacting Dark Matter," Phys. Rev. **X9** (2019) no. 3, 031020, arXiv:1808.05695 [astro-ph.GA].
- [48] J. Bekenstein and M. Milgrom, "Does the missing mass problem signal the breakdown of Newtonian gravity?," Astrophys. J. **286** (1984) 7–14.
- [49] J. D. Bekenstein, "Relativistic gravitation theory for the MOND paradigm," Phys. Rev. D70 (2004) 083509, arXiv:astro-ph/0403694 [astro-ph]. [Erratum: Phys. Rev.D71,069901(2005)].
- [50] C. Skordis and T. Zlosnik, "A general class of gravitational theories as alternatives to dark matter where the speed of gravity always equals the speed of light," arXiv:1905.09465 [gr-qc].
- [51] R. H. Sanders and S. S. McGaugh, "Modified Newtonian dynamics as an alternative to dark matter," Ann. Rev. Astron. Astrophys. 40 (2002) 263–317, arXiv:astro-ph/0204521 [astro-ph].
- [52] B. Famaey and S. McGaugh, "Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions," Living Rev. Rel. **15** (2012) 10, arXiv:1112.3960 [astro-ph.CO].
- [53] M. Milgrom, "A Modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis," Astrophys. J. **270** (1983) 365–370.
- [54] M. Milgrom, "A Modification of the Newtonian dynamics: Implications for galaxies," Astrophys. J. 270 (1983) 371–383.
- [55] G. W. Angus, H. Shan, H. Zhao, and B. Famaey, "On the Law of Gravity, the Mass of Neutrinos and the Proof of Dark Matter," Astrophys. J. 654 (2007) L13-L16, arXiv:astro-ph/0609125 [astro-ph].
- [56] R. Ibata, A. Sollima, C. Nipoti, M. Bellazzini, S. C. Chapman, and E. Dalessandro, "The globular cluster NGC 2419: a crucible for theories of gravity," Astrophys. J. 738 (2011) 186, arXiv:1106.4909 [astro-ph.CO].
- [57] A. Hees, W. M. Folkner, R. A. Jacobson, and R. S. Park, "Constraints on modified Newtonian dynamics theories from radio tracking data of the Cassini spacecraft," Phys. Rev. D89 (2014) 102002, arXiv:1402.6950 [gr-qc].
- [58] A. Hees, B. Famaey, G. W. Angus, and G. Gentile, "Combined Solar System and rotation curve constraints on MOND," Mon. Not. Roy. Astron. Soc. 455 (2016) no. 1, 449–461, arXiv:1510.01369 [astro-ph.GA].
- [59] E. Babichev, C. Deffayet, and G. Esposito-Farese, "Improving relativistic MOND with Galileon k-mouflage," Phys. Rev. **D84** (2011) 061502, arXiv:1106.2538 [gr-qc].
- [60] L. Blanchet, "Gravitational polarization and the phenomenology of MOND," Class. Quant. Grav. **24** (2007) 3529–3540, arXiv:astro-ph/0605637 [astro-ph].

- [61] L. Blanchet and A. Le Tiec, "Model of Dark Matter and Dark Energy Based on Gravitational Polarization," Phys. Rev. D78 (2008) 024031, arXiv:0804.3518 [astro-ph].
- [62] J. Khoury, "Alternative to particle dark matter," Phys. Rev. D91 (2015) no. 2, 024022, arXiv:1409.0012 [hep-th].
- [63] L. Blanchet and L. Heisenberg, "Dark Matter via Massive (bi-)Gravity," Phys. Rev. D91 (2015) 103518, arXiv:1504.00870 [gr-qc].
- [64] L. Blanchet and L. Heisenberg, "Dipolar Dark Matter with Massive Bigravity," JCAP 1512 (2015) no. 12, 026, arXiv:1505.05146 [hep-th].
- [65] L. Berezhiani and J. Khoury, "Dark Matter Superfluidity and Galactic Dynamics," Phys. Lett. **B753** (2016) 639–643, arXiv:1506.07877 [astro-ph.CO].
- [66] L. Berezhiani and J. Khoury, "Theory of dark matter superfluidity," Phys. Rev. D92 (2015) 103510, arXiv:1507.01019 [astro-ph.C0].
- [67] J. Khoury, "Another Path for the Emergence of Modified Galactic Dynamics from Dark Matter Superfluidity," Phys. Rev. D93 (2016) no. 10, 103533, arXiv:1602.05961 [astro-ph.C0].
- [68] A. Addazi and A. Marcian, "UV self-completion of a theory of Superfluid Dark Matter," Eur. Phys. J. C79 (2019) no. 4, 354, arXiv:1801.04083 [hep-th].
- [69] J. Khoury, "Dark Matter Superfluidity," PoS DSU2015 (2016) 017, arXiv:1605.08443 [astro-ph.CO].
- [70] J. Fan, "Ultralight Repulsive Dark Matter and BEC," Phys. Dark Univ. 14 (2016) 84–94, arXiv:1603.06580 [hep-ph].
- [71] A. Hodson, H. Zhao, J. Khoury, and B. Famaey, "Galaxy Clusters in the Context of Superfluid Dark Matter," Astrophys. & Astron. 607 (2017) A108, arXiv:1611.05876 [astro-ph.CO].
- [72] R.-G. Cai, T.-B. Liu, and S.-J. Wang, "Gravitational wave as probe of superfluid dark matter," Phys. Rev. **D97** (2018) no. 2, 023027, arXiv:1710.02425 [hep-ph].
- [73] L. Berezhiani, B. Famaey, and J. Khoury, "Phenomenological consequences of superfluid dark matter with baryon-phonon coupling," JCAP **1809** (2018) no. 09, 021, arXiv:1711.05748 [astro-ph.CO].
- [74] S. Hossenfelder and T. Mistele, "Strong lensing with superfluid dark matter," JCAP 1902 (2019) 001, arXiv:1809.00840 [astro-ph.GA].
- [75] A. Sharma, J. Khoury, and T. Lubensky, "The Equation of State of Dark Matter Superfluids," JCAP 1905 (2019) no. 05, 054, arXiv:1809.08286 [hep-th].
- [76] S. Alexander, E. McDonough, and D. N. Spergel, "Chiral Gravitational Waves and Baryon Superfluid Dark Matter," JCAP 1805 (2018) no. 05, 003, arXiv:1801.07255 [hep-th].

- [77] E. G. M. Ferreira, G. Franzmann, J. Khoury, and R. Brandenberger, "Unified Superfluid Dark Sector," JCAP **2019** (2019) no. 08, 027, arXiv:1810.09474 [astro-ph.CO].
- [78] L. Berezhiani and J. Khoury, "Emergent long-range interactions in Bose-Einstein Condensates," Phys. Rev. **D99** (2019) no. 7, 076003, arXiv:1812.09332 [hep-th].
- [79] L. Berezhiani, B. Elder, and J. Khoury, "Dynamical Friction in Superfluids," arXiv:1905.09297 [hep-ph].
- [80] C. M. Ho, D. Minic, and Y. J. Ng, "Cold Dark Matter with MOND Scaling," Phys. Lett. B693 (2010) 567-570, arXiv:1005.3537 [hep-th].
- [81] C. M. Ho, D. Minic, and Y. J. Ng, "Quantum Gravity and Dark Matter," Gen. Rel. Grav. 43 (2011) 2567–2573, arXiv:1105.2916 [gr-qc]. [Int. J. Mod. Phys.D20,2887(2011)].
- [82] C. M. Ho, D. Minic, and Y. J. Ng, "Dark Matter, Infinite Statistics and Quantum Gravity," Phys. Rev. D85 (2012) 104033, arXiv:1201.2365 [hep-th].
- [83] D. Edmonds, D. Farrah, D. Minic, Y. J. Ng, and T. Takeuchi, "Modified Dark Matter: Relating Dark Energy, Dark Matter and Baryonic Matter," arXiv:1709.04388 [astro-ph.CO].
- [84] E. P. Verlinde, "Emergent Gravity and the Dark Universe," SciPost Phys. 2 (2017) no. 3, 016, arXiv:1611.02269 [hep-th].
- [85] A. Hees, B. Famaey, and G. Bertone, "Emergent gravity in galaxies and in the Solar System," Phys. Rev. **D95** (2017) 064019, arXiv:1702.04358 [astro-ph].
- [86] B. Famaey and J. Binney, "Modified Newtonian dynamics in the Milky Way," Mon. Not. Roy. Astron. Soc. 363 (2005) 603-608, arXiv:astro-ph/0506723 [astro-ph].
- [87] E. D. Kovetz, V. Poulin, V. Gluscevic, K. K. Boddy, R. Barkana, and M. Kamionkowski, "Tighter limits on dark matter explanations of the anomalous EDGES 21 cm signal," Phys. Rev. **D98** (2018) no. 10, 103529, arXiv:1807.11482 [astro-ph.C0].
- [88] C. Creque-Sarbinowski, L. Ji, E. D. Kovetz, and M. Kamionkowski, "Direct millicharged dark matter cannot explain the EDGES signal," Phys. Rev. **D100** (2019) no. 2, 023528, arXiv:1903.09154 [astro-ph.C0].
- [89] M. Milgrom, "The MOND limit from space-time scale invariance," Astrophys. J. 698 (2009) 1630–1638, arXiv:0810.4065 [astro-ph].
- [90] F. Lelli, C. De Breuck, T. Falkendal, F. Fraternali, A. W. S. Man, N. P. H. Nesvadba, and M. D. Lehnert, "Neutral versus ionized gas kinematics at z 2.6: The AGN-host starburst galaxy PKS 0529-549," Mon. Not. Roy. Astron. Soc. 479 (2018) no. 4, 5440-5447, arXiv:1807.03321 [astro-ph.GA].
- [91] R. Jimenez, L. Verde, and S. P. Oh, "Dark halo properties from rotation curves," Mon. Not. Roy. Astron. Soc. 339 (2003) 243, arXiv:astro-ph/0201352 [astro-ph].

- [92] S. R. Coleman and E. J. Weinberg, "Radiative Corrections as the Origin of Spontaneous Symmetry Breaking," Phys. Rev. **D7** (1973) 1888–1910.
- [93] F. Donato, G. Gentile, P. Salucci, C. F. Martins, M. I. Wilkinson, G. Gilmore, E. K. Grebel, A. Koch, and R. Wyse, "A constant dark matter halo surface density in galaxies," Mon. Not. Roy. Astron. Soc. 397 (2009) 1169-1176, arXiv:0904.4054 [astro-ph.CO].
- [94] Y. Ali-Hamoud, J. Chluba, and M. Kamionkowski, "Constraints on Dark Matter Interactions with Standard Model Particles from Cosmic Microwave Background Spectral Distortions," Phys. Rev. Lett. 115 (2015) no. 7, 071304, arXiv:1506.04745 [astro-ph.CO].
- [95] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, L. David, W. Forman, C. Jones, S. Murray, and W. Tucker, "Direct constraints on the dark matter self-interaction cross-section from the merging galaxy cluster 1E0657-56," Astrophys. J. 606 (2004) 819–824.
- [96] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradac, "Constraints on the Self-Interaction Cross-Section of Dark Matter from Numerical Simulations of the Merging Galaxy Cluster 1E 0657-56," Astrophys. J. 679 (2008) 1173–1180, arXiv:0704.0261 [astro-ph].
- [97] D. Harvey, R. Massey, T. Kitching, A. Taylor, and E. Tittley, "The non-gravitational interactions of dark matter in colliding galaxy clusters," Science **347** (2015) 1462–1465, arXiv:1503.07675 [astro-ph.CO].
- [98] D. Wittman, N. Golovich, and W. A. Dawson, "The Mismeasure of Mergers: Revised Limits on Self-interacting Dark Matter in Merging Galaxy Clusters," Astrophys. J. 869 (2018) no. 2, 104, arXiv:1701.05877 [astro-ph.CO].
- [99] R. Barkana, "Possible interaction between baryons and dark-matter particles revealed by the first stars," Nature **555** (2018) no. 7694, 71–74, arXiv:1803.06698 [astro-ph.CO].
- [100] H. Tashiro, K. Kadota, and J. Silk, "Effects of dark matter-baryon scattering on redshifted 21 cm signals," Phys. Rev. **D90** (2014) no. 8, 083522, arXiv:1408.2571 [astro-ph.CO].
- [101] J. B. Munoz, E. D. Kovetz, and Y. Ali-Haimoud, "Heating of Baryons due to Scattering with Dark Matter During the Dark Ages," Phys. Rev. **D92** (2015) no. 8, 083528, arXiv:1509.00029 [astro-ph.CO].
- [102] J. B. Muoz and A. Loeb, "A small amount of mini-charged dark matter could cool the baryons in the early Universe," Nature 557 (2018) no. 7707, 684, arXiv:1802.10094 [astro-ph.CO].
- [103] A. Berlin, D. Hooper, G. Krnjaic, and S. D. McDermott, "Severely Constraining Dark Matter Interpretations of the 21-cm Anomaly," Phys. Rev. Lett. 121 (2018) no. 1, 011102, arXiv:1803.02804 [hep-ph].
- [104] R. Barkana, N. J. Outmezguine, D. Redigolo, and T. Volansky, "Strong constraints on light

dark matter interpretation of the EDGES signal," Phys. Rev. $\bf D98$ (2018) no. 10, 103005, arXiv:1803.03091 [hep-ph].