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Energy Efficiency in LoRa networks - An Approach from Stochastic Geometry

Lam-Thanh Tu, Abbas Bradai and Yannis Pousset

Abstract—In this letter, the closed-form expression of the energy efficiency (EE) in LoRa networks is formulated and derived based on tools from stochastic geometry. In addition, the insights of EE respect to some essential parameters, i.e., the density of EDs and the transmit power are proven based on the rigorous mathematical frameworks. Our findings show that, the EE monotonically decreases respect to the average number of end-devices (EDs) and a pseudo-concave function respect to the transmit power. Furthermore, the asymptotic framework of EE when the average number of EDs approaches zero is also derived. Finally, Monte Carlo simulations are provided to verify the correctness of our framework.

Index Terms—LoRa, Energy Efficiency, Stochastic Geometry, System-Level Analysis.

I. INTRODUCTION

Recently, low power wide area networks (LPWAN) has emerged as one of the most promising solutions for Internet-of-Things (IoTs) networks thanks to its low power and wide coverage area [1]. Among these available LPWAN technologies, i.e., SigFox, Weightless, NB-IoTs and so forth, LoRa is attracted lots of attention from both academia and industry. The core advantages of LoRa is its patented modulation, i.e., the chirp spread spectrum (CSS) modulation, which is proven to better combat noise and fading compared to other conventional modulation techniques, e.g., QAM and PSK. Furthermore, by actively fine-tuning its parameters, particularly, the spreading factor (SF), the coding rate (CR), the bandwidth (BW) and so on, LoRa is able to serve a wide range of IoTs applications/devices with different quality-of-service (QoS) levels. Stochastic geometry (SGs), on the other hand, is a mathematical tool which investigates the random point patterns. Thus, it is regarded as an appropriate tool to study the performance of wireless networks, i.e., cellular networks, ad hoc networks and so on.

The performance of the LoRa networks based on tools from stochastic geometry was studied extensively in [2]–[5]. In particular, in [2] the coverage probability (Pcov) was studied. However, this work did not take into account the correlation between the noise and interference. The extension of this work was done in [3] by considering multiple antennas at end-devices (EDs) and/or the gateway. However, multiple antennas at either EDs or gateway in LoRa networks seems to be impractical due to the simple and low-cost transceiver. The Pcov based on the conventional signal-to-interference-plus-noise ratios (SINRs) was examined in [4], the Pcov, nevertheless, was computed based on numerical results rather

than on closed-form expression. A recent definition of the Pcov which was not only taken into account the correlation between noise and interference but also computed in the closed-form expression was provided in [5]. The performance of the potential spectral efficiency (PSE) was also derived in [5]. Nevertheless, none of the above-mentioned works investigated the energy efficiency (EE) in LoRa networks. Hence, in this letter, based on the recent definition of the Pcov in [5] as well as tools from stochastic geometry, the EE in LoRa networks is formulated and computed in the closed-form expression. The main contributions and novelties of this letter are summarized as follows: i) the EE is computed in the approximated closed-form expression; ii) the trends of the EE respect to both the transmit power and the average number of EDs are proven based on the rigorous mathematical framework; iii) the asymptotic framework of the EE when the average number of EDs goes to zeros is also derived; iv) Monte Carlo simulations are provided to verify the exactness of the proposed frameworks.

The paper is organized as follows. In Section II, the system model is presented. The performance analysis and the behaviors of the EE respect to some vital parameters are investigated in Sections III and IV. In Section V, Monte Carlo simulations are supplied to clarify the correctness of our framework. Finally, Section VI concludes this paper.

Notations: $\Pr(\cdot)$ and $\mathbb{E}\{\cdot\}$ are the probability and the expectation operators; $\min\{\cdot\}$ is the minimum function; $\exp(\cdot)$ is the exponential function; $\mathbf{1}(x)$ is indicator function which is equal to 1 if $x > 0$ and 0 otherwise; $\log(\cdot)$ is the logarithm function; $F_X(x)$ and $f_X(x)$ being the cumulative distribution function (CDF) and the probability density function (PDF) of random variable (RV) X ; $\dot{f}(x)$ is the first-order derivative of function f over x , $\dot{f}(x) = df(x)/dx$; $\ddot{f}(x)$ is the second-order derivative of function f over x , $\ddot{f}(x) = d^2f(x)/dx^2$.

II. SYSTEM MODEL

A. LoRa Networks Modeling

Let us consider uplink LoRa networks with single gateway located at the center of the disk with radius R and a number of end-devices which are followed Poisson distribution with mean \bar{N} and the positions are uniformly distributed around the gateway. As a result, the EDs is followed an homogeneous Poisson point process (PPP) with density $\lambda = \bar{N}/(\pi R^2)$. We assume that there is no interference from different technologies which are operating at the same industrial, scientific and medical (ISM) frequencies [2]. The packet sent by EDs to the gateway is impaired by both the small-scale fading and large-scale path-loss. The impact of the shadowing does not

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take into consideration as its effects can be implicitly studied by modifying the density of the EDs [6].

1) *Small-scale fading*: Let us denote h_e as the small-scale fading from an arbitrary node e to the gateway which follows Nakagami- m distribution with corresponding fading severity and scaling factor m_e and Ω_e . Consequently, the channel gain, h_e^2 , follows Gamma distribution with shape and scale parameters, m_e and $\theta_e = m_e/\Omega_e$, respectively. We also consider case independent and identically distributed (i.i.d.) random variable (RV), so we have following: $m = m_e, \Omega = \Omega_e, \forall e$. We also assume that time is slotted and the fading remains constant during one time-slot and changes between time-slot.

2) *Large scale path-loss*: Considering a transmission link from a generic node e to the gateway, the large-scale path-loss is formulated as

$$L_e = l(r_e) = K_0 r_e^\beta, \quad (1)$$

where $\beta > 2$ and K_0 are the path-loss exponent and the path-loss constant, respectively; and r_e is the distance from the ED e to the gateway. Here, $K_0 = \left(\frac{4\pi f_c}{c}\right)^2$, f_c , c being the carrier frequency and the speed of light.

B. Power Consumption Modelling

There are two operational modes of the EDs in the considered networks, i.e., the EDs is in transmission mode when it transmits packet to the gateway and is in sleep mode otherwise. The power consumption of the ED under transmission mode comprises of two parts. The first part is the transmit power denoted as P_{tx} and the second part is the static (circuit) power, P_{sta} , which accounts for other dissipation irrelevant to the transmit power such as the power for processing data. The sleep mode, on the other hand, consumes P_{sle} power and is less power than the circuit power, i.e., $0 < P_{sle} < P_{sta}$.

III. PERFORMANCE ANALYSIS

A. Energy Efficiency

The energy efficiency (EE) (measured in bits/Joule) refers to the ratio between the benefit and cost where the benefit is the number of bits successfully transmitted per unit time and area in the network, i.e., the network spectral efficiency, and the cost is the network power consumption per area. Mathematical speaking, the EE can be formulated as follows [7]:

$$EE = \frac{PSE}{P_{con}}, \quad (2)$$

where PSE is the potential area spectral efficiency (measured in bits/s/m²) and is formulated as $PSE = \lambda_A BW \log_2(1 + \gamma_D) P_{cov}(\gamma_D)$; $\lambda_A = p_A \lambda$ is the density of active end-devices and $p_A = \frac{L_{pac} 2^{SF}}{T_{in} SFBWCR}$ is the active probability; L_{pac}, T_{in}, CR being the length of the packet, the inter-arrival time between two transmit packets and the coding rate [1]. $P_{cov}(\gamma_D) = \Pr\{SIR \geq \gamma_I, SNR \geq \gamma_D\}$ is the coverage probability; γ_I and γ_D being the rejection and quality-of-service threshold, respectively [5]; SIR and SNR being the signal-to-interference ratio (SIR) and signal-to-noise ratio (SNR) are defined as $SIR = S/I_S$; $S = P_{tx} h_0^2/L_0$;

$I_S = P_{tx} \sum_{i \in \Psi^{A \setminus \{0\}}} (h_i^2/L_i)$ and $SNR = S/\sigma^2$; $\sigma^2 = -174 + NF + 10 \log_{10} BW$ (dBm) is noise variance of AWGN noise. $\Psi^{A \setminus \{0\}}$ is set of active EDs except for the ED of interest; $\log(\cdot)$ is the logarithm function and BW is the system bandwidth.

P_{con} is the average power consumption of the whole networks per area (measured in Watt/m²); $P_{con} = \lambda(p_A(P_{tx} + P_{sta}) + (1 - p_A)P_{sle})$. The first term of the P_{con} accounts for the power consumption when ED is in transmission mode and the second term is for the sleep mode.

In LoRa, the number of concurrent transmission is limited due to the strict constraint of the duty cycle [1], as a consequence, it is acceptable to approximate the sum of interference by the dominant interferer in LoRa networks [2], [3]. In addition, as proven in [5], the exact closed-form expression of the P_{cov} under the instantaneous small-scale fading is never existed due to the intractable mathematical framework. Thus, the P_{cov} is approximated by the dominant interferer and the mean of the small-scale fading as $\widetilde{P}_{cov}(\gamma_D) = \Pr\{\widetilde{SIR} \geq \gamma_I, \widetilde{SNR} \geq \gamma_D\}$; where $\widetilde{SIR} = \widetilde{S}/\widetilde{I}_M = (P_{tx} \mathbb{E}\{h_0^2\}/L_0) / \sum_{i \in \Psi^{A \setminus \{0\}}} (P_{tx} \mathbb{E}\{h_i^2\}/L_i)$; $\widetilde{SNR} = \widetilde{S}/\sigma^2$, $\widetilde{S} = P_{tx} \mathbb{E}\{h_0^2\}/L_0$ and $\widetilde{I}_M = \sum_{i \in \Psi^{A \setminus \{0\}}} (P_{tx} \mathbb{E}\{h_i^2\}/L_i)$ are the approximated SIR, SNR, intended signal and the strongest interferer. Based on this approximation, the approximated P_{cov} is provided by Theorem 1 as follows

Theorem 1: Let us define the shorthand: $\mathcal{U} = \frac{m\theta P_{tx}}{R^\beta K_0 \sigma^2 \gamma_D}$, $\delta = 2/\beta$, the approximated coverage probability of the arbitrary EDs is computed as follows:

$$\begin{aligned} \widetilde{P}_{cov}(\gamma_D) &= (N_A)^{-1} (\gamma_I)^{-\delta} \left(1 - \exp\left(-N_A \min\left\{(\mathcal{U}\gamma_I)^\delta, 1\right\}\right)\right) \\ &+ \exp(-N_A) \left(\mathcal{U}^\delta - (\gamma_I)^{-\delta}\right) \mathbf{1}(\mathcal{U}\gamma_I - 1) \mathbf{1}(1 - \mathcal{U}) \\ &+ \exp(-N_A) \left(1 - (\gamma_I)^{-\delta}\right) \mathbf{1}(\mathcal{U} - 1), \end{aligned} \quad (3)$$

where $\exp(\cdot)$, $\min(\cdot)$ and $\mathbf{1}(x)$ are the exponential, the minimum and the indicator function.

Proof: See Appendix I. \square

Remark 1: Comparing with the SINR-based definition in [4], the P_{cov} in (3), is not only taken into account the correlation between noise and interference but also computed in closed-form expression.

From Theorem 1, the approximated EE denoted as \widetilde{EE} is computed as following

$$\widetilde{EE} = \frac{p_A BW \log_2(1 + \gamma_D) \widetilde{P}_{cov}(\gamma_D)}{p_A(P_{tx} + P_{sta}) + (1 - p_A)P_{sle}}. \quad (4)$$

In the sequel, the trends of the EE respect to some essential parameters, \bar{N} , P_{tx} , are unveiled based on (4).

IV. PERFORMANCE TRENDS

In this section, the behavior of the EE is proven based on the rigorous mathematical framework. In the following, for clarity, the network parameter of interest is denoted by ϖ . The other parameters are implicitly assumed to be constant. For ease of notation, accordingly, the shorthand notation $\widetilde{EE} = \mathcal{E}(\varpi)$ and $\widetilde{P}_{cov} = \mathcal{P}(\varpi)$, for EE and P_{cov} are used. The impact of \bar{N} , P_{tx} , are provided by two followings Propositions.

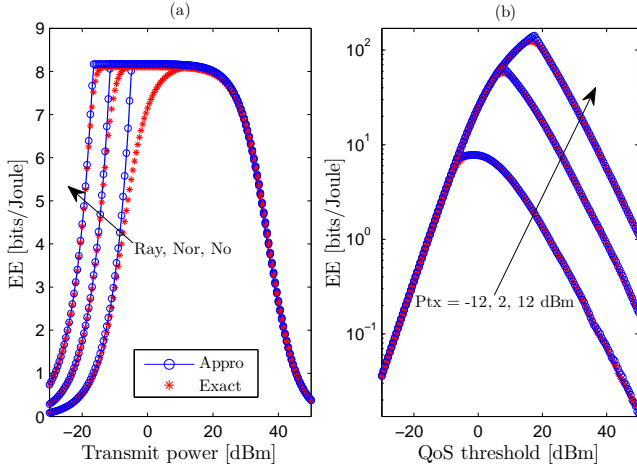


Fig. 1. EE versus Ptx [dBm] (a) and versus QoS [dBm] (b); Solid lines are plotted by using (4) while markers are from Monte-Carlo simulations. The term “Appro” is the proposed approximation framework and “Exact” is sum of interference and instantaneous small-scale fading. The setup for cases “Ray, Nor, No” (Ray=Rayleigh, Nor=Current setup and No =No fading) are followings: ($m = 1, \Omega = 1$), ($m = 4.5, \Omega = 4.5$) and ($m = 15, \Omega = 15$).

Proposition 1: Let us denote $\bar{N} = \varpi$, the EE is a convex function with monotonic decreasing property. The asymptotic of EE when $\varpi \rightarrow 0$ is computed as $\mathcal{E}(\varpi) \stackrel{\varpi \rightarrow 0}{=} \vartheta_3 \min \left\{ (\mathcal{U}^\delta), (\gamma_I)^{-\delta} \right\} + \vartheta_3 \left(\min \{ \mathcal{U}^\delta, 1 \} - (\gamma_I)^{-\delta} \right) \mathbf{1}(\mathcal{U}\gamma_I - 1)$; where $\vartheta_3 = p_A B W \log_2(1 + \gamma_D) / (p_A (P_{tx} + P_{sta}) + (1 - p_A) P_{sle})$.

Proof: See Appendix II. \square

Proposition 2: Let us denote $P_{tx} = \varpi$, the EE is a pseudo-concave function respect to ϖ .

Proof: See Appendix III. \square

V. NUMERICAL RESULTS

In this section, numerical results are provided to verify the accuracy of our mathematical framework as well as to substantiate our findings in Section IV. We consider a class of IoTs devices which is applied to smart city. Particularly, the credit machine is taken into account in this section. Unless otherwise stated, following setups are considered [8], [9]: $\beta = 2.5$; BW = 125 KHz; NF = 6 dBm; $\gamma_I = 6$ dBm; $f_c = 868$ MHz; $R = 3000$ m; $m = 4.5$; $\Omega = 4.5$; $\bar{N} = 5000$; $P_{tx} = 2$ dBm; $P_{sta} = 1.986$ dBm; $P_{sle} = -10.457$ dBm; $\gamma_D = -6$ dBm; $L_{pac} = 24$ bytes; SF = 7; CR = 4/5 and $T_{in} = 1800$ seconds.

Fig. 1 illustrates the performance of the EE versus the transmit power (a) and QoS threshold (b). Particularly, in Fig. 1(a), the EE under three different fading distributions are considered, i.e., Rayleigh, Nakagami- m and no fading, respectively. It is evident that the proposed framework is close to the exact results which are merely obtained based on Monte-Carlo simulations. Fig. 1(a) also confirms our findings in Proposition 2 that the EE is a pseudo-concave function respect to the transmit power. Moreover, this figure also justifies an obvious fact that the no fading distribution attains the highest EE with the smallest transmit power, followed by the Nakagami- m and Rayleigh distribution. It can be explicated that when Ptx is small, the impact of the AWGN noise on the performance of EE is not non-negligible, as a consequence, the better the

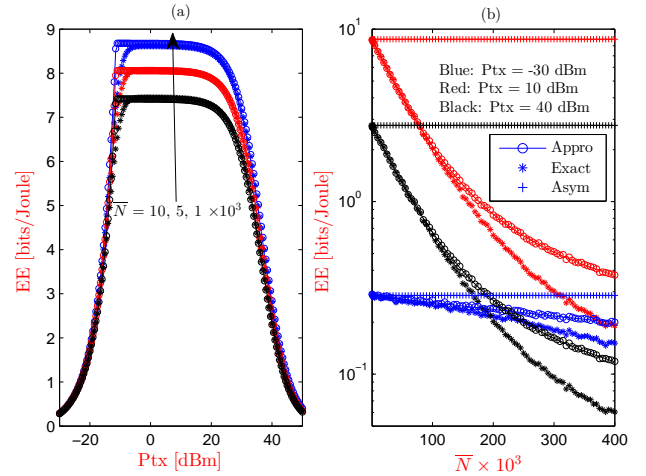


Fig. 2. EE versus Ptx [dBm] (a) and versus \bar{N} (b); Solid lines are plotted by using (4) while markers are from Monte-Carlo simulations. The term “Appro” is the proposed approximation framework; “Exact” is the sum of interference and instantaneous small-scale fading and “Asym” is the asymptotic of EE when $\bar{N} \rightarrow 0$ and is computed in Proposition 1.

channel gain the higher the energy efficiency. However, when the transmit power is sufficient large, the EE of all cases attain the same performance because the system is changing to the interference-limited regime and is independent of the Ptx. Fig 1(b) shows the performance of the EE versus the QoS threshold, γ_D . We observe that the EE is a unimodal function respect to the γ_D , the main reason of this behavior is that the numerator of the EE is dominated by the factor $\log_2(\cdot)$ when γ_D is small and is mainly influenced by the Pcov when γ_D is adequately large. In addition, it is trivial to state that the γ_D solely affects the numerator of the EE and independent of the denominator.

Fig. 2 studies the performance of the EE versus the transmit power (a) and the average number of EDs (b). Fig. 2(a) confirms again our findings in Proposition 2 as well as the accuracy of the framework compared with Monte-Carlo simulations. There is no doubt that increasing the average number of end-devices will decline the system energy efficiency. This statement is substantiated by Fig. 2(b) which illustrates the EE versus the \bar{N} . Indeed, as proven in Proposition 1, the EE is monotonically decreasing with \bar{N} . Furthermore, 2(b) also reveals an interesting fact that the higher transmit power does not necessarily achieve better energy efficiency. In fact, it depends on the system is either under noise-limited or interference-limited regime. To be more specific, when the average number of EDs, \bar{N} , is limited, the considered system is in noise-limited regime, thus, the larger the transmit power the better the EE. On the other hand, when \bar{N} is enormous, the system changes to interference-limited regime and we observe a contrary behavior, the higher the transmit power the worse the EE. The principal reason for this phenomenon is that keep increasing \bar{N} or densifying the network, the numerator of the EE in (4) is almost constant with Ptx while the denominator of the large transmit power will increase with faster pace compared with the low transmit power. As a consequence, the curves with larger Ptx will decrease faster and becomes worse than curves with smaller Ptx. Moreover, Fig. 2(b) also verifies

the correctness of the asymptotic framework in Proposition 1 when \bar{N} approaches zero.

VI. CONCLUSION

In this paper, the performance of the EE in LoRa networks is computed in the closed-form expression based on the approximated coverage probability. Moreover, the trends of the EE respect to both the transmit power and the average number of EDs are also derived based on the rigorous framework. Our findings reveal that the EE is a pseudo-concave function respect to the transmit power and monotonically decreases with the average number of EDs.

APPENDIX I PROOF OF THEOREM 1

In this section, the approximated Pcov is derived as follows:

$$\begin{aligned} \widetilde{\text{Pcov}}(\gamma_D) &= \Pr \left\{ \widetilde{\text{SIR}} \geq \gamma_I, \widetilde{\text{SNR}} \geq \gamma_D \right\} \\ &\stackrel{(a)}{=} \delta \mathcal{A}^\delta \int_{x=\sigma^2 \gamma_D / P_{tx}}^{\infty} x^{-\delta-1} \exp \left(-x^{-\delta} N_A (\gamma_I \mathcal{A})^\delta \mathbf{1}(x - \gamma_I \mathcal{A}) \right) \\ &\quad \times \exp(-N_A \mathbf{1}(\gamma_I \mathcal{A} - x)) \mathbf{1}(x - \mathcal{A}) dx \\ &\stackrel{(b)}{=} (N_A)^{-1} (\gamma_I)^{-\delta} \left(1 - \exp \left(-N_A \min \left\{ (\mathcal{U} \gamma_I)^\delta, 1 \right\} \right) \right) \\ &\quad + \exp(-N_A) \left(\mathcal{U}^\delta - (\gamma_I)^{-\delta} \right) \mathbf{1}(\mathcal{U} \gamma_I - 1) \mathbf{1}(1 - \mathcal{U}) \\ &\quad + \exp(-N_A) \left(1 - (\gamma_I)^{-\delta} \right) \mathbf{1}(\mathcal{U} - 1), \end{aligned} \quad (5)$$

where $\mathcal{A} = \frac{\mathbb{E}\{h^2\}}{R^\beta K_0}$; (a) is obtained by utilising the PDF and CDF of the approximated intended signal and interference, i.e., $f_{\widetilde{S}}(x) = \delta \mathcal{A}^\delta x^{-\delta-1} \mathbf{1}(x - \mathcal{A})$ and $F_{\widetilde{I}_M}(x) = \exp(-N_A \mathbf{1}(\mathcal{A} - x) - x^{-\delta} N_A \mathcal{A}^\delta \mathbf{1}(x - \mathcal{A}))$ [5], and (b) is attained by dividing the integration into three non-overlap regions of the transmit power: i) $P_{tx} \leq \frac{\sigma^2 \gamma_D}{\gamma_I \mathcal{A}}$; ii) $\frac{\sigma^2 \gamma_D}{\gamma_I \mathcal{A}} < P_{tx} \leq \frac{\sigma^2 \gamma_D}{\mathcal{A}}$ and iii) $\frac{\sigma^2 \gamma_D}{\mathcal{A}} < P_{tx}$ and using following results: i) given $0 < a < b < +\infty$ and $0 < c < 1$, we have $\int_{x=a}^b (x)^{-c-1} dx = c^{-1} \left((a)^{-c} - (b)^{-c} \right)$; ii) given $0 < a, b < +\infty$ and $0 < c < 1$, we have $\int_{x=a}^{\infty} x^{-c-1} \exp(-bx^{-c}) dx = (bc)^{-1} (1 - \exp(-ba^{-c}))$ and we conclude the proof here.

APPENDIX II PROOF OF PROPOSITION 1

The behaviors of the EE respect to the average number of EDs is derived in this section. First, let us rewrite the EE as follows:

$$\mathcal{E}(\bar{N} = \varpi) = \frac{p_A \text{BW} \log_2(1 + \gamma_D) \mathcal{P}(\varpi)}{p_A (P_{tx} + P_{sta}) + (1 - p_A) P_{sle}}. \quad (6)$$

From (6), it is obvious that the EE has the same behavior as Pcov respect to \bar{N} . Thus, taking the first-order derivative of Pcov respect to $\bar{N} = \varpi$, we have followings:

$$\begin{aligned} \dot{\mathcal{P}}(\varpi) &= -p_A (p_A \varpi)^{-2} (\gamma_I)^{-\delta} (1 - \exp(-p_A \varpi \vartheta_1)) - (p_A \varpi)^{-1} \\ &\quad \times (\gamma_I)^{-\delta} p_A \vartheta_1 \exp(-p_A \varpi \vartheta_1) - \vartheta_2 p_A \exp(-p_A \varpi) \leq 0, \end{aligned} \quad (7)$$

where $\vartheta_1 = \min \left\{ (\mathcal{U} \gamma_I)^\delta, 1 \right\}$, $\vartheta_2 = \left(\mathcal{U}^\delta - (\gamma_I)^{-\delta} \right) \times \mathbf{1}(\mathcal{U} \gamma_I - 1) \mathbf{1}(1 - \mathcal{U}) + \left(1 - (\gamma_I)^{-\delta} \right) \mathbf{1}(\mathcal{U} - 1)$ and $\dot{\mathcal{P}}(\varpi) = d\mathcal{P}(\varpi)/d\varpi$. From (7), we are able to conclude that the EE monotonically decreases with \bar{N} . The convexity

property of the EE is derived as follows: by taking the first-order derivative of $\dot{\mathcal{P}}(\varpi)$ in (7) respect to the ϖ , we have following

$$\begin{aligned} \ddot{\mathcal{P}}(\varpi) &= 2(\varpi)^{-3} (p_A)^{-1} (\gamma_I)^{-\delta} \\ &\quad \times (1 - (1 - \varpi p_A \vartheta_1) \exp(-\varpi p_A \vartheta_1)) + \vartheta_2 (p_A)^2 \exp(-\varpi p_A) \\ &\quad - (\vartheta_1) (\varpi)^{-2} (\gamma_I)^{-\delta} \exp(-\varpi p_A \vartheta_1) (2 - \varpi p_A \vartheta_1) \geq 0. \end{aligned} \quad (8)$$

The asymptotic of the EE when $\varpi \rightarrow 0$ can be derived straightforwardly by applying the l'hpital rule and we close the proof here.

APPENDIX III PROOF OF PROPOSITION 2

In this section, the behaviors of EE respect to the transmit power, P_{tx} , is identified. In particular, let us re-write the EE as follows:

$$\mathcal{E}(P_{tx} = \varpi) = \text{BW} \log_2(1 + \gamma_D) \frac{\mathcal{P}(\varpi)}{\mathcal{D}(\varpi)} \quad (9)$$

where $\mathcal{P}(\varpi)$ is the Pcov and $\mathcal{D}(\varpi) = \varpi + \alpha$; $\alpha = \frac{(1-p_A)}{p_A} P_{sle} + P_{sta}$. Let us take the first and second-order derivative of $\mathcal{P}(\varpi)$ respect to the transmit power $P_{tx} = \varpi$ as follows:

$$\begin{aligned} \dot{\mathcal{P}}(\varpi) &= \delta \varpi^{-1} \mathcal{U}^\delta \exp \left(-N_A (\mathcal{U} \gamma_I)^\delta \right) \mathbf{1}(1 - \mathcal{U} \gamma_I) \\ &\quad + \delta \varpi^{-1} \mathcal{U}^\delta \exp(-N_A) \mathbf{1}(\mathcal{U} \gamma_I - 1) \mathbf{1}(1 - \mathcal{U}) \geq 0 \quad (10) \\ \ddot{\mathcal{P}}(\varpi) &= -\delta \mathcal{U}^\delta \varpi^{-2} \left[\exp(-N_A) \mathbf{1}(\mathcal{U} \gamma_I - 1) \mathbf{1}(1 - \mathcal{U}) \right. \\ &\quad \left. + \left(1 + \delta \mathcal{U}^\delta N_A (\gamma_I)^\delta \right) \exp \left(-N_A (\mathcal{U} \gamma_I)^\delta \right) \mathbf{1}(1 - \mathcal{U} \gamma_I) \right] \leq 0. \end{aligned}$$

From (10), we are able to conclude that $\mathcal{P}(\varpi)$ is the concave function with increasing property respect to the transmit power. In addition, it is trivial to recognize that $\mathcal{D}(\varpi)$ is an affine function. Hence, EE is a pseudo-concave function respect to the transmit power [7, Proposition 2.9] and we close the proof here.

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