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# Online discursive interactions concerning mathematical issues within digital interactive storytelling

Giovannina Albano<sup>1</sup>, Cristina Coppola<sup>1</sup>, Umberto Dello Iacono<sup>2</sup>, and Anna Pierri<sup>1</sup>

<sup>1</sup>University of Salerno, Italy; {galbano@unisa; ccoppola; apierri}@unisa.it

<sup>2</sup>University of Campania “Luigi Vanvitelli”, Italy; umberto.delloiacono@unicampania.it

*This paper focuses on analysis of online discursive interactions among students from the first years of high school in problem-solving situations requiring the production of conjectures, arguments, and proofs. The study is framed in Vygotskian perspective, focusing on the language used to communicate and to manipulate and solve the tasks. We refer to the one-line proof model in order to analyze the students' path towards argumentation and proof and to discuss the outcomes of an experiment.*

*Keywords: Language, online interaction, proof, argumentation.*

## Introduction and theoretical background

This paper aims to analyze the impact of online discursive interactions among students in problem-solving situations, requiring the production of a conjecture as well as arguments to support and prove it. The study is part of a larger project framed in a Vygotskian historic-cultural perspective (Vygotsky, 1934), assuming that cognitive development is a social process and reasoning capabilities increase through interactions among peers and with an expert (e.g., a teacher). The children's participation in social interactions and communication is pivotal to their evolution and appropriation of cultural tools. Language is one of the most important cultural artifact and it has a fundamental role in mathematics learning processes (Planas, Morgan, & Schütte, 2018; Ferrari, 2004). On the one hand, it has a basic role as a communication and thought-organization tool in the social practices of the classroom. Language has a prominent communicative role in the development of mathematical thinking (Sfard, 2001): “Thinking may be conceptualized as a case of communication, that is communication with oneself” and it “arises as a modified private version of interpersonal communication” (p. 26). On the other hand, language assumes a key role in the process of objectifying and manipulating linguistic objects and symbols that is typical of mathematical activity (Coppola, Mollo, & Pacelli, 2019). This is still more true in our case, as the students predominantly interact through written communication via an online chat room. What may seem a constraint can instead be assumed to be a strength. In fact, writing is considered a semiotic tool of objectification, used by individuals in social processes of producing meaning in order to achieve a stable form of awareness, make their thoughts explicit and visible, and perform actions (Radford, 2002).

The problem-solving activities in our study engage students in producing conjectures, arguments, and proofs, which characterize mathematical activity. It is widely recognized that they are worth considering from an educational viewpoint. In this respect, we refer to *one-line proof* model, by Gholamazad, Liljedahl, & Zazkis (2003), for the fine-grained analysis of students' short proofs and the diagnosis and remediation of their productions, involving the following steps:

- *Recognizing the need for proof*: It is essential that students understand that the production of examples may not be sufficient to establish the validity of a statement and that they recognize that proof is essential.
- *Recognizing the need for representation*: The need for proof inevitably leads to the need for generalization, which requires students to select some form of representation.
- *Choosing correct and useful representation*: It is not enough for students to recognize the need for representation, but they also should select one that is correct and useful in order to arrive at a proof.
- *Manipulating the representation correctly*: Having chosen a representation, students should be able to work with it, thus carrying out the necessary manipulations in order to reach the proof.
- *Interpreting the manipulation correctly*: At the end of the manipulative process, students should be able to interpret the result of their manipulation and understand whether the proof is complete or not.

In the above framework, we want to investigate:

RQ1: How and to what extent can online discursive interactions foster the processes of explanation and argumentation in order to arrive at a proof?

RQ2: Furthermore, is it possible – and if so, to what extent – to transfer the teacher’s expert role to a suitably designed digital environment?

## **Methodology**

### **The design of the activity**

The activity was designed within a digital storytelling platform (Albano, Dello Iacono, Fiorentino, & Polo, 2018). The tasks are embedded in a narrative in which there are characters who will act as the avatars of the various participating students. Each student takes a role and has actions to perform, sometimes individually, sometimes together with others. There is also an expert who interacts with the students by orchestrating discussions on general topics.

Each task will develop according to a scheme that provides the following actions:

*Inquiry & Conjecture*, referred to as Episode 1, is related to the exploration of a mathematical situation. Starting from an event that relates to the mathematical problem, through the investigation, the student arrives at the formulation of a personal conjecture that she is called to share with her peers. Subsequently, starting from the shared findings, a comparison between peers opens up, with the explicit aim of formulating a common conjecture and then communicating it to the expert. While the comparison also takes place in the Chat tool, the communication with the expert takes place through the Forum tool, which encourages the students to produce a response that is expressed in a more evolved, literate register.

*Arguing & Proof*, referred to as Episode 2, leads, through discussion with the expert, to the comparison of the conjecture that emerged in the previous action with a formal proof. The expert manages this discussion with the aim of guiding the students towards the development of a formal proof, if necessary. At the end of the activity, the expert is responsible for institutionalizing what they have found.

*Summing Up & Refining*, referred to as Episode 3, draws the activities to a close. After this, the students, either alone or assisted by the expert, will have achieved a formal proof, providing the solution to the mathematical problem.

### The problem

The following mathematical problem is posed to the students (Mellone & Tortora, 2015, p.1436): Choose four consecutive natural numbers, multiply the two intermediate numbers, multiply the two extremes, and subtract the results. What do you get?

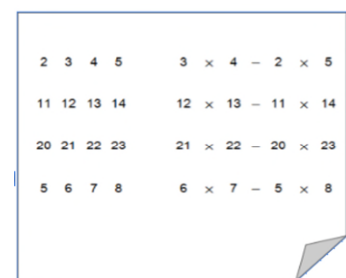
The aim of the problem is to introduce students to algebraic modeling and to develop their argumentative and proving skills. Students should conjecture that the result is always 2 and prove it using suitable algebraic representation, e.g.,  $n, n + 1, n + 2, n + 3$ , obtaining the expression  $(n + 1)(n + 2) - n(n + 3)$  and manipulating it in order to obtain the constant 2. It could also promote linguistic and logical discussions about terms such as “all” and “always”, as well as stimulating students’ thinking about key mathematical concepts such as the meaning of “consecutive” numbers.

This problem has been reframed in a narrative framework (Zan, 2012): a group of four friends, Marco, Clara, Federico, and Sofia, receive mysterious messages from aliens and collaborate with each other to understand it. They ask for help from Federico’s uncle, Gianmaria (Figure 1).



**Figure 1: The four friends and Gianmaria**

In the story, the original problem has been modified as follows. The students see a sheet showing some quadruplets and operations corresponding to the subtraction of the product of the second and third terms from the product of the first and fourth terms (Figure 2):



**Figure 2: The sheet**

In Episode 1, students receive the sheet in Figure 2, with the request to explore in order to try to understand the meaning of these numbers. They should note that these are groups of 4 consecutive numbers and should conjecture that when the operation to the right of each group of 4 numbers is carried out, the result is always 2. In Episode 2, the conjecture is institutionalized and students are asked to come up with a general proof. Various possibilities to be examined are presented through the story and its characters: “large numbers”, “multiple symbols”, “one symbol”. The comic, therefore, acts as scaffolding for recognizing the need for a proof and a representation. In this episode, the students should come up with an adequate formalization of the mathematical problem and prove the conjecture, with Gianmaria’s help. Finally, in Episode 3, students should send a response to the aliens that proves that they have understood their message; namely, the mathematical statement underlying the numbers received and its proof. The start of the task resolution process is scaffolded by a comic strip that lists

several kinds of messages to be sent, such as: “let’s send 2”, “let’s send a formula”, “let’s send a 4-upla”, “let’s send a proof”.

### The experiment

The experiment carried out and analyzed in this paper involved 24 students from the first year of high school. Students worked at a distance on Episode 1 and Episode 2, while they worked on Episode 3 in class, a month after Episode 2. They worked in groups of 4 students. In each group, each student assumed a specific role: Boss, Pest, Nerd, and Blogger. The Boss is task- and group-oriented; the Pest insinuates doubts and asks questions; the Nerd supports the friends in their use of computer tools, and the Blogger takes notes and summarizes the group’s answers. The role of the expert, Gianmaria, was played by a researcher in Episodes 1 and 2, while it was played by the class teacher, assisted by a researcher, in Episode 3. The story was implemented using the Moodle platform; specifically, its collaborative Chat and Forum tools. All data – that is, student interactions – were collected on the platform. The story was implemented through comics, integrated into the Moodle platform.

### Data analysis and discussion

In this section, we analyze the interaction of two groups of students, using the one-line proof model. The first excerpt is taken from group G1’s interactions in the Chat during Episode 2 (section 2.2):

- 1 Federico G1: I thought we could send either the quadruplets we came up with (even if we don’t know if they are correct or not) or the formula which we found, that is  $= 2$
- 2 Sofia G1: I thought we could send “middle items-external items = 2”
- 3 Federico G1: I thought we could send one of these things because they are the things we have developed and reflected about the most
- 4 Sofia G1: Yes, I agree, maybe the second one is better
- 5 Federico G1: If we send that one, we should also add a proof and maybe a reason to justify this choice
- 6 Sofia G1: We should say that we have decided to send this proof because when we calculate the quadruplets that the aliens sent us, the result is always 2.  
That is,  $a, b, c, d = bxc - axd$
- 8 Federico G1: Sending number 2 alone is useless without explaining how we found it. The proof without the formula is similarly useless
- 9 Sofia G1: Summing up, we thought that we could send the aliens the proof of why it is always 2, that is:  $a, b, c, d$  are generic letters, so they assume the value that we want to give them, that is consecutive numbers. So if we find the product of the middle items minus the product of the external items always is 2. So we should send:  $a, b, c, d: bxc - axd$

At the beginning, the students discuss whether they should choose an example or a formula (lines 1–4). It is noteworthy that they agree that the formula is better (line 4), which seems to highlight the awareness that an example is not enough to prove what they have understood. Moreover, the need to justify their choice emerges (line 5). In fact, Federico G1 seems to distinguish between giving a proof

and giving a reason for their decision, and he later underlines the need for completeness (line 8). Then, they try to formalize what they have observed (lines 6) and Sofia (as the Blogger) sums up what they have found (line 9).

We note that the students have grasped the fact that the numbers in the given quadruplets are consecutive, but that they use the representation “a, b, c, d”. They make use of an isomorphism that matches the numbers’ consecutiveness with the order of the letters in the alphabet. The analysis of further Chats from this group shows that they were not able to choose a correct and useful representation and thus they did not produce any proof of their conjecture.

The following two excerpts are about another group’s interactions. The first concerns the Forum in Episode 2, after they verify their conjecture in various examples using both small and large numbers:

- 10 Clara G2: It is generally true because if we do  
 $(9 \times 8) - (10 \times 7) = 2$   
 $(3 \times 4) - (2 \times 5) = 2$  [*some more examples are given*]  
 we can note that in each line the result is 2, then the answer is yes, it is always true. We can note one more thing: the operations always occur according to the pattern “even  $\times$  odd – even  $\times$  odd” or vice versa  
 Let’s try using symbols!  
 $a = \text{even number } b = \text{odd number } c = \text{even number } d = \text{odd number}$   
 $(b \times c) - (a \times d) = 2$
- 11 Marco G2: We can prove it using letters  $a = 12, b = 13, c = 14, d = 15, e = 16, f = 17, g = 18$ , for instance,  $(e \times f) - (d \times g) = 2$
- 12 Sofia G2: For instance, let us prove it with large numbers  
 200, 201, 202, 203  
 $(201 \times 202) - (200 \times 203)$  gives exactly 2 as a result! So I would say that with any sequence of numbers, the result is always the same

Note that the students focus their attention on the pattern relating to the order of the operations; that is, the difference between products such as an even number times an odd one and subsequently the alternation of even and odd numbers in the sequence (line 10). There is no recognition of the fact that the numbers are consecutive, except in the examples (line 11). It is worth noting that the students seem to recognize the need to go beyond the example by using letters, but they are unable to do so: they merely substitute the letters for some specific numbers. Indeed, in the following line, they again “prove” their conjecture using numbers, although they use large numbers. From the fact that it works with large numbers, they conclude that it always works: note “any sequence of numbers” (line 12); that is, their conjecture is proved. In their discourse, the students do not refer to “consecutive” numbers in the quadruplets. Their awareness of this aspect is revealed by the use of the above isomorphism by means of consecutive letters, as well as in the colloquial use of the words “sequence of numbers”, “sequence” meaning “consecutive”.

Let us see how they continue the discussion in the Chat during Episode 3.

- 13 Federico G2: Let's send a formula
- 14 Clara G2: A proof would not be a bad idea
- 15 Marco G2: We should choose from formula, proof, number
- 16 Clara G2: Let's send them a formula using some numbers
- 17 Federico G2: I prefer a formula made of symbols
- 18 Sofia G2: Mhm...a formula about what?
- 19 Marco G2: A formula made of natural numbers, such as n
- 20 Clara G2: We need to use the generic ones because the numbers are infinite
- 21 Sofia G2: What kind of formula do we want to use?
- 22 Clara G2:  $n + 3, n + 4, n + 5, n + 6$
- 23 Marco G2:  $(n)(3 + n) - (2 + n)(1 + n)$
- 24 Sofia G2: A proportion that corresponds to the given rule; in a proportion where the product of the middle items is equal to the product of the external items
- 25 Federico G2:  $n : n1 = n2 : n3$
- 26 Sofia G2: Actually, they use a sequence of numbers that we should denote as A B C D  
 $(A \times D) - (C \times B) = 2$
- 27 Marco G2: We should justify the reason for our choice
- 28 Clara G2: But you have used 4 random letters not in a sequence  
 In my opinion A could be 300, B 3, C 6, and D 90000  
 They should be a sequence, they must be consecutive  
 They must have a sequence, they must be consecutive numbers
- 29 Sofia G2: Using the exact numbers is wrong
- 30 Marco G2: Then we should use either the letters or n
- 31 Federico G2: It is an equation
- 32 Sofia G2:  $(n + 1)(n + 2) - (n)(n + 3) = 2$   
 Let's try to replace n with other numbers, it should always be correct
- 36 Federico G2: But what is n?
- 37 Marco G2: Any natural number
- 38 Clara G2: n is all the numbers

Looking at the beginning of the discussion, we see that the students are looking for a representation (lines 13–17). This need is born within the story's context, so the comic strip acts as scaffolding. Then someone makes explicit the link between the need for a representation and the need for proof, without merely relying on the examples (lines 18–20). Later, the students recognize that there can be various representations, so they discuss looking for the most useful one (lines 21–24) and find they need to justify their choice of representation (lines 25–27). It is worth noting that Sofia G2 uses the word “sequence” linked to “A B C D” (line 26), probably meaning consecutive numbers, but Clara G2's reply highlights that A, B, C, and D are merely variables for identifying numbers and not consecutive

numbers (line 28). It is worth noting that this remark is consistent with Clara's role (that is, the Pest). Going on, the students recognize that they cannot use examples, but need a general representation (lines 29–30). Finally, they reach a correct and useful representation and give an equation (line 32). They stop here, without manipulating the equation in order to show why it works. Sofia G2 merely suggests verifying the equation by replacing  $n$  with numbers. This invitation stimulates a very interesting interaction concerning the values with which  $n$  can be replaced and the universal quantifier naturally comes into play (line 37–38), which is one of the issues we expected the problem to pose.

## Results and conclusions

Concerning RQ1, from the analysis of the students' interactions in the Chat and Forum tools, supported by comic strips acting as scaffolding at the beginning of the solving process, a co-construction of explanations and arguments seems to emerge. Students work together, each one in turn, starting from what other students in the group have said, answering and rebutting, and together they attempt to construct the solution to the proposed task. This process seems to support the path through the steps of the *one-line proof* model (Gholamazad et al., 2003), which also allows them to individuate the failure points. Indeed, the students move along the first step on the left-hand side of the model, recognizing the need for proof: line 5 shows the emergence of the need to justify their choice, and thus the need for representation (the second step in the model). Unfortunately, they fail this step by choosing an incorrect – or at least unhelpful – representation (line 9 – the consecutive numbers are indicated by the letters a, b, c, d) and consequently manipulating them incorrectly (line 11 – the letters are substituted with numbers). The conducted analysis thus seems to suggest, with respect to RQ2, that neither the group of peers nor the technological environment is sufficient to guide students to manipulate the chosen representations in order to correctly interpret them and arrive at a proof of their conjectures. Moreover, we observe that when students talk to each other in the Chat function, they often use a colloquial register (Ferrari, 2004), even though they have to write their answers. This is probably because the chat is perceived as a shared context, almost as if they were in each other's presence. A change can be observed when they have to communicate their answers to Gianmaria (or to the aliens); that is, asymmetric communication. In this case, the environment also changes, from Chat to Forum: they no longer have the same shared context, so they need to be more "formal" and to use a literate register (Ferrari, 2004). The interaction with the expert and the consequent change of register could spur the process towards a proof.

These remarks suggested a change in the design, involving the possibility of an expert intervening in the Chat. The work on this new design is in progress. Another suggestion that emerged from this analysis is that the different characters played by the students seem to work well, in accordance with the characteristics of the roles (Albano, Pierri, & Polo, 2019). Further analysis is in progress.

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