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Lalina Coulange, Grégory Train²

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The School Mathematical Discursive Community: Diversity and the role of language in meaning-making

Lalina Coulange¹ and Grégory Train²

¹Université de Bordeaux, Lab-E3D, Bordeaux, France; lalina.coulange@u-bordeaux.fr

²Université de Bordeaux, Lab-E3D, Bordeaux, France; gregory.train@u-bordeaux.fr

Our theoretical framework allows us to consider the mathematics class as a School Mathematical Discursive Community, relying on Bakhtin's theory of genres of discourses and heteroglossia. In our paper, we question how the teacher can orchestrate heteroglossia in order to establish a Relevant School Mathematical Discursive Community. We show that such an orchestration, which relies on a secondarising process, is productive for teaching and learning mathematical concepts (in this case, decimals and fractions) at primary school.

Keywords: (Relevant) School Mathematical Discursive Community, heteroglossia, secondarising process.

Theoretical framework: The School Mathematical Discursive Community

Our theoretical framework is based on both Vygotskian and Bakhtinian perspectives. In particular, we use Bakhtin's distinction between first and second of discourse in order to analyse the language practices of teachers and students. First genres of discourse relate to everyday or informal contexts and refer to spontaneous or commonplace activities. Second genres of discourse are more elaborate and formal and relate to specific knowledge. Like other researchers (Pimm, 1987; Barwell, 2012; Barwell & Pimm, 2016; Planas & Setati, 2009; Prediger & Wessel, 2011), we are interested in the evolution from first genres of discourse to second genres of discourse in the mathematics classroom. From our theoretical perspective, we tackle this evolution through a specific dynamic called the *secondarising process* of language practices (Bernié, 2002, 2004; Jaubert & Rebière, 2012). The secondarising process also relies on the idea that the students' and teacher's activities always reference diverse contexts. This plurality of contexts potentially provides diverse meanings to their activities. The resulting heteroglossia (Barwell & Pimm, 2016) includes different voices carried by the students' and teacher's discourses, seen as a language diversity that is inherent in teaching and learning. Thus, we consider the mathematics class to be a *School Mathematical Discursive Community*. Within a teaching and learning perspective, it is necessary orchestrate heteroglossia through interactions between students and teacher. In order to establish a *Relevant School Mathematical Discursive Community*, this orchestration also needs to come about via a *secondarising process*, gradually leading the students' language from first genres of discourse to second genres of discourse in order to mediate their access to mathematical concepts.

Methodology

Our research study took place over four years (2015–2018) in a French primary school with students (grades 4 and 5) from socially disadvantaged backgrounds. The table below describes the data collected and presented in this paper and examples of the phenomena we will discuss thereafter.

Grade – Mathematical content		Data	Phenomena
Grade 4	Decimal numbers Positional notation	3 sessions, videotaped (October 2015) and transcribed	Heteroglossic linking (local event)
Grade 4 Grade 5	Fractions Diagrams and problems	Questionnaire (January 2018 – grade 4) – students’ written productions 6 sessions, videotaped and transcribed (February–March 2018 – grade 4) Questionnaire (September 2018 – grade 5) – students’ written productions	Establishing a Relevant School Mathematical Discursive Community (several steps)

Table 1: Data collected and analysed – Phenomena

The analyses of the collected data aim to highlight two kinds of phenomena. In the first section, we will show how we tackle *heteroglossia*. We will illustrate the role played by *heteroglossic linking* in (re)constructing the mathematical meaning of decimals (positional notation) through an analysis of an episode from the end of the first videotaped session. In the second section, we will study dynamics related to the orchestration of *heteroglossia* in order to establish a *Relevant School Mathematical Discursive Community*. Our analyses of several sessions and of students’ written work will allow us to illustrate a *secondarising process* in the teaching and learning of fractions (diagrams, equitable sharing problems).

Heteroglossic linking and constructing the meaning of decimal positional notation

During the session, the teacher asked grade 4 students the following question: “If you know that the length of a given line is 1 cm, how can you give its length in dm?” Students had several rulers: each ruler was graduated in a given unit (cm, dm, mm) and they had previously used them to measure the length of several lines. They had also studied simple fractions ($\frac{1}{2}; \frac{1}{4}; \frac{3}{8}$) as “unit-splitting”, but they had not yet encountered decimal fractions ($\frac{1}{10}; \frac{1}{100}$) or decimal positional notation (0,1; 0,01). The teacher aimed to discover whether students would manage to extend the “unit-splitting” in such a problem: by “reversing” the idea of dividing a group of units by 10 (10 cm = 1 dm) in order to reach the idea of dividing a unit into 10 (1 cm = $\frac{1}{10}$ dm). This is quite a new perspective towards measurement units (grouping/splitting units) which seems to be difficult for students to conceptualise (Coulange & Train, 2019).

One of the students, JOS, managed to envisage that as “one dm was equal to ten cm”, so “one cm was ten times smaller than one dm”, and then “one dm shared by ten or one-tenth of a dm”. Another student, TAR, wrote a decimal notation, “0,1 dm”, which relied on a kind of written literacy. TAR used units on a measurement place value chart and entered “1” in the cm column, then “0” in the dm column. TAR added a comma between the two digits. At the end of the session, the teacher spoke to both JOS and TAR. Our analysis focuses on this final episode:

- JOS: You say that one centimetre is ten millimetres. So, it is ten times larger. You have to make it ten times larger, so, if you want to do it in decimetres, it should be ten times smaller [...].
- Teacher: So how many many centimeters we can put how many in one decimetre?
- Students: Ten.
- Teacher: Then, a centimetre is how many times smaller? So I will wait for your answers [...].
- JOS: One-tenth.
- Teacher: One-tenth of a decimetre, does everybody agree? [Students: Yes.] So you had another suggestion, what did you suggest? Everybody, you have to look. [...] TAR suggested another notation.
- TAR: As there are zero decimetres, I put a zero, but comma one. I think this is the same as this [pointing at JOS's $\frac{1}{10}$ – at the same time, JOS points to the “1” of the “0,1” written by TAR].



Figure 2: Linking decimal notation “0,1 dm” and fractional notation “ $\frac{1}{10}$ dm”

- Teacher: You think they are the same?
- JOS: Yes, because here it is one centimetre and ten millimetres, and here we say it is one-tenth. So, if he puts ‘zero comma one’, it means it is one-tenth. As you add the tenth, it makes one-tenth.
- Teacher: This one, it could be the one-tenth. I don’t know, we will have to speak more about this

During this collective episode, the two students managed to simultaneously construct coherence between two voices, one relating to decimal notation, “0,1 dm” (relying on written literacy), and one relating to “one-tenth of a decimetre” (relying on the mathematical concept of “splitting units”). From our point of view, this is a heteroglossic linking (Lhoste et al., 2011) which was initiated by TAR (“I think that this is the same as this”) and completed by JOS (“zero comma one, it means it is one-tenth”). This heteroglossic linking seems to allow meaning-making of decimal positional notation through a secondarising process. The teacher addressed this issue (“we will have to speak more about this”) and pursued this question further during the following sessions.


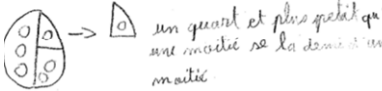

Moreover, during the following years, this strategy of making meaning of decimal positional notation through heteroglossic linking was repeated. Students investigated the relationships between two systems of symbolic notation used for expressing measurements of length: decimal positional

notation (18,2 cm) and complex numbers and metric units (18 cm 2 mm). Then, they had to (re)construct the meaning of the decimal positional notation “18,2 cm” in relation to decimal fractions (the “2” of 18,2 cm interpreted as 2 tenths of a unit) (Coulange & Train, 2019). This teaching strategy relies on the orchestration of diverse voices regarding mathematical notation within the School Mathematical Discursive Community. These voices come from diverse contexts, such as social use, written literacy, and mathematical concepts. Even though it is not easy to orchestrate such a mathematical discussion in a productive way, it nevertheless allows the secondarising process of the students’ discourses and the conceptualisation of decimal (positional) notation to take place.

Establishing a Relevant School Mathematical Discursive Community regarding fractions and diagrams

First step: A strong *heteroglossia* related to words and diagrams

We had observed that students seemed to use familiar words, such as “quarter” or “half”, before studying fractions at school. In order to learn more about the meaning that students attached to these words, we formulated a questionnaire and collected written productions from students (grade 4, before being taught fractions): students were how they understood terms such as “a half”, “a quarter”, “two quarters”, “a half of a quarter”, and “a quarter of a half”. The results of this study revealed diverse understandings of such terms: for example, some students considered a quarter as “a half of a half” or “sharing into four equal parts”, while others described it as “a piece smaller than a half” or “sharing into three parts” (with some different arguments related to sharing actions or the results of these actions). Moreover, a strong heterogeneity of writing practices regarding diagrams was revealed. Some students made drawings (of a pizza or a slice of pizza), sometimes without information about “a unit”, while others produced more elaborated models of “unit-splitting” and used them to make a mathematical argument (for example, in order to form relationships between “a half of a half” and “a quarter”).

	<p>“Here it is as a pizza / A fourth is when you share into three / I said it was like a half / Actually I think it is not that”</p> <p>Drawing of a pizza (unit) to share – Quarter as “sharing into three”</p>
	<p>“A fourth is maybe smaller than a half? It is maybe a part of a half / I think it is a half / a half of a half [...] it is the same as a clock / Actually a quarter of an hour”</p> <p>Drawing of a pizza (unit) to share – Quarter as “the half of a half”</p>
	<p>“A quarter is a small piece”</p> <p>Circle diagram with a unit. Quarter as a “small” part</p>



	<p>“A slice of pizza / As it is cut like this to eat it”</p> <p>Drawing without the pizza (unit) – Quarter as a “small” part</p>
<p>Un quart est quelle que chose qui se divise en 4. ex: 8 est le quart de 32.</p> 	<p>“A quarter is something you divide into four [...] An example / I am not sure because eight, eight plus four, it makes twelve / I think eight is a quarter of twelve”</p> <p>Circle diagram with a unit. Quarter as splitting a unit into four (diagram) and as adding four (numerical example)</p>

Table 3: “What is a quarter?” Students’ answers and diagrams or drawings

We used these diverse language practices relating to both the meaning of “a quarter” (and also to “two quarters” or “a half of a quarter”) and the diagram of “equal sharing” or “splitting a unit” in order to design activities about fractions. Our aim was to orchestrate this kind of *heteroglossia* and to establish a *Relevant School Mathematical Discursive Community* through a *secondarising process*. In particular, as previous results (Champagne & Coulange, 2019) led us to find that drawing or interpreting diagrams (circles or rectangular models) related to fractions may help to resolve persistent difficulties for some students, we especially focused on this point. We designed activities with the teacher in order to help students to conceptualize graphical models of fractions.

Second step: Question about “equitable sharing – splitting a unit” drawing

In a previous session, students had to split a semi-circle into six equal parts. They had produced diverse drawings and diagrams, but they did not manage to conclude whether their propositions were valid or not during the discussion. The teacher came back to this question.

Students: Splitting into six parts.

Teacher: Splitting into six // what was your problem / you had made drawings, but what was the question [pointing on drawings on the whiteboard]?

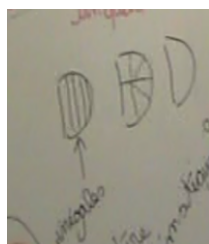


Figure 3: Splitting a semi-circle into six equal parts – First diagrams proposed by students

Student: The problem was / at first we had big parts and then smaller.

Teacher: Actually, it seems we didn’t have equal parts / if I colour this part in green and this part in red / we agree / it is not equal.

Student: The other one / there are equal parts.

Teacher: What do you think? These parts could be equal [*colouring parts on the second drawing*] / You think that / Everybody look / it is important / Do you think this part could be the same as this one? [...]

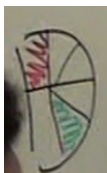


Figure 4: Splitting a semi-circle into six equal parts – Discussion of a diagram

Students: Yes / No / Not necessary because what you coloured in red is small [...].

Teacher: So I will propose something to you / We will continue with this / Today, our question is how we can split this semi-circle [*showing some semi-circular surfaces made of paper – one for each student*] into /// At first, we will try to split it into three equal parts.

Students produced several drawings in order to answer to this question (splitting a semi-circle into three).



Figure 5: Splitting a semi-circle into three equal parts – New diagrams proposed by students

In a new discussion, they seemed more or less in agreement that only the drawing on the left could be the right answer (a student argued “the point should be in the middle [of the diameter]”) In order to check this drawing, a student suggested folding the paper into three. His proposal allowed him to establish relationships between graphical representations and material gestures (folding and superposing). This session played a crucial role in the students’ comprehension of the constraints on “equitable-sharing/splitting units” drawings. Then, the teacher encouraged them to use diagrams for reasoning, for showing their argument, and so on. The students gradually seemed to adopt such written practices for themselves and to become particularly proficient in this kind of use.

Third step: Establishing a Relevant School Mathematical Discursive Community – Some consequences

The following written productions come from the same students (at the beginning of grade 5). They had to solve the following problem: “The teacher has 5 biscuits. Could you help her share all the biscuits among 5 students?”

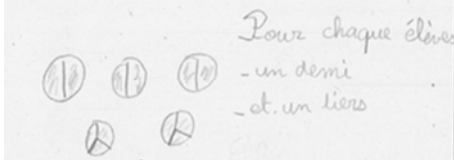
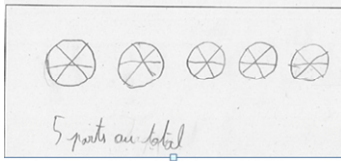
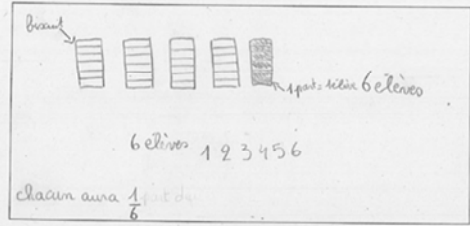
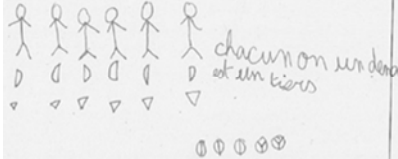
 <p>Correct answer – valid diagram, “6 halves” and “6 thirds”</p>	 <p>Correct answer – valid diagram, “5 sixths”</p>
 <p>Wrong answer – valid diagram, “5 sixths”</p>	 <p>Correct answer – valid diagram, “6 halves” and “6 thirds”</p>

Figure 6: Diagrams proposed by students in order to solve a sharing problem

The analysis of these written productions shows that the students were able to use models in order to represent fractions and to solve “equitable sharing” problems. It also highlights that this secondarising process did not completely reduce the diversity of the students’ language practices: they produced diverse diagrams (circular or rectangular models) and reasoning. However, the students’ written productions also have common features that are relevant to mathematical concepts of “equitable-sharing/splitting units”. Therefore, we can consider that a Relevant School Mathematical Discursive Community was established through a secondarising process: it allowed all the students to evolve their language practices about “equitable-sharing/splitting units” and to access new concepts related to fractions.

Conclusion and discussion

In this paper, we have highlighted phenomena relating to establishing a School Mathematical Discursive Community. Our first example gives a local illustration of how the orchestration of heteroglossia relying on heteroglossic linking is likely to make meaning of decimal positional notation. It also shows that taking heteroglossia into account may be useful for designing efficient teaching and learning trajectories. The second example investigates the steps involved in such a perspective. It shows how establishing a Relevant School Mathematical Discursive Community relies on consideration of the students’ prior language practices in the mathematics classroom. This initial language diversity needs to be gradually orchestrated through a secondarising process guided by relevant mathematical questions acknowledging initial heteroglossia. Such a dynamic process encourages the evolution of students’ language practices in order to mediate mathematical concepts (related to fractions in this case). Therefore, we consider that establishing a Relevant School Mathematical Discursive Community may provide new directions for mathematics teaching and learning. We hope that our contribution highlights some of these perspectives. From a more theoretical point of view, it may be interesting to how Bakhtin’s theory may contribute to studies of language and mathematics learning and teaching (Barwell & Pimm, 2016).

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