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Developing new discourses to deepen students' conceptual understanding in mathematics

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Previous studies have led us to identify some objects or aspects of mathematical activity as being “transparent” for teachers and hence as causing some learning difficulties. This paper examines perspectives and challenges in explicating these transparent mathematical objects and developing new mathematical discourses to increase students' conceptual understanding of geometry and algebra. We present here the first results of an exploratory study conducted in collaboration with a group of teachers and which show similar phenomena in geometry and algebra.

Keywords: Language role, mathematics activities, discourse, measurement, algebra.

Introduction

The work of Lev Vygotsky highlighted the fundamental role of language in mediating individuals' access to concepts, thereby pointing at its epistemic role together with its social, historical, and cultural dimensions. Language constitutes one of the main supports of mathematics teaching and learning activities in classrooms, which has made it an important object of research in mathematical education for some time now (Austin & Howson, 1979). Mathematical and mathematical education discourses (Pimm, 2004) thus constitute a resource for mathematical thinking in classrooms, but also an object of learning: learning mathematics includes learning how to talk like a mathematician. This crucial issue has been expressed and explored from various theoretical perspectives, which vary by the way they link mathematics and language: for example, Vergnaud (1998), who states that “mathematics is not language but knowledge”, or Pimm (2004) or Sfard (2008), who base their approach on the idea of “mathematics as a language”.

This has led many researchers in mathematics education to focus on the development of discourse (e.g. Moschkovitch, 2010; Planas, Morgan, & Schütte, 2018; Radford & Barwell, 2016), considering both the epistemic role of language, based on Vygotsky's theory, and the dialogical form of its development from a more Bakhtinian perspective (Barwell, 2016; Barwell & Pimm, 2016). The teacher's role in the learning process thus appears to be crucial, and it includes the fact of offering opportunities for discussion by the tasks s/he chooses, by the way s/he implements these tasks, and especially by the way s/he organizes discussions; finally, what also appears to be important is discourses that the teacher her/himself produces (reformulations, new words or grammatical forms s/he introduces, etc.). The resulting difficulties for the teacher can be formulated in terms of tensions and dilemmas (Schleppegrell, 2010). The “dilemma of mediation” (Adler, 1997) concerns the fact of letting students develop their own language within the conceptualizing process on the one hand and driving them towards the use of conventional expert language on the other, this latter language being socially and historically constructed, meaning that it is unlikely to appear spontaneously. The crucial “dilemma of transparency” (Adler, 1999) concerns the difficulty teachers experience in making things

clear for students in their discourse and, at the same time, the importance of not losing focus on mathematical activity by paying too much attention to language.

Our contribution to the investigation of these questions has previously consisted in linking some learning difficulties to elements that remain implicit in mathematics classroom discourses. We hypothesized that these elements remain implicit because they correspond to ‘naturalized’ practices for teachers: Chesnais (2018, 2019), Munier and Chesnais (2016), and Chesnais and Munier (2016) highlighted the implicitness in the language used by teachers in geometry lessons, while Constantin (2014) identified some implicitness relating to some aspects of the manipulation of expressions in algebra. Our postulate (supported by classroom observations) is that the implicitness of some objects or aspects of mathematical activity might prevent some students – especially the ones whose sociocultural background is distant from school – from achieving a conceptual understanding of mathematics. Our intention in this paper is to present some first attempts to investigate ways of enriching mathematics classroom discourse in order to remedy some students’ difficulties concerning these two subjects. After revealing the context of our research and the research question we are tackling in this paper, we will present our theoretical framework and methodological tools. The next part will be devoted to the exposition of the first results. The final discussion will lead us to expose the questions raised by these first attempts and the need for further research.

Context of the research

Our previous work has led us to identify mathematical objects and procedures which are “concealed” in most classroom discourses where implicitness seems to be causing difficulties for some students. In particular, we identified elements related to the role of measurement in geometry at the beginning of secondary school (6th grade) and the distributive law in algebra in 8th grade.

Concerning geometry, we identified that the word *measure* is sometimes used by teachers to label either values which are obtained using an instrument (which are decimals and subject to uncertainty) or values which are obtained by calculations (which are real numbers and exact values) (Chesnais & Munier, 2016). This polysemy appeared to cause some difficulties for students when, for example, the two values are not equal or when they do not understand why teachers will not accept measuring with an instrument in some situations (situations where a deductive proof is expected). Most teachers’ discourse³ on this point is often limited to some “rules” such as “you cannot prove a property by measuring a drawing”. This led us to suppose that explicitly tackling the distinction and the relationship between the empirical and theoretical aspects of measuring is necessary to support a conceptual understanding of the role of measurement and proof in geometry. It then necessitates new discourses, including words to label these two “kinds” of measures differently. As researchers, we labeled them “empirical measure” and “theoretical measures”. The distinction hence appeared to be a promising tool for didactic analyses (Chesnais & Munier, 2016), but the matter of its operationalization in mathematics classrooms remained an open question.

³ Such an allegation is supported by multiple observations in classrooms but also by textbook surveys, where discourses in textbooks are considered to approximate teachers’ discourses in classrooms.

Concerning algebra, we identified implicit substitutions in teachers' discourse during transformational activities (Constantin, 2014). When stressing the use of formulas such as $k(a + b) = ka + kb$ for factoring or expanding expressions such as $3x(4x + 2)$ or $3(a - 6b + 9)$, teachers do not consider substitution as a potential tool for explaining what is at stake in these computations. Some difficulties may appear for students when k and a denote an expression instead of a number, such as $3x$ or $a - 6b$. These expressions must no longer only be considered to represent a computational operation; now, they also represent the result of such a procedure. Sfard and Linchevski (1994) use the term *reification* “to denote the switch in the pupil's conception which is necessary to turn a process into an object” (p. 8). Their study yields evidence that this is a long and difficult process and that many pupils see algebraic manipulations as being governed by arbitrary rules. Several studies in France or Lebanon (Abou Raad & Mercier, 2009; Coulange et al., 2012) have also shown how teachers' discourses rely on syntactic descriptions, talking about “taking what is before the brackets” or “after the plus sign”, without linking them to the properties of the operations. Mok (2010) also points out that when they achieve conceptual understanding of the distributive law, pupils are able to justify their claims with different strategies, using substitutions to examine whether a distributive pattern is correct or not and explaining their ideas using functional meanings of symbols and explicit properties. Following this research, our claim is that substitution may contribute to enriching mathematical discourse about the distributive law. These new discourses might necessitate new names such as “substituent” and “substituted” to specify sub-expressions.

Our research question is based on the hypothesis that the options we have identified may potentially constitute good “tools” for fostering mathematical activity and discourse in classrooms that would be more likely to support conceptual understanding than the teacher's usual methods. The experiments we present constitute a first attempt to investigate how talk and argument progress when substitution and a distinction between theoretical and empirical measures are explicitly used.

Theoretical framework

Following Vergnaud and including a Vygotskian perspective, we consider that managing specific linguistic signs – particularly verbal language (oral or written, using words) – is an integral part of the conceptualizing process. From both a Vygotskian and a Bakhtinian perspective, we postulate that new discourses emerge in dialogical forms of activity (Barwell, 2016; Barwell & Pimm, 2016) before they are interiorized by individuals to support their own mathematical activity. Conceptualization and discourse development thus constitute dialectical processes that are highly complex and subject to dilemmas and tensions (Adler, 1997, 1999; Schleppegrell, 2010).

We consider the development of discourse supporting conceptualizing as an evolution from informal genres towards “more formal” genres of discourse (Bakhtin, 1981; Barwell, 2016). This secundarizing process (Bernié, 2002; Jaubert & Rebière, 2012) refers to a “densification” of discourse. In particular, new common nouns appear together with new objects in mathematical activity as a necessity: this is what Vergnaud (1998) calls “substantification”. New discourses may also include new expressions or evolution in the usage of some words in order to support the conceptual complexity in intermediate forms, thus explicating what is transparent. Those intermediate forms might suppose a high syntactical complexity (Prediger & Sahin-Gür, 2019).

Experiments and methodology

Our collaborative research project involves researchers and teachers in iterating processes of elaboration-implementation-analysis. The project is intended to last for at least three years, but we will present only the first step in this paper. The project in geometry included five teachers teaching in the sixth grade. The project in algebra concerns only one class in the eighth grade. In geometry, the experiments concern the subject of angles and last for about 10 sessions. This choice was dictated by the fact that it is an emblematic subject for the distinction and articulation of empirical and theoretical aspects of measure: the French curriculum for the 6th grade includes the use of the protractor, but also work on short proofs concerning theoretical measures (like determining whether angle ABC is a right angle if one knows the size of two adjacent angles ABD and DBC).

Our discussions with teachers about the conceptual challenges that we detailed in the first part of this paper revealed that they correspond to actual difficulties that they had identified in their classes which they did not know how to resolve. Researchers and teachers then developed ideas in order to remedy these difficulties based on what teachers usually do with their classes while attempting to enrich it with elements brought into the discussion by researchers: the explicit distinction between empirical and theoretical measures in the 6th grade, and the idea of substitution in the 8th grade.

Lessons were videotaped and students' written productions were collected. Videos were transcribed and analyzed in order to identify the emergence of new discourses on the two critical points (measure and substitution). This included detecting opportunities for discussions on these points and collecting all the associated discourses, then identifying changes. Our indicators of an evolution were related to lexical and syntactical aspects of the discourse.

Two common findings

Although experiments were conducted on different subjects and levels, some critical phenomena appeared in both contexts. Because of the lack of space, we have decided to illustrate our findings on one particular task for each subject. In geometry, one teacher decided to propose the following task: "Here is an exercise proposed to sixth-grade students and one student's answer: In the figure below [showing an angle DBE which appears close to a right angle], is the angle DBE a right angle? Student's answer: 'I have measured $\angle EBD = 90^\circ$, and therefore EBD is a right angle'. Is the student's answer correct?" In algebra, two sessions were devoted to explicitly making substitutions in $k(a + b) = ka + kb$ in order to expand the expressions. In the first exercise, pupils were asked to substitute k for $2x$, a for 7 , and b for $3x$, while in the second exercise, they had to find possible substitutions in the formula in order to expand $-5(3n + 2)$ or $(4n + 3)(7n + 1)$.

Our first main finding is that while new discourses appear within students' and teachers' discussions, learners invent new words or expressions which do not necessarily correspond to those anticipated by researchers and teachers. While "theoretical measure" and "measured value" were the expressions that teachers had decided to use, and which were more or less used by students at some points, some other expressions appeared within the lessons. Here is an excerpt from a discussion about the task we mentioned above:

Student 1: It could be 91° [...] it is not a precise measure.

Teacher: So, what kind of measure is it, according to you?

Student 1: It is a *measured measure*.

Teacher: A measured measure. Ok. So it is not a theoretical measure, at any rate.

We interpret this excerpt as revealing that students are able to invent their own – ingenious and mindful – words (“measured measure”) to make sense of the situation, even if the expression does not seem completely acceptable to the teacher. We consider this nominalization to be a significant step towards substantification and conceptualization. Working on some other tasks also led one of the teachers to label the geometrical figures in order to distinguish them from their representations. She decided to talk about the “imaginary, ideal, and perfect figure” which is “schematized by the drawing” and she had the students rewrite the questions in exercises more explicitly in order to have them understand that measuring using a protractor was not the correct procedure because the question was about the theoretical value.

In the substitution experiment, pupils were asked to find possible substituents in order to use substitution in $kx(a + b) = ka + kb$ to expand $-5(3n + 2)$. Pupils suggested replacing k with -5 , a with $3n$, and b with 2 . When the teacher asked them how they had found the substituents, a student explained: “We look at the given computation [and] because k is the first, we take the first number, a is the second so we take the second number, and [for] b we take the third number”. Students also discussed about how to produce an equality using substitution:

S 1: As k is hmm, what do you call it? Well the 2, the $2n + 3$, I don’t remember what it’s called.

S 2: k .

S 3: The factor [...].

S 1: The factor, well, as k is the factor, well, hmm, we will multiply it with the terms.

These episodes show the necessity of labeling the different sub-expressions in relation to the identity to be used in the transformational activity. In doing so, two different methods appear: talking about a number (including $3n$), or a factor referring to either what the sub-expression denotes or its syntactic function. This leads us to the idea that substitution may play a role in the process of reification considering both systemic and syntactic structure in algebraic expressions (Kieran, 1989). It also contributes to a certain densification of the discourse, all the more so as several students show how difficult it can be to identify a factor in an expression such as $(4n + 3)(7n + 1)$, saying at first that there is no k or no factor. When expanding $(4n + 3)(7n + 1)$, the question of the choice of the substituent for k was then raised by the students when they were asked about how to find the factor, or “the k ”. One of the pupils argued that it is always at the beginning of the expression. Another challenged this statement by saying that $(7n + 1)$ could as well have been chosen. Two students then asked why the other was chosen at first and put forward the idea that it might not have given the same result, leading them to try another substitution, comparing and discussing the validity of both transformations in relation to the distributive and commutative law.

Our second finding is also illustrated by the two episodes presented above: introducing new tasks generated discussions which do not usually appear in classrooms since, according to teachers who

participated in the experiments, “things are usually swept under the carpet”. Indeed, discussions like the ones we have just evoked seem far from the idea of “applying rules”.

Discussion and conclusion

Though concerning different domains, our findings suggest that fostering the development of new discourses based on new language artifacts may help deepen students' conceptual understanding.

However, developing new discourses is not easy for teachers. Unsurprisingly, introducing new objects destabilizes teachers' practices and some confusion still occasionally appears in their discourse; for example, between theoretical and empirical measures or between substituents and substitutions. Our first experiments also show how difficult it can be to seize pupils' ideas in intermediate forms and link them to concepts or identify potentials for new lexical forms. In the case of substitution, pupils use *number* to refer indiscriminately to either 3 or $3n$. It appears to be both a lever for generalization and a hurdle for others, who will only consider 3 or n as substituents. The teacher also misses an opportunity to talk about a property of substitution: students use reverse substitutions to recognize the identity and to control their choice of substituents, but neither teacher nor students ever put into words what is happening here. Substantification takes some time: substitution is at first considered as a technique (or gesture) and not an object. These observations point out the necessity of long-term processes to provoke evolutions.

However, new names or labels are not sufficient conditions for developing conceptual understanding. Focusing pupils' attention on “the right word”, without relating it to the mathematical activity at stake, probably prevents the students from developing their own discourse.

Our case study also raises methodological questions about the intermediate forms: Since they do not appear in expert practices, where could they have come from? The idea we would like to explore more deeply is to investigate the way in which these mathematical objects emerged historically. Two other limitations of our research still need deeper investigation: first, we need to find ways to appreciate the relationship between discourse development and learning; second, we need to identify some indicators of the secundarizing process which would correspond not only to lexical and syntactical elements, but also to each of the three items identified by Pimm (2004), and especially the question of the voice. What appears certain, however, is that this kind of research necessitates long-term collaborations between researchers and teachers.

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