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Leila Salekhova

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Leila Salekhova

Kazan Federal University, Russia; salekhova2009@gmail.com

Two opposite trends in evaluating the bilingualism effect on student's academic achievements in math are analyzed in the paper: bilingualism as a resource or as a problem. As the linguistic features and practices of bilinguals form a unitary linguistic system which interacts in dynamic ways, bilingualism should be considered a human's resource with a large potential. From this theoretical framework, the basic qualities of mathematical communication in L1 and L2 (correctness, accuracy, coherence, way of translanguaging) and their descriptors are described. These qualities are used to describe how bilingualism influences the mathematical communication skills of pre-service bilingual Tatar-Russian mathematics teachers.

Keywords: Learning and teaching mathematics, bilingualism, bilingual students, mathematical communication, qualities of mathematical communication.

Mathematical communication and bilingualism

The abstract character of mathematics highlights the important role of language in the teaching, and learning mathematics, because it is impossible to physically show the abstract nature of most mathematics concepts. They can be described only through language. Monaghan (2000) introduced the term "mathematical langscape" to denote the combination of mathematical meanings and the resources for communicating these meanings that make up the mathematical curriculum. Similarly, Sfard (2001) argued that thinking was communication and to consider learning mathematics as being equivalent to developing a mathematical discourse. According to Sfard (2001) discourse can be defined as an activity of communication with oneself and others.

The study of bilingual mathematical discourse is of particular interest since it is more complicated because of interacting linguistic features and practices of bilinguals. Many students are currently studying mathematics in their second or third language and this phenomenon is gradually becoming the norm in many countries around the world. One reason for this is the migration process into developed countries, as well as the legacy of colonialism and the diverse plurality of local languages in developing countries. The second reason is that the language of science, technology and the Internet is slowly narrowing down to several international languages, such as English, therefore, textbooks and other teaching materials are often provided only in these selected languages.

The effect of bilingualism on cognitive development and student's academic achievements in mathematics is often seen as being at one or other end of a continuum in which bilingualism is considered either as a resource or as a problem (Planas, 2014).Many researchers state that a lack of fluency in the language of instruction is one of the main reasons for the poor performance of many students in mathematics, especially those who are bilingual and multilingual (Secada, 1992; Salekhova & Danilov, 2016). From this perspective, fluency in the language of instruction in regard to in mathematical discourse requires fixing.

Other research has found that knowledge representations are often cultural artefacts, closely connected to the language of instruction (Spelke & Tsivkin, 2001; Campbell, Davis, & Adams, 2007). Negative effects on academic achievement have been observed when the languages of instruction for learning mathematics and languages of assessment differ because language is a necessary condition for understanding mathematics concepts. Bilingual students that are weak in the language of instruction tend to have poor comprehension and participation in classroom discourse (Setati, 2005). Consequently, they cannot gain the desired objectives of their studies due to a lack of communication skills.

The aspects of the language which are specific for mathematics have also been found to be another source of difficulty and confusion for bilingual students, who are learning both the language of instruction and mathematics in a new language. In particular, words used in mathematical terminology are often endowed with meanings that in most cases are completely different from their everyday meaning. For example, the words: root, similar, space, even or odd have a different meaning when they are used in mathematics. Sometimes it can be difficult, even for students who are not bilingual, to determine what the intended meaning of is "odd" is in a problem. The research of Durand-Guerrier and Ben Kilani (2004) in the Tunisian context showed the difficulties students experienced in understanding mathematical negation.

However, in recent years, more and more researchers consider bilingualism as an intellectual resource with cognitive benefits. This is considered to be because the experience of using more than one language can create unique opportunities in the bilingual brain. For example, the constant switching of bilinguals from one language to another leads to increase in executive function. Bilinguals sometimes have an advantage in inhibitory control, in selection, switching, working memory, representation and retrieval, which play an essential role in learning mathematics (Bialystok, Craik, & Luk, 2012). Planas (2014) also argued that bilingualism can create advantages for learners to deal more deeply with mathematics concepts. In alignment with this, Alòs i Font and Tovar-García (2018) analyzed a sample of 709 ethnic Tatar school students from Tatarstan and showed that those who spoke Tatar at home tended to outperform in mathematics their schoolmates who had Russian as the home language. The results of my previous research showed a significant difference emerged in favor of bilinguals in solving language-independent, symbolic mathematics tasks of high complexity (Salekhova, 2019). Similarly, Mielicki, Kacinik, and Wiley (2017) found that bilingual USA college students solved mathematical problems that required advanced abstract thinking better.

Research and experience of teaching mathematics to bilingual students in Tartarstan is that student's bilingualism is a cognitive and linguistic resource. Nevertheless, there is a need to ensure that this potential is developed and used.

Context of mathematics learning and teaching in Tatarstan Republic

Russia is one of the countries with the highest language diversity in the world; representatives of more than 200 ethnic groups live here. Tatarstan is one of the ethnic republics of Russian Federation, and Tatars constitute 54,6 % of its population. Tatar is spoken by most of the people in Tatarstan either as the dominant (L1) or as the second language (L2). Russian–Tatar and Tatar–Russian bilingualism is widespread in Tatarstan. Both languages are used for teaching almost all school

subjects and the choice of the language of instruction depends on the school location (rural or urban) or the model of bilingual education used in the schools.

There are currently two main trends related to using languages in mathematics teaching and learning contexts can be observed in the Tatarstan schools - submersion and immersion. They lead to supportive and unsupportive bilingualism. Unsupportive bilingualism occurs when a majority Russian language replaces a minority Tatar language, in this case students, whose dominant language is not Russian must adapt to mainstream education where the Russian language is used as the medium of instruction. In the case of supportive bilingualism, the Tatar mother tongue of the child is the majority language of instruction at school, and he or she is learning the Russian language as a second one and has some subjects taught in it. This situation is the case in rural areas of Tatarstan, where the majority of the population is Tatar.

The influence of bilingualism on the mathematical thinking of bilingual students can be both negative and positive depending on the conditions under which the interaction of the two languages in the educational context occurs.

One of the goals of studying mathematics according to the new «Federal state educational standard of basic education in Russia» (2016) is developing methods of thinking (analysis, synthesis, comparison, classification, generalization). The role of language and communication in teaching and learning mathematics is noted in the "Concept of development of mathematical education in the Russian Federation" (2013). The document states that it is necessary to facilitate communication in teaching and learning mathematics, to encourage students to speak, write, read and listen in a math class.

Similar requirements are formulated in "Principles to Actions: Ensuring Mathematical Success for All" of National Council of Teachers of Mathematics (2014). NCTM states that mathematics learning program should give opportunity to students to (1) arrange and link their mathematical thinking through communication, (2) communicating their logical and clear mathematical thinking to their friends, teachers and others; (3) analyze and assess mathematical thinking and strategies used by others; (4) using mathematical language to express mathematical ideas correctly. Such endorsements and recognition of the importance of language in teaching and learning mathematics can be found in education documents of many countries, especially in such multicultural countries as the United States, New Zealand, Australia and South Africa (Ellerton & Clarkson, 1996).

Despite the multiplicity of languages used in the Russian educational system and the issues they raise, the influence of bilingualism on mathematical thinking and mathematical communication is an under-researched field in Russia. The national educational system of the Russian Federation supports monolingualism and assimilation. This can be seen in the fact that state exams in mathematics must be conducted on the territory of the Russian Federation, including Tatarstan, only in the Russian language. Moreover, starting from grade 4 to 11 final tests in all subjects, including mathematics, are only in Russian. The purpose of the final testing exams is to ensure the unity of the educational space of the Russian Federation and support the implementation of Federal educational standards.

70% of mathematics tasks in the final testing are word problems. In order to solve word problems successfully, students need to understand the essence of the mathematics task, the lack of language proficiency may be a problem in comprehension.

Tatar parents want their children to study mathematics in Russian, as they understand that success in passing exams is associated with knowledge of the Russian language and mathematical terminology in Russian. Mathematics teachers also want their pupils to have high scores in mathematics exams; therefore they choose the bilingual approach using the Russian and Tatar languages in teaching mathematics. However, they implement bilingual instruction spontaneously, without experience or knowledge of scientifically developed teaching methods.

The Ministry of Education for the Republic of Tatarstan launched two projects to preserve national identity and education in the Tatar language in 2018. The goal of the first project "Adymnar - the path to knowledge and harmony" is to build multilingual schools in the Republic of Tatarstan in which teaching subjects will be in three languages (Tatar, Russian and English).

The second project, "Bilingual Teacher", is aimed at pre-service teacher training in order to ensure that teachers have the competency to teach subjects in two languages. The preparation of bilingual teachers of mathematics, physics, music, foreign languages for these schools has begun at Kazan Federal University; at this stage, 150 students are studying in the first and second years of the courses.

Teacher educators working with pre-service bilingual teachers have many methodological issues because of little preparation in the development and use of bilingual materials and methodologies, in locating appropriate instructional materials etc. Since the teaching methods are not perfect, pre-service teachers have difficulties, in particular, associated with a limited vocabulary in one of the languages and, as a result, with academic communication in the subject-specific area, such as mathematics.

Theoretical framework and literature review

From a theoretical perspective (Vygotsky, 1986; Sfard, 2001), bilingualism should be seen as a resource for mathematical communication which is a window into mathematical thinking. The aim for our research is to identify how bilingualism influences the mathematical communication skills of Tatar-Russian bilingual pre-service teachers of mathematics. To do this, it was decided to highlight the basic qualities of mathematical communication, then to develop their descriptors and a scale for their assessment. In this paper, this assessment scheme is described.

Ben-Yehuda (2005) proposed four distinctive features of mathematical discourse: (1) the use of words that count as mathematical; (2) the use of uniquely mathematical visual mediators in the form of symbolic artefacts that have been created specifically for the purpose of communicating about quantities; (3) special discursive routines with which the participants implement well-defined types of tasks; (4) endorsed narratives, such as definitions, postulates, and theorems, produced throughout the discursive activity. (p. 182)

The analysis shows that the most frequently encountered qualities of mathematical communication are correctness, accuracy and coherence (Salekhova & Spiridonova, 2018). Therefore, they can be defined as the basic qualities of mathematical communication in the Russian language.

The bilingual mathematical discourse is more complicated than monolingual discourse as two languages are interacting in each utterance. One of the peculiarities of bilingual student's mathematical communication is that to express thoughts they can start a sentence in one language and end it in another. This phenomenon is known as translanguaging. Baker (2011) describes translanguaging as "the process of making meaning, shaping experiences, understandings and knowledge through the use of two languages" (p. 288).

García (2009) introduced the concept of dynamic bilingualism as enacted in translanguaging. Dynamic bilingualism does not simply refer to the addition of a separate set of language features, but acknowledges that the linguistic features and practices of bilinguals form a unitary linguistic system that interact in dynamic ways with each other. Despite the growing number of scholars using the term translanguaging, it is difficult for teachers, steeped in the monoglossic language ideologies that schools often promote, to accept it fully.

As García (2009) writes:

> Using translanguaging theory would mean that we would be able to separate the two types of performances. We would be able to assess if a bilingual student uses the lexicon and linguistic structures of a specific-named language in socially and academic appropriate ways—the named language-specific performance. And we would be able to assess if he or she is able to perform linguistically to engage in academic and social tasks regardless of the language features used—the general linguistic performance. (134).

According to Shohamy (2011), bilingual assessment can be placed in a continuum. At one end of the continuum, teachers can use multiple languages in the same assessment, but only responses in the target language are evaluated. On the other hand, all the student's languages are considered part of a single system-their linguistic repertoire, and students can use any language in the tests and even mix them.

The question is how can the basic qualities of mathematical communication (correctness, accuracy and coherence) be assessed in bilingual students' mathematical communication if two languages (L1, L2) are interacting in their speech? The answer is to introduce one more quality of mathematical communication - how translanguaging is conducted. By viewing translanguaging in mathematic classroom communication as positive, it is important to acknowledge that the linguistic features and practices of bilinguals form a unitary linguistic system that interact in dynamic ways with each other (García, 2009). Students can use their full language repertoire in explanations, but it is essential to pay attention to how translanguaging is conducted. Translanguaging needs to be conscious, in that there should be an awareness of the speech, characterized by the validity of the reasoning, and an ability to select language tools (Russian or Tatar), which meet the goals and conditions of communication.

Potential framework for evaluating the mathematical communication

In order to develop support mechanism for preservice bilingual teachers of mathematics, it was deemed important to provide descriptors of the basic qualities. Table 1 provides the descriptors for basic qualities of mathematical communication in L1 and L2.

Basic qualities	Descriptors
correctness	correct usage of mathematical terms, symbols and notations, correct spelling of mathematical terms and symbols, words and expressions of the natural language (Russian or Tatar), correct design of graphic images and drawings
accuracy	ability to choose a rational way to solve the problem (proof of the theorem), to present the mathematical material precisely, to document the process of solving the problem accurately and efficiently
coherence	knowledge of the basic verbal and logical constructions of the mathematical language, ability to present the material consistently, to build a text following its semantic structure (break into sentences, paragraphs, etc.).
way of translanguaging	translanguaging must be conscious which is understood as an awareness of the speech, characterized by the validity of the reasoning, ability to select language tools (Russian or Tatar) that meet the goals and conditions of communication

Table 1: Basic qualities of mathematical communication

In order to evaluate the preservice bilingual teachers' competencies so that support could be targeted to them, a four-point scale was developed to evaluate the level of each of the qualities: "high" (4 points), "average" (3 points), "low" (2 points) and "very low" (1 point). For example, the level of coherence of mathematical communication can be evaluated in the following way:

4 points: Text content is presented sequentially; splitting a text into meaningful units (sentences and paragraphs) is made clearly; verbal-logical structures of the natural language, namely, comparative quantifiers, conjunctive, disjunctive, implicative of the design and construction of negation are used correctly; there is a variety of used words and expressions denoting the verbal-logical design; one inaccuracy is allowed;

3 points: there are insignificant violations in the sequence of ideas presented, in breaking the text into meaningful parts; verbal-logical constructions are used correctly; there is a sufficient variety in the use of words and expressions denoting the verbal-logical structures; 2-3 errors may occur;

2 points: there are some violations in the sequence and the use of verbal and logical structures; the division of the text into semantic parts is not clear; the written text is characterized by the monotony of words and phrases denoting verbal and logical constructions; 4-5 errors are possible;

1 point: the sequence of presentation of thoughts is broken in all parts of the text, thoughts are not presented in the form of structural units of the text; there are numerous violations in the use of verbal and logical constructions.

Similarly, descriptors for other basic qualities of mathematical communication were developed.

Conclusion and perspective

Based on the view that mathematical communication is a window into mathematical thinking, that dynamic bilingualism is a human resource, that translanguaging in mathematic classroom communication is positive, the basic qualities of mathematical communication (correctness,

accuracy, coherence and way of translanguaging) were highlighted and corresponding descriptors were developed. These descriptors and the accompanying rating scale will be used to evaluate pre-service teachers' solutions to mathematical problems presented in written form. From this evaluation, the basic qualities of mathematical communication in L1 and L2 will be used to explore whether bilingualism influences the mathematical communication skills of bilingual pre-service teachers', and if it does to what extent and why. Our plans are to use the results in order to design task and teaching-learning possibilities that are targeted at improving bilingual learners' language repertoires so that they have the best possibilities for deepening mathematics thinking and learning.

There are many studies on the communicative qualities of mathematical speech, but there are few that raise the question of how these qualities in bilingual pre-service teachers can be developed. Many researchers both within and outside Tartar and Russia along with our experience indicate that the mathematical achievement of bilingual students depends on the knowledge of the language of instruction. Therefore, it is important that future mathematics teachers are aware of specially designed teaching methods and techniques that will support school students' possibilities to learn.

Acknowledgement

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