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Bilingual undergraduate mathematics teaching examined through a commognitive lens

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In this paper we examine key principles of visual mediators and endorsed narratives as relating to the commognitive approach. We apply these concepts to the analysis of teaching in a bilingual undergraduate mathematics context. A central focus is on the lecturer's communicative practices and ways of facilitating discursive shifts (colloquial to literate), while examining which language(s) this occurs in. The findings suggest that the use of graphs as well as accompanying gestures, deictic expressions and symbols substantiate the claims about functions that the lecturer is making. The primary language of communication was Irish, establishing an expectation of utilising Irish when engaged in mathematical narratives relating to functions. English was utilised (with Irish) to clarify meaning, primarily when a discursive shift from an object to meta-level discourse was being established.

Keywords: Commognitive approach, bilingual university teaching, language use, functions.

Introduction

A primary mathematical objective in a first undergraduate year of study is to facilitate and develop advanced mathematical thinking. The focus of this paper is on a bilingual (Irish and English) lecturer's teaching of a variety of function constructs, explicitly graphing and limits to a group of bilingual learners. These students were participating in a bilingual first year undergraduate Calculus module at the National University of Ireland, Galway. The data presented in this paper forms part of a larger study examining mathematical meta-level discourses in the English and Irish languages in this bilingual undergraduate context. Previous findings from a student perspective (Ní Ríordáin & Flanagan, 2019), and adapting Sfard's Commognitive framework (2008) for data analysis, suggest that evidence of employing an advanced language trajectory (in either language) does not correlate with mathematical meta-level thinking for the students who participated in the study. Further, language preferences for communicating mathematics learning are situated and can impact the discursive processes of bilingual learners. It was found that when students considered their individual problem-solving routines as incompatible and/or dichotomous this impacted their mathematical discourse practices and contributed to a lack of language negotiation and meta-level discourse development.

Following on from this, we are interested in examining the teaching practices in this bilingual undergraduate mathematics context. A central focus of this paper is on the lecturer's communicative practices and ways of facilitating discursive shifts, while examining which language(s) this occurs in. We utilise Sfard's (2008) commognitive framework as an analytical lens to support our investigation of bilingual teaching practices. Nardi, Ryve, Stadler, and Viirman (2014) illustrate how the commognitive approach can be applied to university mathematics education, while emphasizing that 'this approach can be extended to topics hitherto untouched' (p.183). Examination of teaching

practices in bilingual undergraduate mathematics education remains under-researched. Barwell, Wessall, and Parra (2019) acknowledge that much has been done to advance the field but that gaps still remain. They highlight the need for further research into how “multilingual students’ language repertoires can be activated and developed in order to achieve deeper mathematics learning.” (p.116). In particular, they highlight the need for discursive research in multilingual classrooms which examines mathematical thinking and interaction. Our primary aim in this paper is to illustrate the commognitive perspective in a naturally occurring bilingual undergraduate mathematics education context, and accordingly to contribute towards the use of the frameworks analytical tools to provide analyses of bilingual mathematics teaching and learning contexts.

Sfard’s Commognitive Framework

Sfard’s (2008) framework for analyzing mathematics discourse (both instruction and learning) was employed to explore a university lecturer’s teaching of functions and in which language(s) (English and/or Irish) this occurred. The neologism *commognition* comprises communication and cognition and recognises the varied, purposive and unique nature of discursive approaches to examining mathematics teaching and learning. Sfard (2008) distinguishes between colloquial (everyday or spontaneous discourses) and literate mathematics discourse (requires deliberate teaching). The focus of the M²EID study was how objectified talk signifies discursive shifts between colloquial to literate mathematical discourses and in which language this occurs. Therefore, we observed the meaning-making processes that the lecturer engaged in when mathematizing and in particular the relationships between object- and meta- level discourse and the language(s) of use. In particular, we aim to illustrate the potential of the commognitive approach (Nardi et al., 2014) in examining bilingual undergraduate mathematics teaching. The four tenets of Sfard’s (2008) approach include 1) word use, 2) visual mediators, 3) routines, and 4) endorsed narratives. For the purpose of this paper we focus on visual mediators and endorsed narratives as these were the dominant themes emerging from the analysis.

Sfard (2008) describes visual mediators as visible objects that embody interlocutors’ mathematical communication (e.g. concrete objects, symbols, tables, graphs, drawings, diagrams, charts, formulae). As a representational tool, such visual mediators can influence interlocutors’ mathematical thinking and consequent actions (Tabach & Nachlieli, 2011). Sfard categorised visual mediators as: symbolic (expressions), iconic (graphs) and concrete (protractors). Symbolic mediators can be utilised to interpret the global properties of the mediator or they can be a basis for substitution process to, for example, simplify an equation, through a clearly defined set of rules or steps. Iconic mediators can be observed (graphs) or constructed (graphing). Concrete mediators are physical objects that can be manipulated, or can refer to mental images of objects. Visual mediators are a key symbiotic feature of mathematics discourse and consequently this research explored how this tenet was employed towards objectification, particularly with respect to the language(s) in which the lecturer discussed the mediator.

Narratives are descriptions (written or verbal) of objects, including relationships between objects and the processes involving objects and therefore, these narratives can be substantiated or rejected according to discourse-specific substantiation procedures (Sfard, 2008). These are the widely accepted rules within mathematical discourse that are sanctioned by the mathematical community (e.g. definitions, theorems, etc.). Narratives are comprised of construction, recall and substantiation

procedures. Constructions result in new narratives, recall is the memorisation of endorsed narratives and substantiation involves the actions leading to the endorsing of a narrative. This paper examines how visual mediators support the development of the endorsed narrative, while being cognizant of which language(s) are utilised.

Context and Methods

At the National University of Ireland, Galway (NUI Galway), first year undergraduate mathematics students are afforded the opportunity to study honours mathematics through a bilingual approach. Modules offered bilingually are Calculus and Algebra. Four weekly lectures are provided through the medium of Irish, with all mathematics terminology offered bilingually (Irish and English). In addition, lecturers may opt to describe more complex concepts (such as the limit of a function) bilingually. The lectures are supplemented by the provision of a weekly tutorial in English in addition to an Irish-medium tutorial. It is important to note that the mathematics register through the medium of Irish has not been developed beyond this first year of undergraduate education and this is the only institution in Ireland that offers the opportunity to learn mathematics through this medium. A change in one's discourse practices is employment of a mathematics discourse that resembles the features of the discourse practiced by the mathematical community (Sfard, 2012). So as such, a mathematics community of which Irish bilingual learners can become a proficient member of, does not exist and is very much restricted to the context of these lectures/tutorials.

Given the scope of this research the authors required evidence of the lecturer's language use and the instructional methodologies employed within a natural educational context. Therefore, all lectures were video recorded. Video research is an effective method of examining teaching and learning experiences in naturalistic contexts and the affordances of modern technologies facilitate the documentation, sharing and intensive analysis of in-situ learning (Derry et al., 2010). In excess of 22 hours of video-recorded lectures were captured and analysed as part of a regular undergraduate mathematics programme.

Our analysis involved two key stages: (1) narration, preparation and immersion and (2) coding and categorization. The first stage involved providing a rich account of the research context (Huberman & Miles, 2002). Verbatim transcriptions of salient video vignettes were developed and subjected to detailed analysis. Immersion in the data facilitated understanding the diverse mathematical discourses generated by the lecturer in the bilingual context. Systematic selection of the vignettes identified key learning events that were characterized as illustrations of mathematical meta-level developments. The second stage is coding and categorization and involves first fragmenting the data into units for analysis (Denscombe, 2010). The codes employed in this study were Sfard's discourse tenets (word use, visual mediators, routines and endorsed narratives), type of discourse (colloquial/object-level or literate/meta-level) and language use (English and/or Irish). Therefore, data was analysed four times initially in accordance with Sfard's four discourse tenets while concurrently considering evidence of object- or meta- level developments and importantly, in which language this occurs (English and/or Irish). Erickson's (2006) deductive, part-to-whole approach to inquiry is a useful method for tracking patterns across codes and categories. Themes and relationships (such as between discourse tenets and language use, e.g. visual mediator and Irish language use) were identified and aligned with the overall analysis framework (Ní Ríordáin & Flanagan, 2019).

Findings

Analysis of the video recordings reveals the lecturer employs commognitive constructs such as visual mediators in an attempt to convey her meta-discursive expectations (as experienced interlocutor) to students towards the aim of producing an endorsed narrative about the topic of functions (Sfard & Kieran, 2001, p.49). Two examples of the use of such visual mediators employed by the lecturer are outlined below and include the explanation of mathematical symbols and the use of graphs. First, the lecturer explains the use of two symbols, one for infinity (∞) and one to signify ‘there exists’ (\exists). The transcript below depicts how the lecturer explains the infinity (∞) symbol in relation to solving the mathematical problem: $f(x) = \frac{1}{x^2}$ (Video: W4L2 4.13mins):

- L1: It looks like x is approaching 0 [writes $x \rightarrow 0$] from the right side; that
 L2: this function is increasing. The values are always positive because this
 L3: is squared [pointing to x^2]. We write this as such: [writes ∞ symbol on board].

[L1: Tá an cuma ar agus x ag druim le 0 [writes $x \rightarrow 0$] ón taobh

L2: dheis; go bhfuil an fheidhm seo ag pleascadh suas. Bíonn na

L3: luachanna i gconáí deimhneach mar gheall ar go bhfuil seo cearnaithe

L4: [pointing to x^2] [...] Scríobhann muid é síos mar seo [writes ∞ symbol on board]

In the example above the lecturer used a combination of written and graphical communication to explain the concept of infinity as continuous or endless. The use of visual mediators supports the development of the endorsed narrative, but the lecturer also places emphasis on explaining the meaning of the symbols and their use. The explanation was provided through the medium of Irish which we expect relates to the fact that the students would have encountered these symbols at post-primary education and no major discursive shifts were evidenced (remained at object level).

Second, the lecturer explains the ‘there exists’ symbol (\exists), which is outlined in transcript below as follows (Video W4L3 32.30mins):

- L1: Do you know this symbol [pointing to \exists]? [The students shake their
 L2: heads to signify they do not know this symbol]. This means, in English,
 L3: ‘there exists’ or ‘there is’. So when I write that it means, ‘there exists’.

[L1: An bhfuil an siombal sin [pointing to \exists] ar eolas agaibh? [The students

L2: shake their heads to signify they do not know this symbol].

L3: Ciallaíonn seo i mBéarla ‘there exists’ or ‘there is’. So when I write that it means, ‘is ann do’.]

The lecturer explains this symbol by switching to English to clarify its meaning as ‘there exists’. This is translated as ‘is ann do’ in Irish, with a literal translation of ‘it is in it, or ‘there it is’. In comparison to the previous example, students would not be as familiar with the symbol \exists and requires a discursive

shift from object to meta-level. The lecturer employs both languages when engaged in such a discursive shift and to aid clarification of meaning.

Graphs are another recurrent visual mediator employed by the lecturer in the process of endorsing the narrative regarding functions. In the following example (Video W4L2 23.40) the lecturer is graphing the function $\frac{-2}{(x-3)^2}$, with $\lim x \rightarrow 3$, to investigate and to illustrate if the function crosses the x -axis.

L1: So, find the limit in this case: $\lim_{x \rightarrow 3} \left(\frac{-2}{(x-3)^2} \right)$,

....

L53: [*Progressing the topic the lecturer now introduces the concepts of*

L54: *asymptotes*]. So I suppose, that you both have heard of things called

L55: asymptotes? [*Repeats the word in English*].

V7: Just like, when the line gets very close [*using hand gesture to signify*

V8: *vertical line*] but it does not hit the line [*referring to y-axis*].

L56: In this case [*referring to the graph from the previous example*

L57: *explained in class*] this line [*pointing to the y-axis*], is an asymptote, a

L58: vertical asymptote. Why? Well, as you [*referring to V*] said, the graph

L59: does not cross the vertical line, but the graph comes very close to the

L60: line [*pointing to the y-axis*] [*The lecturer then refers students to the*

L61: *explanation provided in the notes*]. So, when things like this happen

L62: [*pointing to the previous two examples that were utilised:*

L63: *$\lim_{x \rightarrow 3^+} f(x) = -\infty$ and domain $x \rightarrow 3^- f(x) = \infty$*] at a

L64: certain number, so in that case [*pointing to the example in the notes*]

L65: c is the equivalent of 3, we say that there is a vertical asymptote at

L66: $x = 3$. If you were to draw a graph, at 3, of course it is not defined,

L67: you need more information than that. [*L proceeds to graph the*

L68: *vertical asymptote*]...

....

L90: *running parallel with the x-axis*] [...] This demonstrates what is meant

L91: by vertical asymptotes. So, it is a vertical line and the function tends

L92: to infinity or minus infinity around it. [*L explains the various*

L93: *manifestations of the asymptotes through the use of graphs,*

L94: *and writes the explanation:*

L95: $\lim f(x) = \infty$ nó – ∞ agus $x \rightarrow c^{+/-}$].

In this investigation of the problem: $\lim_{x \rightarrow 3} \left(\frac{-2}{(x-3)^2} \right)$, depicted in the above transcript, the lecturer commences with an investigation of whether the function crosses the x - and y -axes. In this regard, a substitution method is utilised in which the equation is written out, various values are substituted into the equation (L2-L18) and explained verbally. The lecturer also illustrates these explanations with graphs to highlight if and where the function crosses the axes (L58-61) and also to introduce the concept of asymptotes (L66-76). The primary language of communication utilised by the lecturer was Irish and was observed that mathematical language employed by the lecturer progressed from basic to more advanced. English was only utilised when providing key terminology – both the Irish and English versions were provided to students. First, the lecturer employed a process of calculating limits through substitution without necessarily referring to continuity. Then, this concept was developed further through the graphing of asymptotes to convey conceptual understanding of the notion of alternative limits that comprise variables tending to infinity or negative infinity. Therefore, the use of visual mediators facilitated a discursive shift from an object to a more meta-level (literate) discourse, with evidence of the use of both languages in this discursive shift.

Conclusion

A discourse is made clear by a community's word use, visual mediators, endorsed narratives and routines (Sfard, 2008). By employing Sfard's approach to research in the fields of mathematics learning and language use it provided us with an opportunity to examine a bilingual undergraduate lecturer's approach to teaching Functions in a naturally occurring bilingual context. In the examples provided above, the lecturer shifts between verbal and visual mediation of functions. Mathematical words and phrases such as *positive*, *squared* and *there exists* are employed throughout the developing discourse relating to functions to convey meaning such as the continuous nature of functions and tending towards infinity. Word use is then progressed by the utilisation of visual mediators such as gestures (pointing, upward and downward hand gestures signifying increasing/decreasing values), symbols (∞ , \exists) and graphs. In the third example, a graph is employed by the lecturer to visually mediate the students' investigation into whether the function intersects the x -axis in the problem: $\lim_{x \rightarrow 3} \left(\frac{-2}{(x-3)^2} \right)$. Visual mediators are of immense importance in mathematics because "With the help of symbolic records, the inherently transient spoken discourse acquires permanence and the different discursive elements become simultaneously present" (Sfard, 2008, p. 159). The "symbolically encoded mathematical discourse" has the capacity to "become an object of metadiscursive activity" (ibid, p. 159). Of additional interest to us was which language(s) was utilised. In general, the lecturer switched to English to clarify meaning or to express the meaning of a particular visual mediator in its English form, particularly when progressing the discourse from object to meta-level (literate). This perhaps suggests that the endorsed narrative for meta-level mathematics requires initiation into the mathematics English-medium community, ensuring that students are in a position to participate in this community given that the Irish mathematics register has not been developed to support further study in mathematics.

The use of graphs as well as the aforementioned accompanying word use/symbols substantiate the claims about functions that the lecturer is making. Validation of a narrative is the means through which we “become convinced that a narrative can be endorsed” (Sfard, 2008, p. 231) and often forms a sequence “which is deductively inferred from previous ones and the last of which is the narrative that is being endorsed” (ibid, p. 232). Students do not offer any commognitive conflicts (challenges to or rejections of the narrative) in terms of these claims and their collective agreement (signified by head-nodding gestures, uttering ‘yes’ and subsequent performance (see Ní Ríordáin & Flanagan, 2019) endorses the particular narrative the lecturer is creating. This also highlights students’ awareness of the meta-discursive expectations of the lecturer, which they will be required to negotiate when engaged in problem-solving activities. The primary language of communication was Irish, establishing an expectation of utilising Irish when engaged in mathematical narratives relating to functions within this context.

The research and analysis presented here is an initial examination and we fully appreciate that further analysis and depth is required. The insights presented here suggest that the lecturer primarily employed the Irish language in their teaching and used English to clarify meaning, thus setting the expectation of utilising Irish when engaged in mathematical narratives relating to functions. However, English was utilised for clarification when a discursive shift from an object to meta-level discourse was being established. We propose that there is a need to examine in greater detail language use as relating to discursive shifts in the study of university mathematics and bi-/multi-lingual learners. Sfard’s Commognitive Framework (2008) has been utilised in many ways and contexts. However, it has not been adapted and utilised specifically to examine bilingual students and their language use when learning mathematics. Therefore, our research contributes to expanding this framework and its use, particularly within a university context.

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