Linguistic difficulties in the transition to the university: Learning mathematics in three structurally different languages

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This paper reports a study of the structural differences of the three languages used in Algeria: Arabic, dialect and French. These languages are used by both students and teachers at the university: French for written mathematics and the two others for oral expressions and thinking. The use of Arabic and dialect is meant to help students in learning mathematics in French as their mastery of French is weak, but the many differences concerning logical structures might be a source of difficulty for students, when they use the three languages. Some research questions are stated together with how to validate them in a further research.

Keywords: Arabic, dialect, French, undergraduate mathematics.

Introduction

Many studies in the literature show positive aspects of teaching and learning mathematics in multilingual contexts (Poisard, Nì Rìordaìn, & Le Pipec, 2019; Adler, 2001). I support this position and I think that mastering many languages can be an advantage as it allows expressing meanings, concepts and thoughts in many ways, according to different linguistic structures and words of the different languages used, which would enhance mathematical performances. However, this nice image fades in the case of teaching and learning mathematics in multilingual contexts in university (Durand-Guerrier, Kazima, Libbrecht, Njomgang Ngansop, Salekhova, Tuktamyshov, & Winsløw, 2016), where mathematics is taught in a second or a third language that students and teachers do not master very well. The situation is even worse when the different languages have 'different modes' of expressing logical relationships in mathematics, which might introduce ambiguities or misunderstanding between teachers and students and in students' mathematical activities, thus affecting mathematical learning (Edmonds-Wathen, Trinick, & Durand-Guerrier, 2016). According to the linguistic relativity hypothesis, the structure and the vocabulary of a language influences the way we think in that language (Whorf, 1956); however exploring the link between the grammatical differences of languages and the mathematical thinking process is not easy (Nì Rìordàin, 2013; Edmonds-Wathen et al., 2016).

Another motivation for the content of this paper is that literature presents many studies of teaching and learning mathematics in multi-linguistic contexts. However, specific studies investigating the mathematical difficulties of undergraduate students who are native Arabic speakers when learning mathematics in a second language, are very few (Yushau & Hafidz Omar, 2015; Yushau & Bokhari, 2005) and they are not focused on logical aspects.

The research performed in my PhD thesis that concerned some difficulties with proof and proving at the undergraduate level, together with the findings of Azrou and Khelladi (2019) revealed linguistic difficulties of Algerian students caused by the fact that they learn mathematics in a second language (French). The subsequent study (Azrou, 2019) concerned Algerian university students who enter the university and start to learn mathematics completely in French after having learnt mathematics, in all

previous school levels, with verbal expressions in Arabic and written formulas in French. It investigated students' linguistic difficulties as they use three different languages: French to write and read, Arabic and dialect for discussions between teachers and students, and among students, and also for thinking. Findings from interviews showed a complete dependence of students' learning mathematics on translation to Arabic (to think) and from Arabic (to write in French). To understand mathematics when translating from one language to another, students must translate the mathematical statements from one language and express it in the other language. If this translation would be hindered by different linguistic structures of the three languages, students might meet problems when understanding and expressing mathematical statements.

An in-depth investigation of the linguistic structures of some logical aspects in the three languages is thus necessary in order to uncover the differences; this paper presents some findings concerning logic-linguistic aspects that are relevant in mathematical activities. Previous studies helped us to identify those aspects. Selden and Selden (1995) reported that undergraduates in the USA showed significant difficulties to translate informal mathematical statements (written in ordinary language) to formal language (using predicate logic): in particular, the logical connector 'or', 'if-then' statements and 'if-only-if' statements are generally misunderstood by students (Epp, 2003), Durand-Guerrier (2008) observed a semantic ambiguity with both French students and teachers, and Tunisian students about the statements using quantifiers and negation like: 'all As are not B'.

The next part, which is the core of this paper, is a comparison of the three languages (Arabic, dialect and French) regarding the aspects identified as being problematic for students in the literature; the last part of the paper is an outline of further research.

Differences concerning the logical structures of Arabic, dialect and French

The language used for writing mathematics and reading mathematics in textbooks is French; but teachers and students do not speak French in the class. Both Arabic and Dialect are used for explanation and discussion, due to the weak mastery of French language by students, but also by several teachers. By using Arabic or Dialect, teachers try to help students to understand better the concepts and encourage them to ask questions and share the discussions. But the three languages have different features concerning the way of expressing logical and conditional expressions and this results in relevant differences when mathematical reasoning needs to be expressed by words.

- The conditional structure 'if … then', which plays a crucial role in mathematics, is presented in different ways in the three languages.

In mathematics, the only Arabic form taught in schools (from upper secondary) is expressed 'idha…idhen' ('dh' is pronounced as 'th' in then), in the plain case of expression like this: *if* a function is derivable, *then* it is continuous. However, this structure (using idha...idhen...) is not regularly used in Arabic and does not sound good. In Arabic, like in other languages, the conditional sentence is composed by two parts: the protasis, i.e. the sentence of the condition (with the verb of the condition), as 'the function is derivable' in the previous example, and the apodosis, i.e. the consequence of the condition, as 'the function is continuous' in the previous example (with the verb of the consequence). The terms put in the place of 'if' and 'then' are many and are not used two by two necessarily (like 'if' is only used with 'then'), some term1 (as if) is used with some term2 (as then) but is also used

with term 3 (as then). Focus in Arabic is on the conditional terms put at the beginning, there are many and fall into two groups: those that change the ending of the first verb and those that do not. Among those that belong to the first group, there are seven (inna, idhma, mahma, kayfama, haythouma, ayy). The second group is composed by (idha, law, lawla).

- ‘ithen’ (then in English) is rarely used, there is another word used more often, which is ‘fa’, or we use nothing, so the conditional structure is often expressed as: ‘*idha*…*fa*’…’, or ‘*idha*...nothing...’

- There are other words for if, which are in, law and lamma (synonym of indama:when), that would give: ‘in…fa…’, ‘law…fa…’.

The teaching is focused more on how to use these terms syntactically (with care for how ‘the ending’ of the verb of the condition and the verb of the response of the condition are pronounced). There is no correspondence with what happens in the same situations expressed by the conditional ways in French (and also in English) to express possible, probable or impossible situations, by changing the tense and the mode of the first verb and the second verb. Let us see some examples:

‘If you turn left, you will end up in the next street’ (expressing possibility) would be expressed in Arabic by using: *Law*+ verb+money *la* +verb+car, ‘in ...la...’, or ‘in ...sa ...’,

To express a condition that has never been realized, in Arabic we use ‘law…la…’, with the past tense for the verb of the first sentence and a particular mode for the second sentence. Even in mathematics, this situation would have been expressed so, and not by using the ordinary expression ‘idha...ithen’ for ‘if-then’.

Even though, in high school mathematics, we have introduced one structure to express a condition, which is ‘idha…ithen…’, it is not sure that students would consider other conditional sentences (formed with other terms, cited above) as being the same as this one. But more importantly, students might not be able to translate Arabic sentences with different conditional terms (other than ‘idha’ and ‘idhen’) to French sentences with ‘si… alors…’ (if…then…) and when they translate from French the conditional sentence (si...alors...) to Arabic, students might not consider the multiple Arabic terms to express the condition other than ‘idha...idhen...’, particularly in the oral form.

In the dialect, the words are different, the analogous of ‘if’ is ‘ki’ and there is no words for ‘then’ in the present; while for a condition that has not been satisfied, the analogous for if is ‘lukane’ and no words for ‘then’ or we use ‘lukane’. So we say: ‘lukane…(nothing)…’, or ‘lukane…lukane…’. In this structure, the verbs for the first and the second sentence are always in the past. Again here, we wonder if students, when thinking in dialect or listening their teacher who speaks in dialect would translate their conditional structure to the French one with ‘si…alors’ and when they translate from French (for instance when they read a textbook), whether they identify their corresponding structure in dialect. No need to mention how it is confusing and complicated when a student thinks in dialect speaks in Arabic and translates into French to write a text, or when a student listens in dialect, thinks in Arabic and writes in French.

Here, it is worth to consider also the meaning of the implication presented by the arrow at the university level, but presented by the verb ‘implies’ before in the different school levels. When we associate always the same conditional structure and the same markers (if…then…) to the form P⇒

Q, there will be no confusion with students, and this is what it is supposed to happen with the form ‘idha...idhen...’ when translated to or from the French form ‘si…alors’. The problem is how would students deal with other conditional structures in Arabic using other terms than ‘idha’ and ‘idhen’? How would they translate them to French, and are they aware that they would all fall under the form ‘$P \Rightarrow Q$’?

The mathematical negation structure in English is expressed like in natural language; it is formed by do (in the corresponding tense: present or past)+ not+verb (example: I do not eat meat). In French natural language, the negation is formed by ‘ne+verb+pas’ (example: ‘je ne mange pas de viande’ ‘I do not eat meat’). In classical Arabic, the situation is almost the same; we say in mathematics ‘nefye P’ for ‘negation of P or non P and in the natural language, we negate verbs by using only ‘la’ before the verb (la+verb), ‘la’ is also the opposite of yes in Arabic. I also negate by using ‘laysa’ which comes before a noun while ‘la’ comes before a verb. But in dialect, it is a little bit different: when we negate, we use a prefix ‘ma’ attached to the verb followed by a suffix ‘esh’ (ma+verb+esh: ma+eat+esh means ‘I do not eat’), while the word for ‘no’ is ‘lala’ or ‘la’ (according to the different regions in Algeria).

Moreover, when we begin a proof by contradiction, we say ‘if not’ (about the negation of the hypothesis), in French there is one word for ‘if not’ (‘sinon’), used also in natural language and very used in mathematical language (for instance when a function is equal to 1 for x=0, and 0 if not). In Arabic, there is one such word, which is ‘lawla’, but used at the beginning of the sentence and not at the end. For instance, if someone says in English: if I find it I will buy it, if not I will find someone who could lend it to me; in Arabic, the word ‘lawla’ cannot be used here as ‘if not’, the sentence would be expressed as follows: ‘idha (or any other similar conditional term) +verb (find) it, I will buy it, idha (or in case) I do not find it I will find someone to lend it to me.

Lawla is used as follows: verb1+lawla+verb2 means verb2 impeded the action of verb1, or because verb2, verb1 did not happen, for example: ‘I missed the flight ‘lawla’ it had been delayed’, means ‘because the flight had been delayed, I could take it’, or expressed fully in English as: if the flight had not been delayed, I would have missed it. We note here that this word (lawla) is used in very developed literature and is not used in current written Arabic language, thus this formulation might not be known by many students. There is another word used more often, in the oral language, which is ‘illa’ a contraction of ‘inla’ (if not) used with wa ‘and’. Example: ‘call me, if not I will leave’ would be translated as ‘call me+wa inlla+ I will leave’ which is ‘call me and if not I will leave’. This situation that is more similar to the French one and the English one, is not used in mathematics. In dialect, the similar word as ‘lawla’ does not exist; we rather use the word ‘wa illa’ contracted to the word ‘wella’. If we want to express a similar situation as (call me otherwise (or if not) I leave), we would say: call me ‘wella’ I will leave. We note here that wella is the same word as ‘or’, which is an exclusive choice.

For the logical connector ‘or’, in French we use only one word (‘ou’). In Arabic, we have ‘awe’ to make a choice. The choices might be exclusive or inclusive, like in French. But in dialect, we use ‘wella’ which is an exclusion of the precedent choice, which means that in dialect the choice is always expressed in an exclusive way. For example, if we want to say, ‘take the black or the blue’, in Arabic would be ‘take the white awe the blue’, in dialect would be ‘take the black wella the blue’ which is

take the black and in case you did not take the black take the blue. The inclusive case is not expressed in dialect.

For the articles of generality and particularity ‘the’ and ‘a’, in French they are (‘le’ or ‘la’) and (‘un’ or ‘une’) (according to the gender); in Arabic the definite article is a prefix ‘al’ attached to the substantive, and without it, the word is considered indefinite, but also, if it is definite by belonging to anyone or anything, it is expressed without it (like saying: the car of my father, in Arabic we do not use ‘al’ because it is considered definite as it belongs to my father). In French, we can express the generality and the particularity with both ‘le’ and ‘un’; we might say in French:

- ‘la voiture est un moyen de transport’ (the car is a mean of transportation) which expresses generality. And ‘la voiture de mon voisin est rouge’ (the car of my neighbour is red) which expresses particularity. When using ‘un’ (or une), we might say in French:

- ‘un élève doit respecter son professeur’ (a student must respect his professor) which expresses the generality. And ‘un élève de John a réussi’ (a student of John has succeeded) which expresses the particularity.

In Arabic, with the first case, we would say ‘al+car...’ for the generality and ‘car of my...’ without any article, for the particularity. With the second case, we would say in Arabic ‘al+student....’ for the generality and ‘student of John ...’, without any article, for the particularity. It seems easier and simpler for Arabic, we use al+substantive to express the generality and use nothing (no ‘al’) to express the particularity. In dialect, it is the same as with the Arabic. The problem might happen with dialect when the pronunciation is contracted and the ‘al’ cannot be heard, so apparently no difference exists between the pronunciation of the word with and without ‘al’.

With respect to universal and existential quantifiers, students encounter, for the first time, in the university the symbols ∀ and ∃. They already write the full expressions (for all… and there exists ...) in the upper secondary school, but do not use the symbols. In this case there are equivalent expressions in Arabic and in dialect.

About mathematical terms: when students enter the university, they already master the mathematical terms used in the past three years, like: limit, function, continuous, derivative, set, line, equation, inequality, etc. When students start to learn mathematics in French at the university, they try to translate French into Arabic (Azrou, 2019) and would feel secure when they are given a mathematical term in French, for which they already know its translation into Arabic. The problem happens when they begin to learn new mathematical terms related to concepts and definitions of undergraduate mathematics not already met in high school (example: group, intersection, injection, morphism, restriction, inferior bound, etc). In fact, this is the case with students all over the world, but the difference is that with any other student (Italian, English, French, …), a new mathematical term is accepted as such. With our students, who base their learning on translation from French into Arabic, when they encounter a new mathematical term in French whose translation in Arabic does not correspond to a known term (or not known by the teacher), they panic, feel stuck and continue to look for a similar word in Arabic in order to be able to understand the meaning.

The mathematical terms formed by adding prefixes and suffixes inform about their meaning, for example: ‘iso’ is a suffix to express the similarity (‘isomorphism’), ‘dis’ (discontinuity) to express

the opposite, 'sub' (subset, subgroup) to express a part. These processes (consisting in adding prefixes and suffixes that inform about the meanings of the words) are not usual in Arabic. As a consequence of the lack of familiarity with the prefix – suffix processes, when students read terms like isomorphism or mono-morphism, they might tend to memorize them without engaging in finding the meaning suggested by the prefix (or the suffix), and would often confuse them; which is another difficulty for them.

An outline of further research

The analytic comparison of the three languages (classical Arabic, dialect and French) with respect to some of their structural-logical aspects has put into evidence differences that might affect the learning of mathematics and mathematical activities. We have seen how the mastery of the three languages looks very different, related to how students meet and use the three languages before their entering the university: Arabic as the teaching and learning language for all disciplines in all the pre-university levels, French as a "second language" learnt as such since degree three, dialect as the commonly used language in everyday life. These considerations result in three research questions that should be dealt with in further research to be performed with first year university students in scientific areas:

1. How is students' mastery of written French as an ordinary communication language?

This question is motivated by the fact that the teaching of French as a second language might not result in a sufficient mastery of that language for some students to interpret ordinary texts (thus preventing them from the access to verbal explanations provided by scientific textbooks, which are written in French), and it is also motivated by the requests of many students during the exercise sessions to explain what is written in French.

2. How is students' mastery of Arabic at the academic level (Cummins, 2005), in particular in mathematics, by students who enter the university in scientific disciplines?

This question is directly related to the role that should be played by Arabic language in the teaching and learning of all the disciplines at all pre-university school levels, and to Cummins' theory. Cummins distinguishes between different levels of mastery of a language: the Basic Interpersonal Communication Skills (BICS), which are the surface skills of listening and speaking and Cognitive Academic Language Proficiency (CALP) which are the basis for a child's ability to cope with the academic demands in various subjects. Cummins states that while many children develop native speaker fluency (BICS) within two years of immersion in the target language, it takes between 5-7 years for a child to reach an academic level with a second language.

Moreover, according to Cummins' interdependence hypothesis (2005), the more developed or proficient a language 1 is, the easier is to develop a language 2. As Cummins (2000) also states: 'Conceptual knowledge developed in one language helps to make input in the other language comprehensible', in other words if a student understands a mathematical concept (like continuity or function) in Arabic, all she (or he) has to do is acquire the new corresponding French terms. However, a student has a more difficult task if he (or she) has to acquire both the term and the concept in the second language.

3. How is students' awareness of the semantic and syntactic aspects of logical structures of the three languages, and the related differences?

This question is motivated by the fact that (according to previous research) most students move to/from French – the official language of textbooks and of exam tests in mathematics – from/to the ordinary languages of communication and of thinking. In these translations/conversions students need, in particular, to deal with the differences concerning those logical structures of the three languages, which are relevant for mathematics.

In order to answer the three questions a suitable experimental investigation should be planned.

In the three cases, a crucial role should be played by written texts for the same and enough large population (e.g. all the students of the first year in Mathematics – about 150 each year) and subsequent interviews of a sample of them.

The first question might be answered through the choice of a plain text in French and related items aimed at ascertaining the students' understanding of it; interviews might contribute not only to identify the reasons for possible misunderstandings or blockages, but also the mastery of ordinary French at the oral level.

The second question might be answered through the choice of short mathematical texts in Arabic, e.g. a simple definition, a simple statement of a property (or a theorem), a simple proof for a given statement – and related items to ascertain understanding. Also non-mathematical Arabic texts with logical structures should be used, in order to distinguish between difficulties depending on the level of mastery of Arabic needed to interpret them, and difficulties depending on the knowledge of mathematical notions.

The third question might be answered by asking students to translate a text containing some logical content (not necessarily in mathematics) from Arabic to French, and the same from French to Arabic, with items focused on making explicit the conversion from one structure in the first language to another in the second language. The interviews might provide information on the use of dialect as an oral communication means for logical content.

References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht, the Netherlands: Kluwer.

Azrou, N. (2019). Learning mathematics in French at the undergraduate level in Algeria. In U.T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the eleventh congress of the European Society for Research in Mathematics Education (CERME11)* (pp. 1598–1605). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Azrou, N., & Khelladi, A. (2019). Why do students write poor proof texts? A case study on undergraduates' proof writing. *Educational Studies in Mathematics*, *102*, 257–274.

Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire*. Clevedon, UK: Multilingual Matters.

Cummins, J. (2005). *Teaching for cross-language transfer in dual language education: Possibilities and pitfalls.* Paper presented at the TESOL Symposium on Dual Language Education: Teaching and Learning Two Languages in the EFL Setting. Bogazici University, Istanbul, Turkey.

Durand-Guerrier, V. (2008). Truth versus validity in mathematical proof. *ZDM Mathematics Education*, 40, 373–384.

Durand-Guerrier, V., Kazima, M., Libbrecht, P., Njomgang Ngansop, J., Salekhova, L., Tuktamyshov, N., & Winsløw, C. (2016). Challenges and opportunities for second language learners in undergraduate mathematics. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J.N. Moschkovich, N. Planas … M. Villavicencio Ubillus (Eds.), *Mathematics education and linguistic diversity: The 21st ICMI study* (pp. 85–102). Cham, Switzerland: Springer.

Edmonds-Wathen, C., Trinick, T., & Durand-Guerrier, V. (2016). Impact of differing grammatical structures in mathematics teaching and learning. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J.N. Moschkovich, N. Planas … M. Villavicencio Ubillus (Eds.), *Mathematics education and linguistic diversity: The 21st ICMI study* (pp. 23–46). Cham, Switzerland: Springer.

Epp, S.S. (2003). The role of logic in teaching proof. *American Mathematical Monthly*, *110*, 886–899.

Nì Rìordaìn, M. (2013). A comparison of Irish and English language features and the potential impact on mathematical processing. In B. Ubuz, C. Haser, & M.A. Mariotti, (Eds.), *Proceedings of the eighth congress of the European Society for Research in Mathematics Education (CERME8)* (pp. 1576–1585). Ankara, Turkey: Middle East Technical University and ERME.

Poisard, C., Nì Rìordaìn, M., & Le Pipec, E. (2015). Mathematics education in bilingual contexts: Irish-English, Breton-French. In K. Krainer & N. Vandrovà (Eds.), *Proceedings of the ninth congress of the European Society for Research in Mathematics Education (CERME9)* (pp. 1468–1474). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

Selden, J., & Selden, A. (1995). Unpacking the logic of mathematical statements. *Educational Studies in Mathematics*, *29*(2), 123–151.

Whorf, B.L. (1956). *Language, thought and reality* (J.B. Carroll, Ed.). Cambridge, MA, USA: MIT Press.

Yushau, B., & Bokhari, M.A. (2005). Language and mathematics: A mediational approach to bilingual Arabs. *International Journal of Mathematics Teaching and Learning*, 13 April. Retrieved from http://www.cimt.org.uk/journal.yashau.pdf

Yushau, B., & Hafidz Omar, M. (2015). Mathematics performance and its relation to English language proficiency level of bilingual Arab university students. *Indian Journal of Science and Technology*, *8*(13), 1–15.