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Time Intervals of Symmetric Signal Transfer in the Presence of Relative Motion

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Abstract: The current scientific framework contemplates substantial modifications in the special relativity (SR). Its incompatibility with quantum gravity and the research that has triggered the violation of the Lorentz Invariance is a clear example of the emerging controversy. In the light of the modifications that have been proposed to the SR, this paper suggests a new perspective of relativism. We study the time interval required in the transmission of an electromagnetic wave between two particles with a relative movement. Using the symmetry of the system, a factor that relates the sending and receiving time intervals is obtained, showing that they are not equal according to the observers. We study the compatibility of these surprising results with the SR, posing a novel notion of time that until now may have gone unnoticed.

Keywords: special relativity; time intervals; velocity; kinematics; information transfer; electromagnetic signals

1. Introduction

The coherence between special relativity (SR) and the theories of quantum gravity is currently a topic of research. The main problem, as it is well known, concerns the Lorentz invariance, the fundamental symmetry of SR. Which is violated in entangled particles at the Planck energy scale, [1–3]. In [4–9] manifestations of these very small Lorentz-violating signals are studied. In the context of searching traces of these signals, Kislat [10], analyzes the optical polarization measurements from 63 Active Galactic Nuclei and Gamma-ray Bursts. The most recent astrophysical limits on the Lorentz Invariance Violation (LIV) are presented in [11]. Other authors have considered implementing "minimalist" violations of Lorentz invariance([12]), which results into physically compelling models. Nevertheless, the quantization of the ADM decomposition (3+1) of GR ensures the compatibility of the Lorentz invariance [13].

An important concept, yet not trivial, is understanding the role of the observer in the entanglement. In this regard, [14] shows that the entanglement is observer-dependent. The dependence of the observer is the cornerstone of the theory developed in the present paper. Where the symmetric nature of a dual-particle system is leveraged to combine the results obtained from each observer.

Following this train of thought, several reformulations of the foundations of SR have emerged recently [15–17]. An example of that is the "Very Special Relativity" [18–20], which changes some of the SR postulates with the intend to solve this issue. In [21,22] are the experimental results that quantify the low-energy modifications the SR would need for it to be compatible with the observations.

Doubly or deformed special relativity (DSR) is another example of a modified theory of SR, where an observer-independent maximum energy scale for the possible states of a particle is defined [23–26]. In [27–29], the origin of DSR and the interpretation of its different bases are studied. Another topic of research inside the DSR is the construction of the position space. In [30] this is deeply studied for a particular case of Magueijo-Smolín DSR. It is important to note that this deformation of special relativity leads to an altered notion of spacetime. In [31] the propagation of a particle over the distances that emerge from this new spacetime is discussed. Overall, deformed kinematics can either

be compatible with a relativity principle, or represent a LIV. But if the relativity principle is maintained, the deformation of the Lorentz transformations is required [32,33].

There are several other authors that have studied alternative modifications to the SR. One example of that is the enrichment of SR by incorporating Lorentz transformation groups, [34]. In [35] the orbits of two interacting particles are described by a fully relativistic classical mechanical Hamiltonian. Where it is also shown numerically a failing case of the kinematic Lorentz space-time transformations. Coumbe, [36], provides a numerical evidence that time also dilates according to the relative scale. Even the ability of an observer to localize events defined in a different observer's reference frame is questioned ([37]). The consequences of event description for different observers is also studied in [38].

The symmetry has a transcendental importance in the context of SR. Friedman and Scarr study this in [39]. Similarly, Chao ([40]) discusses the relevance of frame-exchange time-inversion symmetry in the Lorentz transformation. As we will see in section 2, the present paper leverages the inherent symmetry of relative motion to yield a simple but self-contained theory. We will first develop a rigorous mathematical formulation for the inertial case in section 2, but then generalize the results for a non-inertial scenario in section 5. The kinematics of accelerated observers in relativistic physics are illustrated in [41]. The equivalence principle, which postulates that accelerated observers are always pointwise inertial, is a viable alternative for several problems. However, it presents some inconvenience when dealing with the wave-particle duality, but Mashhoon shows a procedure in [42] that solves this conflict. In regard to this, in the present paper an experimental methodology will be explored for the generalization of the findings to the non-inertial system.

Additionally, experimental methods that confirm SR in certain fields of application have improved substantially during the last decade ([43–46]), making clear the validity of SR in those cases. Even more, some of these methods have marked an era in certain areas. As for example in communication satellites [47–49]. In section 4 we delve into a real-world problem by considering an application case for a system consisting of a Low Earth Orbit satellite and a ground segment.

There is a vast area of research dealing with one and two way light velocity measurements between distant observers in uniform translation motion relative to one another [50–52]. But this article considers an alternative visualization of the information transferring that accounts for all the possibilities in a dual particle system. These geometric visualizations and the formulation that follows enables the calculation of a motion-dependent factor that relates the sending and receiving time intervals in the transmission of the electromagnetic wave.

2. Methodology

Let the masses m_1 and m_2 be in a relative motion in a two-dimensional space. We consider a given point A of mass m_1 , and a point B of mass m_2 . A reference frame is attached to point B . For an observer located in this reference frame the point A is moving with a vector velocity \mathbf{v}_{A_rB} (Figure 1 (a)). Symbol v denotes the module of \mathbf{v}_{A_rB} . Its direction is defined by the unit vector $\hat{\mathbf{u}}_{A_rB}$, such that

$$\mathbf{v}_{A_rB} = v \hat{\mathbf{u}}_{A_rB}. \quad (1)$$

In this section, all the reference frames considered are inertial. Thus, the velocities described will be constant in module and direction. Let, the point B be a light source constantly emitting photons in all directions. For simplicity, there is no substance between the two masses. Thus, the information containing the position of B travels in the form of light at a speed of c . The transmission of information requires an interval of time $\Delta t_{B \rightarrow A}$ to get from B to A . But in that period of time point moves from A_0 to A_f (Figure 1 (b)). The photon leaving B travels a distance of $c \Delta t_{B \rightarrow A}$, while point A travels a distance of $v \Delta t_{B \rightarrow A}$ (Figure 1 (c)).

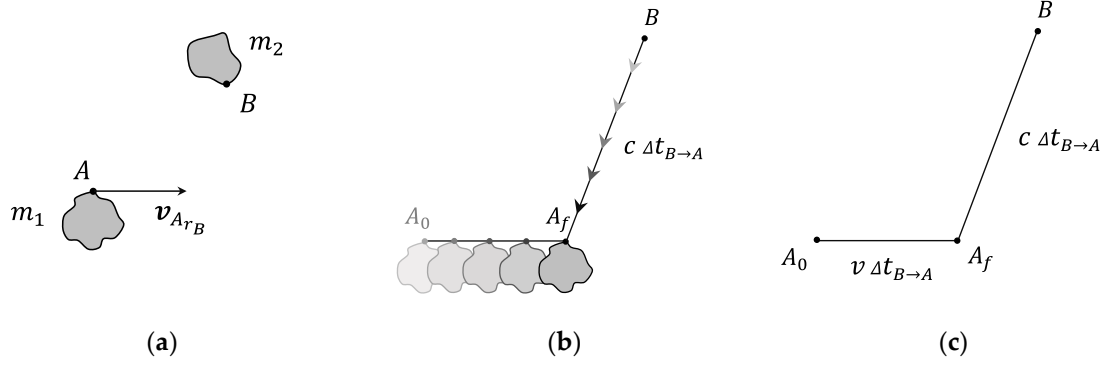


Figure 1. Relative motion between points A and B : (a) Relative velocity of the point A with respect to the observer B ; (b) Distance travelled by the information of B from B to A_f ; (c) Distance travelled by the information of B from B to A_f and by point A from A_0 to A_f .

Similarly, we assume that point A is also emitting photons, some of which are travelling towards B . The information that is sent when A is at A_0 reaches B after a time interval $\Delta t_{B \leftarrow A}$ (different from $\Delta t_{B \rightarrow A}$). The length of path travelled by this photon is $c \Delta t_{B \leftarrow A}$. Combining these three distances, the triangle shown in Figure 2 (a) is obtained. It can be inferred by observation that the time intervals $\Delta t_{B \rightarrow A}$ and $\Delta t_{B \leftarrow A}$ are different in a generic case. The first ($\Delta t_{B \rightarrow A}$) represents the time, according to B , in which a signal travels from B to A . The second ($\Delta t_{B \leftarrow A}$) represents the time, also according to B , in which a signal travels from A to B . It is important to note that B is the observer in both cases, and therefore even if the photon is transmitted by or received from B , is the time interval perception of the observer. The triangle of distances has been parameterized in Figure 2 (b).

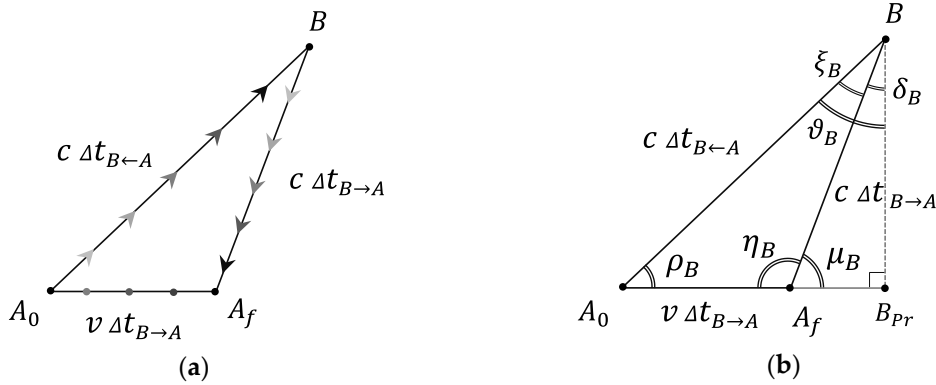


Figure 2. Triangle of distances for the observer B : (a) Directions of the transfer of information and direction of the movement of A ; (b) Parametrization of the triangle of distances for the observer B .

Figure 2 (b) also shows an auxiliary triangular construction (triangle formed by A_f , B and B_{Pr}). The point B_{Pr} is obtained by the intersection between the direction of velocity $v_{A_r_B}$ and its perpendicular traced by B . The subscript Pr is an abbreviation of projected (B_{Pr} is the actual projection of B on the line defined by $v_{A_r_B}$). The angular variables have a subscript B to indicate that they are referenced at observer B . The angles ξ_B , ρ_B , η_B , δ_B and μ_B will be used for the geometric relationships between the three distances. The angle ϑ_B is an independent variable of the problem. The angles of the triangle formed by A_f , B and B_{Pr} are related by

$$\mu_B + \delta_B = \pi/2. \quad (2)$$

Therefore,

$$\eta_B = \pi - \mu_B = \pi - (\pi/2 - \delta_B) = \delta_B + \pi/2. \quad (3)$$

Additionally,

$$\xi_B = \vartheta_B - \delta_B, \quad (4)$$

$$\rho_B = \pi/2 - \vartheta_B. \quad (5)$$

From the triangle formed by A_0 , A_f and B ,

$$\frac{c \Delta t_{B \leftarrow A}}{\sin(\eta_B)} = \frac{c \Delta t_{B \rightarrow A}}{\sin(\rho_B)}, \quad (6)$$

$$\frac{v \Delta t_{B \rightarrow A}}{\sin(\xi_B)} = \frac{c \Delta t_{B \rightarrow A}}{\sin(\rho_B)}. \quad (7)$$

Simplifying,

$$\frac{\Delta t_{B \leftarrow A}}{\sin(\eta_B)} = \frac{\Delta t_{B \rightarrow A}}{\sin(\rho_B)}, \quad (8)$$

$$\frac{v}{\sin(\xi_B)} = \frac{c}{\sin(\rho_B)}. \quad (9)$$

Using (3), (4) y (5), the following trigonometric relationships are obtained,

$$\sin(\eta_B) = \sin(\delta_B + \pi/2) = \cos(\delta_B), \quad (10)$$

$$\sin(\xi_B) = \sin(\vartheta_B - \delta_B), \quad (11)$$

$$\sin(\rho_B) = \sin(\pi/2 - \vartheta_B) = \cos(\vartheta_B), \quad (12)$$

Substituting in (8) and (9),

$$\frac{\Delta t_{B \leftarrow A}}{\cos(\delta_B)} = \frac{\Delta t_{B \rightarrow A}}{\cos(\vartheta_B)}, \quad (13)$$

$$\frac{v}{\sin(\vartheta_B - \delta_B)} = \frac{c}{\cos(\vartheta_B)}. \quad (14)$$

Solving in (14),

$$\delta_B = \vartheta_B - \arcsin\left(\frac{v}{c} \cos(\vartheta_B)\right). \quad (15)$$

Substituting in (13),

$$\frac{\Delta t_{B \rightarrow A}}{\Delta t_{B \leftarrow A}} = \frac{\cos(\vartheta_B)}{\cos\left(\vartheta_B - \arcsin\left(\frac{v}{c} \cos(\vartheta_B)\right)\right)}. \quad (16)$$

Up to now, this development has been considering the point B as the observer. The equation (16) will be used later. But before that, the reversed problem will be studied. Indeed, a similar expression to the one obtained in (16) is expected. To begin with, a reference frame is assigned to point A . In this case, point B has now a speed $v_{B_{r_A}}$ (Figure 3 (a)). The velocity $v_{B_{r_A}}$ can be decomposed as before into module and direction. As it is known from the relative kinematic principles,

$$v_{A_{r_B}} = v_{B_{r_A}} = v \quad (17)$$

is applicable for the case of systems of two points. Where $v_{A_{r_B}}$ and $v_{B_{r_A}}$, modules of the velocities of point A according to B and point B according to A respectively, are equal. Therefore, in the decomposition

$$\mathbf{v}_{B_{r_A}} = v \hat{\mathbf{u}}_{B_{r_A}}, \quad (18)$$

v is used as the module of the velocity. Additionally, the direction defined by the unit vector $\hat{\mathbf{u}}_{B_{r_A}}$ is opposite to that defined by $\hat{\mathbf{u}}_{A_{r_B}}$,

$$\hat{u}_{B_rA} = -\hat{u}_{A_rB}. \tag{19}$$

In the previous scenario, a photon emitted from A_0 reached point B (Figure 2 (a)). That exact same photon could also be considered in this configuration, where A is now the observer. In this case, the point B is the one moving from B_0 to B_f (Figure 3 (b)). Thus, the photon emitted from A , that reaches B_f in Figure 3 (b), is the same photon as the one in Figure 2 (a) that is emitted from A_0 and reaches B . To obtain the speed of the photon in Figure 3 (b), the principle of conservation of the speed of light regardless of the inertial reference frame is considered. In this case, the reference frame is inertial. Thus, as it happened when the observer was B , the information is transmitted at a speed of c . Figure 3 (c) adds to the previous construction the value of the separation between B_0 and B_f , which is the distance travelled by B while the photon reaches point B (in the time $\Delta t_{A \rightarrow B}$).

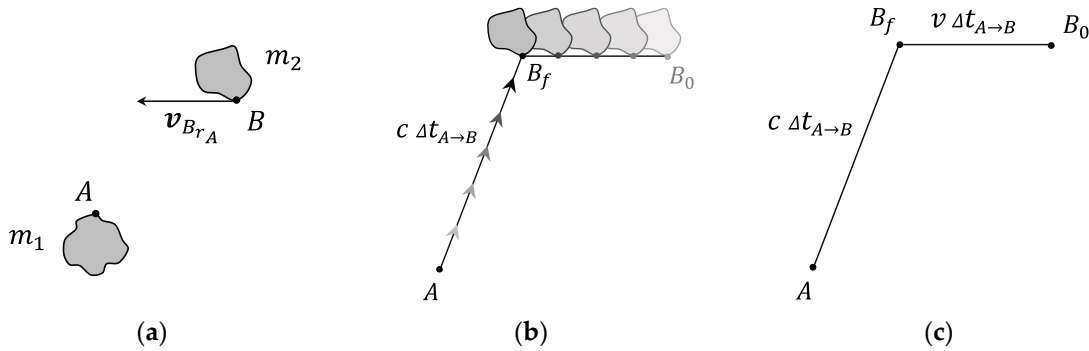


Figure 3. Relative motion between points B and A : (a) Relative velocity of the point B with respect to the observer A ; (b) Distance travelled by the information of A from A to B_f ; (c) Distance travelled by the information of A from A to B_f and by point B from B_0 to B_f .

Similarly, the transmission of the signal that travelled from B to A is also reappraised. Making the photons coincide, the conservation principle can be used again. Henceforth, the photon of Figure 2 (a) that goes from B to A_f is the same as the photon of Figure 4 (a) that goes from B_0 to A . Certainly, the time elapsed in this case ($\Delta t_{A \leftarrow B}$) is different from $\Delta t_{A \rightarrow B}$ (for a generic problem). Next, the triangle of distances of observer A is parametrized Figure 4 (b). This time, the subscript used for the angular variables is A . The point A_{Pr} is obtained from the intersection between the direction of the velocity vector v_{B_rA} and the line perpendicular to said direction that passes through A . As a side note, since v cannot exceed the speed of light, the distance $v \Delta t$ can never be greater than $c \Delta t$.

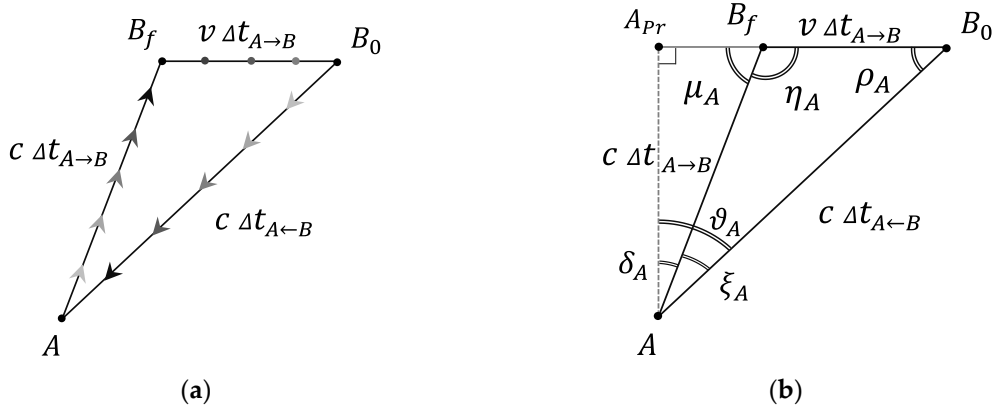


Figure 4. Triangle of distances for the observer A : (a) Directions of the transfer of information and direction of the movement of B ; (b) Parametrization of the triangle of distances for the observer A .

The mathematical analysis derived from Figure 4 (b) is analogous to that carried out in the previous case where the observer was point B , Figure 2 (b). Therefore, instead of repeating the

development, the final result (20) is directly displayed, which refers to the relationship between the time intervals $\Delta t_{A \rightarrow B}$ and $\Delta t_{A \leftarrow B}$,

$$\frac{\Delta t_{A \rightarrow B}}{\Delta t_{A \leftarrow B}} = \frac{\cos(\vartheta_A)}{\cos\left(\vartheta_A - \arcsin\left(\frac{v}{c} \cos(\vartheta_A)\right)\right)}. \quad (20)$$

Multiplying (16) and (20),

$$\frac{\Delta t_{B \rightarrow A}}{\Delta t_{B \leftarrow A}} \frac{\Delta t_{A \rightarrow B}}{\Delta t_{A \leftarrow B}} = \frac{\cos(\vartheta_B)}{\cos\left(\vartheta_B - \arcsin\left(\frac{v}{c} \cos(\vartheta_B)\right)\right)} \frac{\cos(\vartheta_A)}{\cos\left(\vartheta_A - \arcsin\left(\frac{v}{c} \cos(\vartheta_A)\right)\right)}. \quad (21)$$

Remember that $\Delta t_{A \rightarrow B}$ refers to the transfer of information from A to B , as observed by A . In contrast, the interval $\Delta t_{B \leftarrow A}$ represents the same process (transfer of information from A to B) being the reference point B . This entire research is focused on the difference between these two time intervals ($\Delta t_{A \rightarrow B}$ and $\Delta t_{B \leftarrow A}$). To evaluate it, the factor F is defined as

$$\Delta t_{A \rightarrow B} = F \Delta t_{B \leftarrow A}. \quad (22)$$

Along the process, no particular difference has been made between points A and B . Therefore, these points are interchangeable. That is, the problem has symmetry. Then, the factor that relates the variables $\Delta t_{B \rightarrow A}$ and $\Delta t_{A \leftarrow B}$ must also be F ,

$$\Delta t_{B \rightarrow A} = F \Delta t_{A \leftarrow B}. \quad (23)$$

Incorporating this information in (21), it is possible to solve for the value of F ,

$$F^2 = \frac{\cos(\vartheta_B)}{\cos\left(\vartheta_B - \arcsin\left(\frac{v}{c} \cos(\vartheta_B)\right)\right)} \frac{\cos(\vartheta_A)}{\cos\left(\vartheta_A - \arcsin\left(\frac{v}{c} \cos(\vartheta_A)\right)\right)}, \quad (24)$$

$$F = \sqrt{\frac{\cos(\vartheta_B)}{\cos\left(\vartheta_B - \arcsin\left(\frac{v}{c} \cos(\vartheta_B)\right)\right)} \frac{\cos(\vartheta_A)}{\cos\left(\vartheta_A - \arcsin\left(\frac{v}{c} \cos(\vartheta_A)\right)\right)}}. \quad (25)$$

The perspectives of the observers refer to the same system, therefore, as it happened with the velocity, the values of ϑ_A and ϑ_B are equal. ϑ_B and ϑ_A are replaced by ϑ ,

$$F = \frac{\cos(\vartheta)}{\cos\left(\vartheta - \arcsin\left(\frac{v}{c} \cos(\vartheta)\right)\right)}. \quad (26)$$

Developing,

$$F = \frac{\cos(\vartheta)}{\cos(\vartheta) \cos\left(\arcsin\left(\frac{v}{c} \cos(\vartheta)\right)\right) + \sin(\vartheta) \sin\left(\arcsin\left(\frac{v}{c} \cos(\vartheta)\right)\right)}, \quad (27)$$

$$F = \frac{1}{\cos\left(\arcsin\left(\frac{v}{c} \cos(\vartheta)\right)\right) + \frac{v}{c} \sin(\vartheta)}, \quad (28)$$

$$F = \frac{1}{\sqrt{1 - \frac{v^2}{c^2} \cos^2(\vartheta)} + \frac{v}{c} \sin(\vartheta)}. \quad (29)$$

The F factor represents the difference in the elapsed time between two basic actions, sending and receiving, whenever the points of the system have a relative movement. Considering any pair of moving points i and j , for each second it takes to send a signal from i to j (as seen by i), it takes $1/F$ seconds to receive it by j (as seen by j). Indeed, this does not only apply to the transfer of light, but also to any electromagnetic radiation in vacuum (that shares the speed of c), and is a direct consequence of the principle of conservation of c regardless the inertial reference frame.

To better visualize F as a function of both v and ϑ , the term v/c will be replaced by β ,

$$F = \frac{1}{\sqrt{1 - \beta^2 \cos^2(\vartheta)} + \beta \sin(\vartheta)}. \quad (30)$$

Figure 5 shows two perspectives of the evolution of F in terms of its two independent variables. To avoid infinitely big values of F in the visualizations of Figure 5, the variable β ranges only from 0 to 0.99.

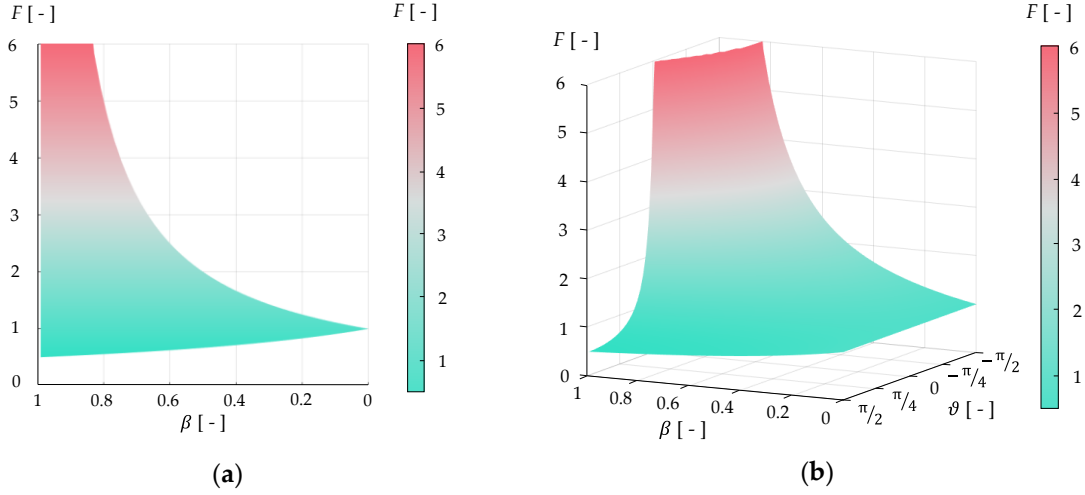


Figure 5. Evolution of F in terms of its two independent variables: (a) Perspective 1; (b) Perspective 2.

Next, the necessary relationship between β and ϑ is studied so that there is no difference between the sending and receiving time intervals. This is true when F has the value of 1. Substituting in (29),

$$\sqrt{1 - \beta^2 \cos^2(\vartheta)} + \beta \sin(\vartheta) = 1, \quad (31)$$

$$1 - \beta^2 \cos^2(\vartheta) = 1 - 2 \beta \sin(\vartheta) + \beta^2 \sin^2(\vartheta), \quad (32)$$

$$0 = -2 \beta \sin(\vartheta) + \beta^2. \quad (33)$$

Thus there are two possible solutions for β . The first is the trivial solution, obviously feasible since that is the case of no relative motion between the objects of the system. The second is $2 \sin(\vartheta)$. Figure 6 is a top view of the surface of F , where these two solutions have been incorporated to the surface.

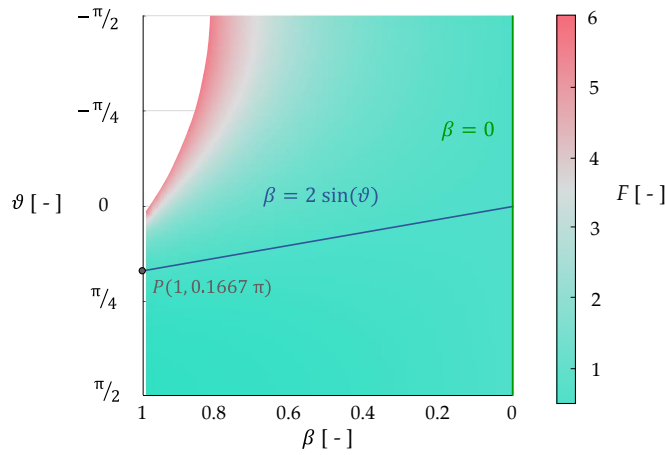


Figure 6. Top view of the surface of F together with both solutions for the case of F equal to 1.

The second solution is constrained for values of β between 0 and 1. In terms of ϑ , those limits are 0 and 0.1667π . Within these values, the sine function can be approximated to a straight

line, resulting into a value of β very close to 2ϑ . The cases where this second solution is satisfied are those with an isosceles triangle of distances (as the example shown in Figure 7).

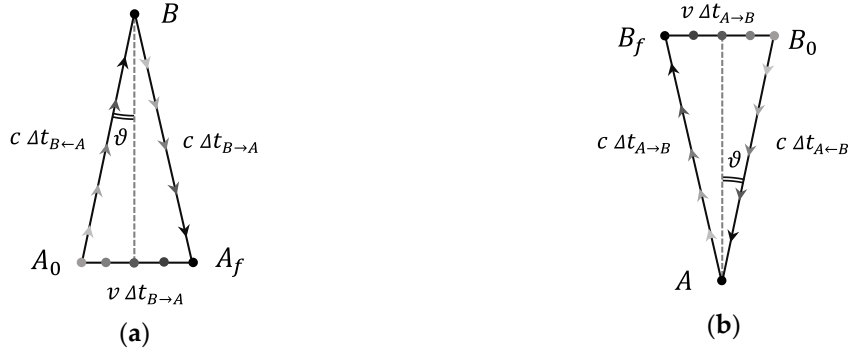


Figure 7. Example of an isosceles triangle of distances: (a) Triangle of distances as seen by observer B ; (b) Triangle of distances as seen by observer A .

3. Discussion

The problem statement considered in the previous section is highly similar to the one covered in the SR. Thus, in this section, the key differences with the SR will be discussed, as well as an interpretation of the prior formulation.

Before that, there are two essential concepts that need to be distinguished: First, the time intervals required for the transfer (in both directions) of an electromagnetic wave according to each observer ($\Delta t_{A \rightarrow B}$, $\Delta t_{B \rightarrow A}$, $\Delta t_{B \leftarrow A}$ and $\Delta t_{A \leftarrow B}$). Second, the time elapsed in the temporal dimension of the particles for any event (note that before we were only focusing on the transfer of an electromagnetic wave), which will be identified in this paper as $\Delta \uparrow$. Such that, $\Delta \uparrow_A$ refers to a time interval that elapsed for particle A , and $\Delta \uparrow_B$ refers to a time interval that elapsed for particle B .

The Lorentz transformations are obtained by making use of the two postulates of relativity and the assumption of homogeneity of space and time. To do so, it is often common to consider the propagation of a spherical wavefront of light from the perspectives of two observers. Figure 8 is a well-known diagram to show the problem that the constancy of the speed of light poses when the system involves relative motion. From a fixed outer perspective, the photon travelling from A to B appears to trace a diagonal trajectory, while from the moving reference frame (attached to A) the photon moves vertically.

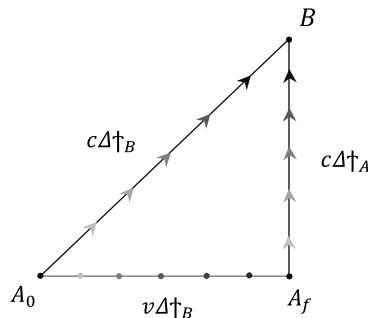


Figure 8. Visual representation of the different trajectories of a photon based on the perspective of the observer, which leads to the obtention of the Lorentz factor.

As it is known, $\Delta \uparrow_A$ and $\Delta \uparrow_B$ can be related by the Lorentz factor,

$$\Delta \uparrow_B = \Delta \uparrow_A \frac{1}{\sqrt{1 - \beta^2}}, \tag{34}$$

where the temporal dimension is dilated in order to account for the difference in the trajectories of the same photon (which moves at the same velocity regardless of the reference). The representation of Figure 8 is however misleading, because the perspectives of both observers are superposed in the

same diagram. We are actually seeing the transfer of the signal twice, from A to B as seen by B (represented in Figure 8 as A_0 to B), and from A to B as seen by A (represented in Figure 8 as A_f to B). For a better understanding, the movement described in Figure 8 is divided into its two perspectives, A (Figure 9 (a)) and B (Figure 9 (b)).

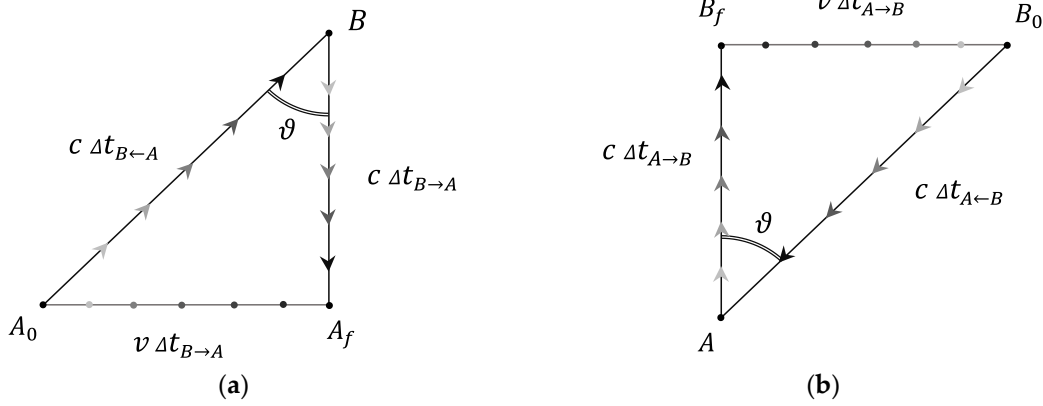


Figure 9. Division of the movement for the two perspectives: (a) Perspective of observer B ; (b) Perspective of observer A .

Figure 9 is the appropriate representation of the problem to apply the formulation developed in the prior section. The first step is to obtain the value of ϑ . To do so,

$$\cos(\vartheta) = \frac{\Delta t_{B \rightarrow A}}{\Delta t_{B \leftarrow A}} = \frac{F \Delta t_{A \leftarrow B}}{\Delta t_{B \leftarrow A}}. \quad (35)$$

Due to the symmetry of the problem, $\Delta t_{A \leftarrow B}$ and $\Delta t_{B \leftarrow A}$ have actually the same value, thus

$$\cos(\vartheta) = F, \quad (36)$$

$$\cos(\vartheta) = \frac{1}{\sqrt{1 - \beta^2 \cos^2(\vartheta) + \beta \sin(\vartheta)}}, \quad (37)$$

$$\cos(\vartheta) = \frac{1}{\sqrt{1 - \beta^2 \cos^2(\vartheta) + \beta \sqrt{1 - \cos^2(\vartheta)}}}. \quad (38)$$

Let x be $\cos(\vartheta)$,

$$x = \frac{1}{\sqrt{1 - \beta^2 x^2 + \beta \sqrt{1 - x^2}}}, \quad (39)$$

$$1 - \beta^2 x^2 + \beta^2 (1 - x^2) + 2 \beta \sqrt{1 - \beta^2 x^2} \sqrt{1 - x^2} = \frac{1}{x^2}, \quad (40)$$

$$2 \beta \sqrt{1 - \beta^2 x^2} \sqrt{1 - x^2} = \frac{1}{x^2} - (1 + \beta^2) + 2 \beta^2 x^2, \quad (41)$$

$$4 \beta^2 (1 - \beta^2 x^2) (1 - x^2) = \left[\frac{1}{x^2} - (1 + \beta^2) + 2 \beta^2 x^2 \right]^2, \quad (42)$$

$$4 \beta^2 (1 - x^2 - \beta^2 x^2 + \beta^2 x^4) = \frac{1}{x^4} + (1 + \beta^2)^2 + 4 \beta^4 x^4 - \frac{2(1 + \beta^2)}{x^2} + \frac{4 \beta^2 x^2}{x^2} - 4 \beta^2 x^2 (1 + \beta^2), \quad (43)$$

$$0 = \frac{1}{x^4} + (1 + \beta^2)^2 - \frac{2(1 + \beta^2)}{x^2}, \quad (44)$$

$$0 = x^4 - \frac{2}{1 + \beta^2} x^2 + \frac{1}{(1 + \beta^2)^2}, \quad (45)$$

$$x^2 = \frac{2}{1 + \beta^2} \pm \sqrt{\frac{4}{(1 + \beta^2)^2} - \frac{4}{(1 + \beta^2)^2}}, \quad (46)$$

$$x = \frac{1}{\sqrt{1 + \beta^2}}, \quad (47)$$

$$x = \frac{\sqrt{1 - \beta^2}}{\sqrt{1 - \beta^4}}. \quad (48)$$

For a case where

$$v \ll c, \quad (49)$$

$$\sqrt{1 - \beta^4} \approx 1. \quad (50)$$

Then,

$$x = \sqrt{1 - \beta^2} = F. \quad (51)$$

From (35), then,

$$\vartheta = \arccos\left(\sqrt{1 - \beta^2}\right). \quad (52)$$

Incorporating (51) in (35),

$$\Delta t_{B \leftarrow A} = \Delta t_{A \rightarrow B} \frac{1}{\sqrt{1 - \beta^2}}. \quad (53)$$

Based on Figures 8, and 9, we can see the parallelism between Δt_B and $\Delta t_{B \leftarrow A}$, as well as for Δt_A and $\Delta t_{A \rightarrow B}$. If equation (53) is compared to (34), it can be seen that they are actually representing the same transfer of information. But their meanings are very different. Equation (53) associates the difference in the time intervals to the transfer of the signal, but (34) associates such difference to the time dimension itself.

In the previous section, we started assuming proportionality of the sending and receiving time intervals (22, 23). Then, we were also able to obtain the mathematical definition of such factor (29), thus proving that the proportionality holds. Now that we decomposed the problem of Figure 8 in two perspectives, we have seen that this is in fact a particular case of our information transfer problem, precisely when ϑ takes the value of $\arccos(\sqrt{1 - \beta^2})$.

The SR considers that the temporal dimension of both particles should be different (thus the time intervals measured from these two particles, Δt_A and Δt_B , also differs, according to (34)). But that statement was introduced in order to solve the problem that the constant speed of light posed when dealing with two perspectives in relative motion.

In this research we solved the problem of the perspective by accounting for the time difference in the time intervals required for the transfer of information, not in the time dimension of the particles. Thus, now that the problem is solved, there is no need to believe that temporal dimension of both particles is different due to the velocity. In other words, regardless of the relative velocity, what this theory suggests is that

$$\Delta t_B = \Delta t_A. \quad (54)$$

Years of experience and uncountable number of experiments have ratified the SR but it might have been the case where the transfer of information made the results to actually match the predictions. Although the development shown has been mathematically rigorous, it requires experimentation to be proven. But if there are the minimal evidences that the prior formulas apply, then we could actually be facing a non-trivial reformulation of time. Thus, it is worth testing it.

4. Example of Application

Up to this point, the development revolved around the two-dimensional space, but in a three-dimensional case, it will be no different. As long as the reference frames are inertial, constant relative velocity vector (direction and module), there will always be a planar surface that encloses the two points and the paths described by them. Thus, all the prior development will be still applicable.

Consider the International Space Station (ISS), which is orbiting the Earth in LEO (Low Earth Orbit), approximately 408 [km] away from the surface. A signal is sent to the ISS from a fixed Ground Segment located in San Francisco, CA. The speed of the ISS relative to the Ground Segment at that moment, v , is roughly 7.66 [km/s]. Suppose the angle ϑ that defines the relative position of the ISS at the beginning of the transmission has a value of -0.25 [rad] or -14.3239 [°]. The satellite is tracing a curved path; thus, it has normal acceleration. Which makes it a non-inertial reference frame. But the separation between San Francisco and the ISS at that point is not big at all compared to the speed c (at which the signal is sent to the ISS). Hence, it can be assumed that the trajectory of the ISS is quasi-rectilinear and its speed v is constant while the information is being transmitted. Which is the necessary and sufficient condition to use (29). In Figure 10, an approximation of this problem is illustrated. Several considerations are neglected in this problem for the sake of simplicity and clarity.

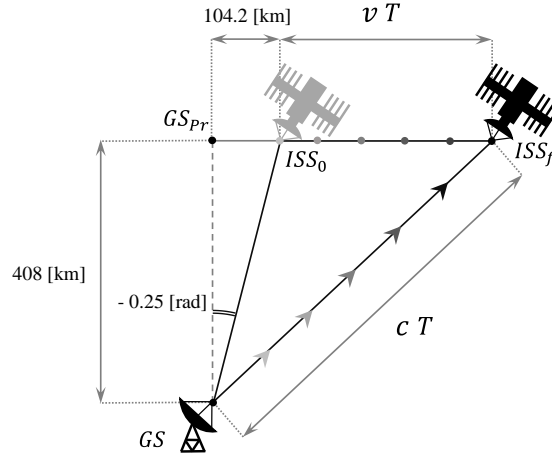


Figure 10. Simplified problem of the signal transmission to a satellite orbiting the Earth.

In a general problem, the value of ϑ is between $\pi/2$ and $-\pi/2$ radians. The sign convention used in this development is such that the value of ϑ is positive for the cases shown in Figures 2 and 4. In this example, since the ISS is not getting closer to San Francisco, ϑ is negative. On the other hand, the direction of the relative velocity vector is already present in the sign of ϑ , in other words, the value of v is always positive. The term GS represents the Ground Segment, the distance between GS_{pr} and GS is assumed to be 408 [km] (although it could be calculated more accurately using the radius of the Earth and the real arc travelled by the ISS between the perpendicular line that joins GS_{pr} and GS, and the start point ISS_0). The variable T refers to $\Delta t_{GS \rightarrow ISS}$, which is the time needed for the information to be sent from the GS to the ISS as seen by the GS. This value is first calculated using basic trigonometry, (note that the distances $c T$ and $v T$ have been extremely exaggerated in the drawing),

$$408^2 + (104.2 + v T)^2 = (c T)^2. \tag{55}$$

Considering c as 299792.458 [km/s], then

$$T = T_{Send} = 0.00140463 \text{ [s]}. \tag{56}$$

The variables v and ϑ are substituted in (29), obtaining a value of F equal to 1.00000632 [-]. T_{Send} is the time interval required to send the signal as observed by the GS. While from the satellite's perspective, the signal travels for $T_{Receive}$ seconds, where

$$T_{Receive} = \frac{0.00140463}{1.00000632} = 0.00140462 \text{ [s]}. \tag{57}$$

That is a very small difference of 10^{-8} [s], 10 nanoseconds. But in the vastness of the universe, the velocities of the different masses are much bigger, and the distances too, hence the impact of F in the calculations is much more significant.

5. Non-Inertial Case

The previous development is valid for inertial reference frames. But in several problems, the relative velocity tends to change in direction and module during the information transmission period. Although in some of the movements that take place in the universe, the relative velocity vector can be assumed as a constant vector without making big errors in the calculation (as shown in the example of the ISS). A method is proposed below that allows the application of formula (29) in the event that the velocity vector varies in direction and/or module. First, a point A with a random trajectory is considered (Figure 11 (a)). The symbol (t) reflects the temporal dependence of both variables \mathbf{v} (velocity vector) and A (position). As before, it is necessary to specify the observer (B) with respect to which point A has that velocity. This trajectory could be studied as the sum of small segments of infinitesimal length ($\mathbf{v}(t) dt$). Therefore, in each of these small sections of the path, the variables v and ϑ can be considered constant. However, it is possible that in such a minuscule time interval (dt) the information is not able to travel between A and B . It would be necessary to define a new observer located between A and B , as shown in Figure 11 (b). This new observer has been defined as I (letter i refers to the fact that is an imaginary observer). It is located on the line that joins points A and B , therefore its position depends on the instant of time ($I(t)$).

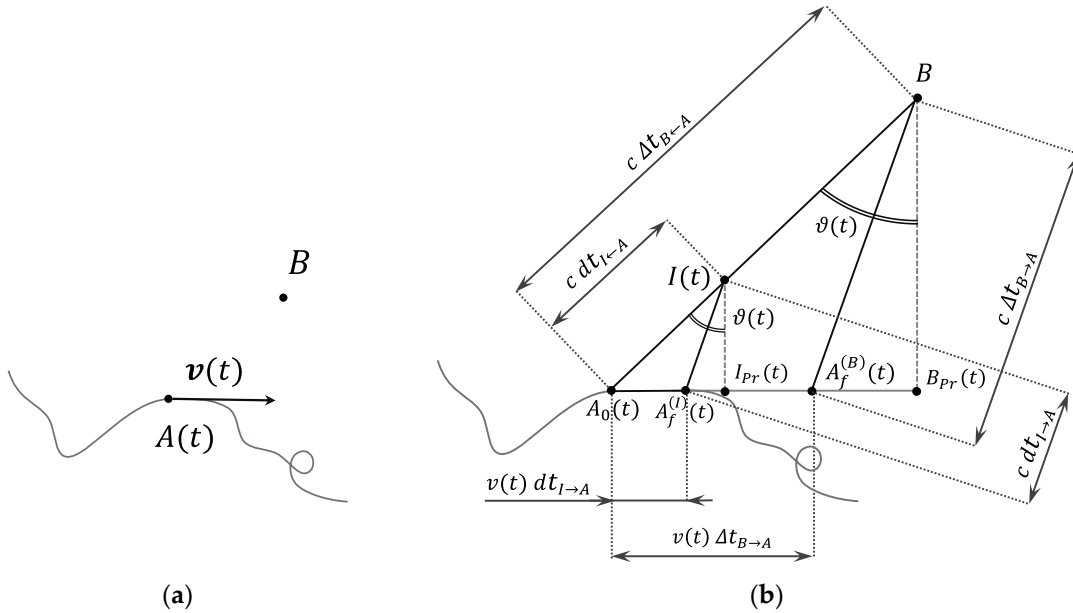


Figure 11. Study of a generic trajectory, non-inertial reference frame case: (a) Complex trajectory traced by point A ; (b) Triangle of distances for observers I and B .

As it can be appreciated, the point A is able to reach point $A_f^{(I)}$ but not point $A_f^{(B)}$. Thus, the formula (29) can only be applied for observer I . Which results in

$$dt_{A \rightarrow I} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2} \cos^2(\vartheta) + \frac{v}{c} \sin(\vartheta)}} dt_{I \leftarrow A}. \tag{58}$$

This method requires taking into account the contribution of each differential advance from point A in the travel of the information until it finally reaches point B . Furthermore, it involves complex integration. Thus, the problem will be solved in a different way.

First, the end point of the trajectory of A is located. That is, point A_f , at which the information emitted by B reaches A (according to observer B). To do so, the propagation of the information is represented as a spherical fringe that gradually increase in radius (because it occurs in all directions of the space) (Figure 12 (a)). Obviously, the information propagates at a velocity of c . We consider small time steps denoted by Δt_i . At each time t_i , the location of the point, $A(t_i)$, is recorded (point A moves with a varying velocity $\mathbf{v}(t_i)$). When the current location of A touches or crosses the limit

defined by the current sphere of light, the end point A_f is obtained (the final real geometry is shown in Figure 12 (b)). The separation between two successive pairs of locations of point A are not necessarily the same. That is, the length of the path between $A(t_i)$ and $A(t_{i+1})$, and $A(t_j)$ and $A(t_{j+1})$ can be different. Oppositely, the spherical fringes always have the same separation, because c is constant.

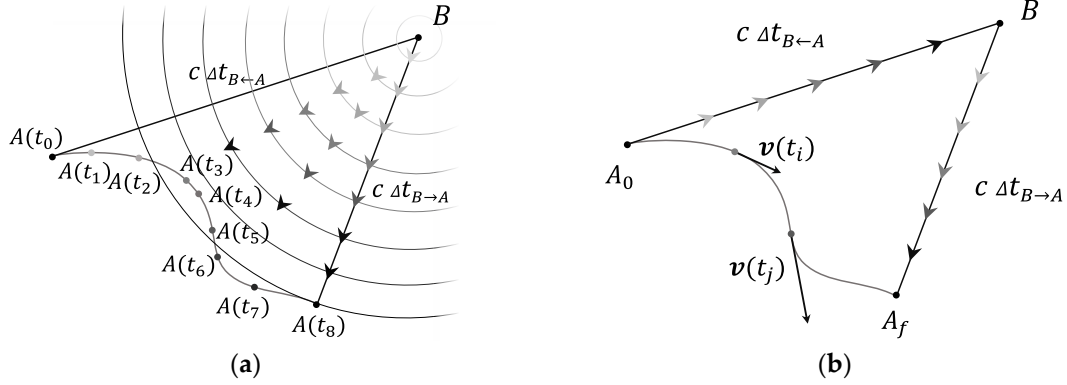


Figure 12. Obtention process of the real geometry of distances for observer B : (a) Method of the progressive propagation of information as spherical fringes to obtain point A_f ; (b) Real geometry of distances for observer B .

Once the above method has been applied, the length of each side of the geometry displayed in Figure 12 (b) is known (Figure 13 (a)). Likewise, the time it took for point A to travel from A_0 to A_f is also known ($\Delta t_{B \rightarrow A}$). Therefore, its equivalent average velocity is defined as

$$v_{eq} = \frac{r_{A_0, A_f}}{\Delta t_{B \rightarrow A}}. \quad (59)$$

Additionally, $\Delta t_{B \leftarrow A}$ can be easily retrieved,

$$\Delta t_{B \leftarrow A} = \frac{r_{B, A_0}}{c}. \quad (60)$$

If the curved path is stretched conserving its length, the equivalent triangle of distances of Figure 13 (b) is obtained. The value of $\vartheta_{B_{eq}}$ is fully defined from

$$r_{B, A_f}^2 = r_{B, A_0}^2 + r_{A_0, A_f}^2 - 2 r_{B, A_0} r_{A_0, A_f} \sin(\vartheta_{B_{eq}}), \quad (61)$$

$$\vartheta_{B_{eq}} = \arcsin\left(\frac{r_{B, A_0}^2 + r_{A_0, A_f}^2 - r_{B, A_f}^2}{2 r_{B, A_0} r_{A_0, A_f}}\right). \quad (62)$$

With these known values of v_{eq} and $\vartheta_{B_{eq}}$ the factor F can be calculated simply substituting in equation (29).

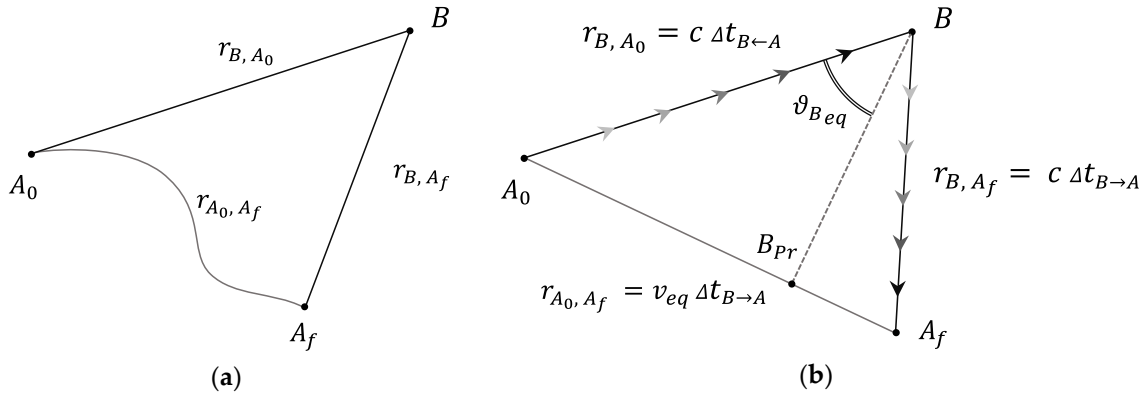


Figure 13. Transformation to the equivalent triangle of distances: (a) Length of the sides do the real geometry of distances for observer B ; (b) Equivalent triangle of distances for observer B .

6. Conclusion

This theoretical development suggests an interpretation of the time that elapses in the transmission of electromagnetic signals. It focuses on the case where relative motion exists between the agents of the system. A factor has been obtained that quantifies the difference between sending time interval (from the sender's perspective) and reception time interval (from the receiver's perspective). The development starts from the hypothesis of an inertial system; however, its application has been generalized for the study of the non-inertial case using a geometry equivalence method. The experimental verification of the proposed theory is within the reach of current technology. To do so, the authors suggest carrying out a series of experimental measurements similar to those shown in the application example of the satellite in LEO and contrasting their values with the theoretical ones. The experiment should focus on the differences found between both observers in the time required for the complete transmission of the signal.

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Abbreviations

The following abbreviations are used in this manuscript:

SR	Special Relativity
LIV	Lorentz Invariant Violation
DSR	Deformed Special Relativity
LEO	Low Earth Orbit
ISS	International Space Station
GS	Ground Segment

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