

# REAL-TIME FLOW ESTIMATION FROM REDUCED ORDER MODELS AND SPARSE MEASUREMENTS

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# CONTENT

1. Context : observer for unsteady aerodynamism
2. Physics, data & reduced order model (ROM)
3. Simulation, measurements & data assimilation
4. Reduced order model under location uncertainty
5. Results



# PART I

CONTEXT :  
OBSERVER FOR UNSTEADY  
AERODYNAMISM

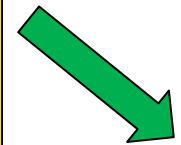
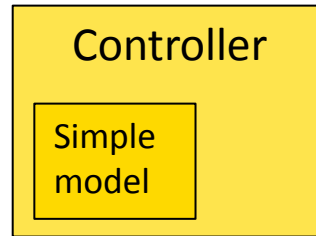
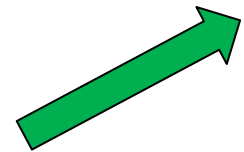
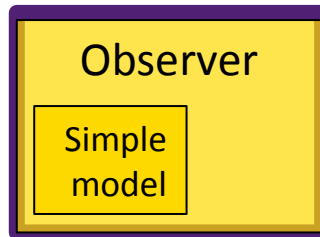
# APPLICATION

*Estimate and predict unsteady aerodynamism and aeroelasticity  
-- in real-time -- for better active control loops*

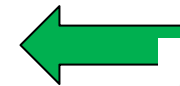
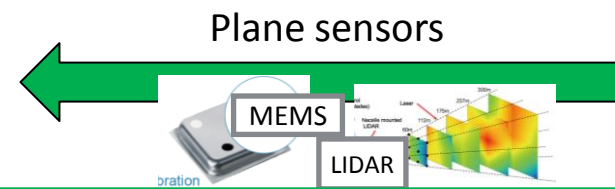
- active flutter suppression
- Passengers comfort
- Drag reduction

Estimation and prediction:

- Flow
- Lift, drag
- ...



- Flaps, slats
- Fluidic actuators
- ...



Which simple model?      How to combine model & measurements?



## PART II

### PHYSICS, DATA & REDUCED ORDER MODEL

# REDUCED ORDER MODEL (ROM)

Solution of an PDE with the form:

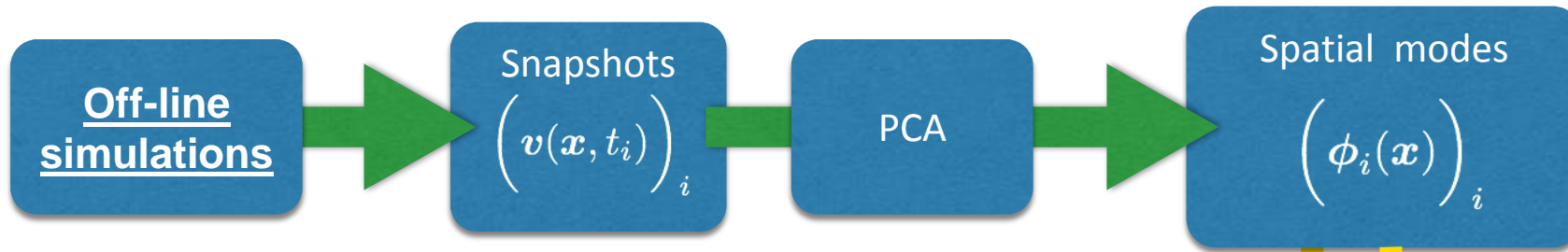
$$v(x, t, \alpha) \approx \sum_{i=0}^n \boxed{b_i(t)} \boxed{\phi_i(x)} \boxed{\gamma_i(\alpha)}$$

Order of magnitude examples in CFD

|                      | Full space             | Reduced space     |
|----------------------|------------------------|-------------------|
| Solution coordinates | $v_q(x_i, t)_{qi}$     | $(b_i(t))_i$      |
| Dimension            | $M \times d \sim 10^7$ | $n \sim 10 - 100$ |

# POD-GALERKIN

- Principal Component Analysis (PCA) on a *dataset* to reduce the dimensionality:



- Approximation:

$$v(x, t) \approx \sum_{i=0}^n b_i(t) \phi_i(x)$$

Resolved modes

- Projection of the “physics” onto the spatial modes :

$$\int_{\Omega} dx \phi_i(x) \cdot (\text{Physical equation (e.g. Navier-Stokes)})$$

→ ROM for very fast simulation of temporal modes

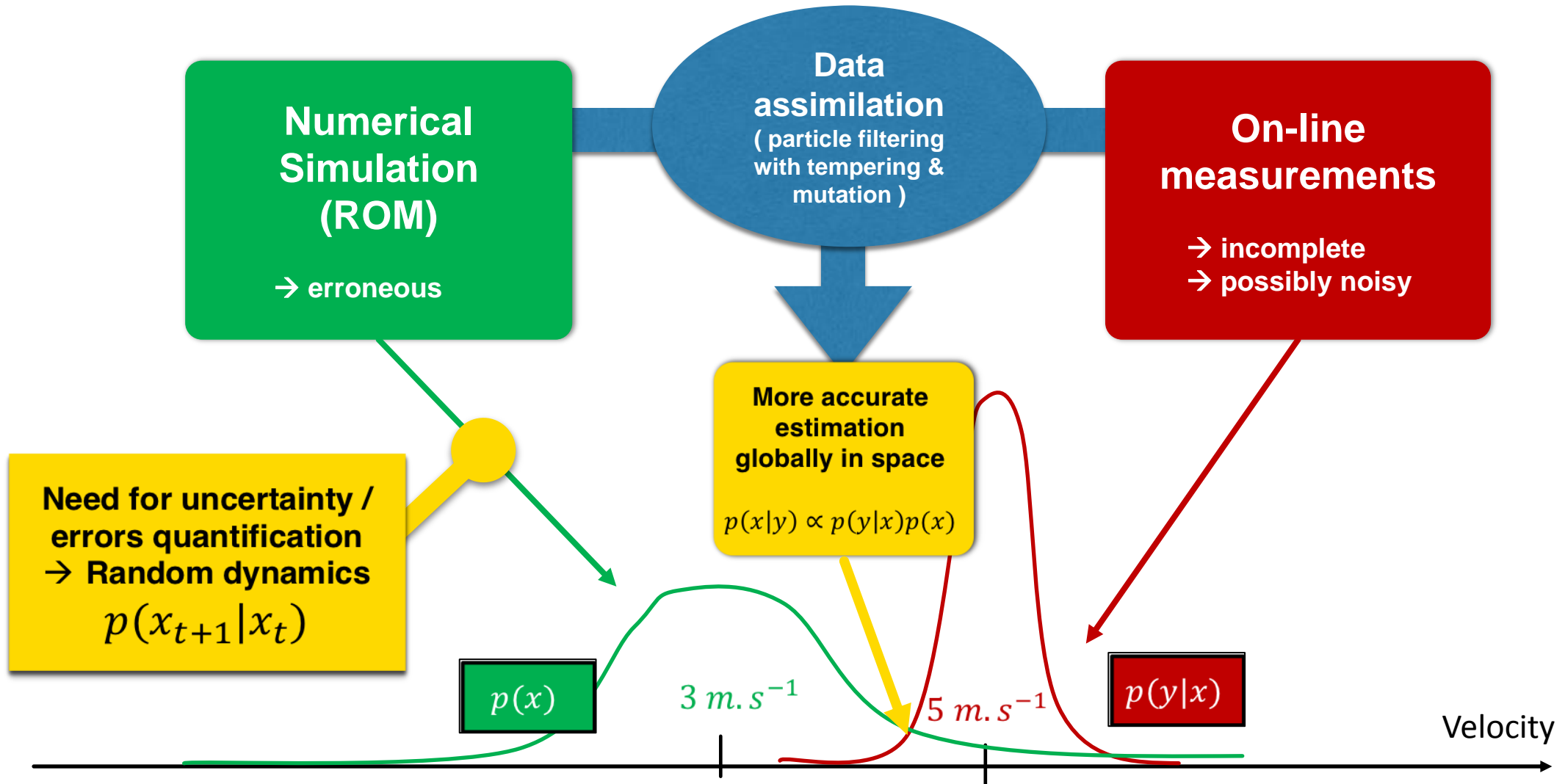


## PART III

### SIMULATION, MEASUREMENTS & DATA ASSIMILATION



# COMBINING SIMULATIONS AND MEASUREMENTS

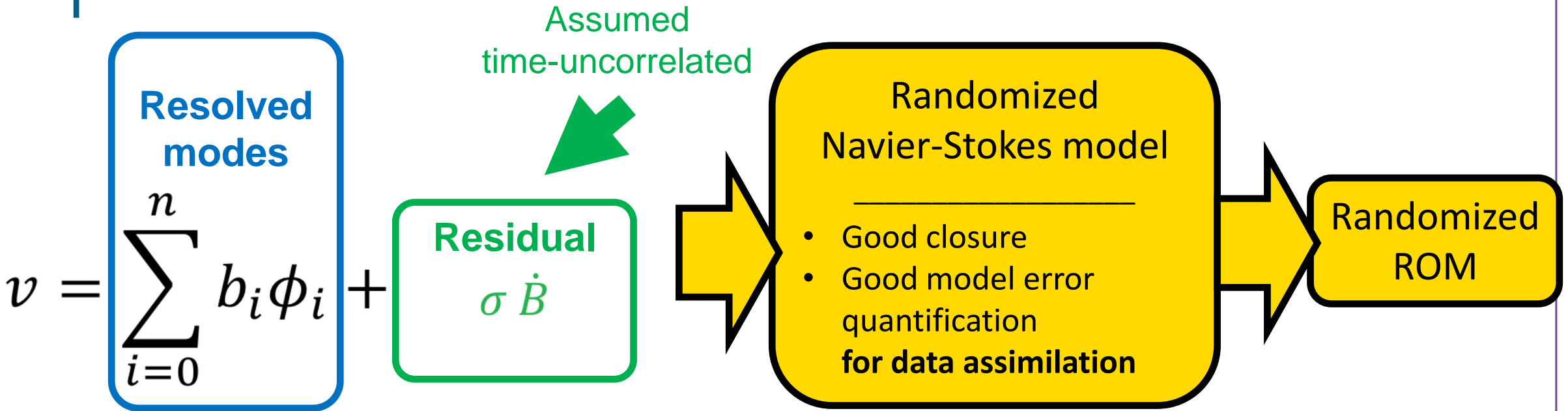




## PART IV

# REDUCED ORDER MODELS UNDER LOCATION UNCERTAINTY

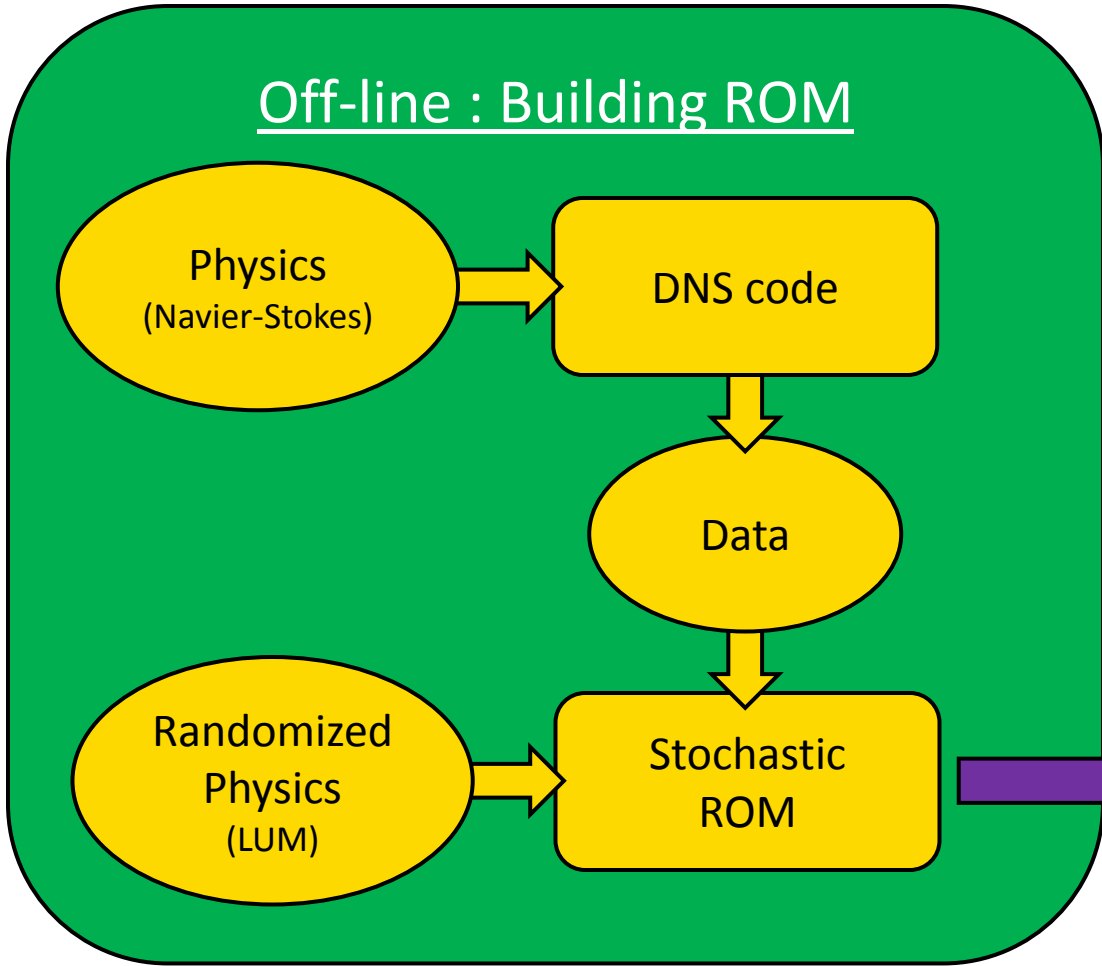
# LOCATION UNCERTAINTY MODELS (LUM)



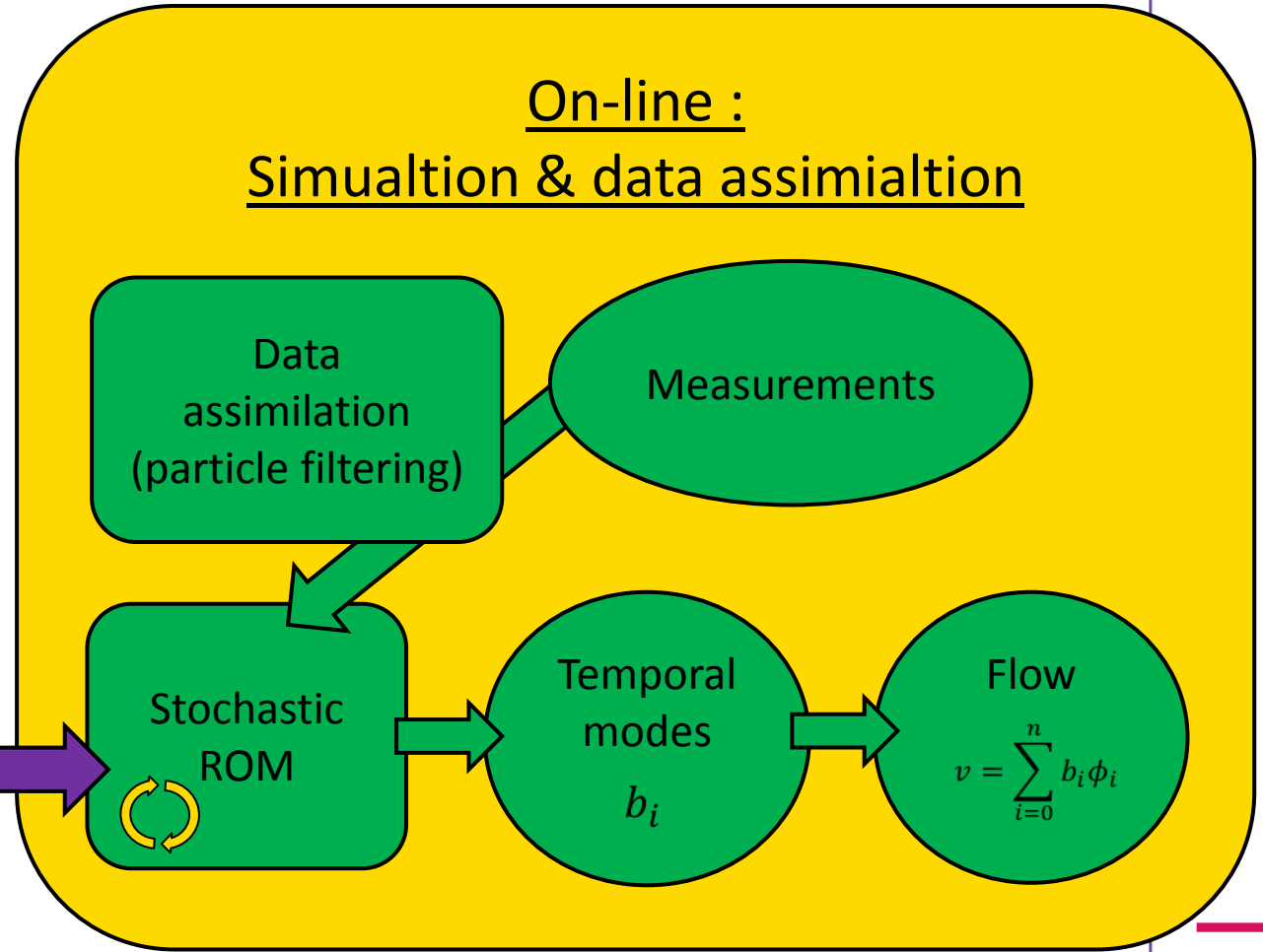
|              | LUM   | SALT  |
|--------------|---|---|
| References : | Memin, 2014<br>Resseguier et al. 2017 a, b, c, d<br>Cai et al. 2017<br>Chapron et al. 2018<br>Yang & Memin 2019 | Holm, 2015<br>Holm and Tyranowski, 2016<br>Arnaudon et al. 2017<br>Crisan et al., 2017<br>Gay-Balmaz & Holm 2017<br>Cotter and al. 2018 a, b<br>Cotter and al. 2019 |
|              | Cotter and al. 2017 Resseguier et al. 2019 a, b   |   |

# SUMMARY

## Off-line : Building ROM



## On-line : Simulation & data assimilation



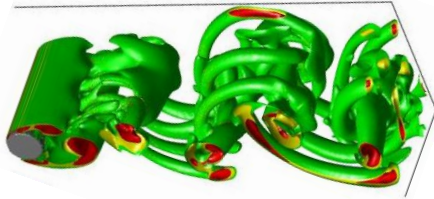


## PART V

RESULTS :  
UQ &  
FAST OBSERVER  
OF THE FLOW

# NUMERICAL RESULTS : 3D WAKE AT REYNOLDS 300

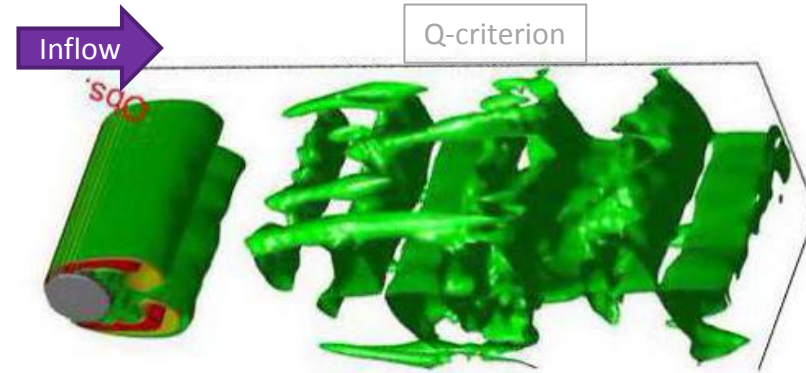
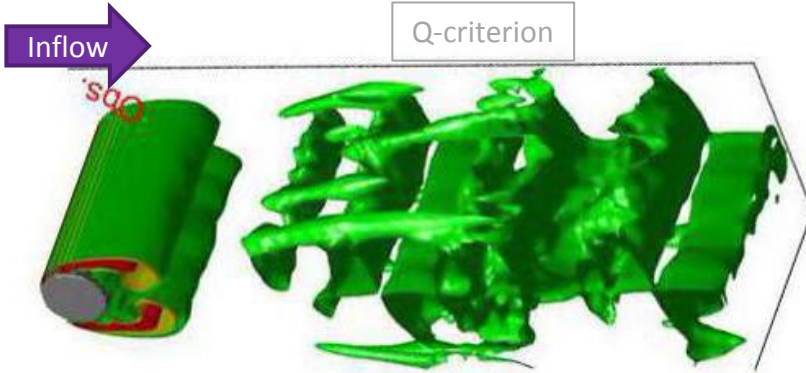
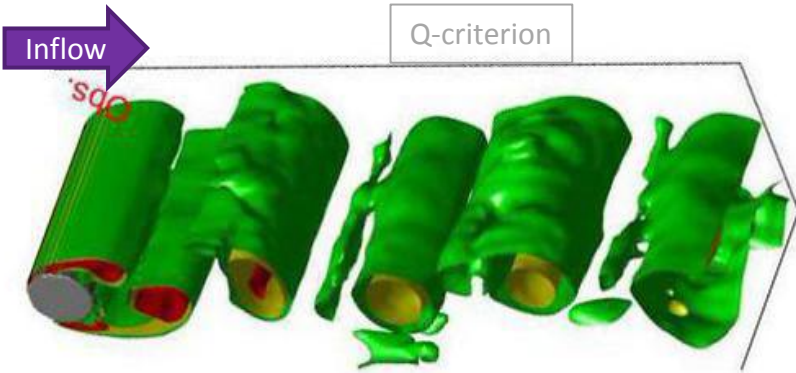
Reduced order models with **8 degrees of freedom**  
 ➤ only 1 PIV spatial point (local, blurred and noisy measure) is assimilated 10 times by vortex shedding cycle



**Reference :**  
 PCA-projection of the  
 “true” simulation ( $10^7$ -dof DNS)  
 (Optimal from 8-dof linear decomposition)

**Our method :**  
 POD-Galerkin with Navier-Stokes  
 under location uncertainty (LUM)

**State-of-art :**  
 POD-Galerkin with Navier-Stokes +  
 optimally tuned eddy viscosity &  
 additive noise



The word 'CONCLUSION' is centered on the page. To its left is a decorative graphic consisting of two perpendicular lines forming an L-shape, with the top horizontal line extending further to the left.

## CONCLUSION

- ▶ Reduced order model (ROM) : for very fast and robust CFD ( $10^7 \rightarrow 6$  degrees of freedom.)
  - Combine data & physics (built off-line)
  - Closure problem handled by LUM
  - Now implemented in OpenFOAM / ITHACA-FV
- ▶ Data assimilation : to correct the fast simulation on-line by incomplete/noisy measurements
  - Model error quantification handled by LUM
- ▶ First results
  - Optimal unsteady flow estimation/prediction in the whole spatial domain (large-scale structures)
  - Robust far outside the learning period

## NEXT STEPS

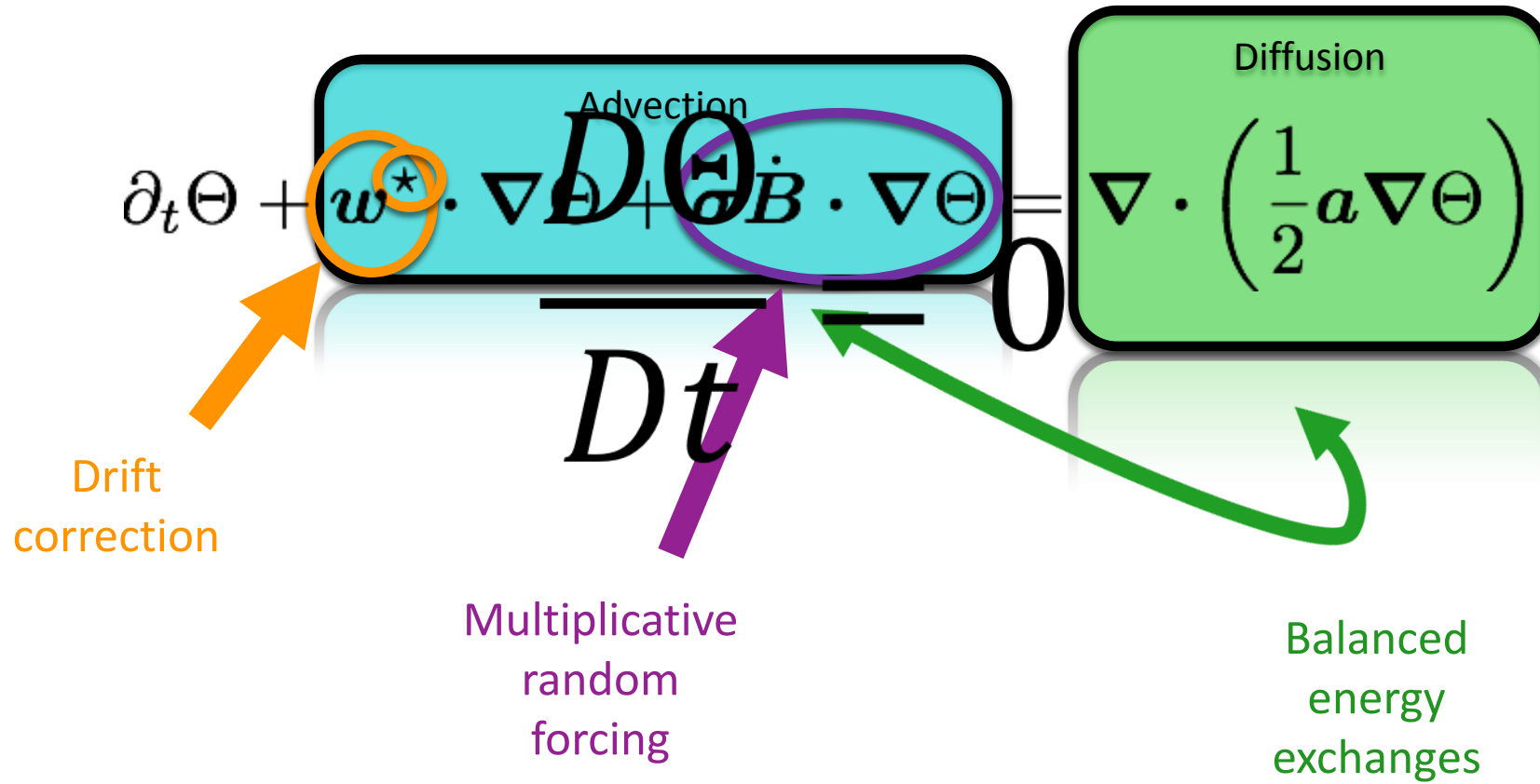
- ▶ Real measurements (PIV, Mems, ...)
- ▶ Increasing the degrees of freedom ( $n$ )
- ▶ Parametric ROM
- ▶ Increasing Reynolds (reduced DNS  $\rightarrow$  reduced LES  $\rightarrow$  reduced DDES )





# BONUS SLIDES

# MODEL UNDER LOCATION UNCERTAINTY, THE TRACER ADVECTION EXAMPLE



Large scales:  
 $w$   
 Small scales:  
 $\sigma \dot{B}$   
 Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

# REDUCED MODELS UNDER LOCATION UNCERTAINTY: GALERKIN PROJECTION GIVES SDES FOR RESOLVED MODES

$$\int_{\Omega} \phi_i \cdot (\text{stochastic Navier-Stokes})$$



$$\frac{db_i}{dt} = \underbrace{F_i(\mathbf{b})}_{n \times 1} + \underbrace{(\alpha_{.i} \dot{B}_t)^T}_{n \times M \times M \times 1} \underbrace{\mathbf{b}}_{n \times 1} + \underbrace{(\theta_i \dot{B}_t)}_{1 \times M \times M \times 1}$$

multiplicative noise                  additive noise

Large scales:  
 $\frac{d\mathbf{b}}{dt}$

Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $\frac{db_i}{dt} = F_i(\mathbf{b}) + (\alpha_{.i} \dot{B}_t)^T \mathbf{b} + (\theta_i \dot{B}_t)$

2<sup>nd</sup> order polynomial:  
coefficients given by physics,

$$(\phi_j)_j \text{ and } a = \overline{\sigma \dot{B} (\sigma \dot{B})^T} \tau$$

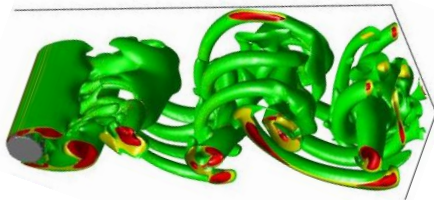
## Correlations to estimate

$$\alpha_{pi} \cdot \alpha_{qj}, \alpha_{pi} \cdot \theta_j, \theta_i \cdot \theta_j$$

# NUMERICAL RESULTS : 3D WAKE AT REYNOLDS 300

Reduced order models with 2 degrees of freedom

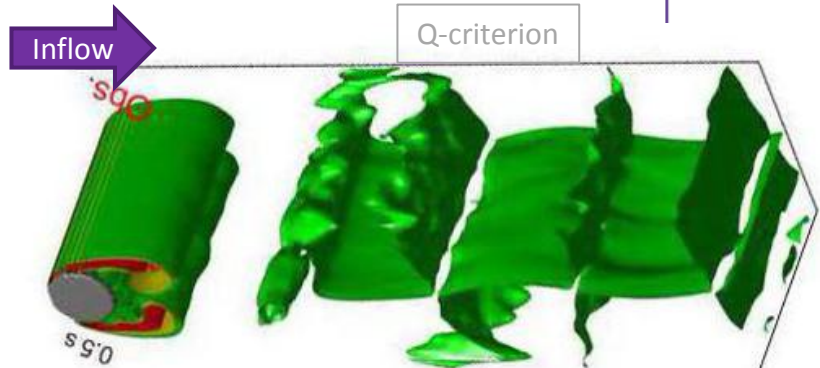
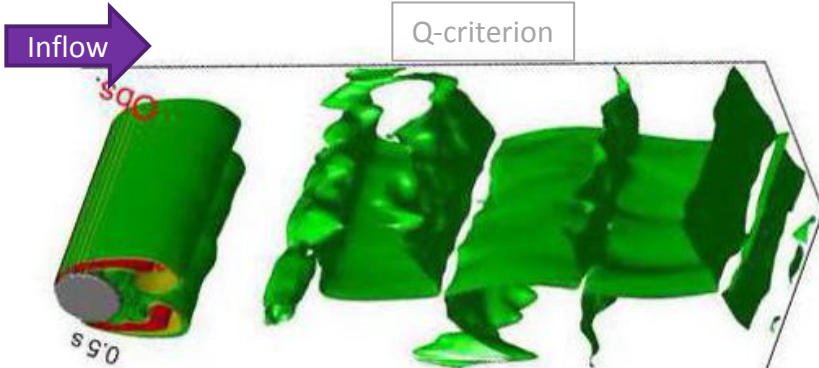
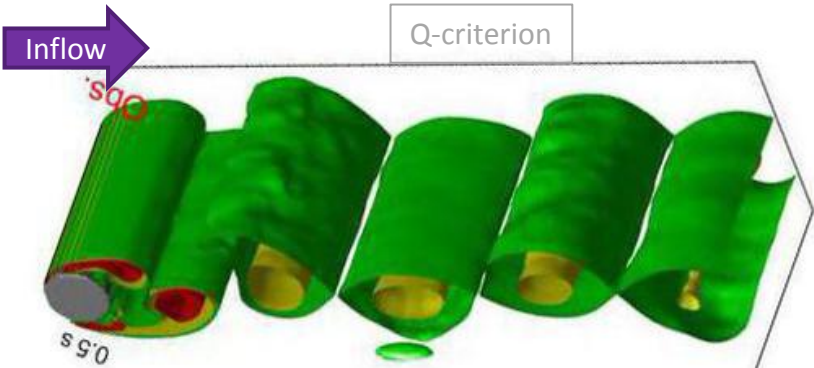
- only 1 PIV point (local, blurred and noisy measure) is assimilated 10 times by vortex shedding cycle



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(Optimal from 2-dof linear decomposition)

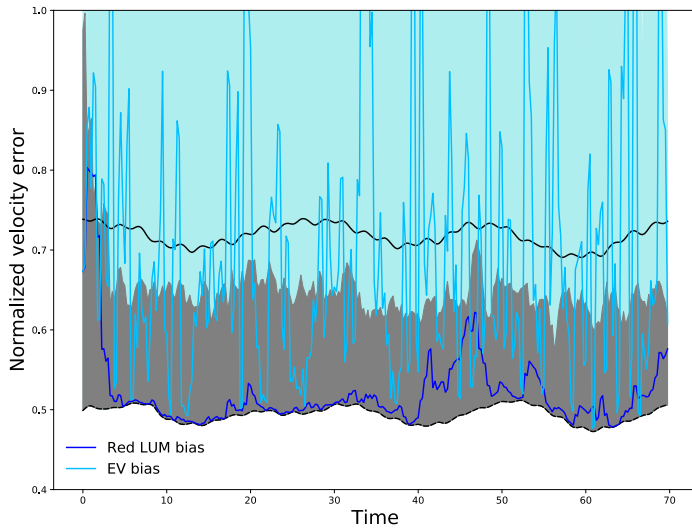
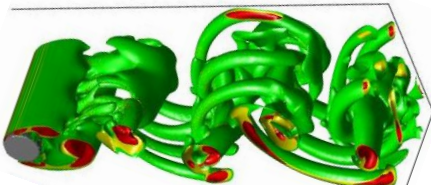
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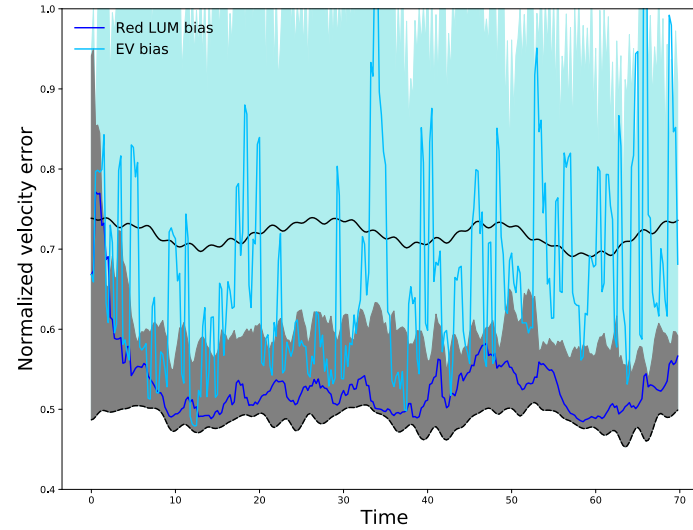


# NUMERICAL RESULTS : 3D WAKE AT REYNOLDS 300

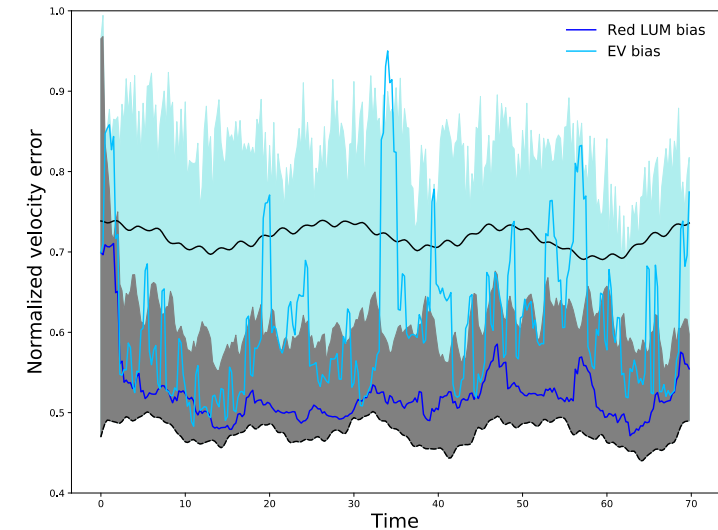
Reduced order models with  $n$  degrees of freedom  
 ➤ only 1 PIV spatial point (local, blurred and noisy measure)  
 is assimilated 10 times by vortex shedding cycle



$n=2$



$n=4$



$n=8$