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# Multiphase coupled converter models dedicated to transient response and output voltage regulation studies

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**Abstract**— In order to study transient response and output voltage regulation in multiphase coupled buck converter, it is proposed two models of interleaved coupled buck converter. These two models provide accurate current and voltage waveforms for any value of duty cycle. In the first part, the two proposed models are described in details. In the second part, it is shown the interest of this approach to study dynamic behaviour and determine compensation filters for voltage regulation in a multiphase coupled buck converter.

**Keywords**— Interleaved converters, Multiphase coupled buck converter, Converter control, Regulation.

## I. INTRODUCTION

High current low voltage power converters with fast response are needed for powering digital systems such as microprocessors which require more than 100 A at under than 1V [1-2]. As special power supply for the microprocessor is the voltage regulator module (VRM). The industry standard VRM topology used is the multiphase buck converter [3], [4] and [5]. Increasing demand for higher current, lower voltage and faster dynamic response imposes a challenge for multi-phase buck converter designs. Coupling phases of interleaved buck has the potential to reduce steady-state power losses while maintaining or even improving dynamic performances [6], [7], [8], [9] and [10]. Coupling phases gives different equivalent inductances for transient response ‘ $L_{tr}$ ’ and steady-state operation ‘ $L_{ss}$ ’ [11] and [12] and it is unclear which one should be used to design filters compensation.

In the literature [6] and [13], only when the duty cycle,  $D_1$  is lower than the inverse of the number of phases ( $D_1 < 1/q$ ,  $q$ : the number of parallel phases), the multi-phase buck (Fig. 1) is equivalent to a single phase buck with the effective duty cycle is equal to ‘ $q$ ’ times the actual duty cycle ( $D_{eq} = q.D_1$ ), the switching frequency seen from the output is equal to ‘ $q$ ’ times the actual switching frequency ( $F_{sweq} = q.F_{sw}$ ) and the input voltage is the actual input voltage divided by the number of phases ( $V_{in_{eq}} = V_{in}/q$ ).

In this paper, we propose two accurate models of interleaved coupled buck converter, for any value of duty cycle, in order to show which equivalent inductance must be used to design compensation filters for voltage regulation. In the first part of this paper, the two proposed

models are described in details. In the second part, it’s shown the interest of these models to study dynamic behaviour and determine compensation filters for voltage closed loop regulation in a multiphase coupled buck converter.

## II. MULTIPHASE COUPLED BUCK CONVERTER DESCRIPTION

Interleaved converters are widely employed in various applications such as automotive 42/14V systems and Voltage Regulator Module (VRM). The well known advantage of this system is an increase of the apparent frequency of the current ripple applied across the input and output filters; associating ‘ $q$ ’ identical commutation cells fed by interleaved control signals (equal duty cycles, phase-shifts  $2\pi/q$ ), the apparent frequency for the input filters and the output capacitor is ‘ $q$ ’ times the switching frequency. In addition, an improvement of dynamic behaviour can be obtained in interleaved converters [6]. Two main solutions can be applied to interconnect interleaved commutation cells: by means of either uncoupled/independent inductors or intercell transformers (Fig. 1). By the first mean of interleaving, the increase of frequency concerns only the input filters

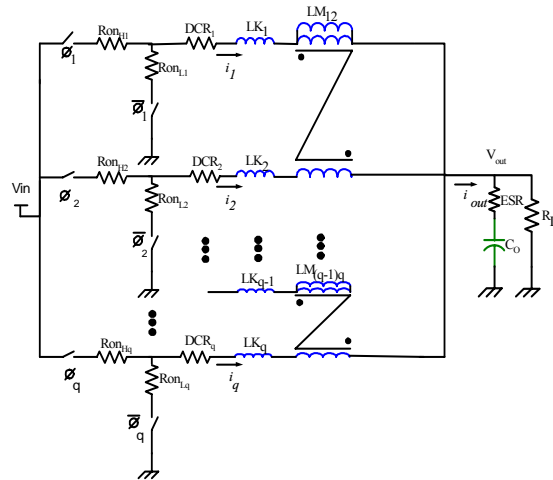


Fig. 1.  $q$ -phase coupled buck converter

and the output capacitor but the phase current is at the switching frequency  $F_{sw}$ . By the second mean of interleaving, also the apparent frequency of the current of phases is multiplied by 'q' [6], [7], [8], [9] and [10]. Fig.2 shows the output current waveform of the multiphase coupled converter: the apparent frequency of the current is q times the switching frequency. From (Fig.2) it can be seen that for each  $(1/q)T_{sw}$ , the same waveform of the output current occurs for different value of duty cycle ( $D_1, D_2, D_3, \dots$ ). It can be noted that any duty cycle can be written as a function of the duty cycle  $D_1 < 1/q$ :  $D = D_1 + (k-1)/q$  (where  $k = 1, 2, \dots, q$ ). The main idea in this paper, is to develop two simple models of interleaved coupled buck converter using the fact that D is equal to  $D_1$  modulo  $1/q$ : the first model is an equivalent uncoupled interleaved buck converter using only equivalent leakage inductances present in each phase. It is dedicated to study transient response. The second model is equivalent to a single buck converter used to simplify filters design for output voltage regulation.

### III. EQUIVALENT MODELS FORMULATION

In this section, it will be demonstrated that for any value of duty cycle, the multiphase coupled buck converter (Fig. 1) can be reduced to a simple uncoupled parallel buck converters or to a simple single buck converter. To simplify explanation a two-phase coupled converter will be analyzed, but the same method can be applied to a greater number of phases.

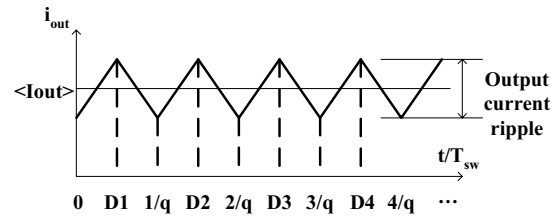


Fig. 2. q-phase coupled buck converter output current ripple

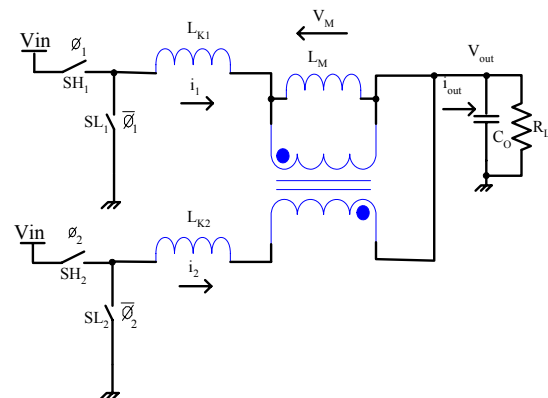


Fig. 3. A simplified schematic of two- phase coupled buck

#### A. Case 1: Duty cycle equal to $D_1 < 1/2$

Fig. 3 is a simplified schematic of two-phase coupled converter topology in which the intercell transformer is represented by two leakage inductances  $L_{K1}$  et  $L_{K2}$ , the ideal transformer and magnetizing inductance  $L_M$ . For the purpose of this analysis, it is assumed that  $L_{K1} = L_{K2} = L_K$  and the following relationship should be noted:

$$i_{out} = i_1 + i_2 \quad (1)$$

$$V_{LK1} = L_K(di_1/dt) \quad (2)$$

$$V_{LK2} = L_K(di_2/dt) \quad (3)$$

$$V_M = L_M(di_M/dt) \quad (4)$$

$$i_M = i_1 - i_2 \quad (5)$$

Equation (5) can be written as the following:

$$V_M = L_M(di_1/dt - di_2/dt) \quad (6)$$

From Fig.4, there are four distinct states of the phase currents and the output current when the duty cycle is equal to  $D_1 < 1/2$ .

1) *State one*: from  $t = 0$  to  $t = D_1 T_{sw}$

State one of the two-phase coupled converter covers the phase-one on time from  $t = 0$  to  $t = D_1 T_{sw}$ . During state one, phase one is connected to the input voltage ( $S_{H1}$  closed and  $S_{L1}$  open) and phase two is connected to ground ( $S_{H2}$  open and  $S_{L2}$  closed) and writing the voltage equations around the two current loops, we have:

$$V_{LK1} = V_{in} - V_{out} - V_M \quad (7)$$

$$V_{LK2} = V_M - V_{out} \quad (8)$$

Substituting (7) and (8) into (6), we have:

$$V_M = V_{in} L_M / (L_K + 2L_M) \quad (9)$$

To simplify the proceeding analysis a ratio,  $p = L_M / L_K$  [8] will be used and (9) reduces to:

$$V_M = V_{in} (p / (1 + 2p)) \quad (10)$$

Substituting (10) into (7) and (8), we have:

$$\frac{di_1}{dt} = (V_{in} / L_K) (1 - p / (1 + 2p) - D_1) \quad (11)$$

$$\frac{di_2}{dt} = (V_{in} / L_K) (p / (1 + 2p) - D_1) \quad (12)$$

$$\frac{di_{out}}{dt} = 2(Vin) \frac{(1-2D_1)}{(L_K)} \quad (13)$$

From (11), (12) and (13), it can be noted that although the coupling ratio ( $p$ ) does not affect the output current ripple, it has a significant effect on the phase current ripple slope. using a perfect coupling, the coupling ratio is increased toward infinity:

$$V_M = V_{in} / 2 \quad (14)$$

Equations (11) and (12) become equal to:

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \left(\frac{V_{in}}{2}\right) \frac{(1-2D_1)}{L_K} \quad (15)$$

$$\frac{di_1}{dt} = \frac{di_2}{dt} = (Vin_{eq}) \frac{(1-D_{eq})}{L_K} \quad (16)$$

Furthermore, using (1) we have an output current of:

$$\frac{di_{out}}{dt} = 2(Vin_{eq}) \frac{(1-D_{eq})}{(L_K)} \quad (17)$$

With:  $Vin_{eq} = V_{in} / 2$  and  $D_{eq} = 2D_1$

From (16), the strong coupling makes the current wave forms in the two phases indistinguishable and each phase is equivalent to a single buck converter with the equivalent inductance is the leakage inductance present in each phase. This formulation does no effect on studying the transient response, as it is shown in [6]: the transient response is determined exclusively by the leakage inductances.

2) *State Two*: from  $t = D_1 T_{sw}$  to  $t = T_{sw}/2$

State two covers the off time from  $t = D_1 T_{sw}$  to  $t = T_{sw}/2$ . During state two, the input is disconnected entirely ( $S_{H1}$  and  $S_{H2}$  open) and both windings are tied to ground ( $S_{L1}$  and  $S_{L2}$  closed). Looking in Fig. 4 and writing the voltage equations around the two current loops:

$$V_{LK1} = -V_{out} - V_M \quad (18)$$

$$V_{LK2} = V_M - V_{out} \quad (19)$$

In state two  $V_M = 0$  and we have:

$$\frac{di_1}{dt} = \frac{di_2}{dt} = (-V_{in} / L_K) D_1 \quad (20)$$

Equation (20) can be written as the following:

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \left(-\frac{V_{in}}{2}\right) \frac{(2D_1)}{L_K} \quad (21)$$

$$\frac{di_1}{dt} = \frac{di_2}{dt} = -(Vin_{eq}) \frac{(D_{eq})}{L_K} \quad (22)$$

Using (1) the output current is equal to:

$$\frac{di_{out}}{dt} = -2(Vin_{eq}) \frac{(D_{eq})}{(L_K)} \quad (23)$$

From (22) the current waveforms in the two phases are identical and each phase is equivalent to a single buck converter

3) *State three*: from  $t = T_{sw}/2$  to  $t = T_{sw}/2 + D_1 T_{sw}$

In this state,  $L_{K2}$  is now connected to the input and  $L_{K1}$  is connected to the ground. By symmetry, we obtain the same waveforms in (15), (16) and (17) as in state one, with just inverting the two currents.

4) *State four*: from  $t = T_{sw}/2 + D_1 T_{sw}$  to  $T_{sw}$

This state is identical to state two and the same waveforms (22) and (23) are obtained.

It can be noted that when the duty cycle  $D_1 < 1/2$  (2: the number of phases) the two phase and the output currents are increasing in state one and decreasing in state two and that this cycle repeats again in states three and four. The 2-phase coupled buck (Fig. 3) is equivalent to a 2 single parallel buck converters where the phase-shift between two phases is equal to zero, with equivalent duty cycle ' $D_{eq} = 2.D_1$ ', equivalent switching frequency ' $F_{sw_{eq}} = 2.F_{sw}$ ', equivalent input voltage ' $Vin_{eq} = Vin/2$ '.

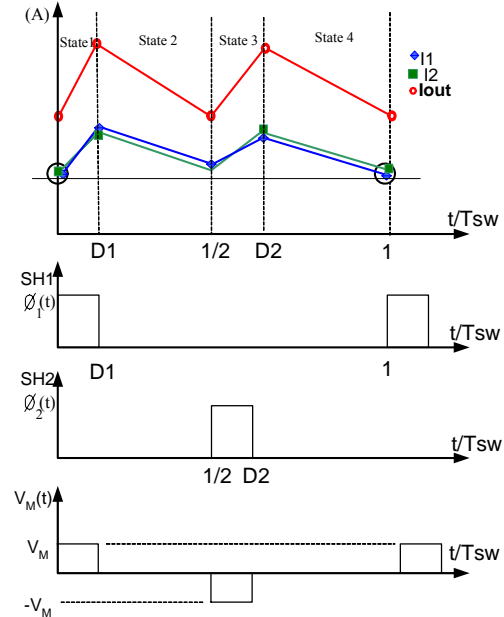


Fig. 4. Phase current ripple and output current ripple for two-phase coupled buck converter for a duty cycle equal to  $D_1 < 1/2$

**B. Case 2: Duty cycle equal to  $D_2 > 1/2$**

Now, the same study will be done with a duty cycle  $D_2 = 1/2 + D_1$

1) *State one*: from  $t = 0$  to  $t = D_1 T_{sw}$

During state one, phase one and phase two are connected to the input voltage ( $S_{H1}$  and  $S_{H2}$  closed) and ( $S_{L1}$  and  $S_{L1}$  open): in this case  $V_M = 0$  and writing the voltage equations around the two current loops:

$$V_{LK1} = V_{in} - V_{out} \quad (24)$$

$$V_{LK2} = V_{in} - V_{out} \quad (25)$$

With  $V_{out} = D_2 V_{in}$ :

$$\frac{di_1}{dt} = \frac{di_2}{dt} = (V_{in} - D_2 V_{in}) / L_K \quad (26)$$

As  $D_2 = D_1 + 1/2$ , (26) becomes:

$$\frac{di_1}{dt} = \frac{di_2}{dt} = (V_{in} / L_K)(0.5 - D_1) \quad (27)$$

$$\frac{di_1}{dt} = \frac{di_2}{dt} = (V_{in_{eq}}) \left( \frac{1 - D_{eq}}{L_K} \right) \quad (28)$$

$$\frac{d_{i_{out}}}{dt} = 2(V_{in_{eq}}) \left( \frac{1 - D_{eq}}{L_K} \right) \quad (29)$$

With:  $V_{in_{eq}} = V_{in} / 2$  and  $D_{eq} = 2D_1$

It can be noted in this state that the obtained expressions of phase and output currents are the same as in state one with duty cycle  $D_1 < 1/2$ .

2) *State Two*: from  $t = D_1 T_{sw}$  to  $t = T_{sw}/2$

$$V_{LK1} = V_{in} - V_{out} - V_M \quad (30)$$

$$V_{LK2} = V_M - V_{out} \quad (31)$$

$V_M = V_{in}(p/(1+2p))$ ,  $V_{out} = D_2 V_{in}$  and  $D_2 = D_1 + 1/2$ :

$$\frac{di_1}{dt} = (V_{in}/L_K)(1 - D_2 - p/(1+2p)) \quad (32)$$

$$\frac{di_2}{dt} = (V_{in}/L_K)(p/(1+2p) - D_2) \quad (33)$$

$$\frac{d_{i_{out}}}{dt} = 2(V_{in}) \left( \frac{1 - 2D_1}{L_K} \right) \quad (34)$$

From (32), (33) and (34), it can be noted that although the coupling ratio ( $p$ ) does not affect the output current ripple, it has a significant effect on the phase current ripple.

If we suppose that we have a perfect coupling: the coupling ratio is increased toward infinity:  $V_M = V_{in} / 2$ , (32) and (33) become equal to:

$$\frac{di_1}{dt} = \frac{di_2}{dt} = -(V_{in_{eq}}) \left( \frac{D_{eq}}{L_K} \right) \quad (35)$$

$$\frac{d_{i_{out}}}{dt} = -2(V_{in_{eq}}) \left( \frac{D_{eq}}{L_K} \right) \quad (36)$$

It can be noted in this state that the obtained expressions of phase and output currents, are the same as in state two with duty cycle  $D_1 < 1/2$ .

3) *State three*: from  $t = T_{sw}/2$  to  $t = T_{sw}/2 + D_1 T_{sw}$

This state is identical to state one and the same waveforms (28) and (29) are obtained.

4) *State four*: from  $t = T_{sw}/2 + D_1 T_{sw}$  to  $T_{sw}$

By symmetry, we obtain the same relations (32), (33) and (34) as in state two, with just inverting the two currents.

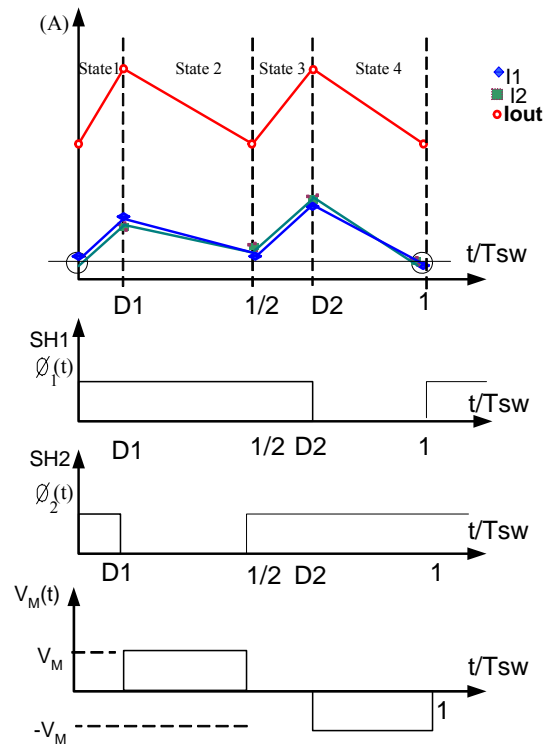


Fig. 5. Phase current ripple and output current ripple for a duty cycle equal to  $D_2 > 1/2$

It can be noted that when the duty cycle  $D_2 > 1/2$ , the two phase and the output currents are increasing in state one and decreasing in state two and that this cycle repeats again in states three and four. The 2-phase coupled buck is equivalent to 2 single parallel buck converters where the phase-shift between two phases is equal to zero, with equivalent duty cycle ' $D_{eq} = 2.D_1$ ', equivalent switching frequency ' $F_{sweq} = 2.F_{sw}$ ', equivalent input voltage ' $V_{ineq} = V_{in}/2$ '.

TABLE I.  
CURRENT RIPPLE FOR TWO DIFFERENT VALUES OF DUTY CYCLE

Duty cycle	$\frac{di_1}{dt} = \frac{di_2}{dt}$ in State 1	$\frac{di_1}{dt} = \frac{di_2}{dt}$ in State 2
$D_1 < 1/2$	$(V_{ineq}) \left( \frac{1-D_{eq}}{L_K} \right)$	$-(V_{ineq}) \left( \frac{D_{eq}}{L_K} \right)$
$D_2 = D_1 + 1/2$ $D_2 > 1/2$	$(V_{ineq}) \left( \frac{1-D_{eq}}{L_K} \right)$	$-(V_{ineq}) \left( \frac{D_{eq}}{L_K} \right)$

In table I (it was considered only state 1 and 2 since there is the same behavior in states 3 and 4) are summarized the different phase current ripple in two-phase coupled buck converter for two different duty cycle:  $D_1 < 1/2$  and  $D_2 > 1/2$  ( $D_2 = 1/2 + D_1$ ). It was shown that for the two different duty cycles the same model can be used

To simplify explanation a two-phase coupled converter was analyzed in this paper, but the same method can be applied to a greater number of phases. For any number of phases ' $q$ ' and for any value of duty cycle  $D = D_1 + (k-1)/q$  (where  $k = 1, 2, \dots, q$  and  $D_1 < 1/q$ ), the multi-phase buck (Fig. 1) is equivalent to a  $q$ -single parallel buck converters where the phase-shift between two successive phases is equal to zero, with equivalent duty cycle ' $D_{eq} = q.D_1$ ', equivalent switching frequency ' $F_{sweq} = q.F_{sw}$ ' and equivalent input voltage ' $V_{ineq} = V_{in}/q$ '. The inductance per phase is the sum of leakage inductances present in each phase. The on-time is unchanged, but the apparent duty cycle is ' $q$ ' time  $D_1$ . It can be noted that  $V_{out} = D.V_{in} = (D_1 + \frac{k-1}{q})V_{in}$

with  $k = 1, 2, \dots, q$ , so to have the same output voltage an additional voltage source  $V_k$  has to be added in the proposed model with  $V_k = (D - D_1)V_{in} = (\frac{k-1}{q})V_{in}$  as it

is shown in Fig. 6. This first model is very helpful to study transient response for any duty cycle  $D = D_1 + (k-1)/q$  but in order to study output voltage regulation with usual tools, a second equivalent model is proposed (Fig.7). With equal leakage inductances ' $L_k$ ', the equivalent multiphase uncoupled buck converter (Fig. 6) becomes equivalent to a single buck converter with only one inductance equal to  $L_k/q$ .

If unmatched phases provide different mean current values in each branch, model of Fig. 6. can be used with additional branch resistors providing effects of mismatch.

Fig. 8. shows this model with expression of resistors.

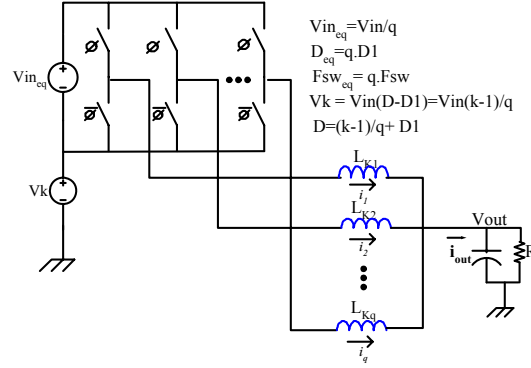


Fig. 6. Equivalent multiphase model of multiphase buck converter using  $V_{ineq}$ ,  $D_{eq}$ ,  $F_{sweq}$  and  $V_k$

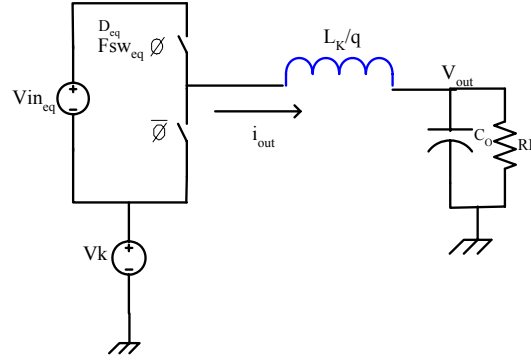


Fig. 7. Equivalent single phase buck converter of multiphase buck converter using  $V_{ineq}$ ,  $D_{eq}$ ,  $F_{sweq}$  and  $V_k$

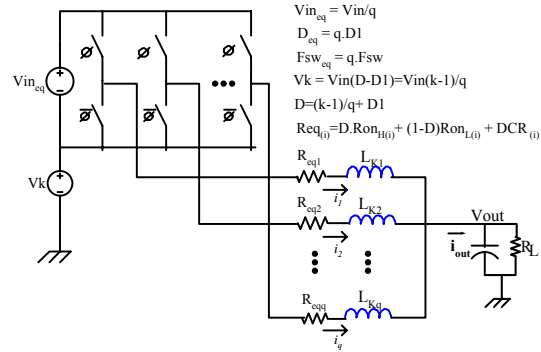


Fig. 8. Equivalent single phase buck converter

#### IV. VALIDATION OF THE TWO EQUIVALENT MODELS

In order to validate the two models presented in the previous section (multiphase and simple buck), comparisons between real coupled system and models

have to be done by simulation. A five coupled-phase buck converter is studied ( $q = 5$ ). In Fig. 9 is shown the electrical scheme of this converter in cyclic cascade configuration [14]. In the simulation,  $V_{in}=6V$ ,  $V_{out}=3.3V$ ,  $I_{out}=15A$ ,  $R_{load}=0.22$ ,  $L_K=1\mu H$ ,  $L_M=150\mu H$  and  $F_{sw}=50KHz$ . In this case, it can be noted that duty-cycle  $D > 1/5$ .

Using general expression of duty-cycle presented in section II,  $D$  can be formulated in this example with the following expression:

$$D = \frac{k-1}{q} + D1 = 0.55 \quad (37)$$

with  $D=0.55$ ,  $k=3$ ,  $D1=0.15$

Fig 10 and Fig 11 show respectively responses of the two models for a fast load transient. It can be noted that voltage and current signal waves are very similar. Current slopes are accurate and transient time responses are the same. Model switching frequency is five time higher than real system frequency (see signals  $V(HS\_model)$  and  $V(HS)$ ).

For all cases with odd number for 'k', comparing the real system and the multiphase model, phase current slopes are the same. With even number for 'k', slopes are inverted. This behaviour has no impact on accuracy of mean values and time responses. For all 'k' value cases, output current waves are exactly the same.

Now, simple buck model can be used to perform closed loop analysis in voltage mode in order to provide a accurate and stable output voltage value. Such a simple model allows writing expression of electrical system to compensate. Overall open-loop gain expression is the following:

$$G(s).H(s) = \frac{V_{in}.K(s)}{V_p} \cdot \frac{1 + esr.Co.s}{1 + \left(\frac{Leq}{R_{load}} + esr.Co\right)s + Leq.Co.s^2} \quad (38)$$

with  $Leq=2.L_K/q$ ,  $K(s)$  : compensation gain stage,  $R_{load}$  : output resistive charge,  $Co$  : output filter capacitor and  $esr$  : equivalent serial resistor of capacitor  $Co$ .

Note that term '2.L<sub>K</sub>' in expression of  $Leq$  represent the sum of leakage inductances present in each phase of converter (Fig. 9).

Expression (38) is a well-known voltage-mode buck open-loop gain equation. A second order behavior providing a resonant frequency at  $f_n$  has to be compensate to guaranty stability.

$$f_n = \frac{1}{2.\pi.\sqrt{Leq.Co}} \quad (39)$$

Fig 12 describes a possible filter  $K(s)$  providing 2 zeros and 1 pole used to get a good phase margin, large bandwidth, good attenuation at  $f_n$  and very high gain at low frequencies. With  $L_K=6\mu H$ ,  $L_M=1.4mH$ ,  $C=5 \times 100\mu F$  and  $R_{load}=0.22\Omega$ , following values are computed:

$R1=1K\Omega$ ,  $C1=10nF$ ,  $R2=10K\Omega$ ,  $C2=3\mu F$  and  $C3=100pF$ .

Fig 13 shows output signal behavior for the closed-loop system. Error voltage 'Verror' reach the good value of 0.75 corresponding to expecting '5.D1' value.

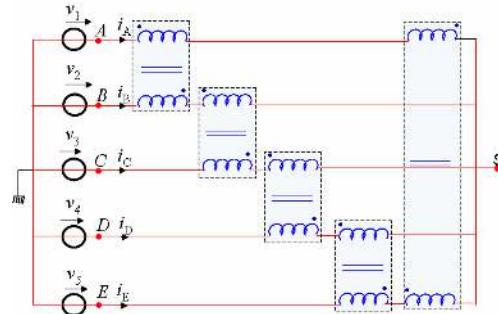


Fig. 9. Electrical scheme of 5-coupled buck converter

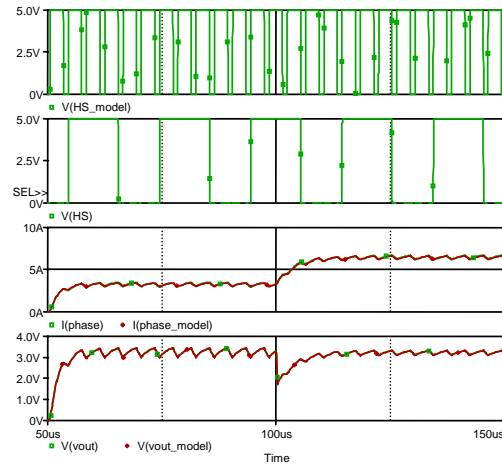


Fig. 10. Load transient response for both real five coupled-phase system and multiphase model.

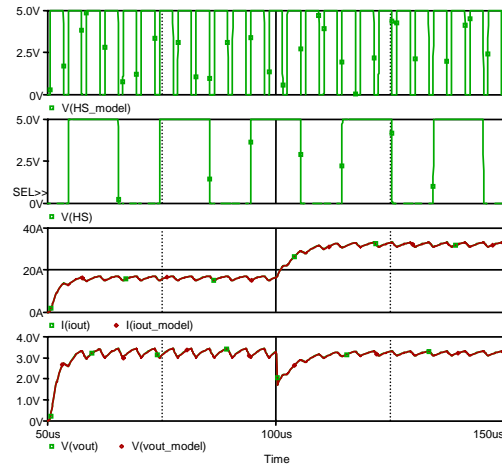


Fig. 11. Load transient response for both real five coupled-phase system and simple buck model

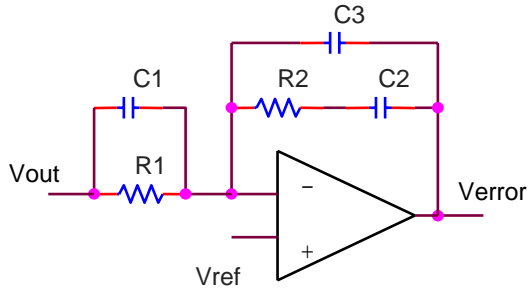


Fig. 12.  $K(s)$  filter used for compensation purpose

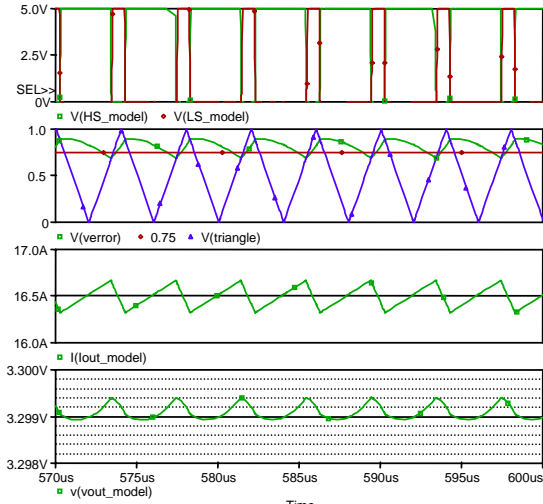


Fig. 13. Steady-state signal waveforms of closed-loop system using simple buck model

Obtained signals are very similar to those provided by the real system of Fig. 9.

## V. CONCLUSION

In this article, two accurate models of complex multiphase coupled buck, for any value of duty cycle, have been described. Method used to develop those models is an analytical approach providing accurate slopes and mean values for all system signals. Simulation results validate them in a steady and dynamic state for a five-coupled buck. Validity range of multiphase model is given by factor ' $p=L_K/L_M$ ' (higher than possible). Simple buck model is accurate for any value of ' $p$ '. It can be noted that using the single buck converter model, it is possible to design compensation filters for any value of duty cycle. Further works have to be done in order to get a dynamic ' $V_K$ ' source, following duty-cycle variation for accurate closed-loop transient load analysis.

## REFERENCES

- [1] B. Rose, "Voltage regulator technology requirements", in 4th Annual Intel Technology Symposium, Sept. 1990.
- [2] E. Stanford, "Device requirements for low voltage, fast transient response regulators used to power future microprocessors and other low voltage logic chips", Proceedings of the Sixteenth International High Frequency Power Conversion Conference, Rosemont, IL, September, 2001, pp. 1-10.
- [3] G. Schuellein, "Current sharing of redundant synchronous buck regulators powering high performance microprocessors using the  $V^2$  control method", in Thirteenth Annual Applied Power Electronics Conf. and Exposition, 1998, vol. 2, pp. 853-9.
- [4] A.M.Wu, Jinwen Xiao, D. Markovic, and S.R. Sanders, "Digital PWM control: application in voltage regulation modules", in *30th Annual IEEE Power Electronics Specialists Conf.*, 1999, pp. 77-83 vol.1.
- [5] Jinwen Xiao, A.V. Peterchev, and S.R. Sanders, "Architecture and IC implementation of a digital VRM controller", in *32nd Annual IEEE Power Electronics Specialists Conf.*, 2001.
- [6] J. Li, C. R. Sullivan, and A. Schultz, "Coupled inductors design optimization for fast- response low voltage DC-DC converters", IEEE Applied Power Electronics Conference, Dallas, March 2002.
- [7] J. Czogalla, J. Li, C. R. Sullivan, "Automotive application of multiphase coupled-inductor DC-DC converter", Proc. IAS'03, vol. 3, pp. 1524, 2003.
- [8] J. Li, A. Stratakos, A. Schultz, and C. R. Sullivan, "Using coupled inductors to enhance transient performance of multi-phase buck converters", APEC'04, vol. 2, pp. 1289-1293, 2004.
- [9] P.-L. Wong, P. Xu, B. Yang, and F. C. Lee, "Performance improvements of interleaving VRMs with coupling inductors", APEC'00, pp. 973-978, 2000. M. Young, *The Technical Writer's Handbook*. Mill Valley, CA: University Science, 1989.
- [10] Pit-Leong Wong, Q. Wu, Peng Xu, Bo Yang, and F.C. Lee, "Investigating coupling inductors in the interleaving QSW VRM", in Proceedings of APEC 2000 - Applied Power Electronics Conf., pp. 973-8 vol.2.
- [11] Y. Dong, M. Xu, and F. C. Lee, "DCR Current Sensing Method for Achieving Adaptive Voltage Positioning (AVP) in Voltage Regulators with Coupled Inductors", PESC '06, 2006.
- [12] P.-L. Wong, P. Xu, B. Yang, and F. C. Lee, "Performance improvements of interleaving VRMs with coupling inductors", APEC'00, pp. 973-978, 2000. M. Young, *The Technical Writer's Handbook*. Mill Valley, CA: University Science, 1989.
- [13] P. Zumel, O. Garcia, J.A. Cobos, and J. Uceda, "Tight magnetic coupling in multiphase interleaved converters based on simple transformers", Proc. APEC'05, vol. 1, pp. 385, 2005.
- [14] I. G. Park and S. I. Kim, "Modeling and analysis of multi-intercell transformers for Connecting power converters in parallel", PESC'97, vol. 2, pp. 1164-1170, 1997.