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NUMERICAL TOOLS FOR EFFICIENT SIMULATIONS OF WAVE PROPAGATION IN DAMPED PERIODIC STRUCTURES

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ABSTRACT

Periodic structures exhibit very specific properties in terms of wave propagation. In this paper, some numerical tools for dispersion analysis of periodic structures are presented, with a focus on the ability of the methods to deal with the dissipative behavior of the systems.

The classical Floquet-Bloch approach is first presented, as a reference. This technique uses proper boundary conditions on the unit cell, it is quite simple to implement but dealing with damping is not easy for 2D or 3D cases. Secondly, the Shifted-Cell Operator technique is described. It consists in a reformulation of the PDE problem by shifting in terms of wave number the space derivatives appearing in the mechanical behavior operator inside the cell, while imposing continuity boundary conditions on the borders of the domain. Damping effects can be introduced in the system and a quadratic eigenvalue problem yields to the dispersion properties of the periodic structure.

Both approaches are first analyzed in terms of mesh convergence, and then a focus is proposed on tools for the post-processing of dispersion diagrams in damped configurations. The group velocity of each branch in the diagram is defined including damping effects, and it is shown that it constitutes a pertinent indicator for the branches tracking from one computational point to another.

1 INTRODUCTION

A periodic medium is a material or a structural system that exhibits spatial periodicity. The study of periodic structures has a long history in the field of vibrations and acoustics. Newton described the propagation of sound in air [1]. He assumed that sound was propagated in air like an elastic

wave would be propagated along a lattice of point masses. This topic has interested researchers over the years, and a growing activity on this field is observed on the last years, with the objective of designing structures exhibiting properties that conventional ones cannot possess. The methods currently used are most of the time based on those derived from wave propagation in crystals [2], where almost no dissipation occurs. Reaching the upper scale for structural dynamics implies that damping effects have to be included in the analyses which are performed. A very detailed review of historical origins, recent progress and future outlook of this topic has been published recently by [3]. The reader is invited to refer to this article and the following discussion by [4] that cover the most important aspects of this topic.

2 STRUCTURE PRESENTATION

The structure is presented in ref. [5]. The system consists in an infinite periodic bidirectional waveguide shown in (Fig. 1). It is a 1 mm thick plate with periodic cylindrical pillars made of isotropic Aluminium 6063-T83 ($\nu = 0.33$, $E = 69e9[Pa]$ and $\rho = 2700[kg/m^3]$). Due to the periodicity, the unit cell is used and the corresponding first Brillouin zone is described in (Fig. 2).

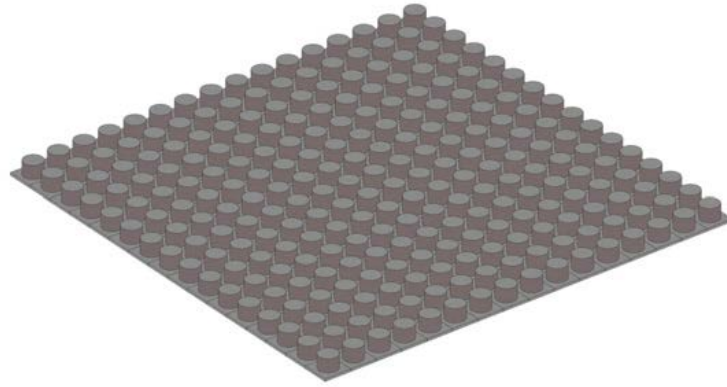


Figure 1. Infinite plate.

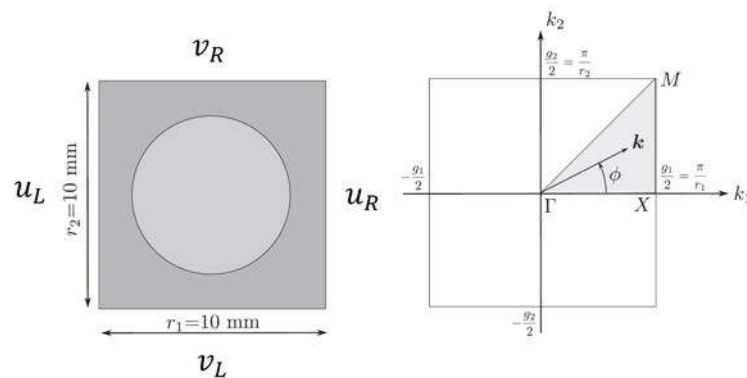


Figure 2. Real and reciprocal lattices.

3 CLASSICAL FLOQUET-BLOCH METHOD

The Floquet-Bloch approach is a method commonly used for the study of periodic structures but its use in a damping context is not easy in 2D and 3D cases.

The material constitutive law is linear, elastic and isotropic. The periodicity is defined on the borders of the domain $u_R = e^{-jk_x r_1} u_L$ and $v_R = e^{-jk_y r_2} v_L$ (Fig. 2) where u_R is the displacement on the right border and u_L is the displacement on the left border in x axis, v_R is the displacement on the right border and v_L is the displacement on the left border in y axis, k_x and k_y are respectively the wavenumbers in the x and y directions. Tetrahedral quadratic elements (TET10) are used to mesh the plate.

The harmonic homogeneous dynamical equilibrium of the system is driven by the following partial derivative problem

$$\begin{cases} \rho\omega^2 u + \nabla\sigma = 0 \\ \sigma = C : \varepsilon, \end{cases} \quad (1)$$

where $u \in \mathbb{R}^3$ is the displacement, σ is the stress tensor, C is the elastic tensor and ε is the strain tensor.

A parametric eigenvalue analysis is performed using the Pardiso solver [6], two parameters (wavenumbers) are considered, namely $k_x = [0 \pi/r_1]$ and $k_y = [0 \pi/r_2]$. The eigen frequencies are obtained by solving the problem. The wave's dispersion curves of the undamped system are plotted on all the first Brillouin zone (Fig. 3) and on the contours (Fig. 4). The frequencies defining the bandgaps can always be found by considering only the contour of the irreducible Brillouin zone [2]. So, (Fig. 4) is sufficient to observe partial bandgaps and complete bandgap in all the directions. It appears that the structure exhibits both partial bandgaps for frequencies around 2×10^4 and 1×10^5 Hz in some specific directions and a complete bandgaps between 114 and 143 kHz.

In the approach, the wavenumbers are parameters of the eigenvalue problem, the solutions being the frequencies. Therefore, this approach is not suitable for systems with frequency-dependent characteristics.

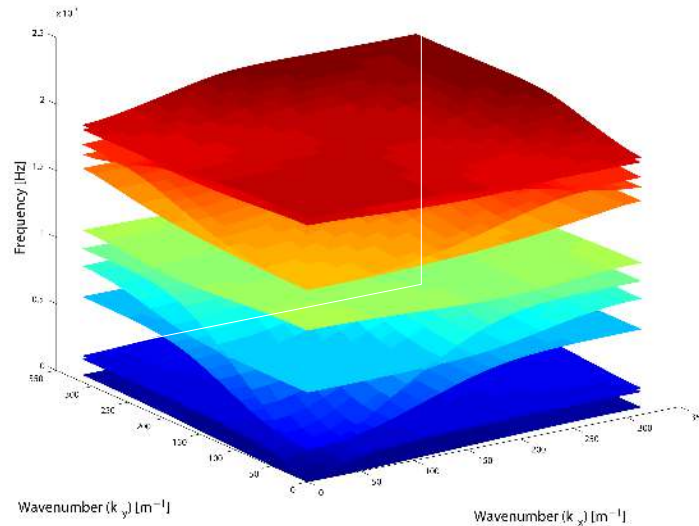


Figure 3. Dispersion surfaces on the Brillouin zone.

4 "SHIFTED-CELL OPERATOR" METHOD

The "Shifted-Cell Operator" technique consists in a reformulation of the PDE problem by "shifting" in terms of wave number the space derivatives appearing in the mechanical behavior operator inside the cell, while imposing continuity boundary conditions on the borders of the domain.

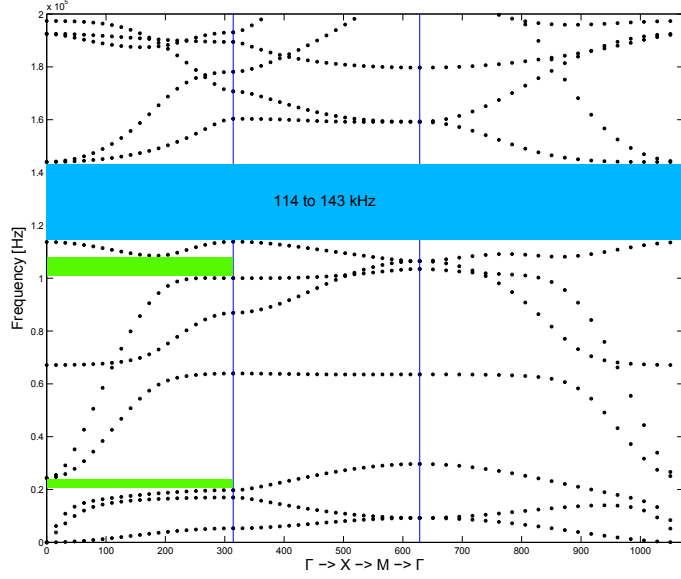


Figure 4: Dispersion relation on the contour of the Brillouin zone. Complete and partial bandgap in the $\Gamma - X$ direction respectively in the blue and green regions.

The formulation leads to the following eigenvalue problem [7]

$$[(K - \omega^2 M) + \lambda_i(L - L^T) - \lambda_i^2 H]\phi_i^r = 0, \quad (2)$$

where $\lambda_i = jk_i$ is the i -th eigen value, ϕ_i^r denotes the right eigenvector associated to λ_i , M and K are respectively the standard symmetric definite mass and symmetric semi-definite stiffness matrices, L is a skew-symmetric matrix and H is a symmetric semi-definite positive matrix. In this formulation, all matrices can depend on ω .

Continuity boundary conditions are imposed on the borders of the domain, namely $u_R = u_L$ and $v_R = v_L$.

A parametric eigenvalue analysis is performed using Pardiso solver. In this problem, the pulsation ω is fixed as real parameter. The wavenumbers $\lambda_i = jk_i$ and the associated right eigenvectors ϕ_i^r are computed by solving the quadratic eigenvalue problem.

This approach allows to introduce damping effects.

The problem being damped, all the wavenumbers are complex, and the distinction between "propagative" and "evanescent" waves becomes difficult. This is why sorting criteria are proposed in the following.

First of all, all the waves are shifted to the first irreducible Brillouin zone. Then, criteria are defined as:

- ratio between real and imaginary part of each wavenumbers: $real(k)/imag(k) > 1$;
- ratio between real and imaginary part of the velocity of energy transport [8] $v = I/E$ where I is the energy flux and E the total energy density, approximated from the kinetic energy $E = 2E_c$: $real(v)/imag(v) > 1$;
- ratio between real and imaginary part of the group velocity [9]: $real(C_g)/imag(C_g) > 1$.

For frequency-dependent damped systems, the estimation of the group velocity is not trivial. We propose to rewrite (Eq. 2) as

$$A_1(\omega)\psi_i^r = \lambda_i A_2(\omega)\psi_i^r \quad (3)$$

with

$$A_1(\omega) = \begin{pmatrix} 0 & I_d \\ (K - \omega^2 M) & L \end{pmatrix}, \quad (4)$$

$$A_2(\omega) = \begin{pmatrix} I_d & 0 \\ L^T & H \end{pmatrix}, \quad (5)$$

and

$$\psi_i^r = \begin{pmatrix} \phi_i^r \\ \lambda_i \phi_i^r \end{pmatrix}. \quad (6)$$

Conversely, a left-eigenvector for the same eigenvalue satisfies

$$\psi_i^{lT} A_1(\omega) = \lambda_i \psi_i^{lT} A_2(\omega). \quad (7)$$

Generally, left and right eigenvectors for the same eigenvalue are not equal. In the resolution of the right eigenvalue problem, the i -th mode ($i \in \mathbb{N}^*$) is defined by its $\lambda_i \geq 0$ and its eigenvector ϕ_i^r . For each mode i , a mode $-i$ is associated with $\lambda_{-i} \leq 0$ such that $\lambda_{-i} = -\lambda_i$ and ϕ_{-i}^r such that $\phi_{-i}^r = \phi_i^l$. So, by solving the right eigenvalue problem, the adjoint solution is found too.

(Eq. 2) is differentiated and multiplied by the left eigenvector such that

$$\frac{\partial A_1(\omega)}{\partial \omega} \psi_i^r + A_1(\omega) \frac{\partial \psi_i^r}{\partial \omega} = \frac{\partial \lambda_i}{\partial \omega} A_2(\omega) \psi_i^r + \lambda_i \frac{\partial A_2(\omega)}{\partial \omega} \psi_i^r + \lambda_i A_2(\omega) \frac{\partial \psi_i^r}{\partial \omega}, \quad (8)$$

$$\psi_i^{lT} \frac{\partial A_1(\omega)}{\partial \omega} \psi_i^r + \psi_i^{lT} A_1(\omega) \frac{\partial \psi_i^r}{\partial \omega} = \psi_i^{lT} \frac{\partial \lambda_i}{\partial \omega} A_2(\omega) \psi_i^r + \psi_i^{lT} \lambda_i \frac{\partial A_2(\omega)}{\partial \omega} \psi_i^r + \psi_i^{lT} \lambda_i A_2(\omega) \frac{\partial \psi_i^r}{\partial \omega}, \quad (9)$$

which gives the expression of the group slowness using $\lambda_i = j k_i$

$$\frac{\partial k_i}{\partial \omega} = \text{real} \left(-j \frac{\psi_i^{lT} \left[\frac{\partial A_1(\omega)}{\partial \omega} - \lambda_i \frac{\partial A_2(\omega)}{\partial \omega} \right] \psi_i^r}{\psi_i^{lT} A_2(\omega) \psi_i^r} \right), \quad (10)$$

The group velocity is the inverse of the group slowness

$$C_g = \text{real} \left(\frac{\partial \omega}{\partial k_i} \right) = \text{real} \left(\frac{j \phi_i^{lT} [(L - L^T) - 2\lambda_i H] \phi_i^r}{\phi_i^{lT} [2\omega M - \frac{\partial K}{\partial \omega} - \lambda_i (\frac{\partial L}{\partial \omega} - \frac{\partial L^T}{\partial \omega}) + \lambda_i^2 \frac{\partial H}{\partial \omega}] \phi_i^r} \right). \quad (11)$$

We now focus on homogeneous cases where the frequency dependency is characterized by a Young's modulus such that $E = f(\omega)E_0$ and a constant Poisson's ratio. Hence $K = f(\omega)K_0$, $H = f(\omega)H_0$ and $L = f(\omega)L_0$.

Therefore, the group velocity is expressed as

$$C_g = \text{real} \left(\frac{\partial \omega}{\partial k_i} \right) = \text{real} \left(\frac{j \phi_i^{lT} [f(\omega)(-L_0 + L_0^T + 2\lambda_i H_0)] \phi_i^r}{\phi_i^{lT} [\omega^2 \frac{\partial f}{\partial \omega} - 2\omega] M \phi_i^r} \right). \quad (12)$$

For frequency-independent cases, a simplification is possible [9]

$$C_g = \text{real} \left(\frac{\partial \omega}{\partial k_i} \right) = \text{real} \left(\frac{j [\phi_i^{lT} (L - L^T) \phi_i^r - 2\lambda_i \phi_i^{lT} H \phi_i^r]}{2\omega \phi_i^{lT} M \phi_i^r} \right). \quad (13)$$

A comparison between the results obtained using the Floquet-Block method and the "Shifted-Cell Operator" is firstly performed on a conservative structure to validate the implementation of

both methods and in particular the implementation of the "Shifted-Cell Operator". Fig. 5 presents the dispersion curves obtained along the $\Gamma-X$ direction when ϕ is equal to zero. Both methods lead to similar results and the shift-cell method is thus validated. Damping effects are then introduced in the system and results obtained using the "Shifted-Cell Operator" are presented with increasing damping on (Fig. 6). An hysteretic damping factor is used as a first step. A complex Young's modulus is obtained such $E^* = E(1 + j\eta)$ with η is the loss factor. Damping effects mainly affect the curves at higher wavenumbers. Moreover, the damping reduces the frequency bandgap and reduces the amplitude resonance. So, from a design point of view, a compromise has to be found.

The method has another advantage, the group velocity constitutes a pertinent indicator for the branches tracking from one computational point to another (see Fig. 7 and Fig. 8). In order to do that, an algorithm allows to compare the computed group velocity $\left(\frac{\partial\omega}{\partial k_i}\right)$ and the post processed group velocity $\left(\frac{\Delta\omega}{\Delta k_i}\right)$. The finite difference method is used to calculate the post-processed group velocity directly with the dispersion relation.

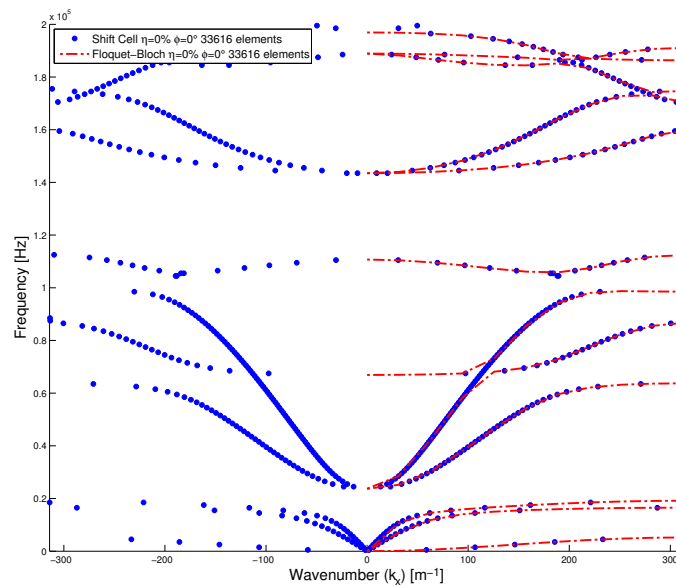


Figure 5. Dispersion curves when $\phi = 0^\circ$ - Methods comparison.

5 CONCLUSION

Periodic structures are interesting for waves propagation as they may exhibit complete or at least partial frequency bandgaps such that the associated waves cannot propagate through the structure. The determination of the dispersion curves is thus necessary to design specific structures for an absorption purpose. The Floquet-Bloch method is a commonly used approach well-adapted for such problems but its application in presence of damping is difficult. Another approach is here presented consisting in the "Shifted-Cell Operator" based on a reformulation of the PDE problem. The method is first applied to undamped periodic structure and the good agreement with the results given by the classical method validate the implementation. The method is then applied to a structure presenting an hysteretic behavior : the impact of the damping on the frequency bandgaps can thus be analysed.

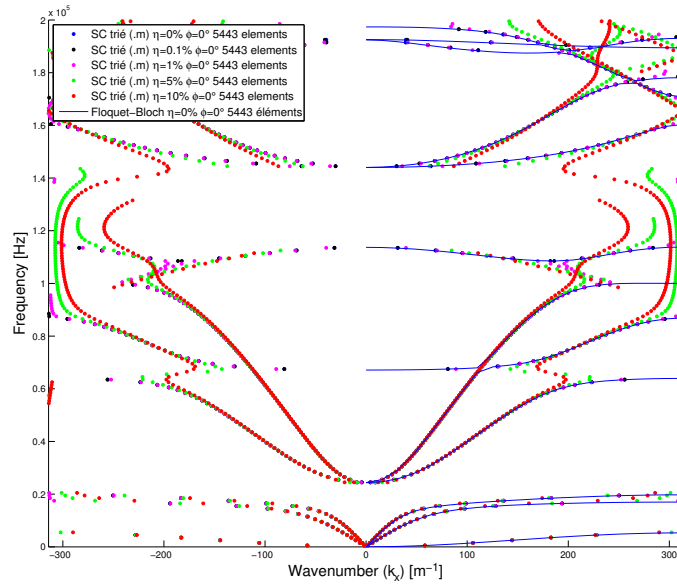


Figure 6. Dispersion curves with increasing hysteretic damping.

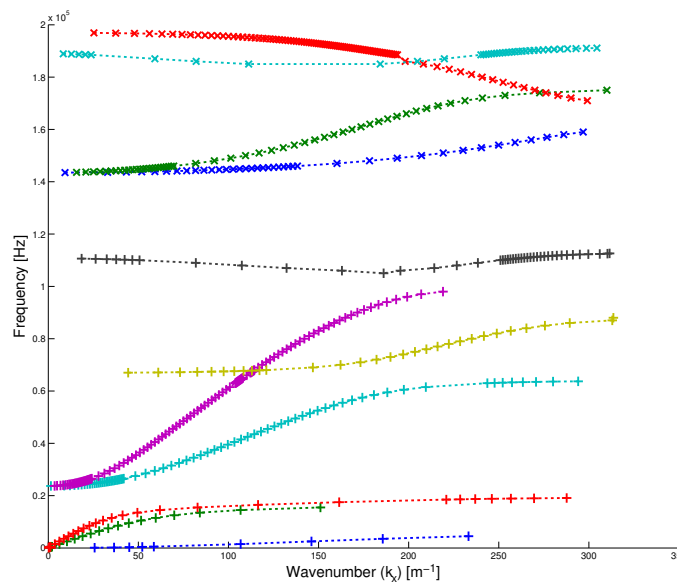


Figure 7. Dispersion curves with branches tracking.

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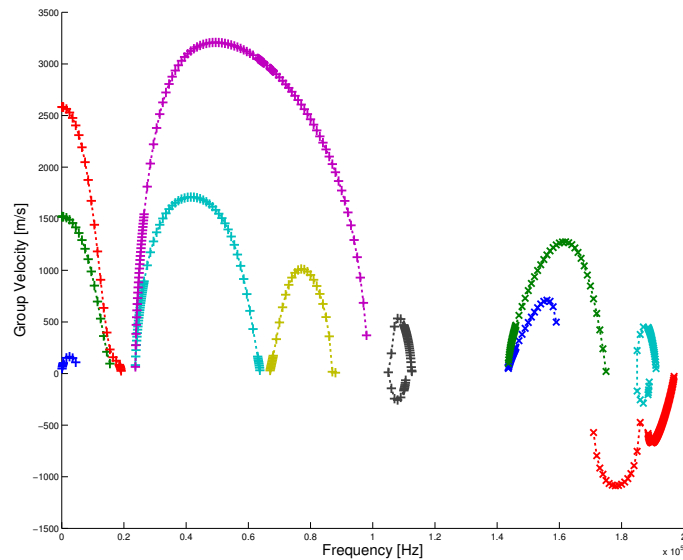


Figure 8. Group velocity with branches tracking.

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