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Accounting for risk factors on health outcomes: The case of Luxembourg*

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Abstract

This paper proposes a two-parameter family of health indicators. First, these operational research indicators are axiomatically derived and allow a Boolean risk factor to be linked to different health dimensions. Second, the behavior of the social planner with respect to the risk factor (risk insensibility, risk sensibility and extreme risk sensibility) is introduced. We demonstrate that these indicators are consistent with a stochastic dominance rule, which is an operational research rule for decision making. Using data from the Survey of Health, Ageing and Retirement in Europe, we show that, among the different childhood circumstances of individuals having an impact on the health status in adulthood, having parents with a migration background and low educated constitutes the risk factors that aggravate the most the overall level of socio-economic health inequality in Luxembourg.

Keywords: Behavioral OR (D); Health; Indicators; Policies; Risk factors.

Classification JEL: D6, I1.

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1 Introduction

In behavioral social sciences, modelling risk factors is crucial to design policy orientations according to those risks that people may face and may (or may not) control for. In the context of health, risky factors encompass multidimensional aspects. This is the reason why, for this purpose, operational research (OR) such as multi-criteria analyses are employed, see for instance Lagravinese, Liberati and Resce (2019). Multi-criteria analysis, however, sometimes lead to nonsensical results in the cases of prioritization of health menaces as they do not come under a good axiomatization when health is concerned (see Montibeller, Patel and del Rio Vilas, 2020). Another multidimensional OR approach consists in implementing the directional distance function to estimate the technical efficiency of the healthcare system (Cheng and Zervopoulos, 2014) or the data envelopment analysis approach (DEA) to obtain composite indicators for the healthcare system (Shwartz, Burgess and Zhu, 2016).

In this paper, we follow the OR approach in employing multidimensional indicators (in the same spirit than the directional distance function) to deal with the behavior of the social planner that depends both on risk factors and inequality aversion. The literature, however, usually focuses only on the latter. In the case of socio-economic health inequality indices, the literature exhibits the necessity to employ some decomposable indices in order to account for inequality aversion. For example, Wagstaff, van Doorslaer and Watanabe (2003) show that, in an unidimensional context, socio-economic health inequality indices are decomposable into the contribution of different attributes such as, among other, the level of education and consumption. Recently, the property of attribute decomposition has been extended to the multidimensional setting. Makdissi, Sylla and Yazbeck (2013), based on the rank-dependent approach, show that this decomposition property is matched when the health problems – which are categorical dimensions – are defined to be Boolean variables. This counting approach, which allows each dimension of health to be gauged in proportion to the overall amount of the socio-economic health inequality index, involves a parameter representing the intensity of health redistribution to be done. The idea of capturing the role of one or many dimensions will be of interest in what follows.

The aim of this paper is to propose a multidimensional socio-economic health inequality index, which depends on different health dimensions and on one risk factor. This index reflects two different, but at the same time complementary, behaviors of the social planner. First, is his/her attitude towards inequality as embodied by the degree of aversion to inequality (see Yitzhaki, 1983), *i.e.* the willingness of the social planner to operate redistributive policies toward less healthy people in order to alleviate overall inequality in a society. Second, the degree of risk sensibility of the social planner. The literature advocates, amongst others, the use of Yaari's (1987) dual social welfare function in order to involve different degrees of risk aversion in the analysis. In our approach, however, the degree of risk aversion is derived from the association between the health dimensions and the risk factor (considered as an additional dimension). In consequence, we consider risk sensibility towards health factors

rather than risk aversion. In particular, our approach introduces a risk sensibility parameter, which allows to properly aggregate the health dimensions. The aggregation process depends, in turn, on the Boolean values inherent to the risk dimension.

The major advantage of the proposed socio-economic health inequality index is threefold. First, it allows a Boolean risk factor to be associated and correlated with other health dimensions (*e.g.* physical and mental ones). Second, the comparison of multidimensional health distributions relies on a simple graphical approach. Indeed, a stochastic dominance criterion provides a non-ambiguous ranking of the health distributions including one risk factor. In this respect, the dominance rule is compatible with either risk insensibility, risk sensibility, or extreme risk sensibility. Thus, the social planner's attitudes to risk are captured for each order of stochastic dominance. Finally, each order of stochastic dominance is properly associated to redistributive health actions (transfers), *i.e.* to the degree of inequality aversion of the social planner. Consequently, the dominance rule depends simultaneously on two parameters, one parameter that represents risk sensibility and another parameter that embodies inequality aversion of the social planner. This proposed approach is a multidimensional extension of Yitzhaki's Gini indices (1983) for which it is possible to calibrate both inequality aversion and risk sensibility. This, in turn, allows to provide not only an index but a two-parameter family of multidimensional socio-economic health inequality indices.

Our paper contributes to the existing literature on multidimensional operational research approaches. Compared with the multi-criteria analysis, our approach is more general for several reasons. First, multi-criteria analysis is sometimes associated with one particular index, which captures a particular view of the social planner leading to a unique possible decision. Lagravinese *et al.* (2019), for example, deal with multi-criteria analysis consistent with the Gini index to measure the degree of inequality in the society. Their index describes the behavior of social planners inclined to perform resources redistribution (rich-to-poor transfers) and, in consequence, it is a good candidate to simulate redistributive policies. In contrast, our approach deals with a family of socio-economic health inequality indices, each one reflecting the intensity of resources transfers to be made by several social planners. Second, dealing with one particular index, as the Gini index, can lead many income distributions to be equal to one given particular value of the index. As a consequence, it is not possible to unambiguously anticipate that some public policies are able to reduce inequality for all individuals in the society. Since our approach is based on stochastic dominance, each index of the family is represented by an *achievement curve* describing all income percentiles of the population. Therefore, simulating redistributive policies such as health policies, may be performed by checking that the corresponding achievement curve lies nowhere below that representing the actual situation of the society. In this case, the health policy allows to reduce the inequality *for all* individuals in the society and *for all* indices included in the family of socio-economic health inequality indices. Finally, the proposed family of indices is axiomatized in order to provide specific indices relevant to risk factors that individuals do not control for. Our approach allows to identify those specific factors that increase socio-

economic health inequalities in the society and are important to simulate public policies and to propose redistributive actions.

Our approach is also comparable to the one adopted in the DEA literature, which measures efficiency scores (*i.e.* achievement scores) using the (directional) distance function on the basis of inputs and outputs.¹ Ravelojaona (2019) and Briec and Mussard (2020) recently show that the directional distance function can be generalized thanks to a real parameter α . Therefore, efficiency may be measured with different DEA models that depend on this parameter (such as α -returns to scale). However, to the best of our knowledge, this parameter cannot be associated with the behavior of the social planner in charge of public policies and, in consequence, it cannot identify the risk factors underlying a potential raise of socio-economic health inequalities.

A central issue in this paper is the notion of risk. A large number of scholars analyze different aspects of factors that can have an impact on the health status of individuals. For example, Gakidou, Murray and Frenk (1999) distinguish risk factors such as genes at birth and choices (*e.g.*, additive habits). Le Clainche and Wittwer (2015) pay a particular attention to risky behaviors. The authors prove, based on experiments, that students are inclined to support the health costs associated to their risky choices. A growing number of articles try to distinguish between legitimate and illegitimate factors, which causes health inequalities. These articles are based on the equality of opportunity theory. The main idea behind this distinction is the fact that the health outcomes can be the consequence of circumstances that are beyond an individual's control, and autonomous choices which are within his/her control. In his article, Roemer (1995) recommends that society compensates only the cases where bad consequences are due to circumstances or brute luck. In other words, these circumstances can be identified as exogenous risks that can increase inequalities in health, these inequalities that should be compensated.

In this line of research, Rosa Dias (2009) implements stochastic dominance tests in order to detect inequality of opportunity in the self-assessment health in adulthood using the United Kingdom (UK) National Child Development Study. Trannoy, Tubeuf, Jusot and Devaux (2010) attempt to estimate the degree of inequality in health opportunity by analyzing the role of circumstances during childhood such as family and social backgrounds and focusing in older adults (see also Jusot, Tubeuf, and Trannoy (2013), Garcia Gomez, Schokkaert, Van Ourti and Bago d'Uva (2015), and Bricard, Jusot, Trannoy and Tubeuf (2013)). More recently, Deutsch, Pi Alperin and Silber (2018) use the Shapley value in order to disentangle the impact of circumstances and efforts on health inequality in Luxembourg on the basis of self-reported health status. In the same vein, based on the Shapley decomposition, Davillas and Jones (2020) propose quantile regressions approaches (with counterfactual analysis) in order to measure the effect of the risk of chronic conditions in UK with a clear distinction between circumstance and effort variables. Also, Carrieri, Davillas and Jones (2020), by

¹The achievement curves issued from our family of socio-economic health inequality indices enable measuring inequality rather than efficiency.

means of the variance decomposition, decompose the effect of efforts and circumstances and show that two third of inequalities in allostatic load (a composite measure of biomarker data) are due to circumstances in UK. In this paper, we follow the line of research base on the equality of opportunity theory and concentrate our analysis on factors, which are not under the individuals' control, the exogenous risk factors.

The analysis relies on data from Wave 5 of the Survey of Health, Ageing and Retirement in Europe. In particular, we are interested to analyze the socio-economic health inequalities in Luxembourg. To summarize the health status of individuals, nine different dimensions of health are aggregated, which represent different aspects of the mental and the physical components of global health. Concerning the risk factors, we investigate four childhood circumstances: parental education, longevity of parents, parents' migration background, and financial situation during childhood.

The paper is organized as follows. Section 2 exposes the family of rank-dependent socio-economic health inequality indices. The properties linked with sensibility to risk factors are presented in Section 3. Section 4 proposes a stochastic dominance criterion associated with the family of two-parameter socio-economic health inequality indices. Section 5 presents the database and the definitions used in the paper. An empirical application is presented in Section 6 and Section 7 concludes.

2 Health Inequality and Health Achievement

This section briefly summarizes the notations and the definitions used in the remainder of the paper.

2.1 Notations

Let y^E be an equivalent income distribution such that $F(y^E)$ is its cumulative distribution function defined over $[0, a]$, where a is the maximum conceivable equivalent income. There are n individuals in the society, where n is a positive integer. The rank p of the individuals are issued from $F(y^E)$, such that $p \in [0, 1]$. Following the literature on the counting approach for measuring poverty, see *e.g.* Alkire and Foster (2011), we adopt the counting approach to gauge inequalities in multidimensional health in line with Makdissi and Yazbeck's (2014) approach. Let $\mathbf{H}(p) := (h_1(p), \dots, h_K(p)) \in \mathbb{R}_+^K$ be the information related to each dimension of health indexed by $k \in \{1, \dots, K\} =: \mathcal{K}$ for an individual at rank p of the equivalent income distribution, and where \mathbb{R}_+^K is the K -dimensional Euclidean space such that K is a (strictly) positive integer.

The set of health information for all individuals is denoted \mathcal{H} . The $n \times K$ health information matrix is $\mathbf{H} \in \mathcal{H}$ such that $\mathbf{H} = (h_1, \dots, h_K)$ where h_k denotes the k th column of \mathbf{H} , whereas $\mathbf{H}(p)$ denotes a row of \mathbf{H} for an individual at rank p . \mathbf{H}_{-p} denotes the health information matrix \mathbf{H} without the row $\mathbf{H}(p)$ and $[0]$ a matrix of zeros.

The health information is derived from an identification function *i.e.* a threshold function τ_k for each dimension k . Then, any given individual at rank p who falls below (above) the threshold τ_k is considered 'deprived' (not deprived) in the health dimension k . Thus, the individual is counted 1 in dimension k , otherwise 0, which indicates that there is total deprivation (no deprivation) in this health dimension:

$$\iota(h_k(p)) := \begin{cases} 1, & \text{if } h_k(p) < \tau_k \\ 0, & \text{otherwise.} \end{cases} \quad (2.1)$$

Accordingly,

$$\Upsilon(\mathbf{H}(p)) := (\iota(h_1(p)), \dots, \iota(h_k(p)), \dots, \iota(h_K(p))), \quad (2.2)$$

provides the K -dimensional situation of an individual at rank p of the equivalent income distribution, and $\Upsilon(\mathbf{H})$ a $n \times K$ Boolean matrix.

2.2 Definitions

In order to aggregate the health dimensions for an individual at rank p , it is necessary to define a normalized aggregator, a map ϕ . The most common aggregator in welfare economics is the generalized mean introduced and axiomatized by Blackorby, Donaldson and Auersperg (1981).

Definition 2.1 – Aggregator – Let $\|\cdot\|_\alpha^\Theta : \mathbb{R}_+^K \rightarrow \mathbb{R}_+$ be a twice differentiable map such that for all $x \in \mathbb{R}_+^K$ and some weight vector $\Theta := (\theta_1, \dots, \theta_K) \in [0, 1]^K$:

$$\|x\|_\alpha^\Theta := \begin{cases} \left(\sum_{k=1}^K \theta_k x_k^\alpha \right)^{\frac{1}{\alpha}} & \forall \alpha > 0 \\ \prod_{k=1, \dots, K} x_k^{\theta_k} & \alpha \rightarrow 0 \text{ (or } \alpha = 0 \text{ by convention)}. \end{cases} \quad (2.3)$$

A normalized aggregator function $\phi : [0, 1]^K \rightarrow [0, 1]$ is,²

$$\phi(\mathbf{H}(p)) = \frac{\|\Theta\|_\alpha^{1_K} - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta}{\|\Theta\|_\alpha^{1_K}}, \quad (2.4)$$

where $\mathbf{1}_K$ denotes the K -dimensional vector of ones. Without loss of generality, one may impose that $\sum_k \theta_k = 1$, and so,

$$\|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta \leq \|\Theta\|_\alpha^{1_K} \leq \max_{k=1, \dots, K} \theta_k = 1. \quad (2.5)$$

In this case, the normalized aggregator may be simply expressed as follows:

$$\phi_\alpha(\mathbf{H}(p)) := 1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\Theta. \quad (2.6)$$

²For instance Makdissi, Sylla and Yazbeck (2013) propose:

$$\phi(\mathbf{H}(p)) = \frac{K - \Upsilon(\mathbf{H}(p))\Theta'}{K}, \text{ where } \sum_{k=1}^K |\theta_k| = K.$$

The aggregator $\phi_\alpha(\mathbf{H}(p))$ represents the average health *achievement* for an individual at rank p . Then, the overall socio-economic health achievement index is written as follows,

$$A_\alpha(\mathbf{H}) := \int_0^1 v(p)\phi_\alpha(\mathbf{H}(p))dp = \int_0^1 v(p) \left(1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\ominus\right) dp, \quad (2.7)$$

where $v(p)$ is a rank-dependent weight function such that $v : [0, 1] \rightarrow [0, 1]$ which embodies the social planner's preferences.³ The index $A_\alpha(\mathbf{H})$ is a natural K -dimensional extension of the concentration index analyzed for instance by Erreygers, Clarke and Van Ourti (2012), amongst others. It represents a weighted mean of the *individual achievements* $1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\ominus$ across all individuals in a society. Choosing $v(p) = \nu(1-p)^{\nu-1}$ in line with Yitzhaki's (1983) extended Gini indices, we obtain a two-parameter family of achievement indices:

$$A_{\nu,\alpha}(\mathbf{H}) := \int_0^1 \nu(1-p)^{\nu-1}\phi_\alpha(\mathbf{H}(p))dp, \quad \nu > 1, \alpha \geq 0. \quad (2.8)$$

In this family of indices, the parameter ν embodies inequality aversion. In particular, if $\nu \geq 2$ the index displays health inequality aversion, whereas health inequality loving is obtained whenever $\nu \in (1, 2)$.⁴ The parameter α is linked to the risk sensibility of the social planner, which is derived from the association between the risk factor and the health dimensions.

Definition 2.2 – Socio-economic health inequality indices – For all $\mathbf{H} \in \mathcal{H}$ and $\mu_{\phi(\mathbf{H})} = \int_0^1 \phi_\alpha(\mathbf{H}(p))dp$, the two-parameter family of socio-economic health inequality indices is given by:

$$\begin{aligned} I_{\nu,\alpha}(\mathbf{H}) &= 1 - \frac{A_{\nu,\alpha}(\mathbf{H})}{\mu_{\phi(\mathbf{H})}} \\ &= 1 - \frac{1}{\mu_{\phi(\mathbf{H})}} \int_0^1 \nu(1-p)^{\nu-1} \left(1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\ominus\right) dp, \quad \nu > 1, \alpha \geq 0. \end{aligned} \quad (2.9)$$

The advantage of dealing with a two-parameter family of socio-economic health inequality measures, denoted in a generic manner from now on as $\{I\}_{\nu,\alpha}$, is the possibility of capturing risk sensibility properties.

3 Index parametrization with risk sensibility properties

In this section, emphasis is put on the notion of risk factors. The different subsections 3.1-3.4 describe the properties of the two-parameter family of socio-economic health inequality indices, $\{I\}_{\nu,\alpha}$ with respect to one risk factor. The subsection 3.5 explores a type of parametrization related to distributional purposes.

³See the seminal work of Yaari (1987).

⁴If $\nu = 2$ a multivariate Gini index is deduced.

3.1 Boolean risk factor

In what follows, we consider that within health information matrix, one of the k dimensions represents a risk factor that may decrease health. In the remainder to facilitate the exposition of the results, when comparing two health matrices $\mathbf{H}, \tilde{\mathbf{H}} \in \mathcal{H}$ we suppose that $\mu_{\phi(\mathbf{H})} = \mu_{\phi(\tilde{\mathbf{H}})}$ without loss of generality.⁵

Definition 3.1 – One-dimensional risk factor – *For all distributions $\mathbf{H} \in \mathcal{H}$, the risk factor potentially correlated to the health dimensions is defined to be dimension $K = \mathbf{r}$, by convention, such that $\iota(h_K) \equiv \iota(h_{\mathbf{r}}) \in \{0, 1\}$, with $\iota(h_{\mathbf{r}}(p)) = 0$ for an individual at rank p not affected by a risk factor and 1 otherwise. The set of health information is accordingly decomposed such that $\mathcal{K} = \{1, \dots, K - 1\} \cup \{\mathbf{r}\} \equiv \mathcal{K}_{-\mathbf{r}} \cup \{\mathbf{r}\}$, with $\mathbf{H}_{-\mathbf{r}} := (h_1, \dots, h_{K-1})$, so that $\mathbf{H} = (\mathbf{H}_{-\mathbf{r}}, h_{\mathbf{r}})$.*

Although $\iota(h_{\mathbf{r}})$ could be a continuous function of many risk factors, we suppose without loss of generality that only one risk dimension is available. An alternative strategy is to consider that $\iota(h_{\mathbf{r}})$ itself depends on r Boolean risk factors such that $r = 1, \dots, R$ with $\iota(h_{\mathbf{r}}) = \|(\iota(h_{\mathbf{r},1}), \dots, \iota(h_{\mathbf{r},R}))\|_{\alpha_{\mathbf{r}}}^{\Theta_{\mathbf{r}}}$, and where $\alpha_{\mathbf{r}}$ and $\Theta_{\mathbf{r}}$ are risk specific weights. Different strategies of averaging risk factors will be discussed in Section 5. In what follows, it is assumed that $\iota(h_{\mathbf{r}}) \in \{0, 1\}$, however for the properties developed below, it is possible to assume a bounded risk factor included in $[0, 1]$.

3.2 Risk sensibility

The traditional notion of risk aversion is deeply rooted in the shape of the utility function of the social planner. In this subsection, we define the *risk sensibility* of the social planner with respect to properties inherent to the association between the health dimensions and the risk factor, without imposing specific utility functions.

The first sensibility property is related to the usual union/intersection approach of the literature on multidimensional poverty, introduced by Atkinson (2003). The union approach ($\alpha \rightarrow 0$) is the less demanding value judgment of the social planner in charge of the identification of healthy individuals across dimensions. It postulates that an individual has to be non-healthy in all dimensions to be considered as non-healthy. In other terms, an individual is considered as totally non healthy when $\phi_{\alpha}(\mathbf{H}(p)) = 1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta} = 0$. In contrast, an individual is totally healthy if $\phi_{\alpha}(\mathbf{H}(p)) = 1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\Theta} = 1$. Suppose that an individual is healthy in at least one health dimension. Then, there exists one dimension $k \in \mathcal{K}_{-\mathbf{r}}$ such that $\iota(h_k(p)) = 0$. In consequence, such an individual is judged to be totally healthy in the same manner as an individual who is healthy in all dimensions.

In particular, if an individual is affected by a risk factor *i.e.* $\iota(h_{\mathbf{r}}(p)) = 1$, this dimension would actually be neutral. Formally, whenever $\alpha \rightarrow 0$, the risk insensibility implies that the

⁵In Section 4 we will show that, for dominance purposes, no condition has to be imposed on the mean.

individual is healthy even if $\iota(h_{\mathbf{R}}(p)) = 1$:

$$\phi_0(\mathbf{H}(p)) := \lim_{\alpha \rightarrow 0} \left(1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\ominus} \right) = 1 - \prod_{k=1}^K \iota(h_k(p))^{\theta_k} = 1. \quad (3.1)$$

Note that a problem may arise whenever an individual is not affected by the risk factor, $\iota(h_{\mathbf{R}}(p)) = 0$, whereas he is non healthy in all dimensions, $\iota(h_k(p)) = 1$ for all $k \in \mathcal{K}_{-\mathbf{R}}$. In this case $\phi_0(\mathbf{H}(p)) = 1$, this would erroneously lead to a healthy state. To avoid such a problem, we impose the following convention :

$$\phi_0(\mathbf{H}(p)) := 0, \text{ if } \iota(h_k(p)) = 1 \ \forall k \in \mathcal{K}_{\mathbf{R}} \text{ and } \iota(h_{\mathbf{R}}(p)) = 0. \quad (3.2)$$

The opposite property, the more demanding one, is known to be the intersection approach ($\alpha \rightarrow \infty$). Suppose that an individual is only affected in the risk dimension. The social planner is going to judge this person as totally non healthy, in the same manner as a person who is non-healthy in all health dimensions. In this case, the risk sensibility is maximal since, for any given health state $\mathbf{H}_{-\mathbf{R}}(p)$ of an individual ranked p , the risk factor $\iota(h_{\mathbf{R}}(p)) = 1$ will provide a non-healthy state. Formally, whenever $\alpha \rightarrow \infty$ (or $\alpha = \infty$ by convention):

$$\phi_{\infty}(\mathbf{H}(p)) := \lim_{\alpha \rightarrow \infty} \left(1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\ominus} \right) = 1 - \max_{k=1, \dots, K} \iota(h_k(p)) = 0. \quad (3.3)$$

As a consequence, the α parameter enables to capture the sensibility of the social planner with respect to the health dimensions associated with one risk factor.⁶

Property 3.1 – Risk insensibility / Risk sensibility – $\mathcal{RI}/\mathcal{RS}$:

The aggregator $\phi_{\alpha}(\mathbf{H}(p)) = 1 - \|\Upsilon(\mathbf{H}(p))\|_{\alpha}^{\ominus}$ yields either risk insensibility or different degrees of risk sensibility, which are characterized by the following sets of health distributions.

(i) *Risk Insensibility:*

$$\mathcal{RI} := \left\{ \mathbf{H}, \tilde{\mathbf{H}} \in \mathcal{H} \left| \begin{array}{l} \tilde{\mathbf{H}} \text{ is issued from } \mathbf{H} \text{ such that} \\ \exists k \in \mathcal{K}_{-\mathbf{R}} : \iota(h_k(p)) = \iota(\tilde{h}_k(p)) = 0, \\ \iota(h_{\mathbf{R}}(p)) = 1, \iota(\tilde{h}_{\mathbf{R}}(p)) = 0, \\ \phi_0(\mathbf{H}(p)) = \phi_0(\tilde{\mathbf{H}}(p)). \end{array} \right. \right\}$$

(ii) *Risk sensibility:*

$$\mathcal{RS} := \left\{ \mathbf{H}, \tilde{\mathbf{H}} \in \mathcal{H} \left| \begin{array}{l} \tilde{\mathbf{H}} \text{ is issued from } \mathbf{H} \text{ such that } \mathbf{H}_{-\mathbf{R}} = \tilde{\mathbf{H}}_{-\mathbf{R}}, \\ \iota(h_{\mathbf{R}}(p)) = 1, \iota(\tilde{h}_{\mathbf{R}}(p)) = 0, \\ \phi_{\alpha}(\mathbf{H}(p)) < \phi_{\alpha}(\tilde{\mathbf{H}}(p)), \alpha \in (0, \infty). \end{array} \right. \right\}$$

(iii) *Extreme Risk sensibility:*

⁶It is noteworthy that in the literature on multidimensional poverty the union and intersection approaches rely on the number of dimensions to be considered as deprived or not, rather than a parameter directly linked to the aggregation of the dimensions (see Alkire and Foster, 2011).

$$\mathcal{ERS} := \left\{ \mathbf{H}, \tilde{\mathbf{H}} \in \mathcal{H} \left| \begin{array}{l} \tilde{\mathbf{H}} \text{ is issued from } \mathbf{H} \text{ such that} \\ \Upsilon(\mathbf{H}_{-\mathbf{R}}) = \Upsilon(\tilde{\mathbf{H}}_{-\mathbf{R}}) = [\mathbf{0}], \\ \iota(h_{\mathbf{R}}(p)) = 1, \iota(\tilde{h}_{\mathbf{R}}(p)) = 0, \\ \phi_{\infty}(\mathbf{H}(p)) = 0 < 1 = \phi_{\infty}(\tilde{\mathbf{H}}(p)). \end{array} \right. \right\}$$

The \mathcal{RI} property displays the risk insensibility of the social planner. When the individuals are healthy in only one dimension, for any given risk level (0 or 1), they are considered as healthy. On the other hand, to be considered non-healthy, the individuals must be non-healthy in all dimensions without any distinction between the risk factor and the health dimensions. The risk sensibility property \mathcal{RS} directly depends on the risk factor, which, *ceteris paribus*, decreases the individual health achievement. In other words, it worsens the individual health status. The extreme risk sensibility view \mathcal{ERS} is the intersection approach explained above. A non-healthy state is assigned to an individual exposed to a risk factor even if he is totally well-off in all health dimensions $\mathcal{K}_{-\mathbf{R}}$.

Property 3.1 outlines a clear separation between the health dimensions and the risk factor. It is noteworthy that this possibility is inherent to the generalized mean aggregator $\|\cdot\|_{\alpha}^{\Theta}$, characterized by the additive separability axiom – see Blackorby *et al.* (1981). As shown below, this property is compatible with the design of the correlations between the risk factor and the health dimensions. In other words, each risk notion ($\mathcal{RI}/\mathcal{RS}/\mathcal{ERS}$) may be clearly associated with properties describing the behavior of the social planner who compares the health profile of two individuals in the society.

3.3 Correlation increasing risk

The degree of correlation between the health dimensions, in a multidimensional framework, has to be studied through the prism of *correlation increasing risk*, introduced by Richard (1975).⁷ This concept has been suitably used for multivariate measurement tools, known as correlation increasing switch, see *e.g.* Atkinson and Bourguignon (1982) and Boland and Proschan (1988) amongst others. The idea underlying those concepts is to capture the interaction between the dimensions, their complementarity as well as their substitutability, in the same manner as the sensibility towards risk discussed above. In this context, the parameter α may be connected to the degree of correlation between health dimensions.

Consider two individuals p_1 and p_2 to denote the individuals at rank p_1 and p_2 , respectively. Suppose that p_1 receives the maximum achievement amount (denoted \vee) among all dimensions $k \in \mathcal{K}$, including the risky factor, between his/her situation and that of p_2 . Suppose that in contrast, p_2 receives the minimum between their achievements (denoted \wedge) across all dimensions. Actually, this *correlation increasing risk* aggravates the inequality between p_1 and p_2 while each individual situation appears to be smoother (all 0's for p_2 and all 1's

⁷This concept was formerly introduced by Meyer (1972) in the unidimensional setting.

for p_1). The inequality is more important because one individual would be non-healthy in all dimensions and furthermore he would support the entire risk factor. If $\tilde{\mathbf{H}}$ denotes the health information after a correlation increasing risk, then:

$$\|\Upsilon(\tilde{\mathbf{H}}(p_1))\|_\alpha^\ominus = \|\iota(h_1(p_1)) \vee \iota(h_1(p_2)), \dots, \iota(h_{\mathbf{R}}(p_1)) \vee \iota(h_{\mathbf{R}}(p_2))\|_\alpha^\ominus \quad (3.4)$$

$$\|\Upsilon(\tilde{\mathbf{H}}(p_2))\|_\alpha^\ominus = \|\iota(h_1(p_1)) \wedge \iota(h_1(p_2)), \dots, \iota(h_{\mathbf{R}}(p_1)) \wedge \iota(h_{\mathbf{R}}(p_2))\|_\alpha^\ominus. \quad (3.5)$$

The concept of correlation increasing risk relies on the idea that the social planner would have the possibility of switching the risk level to which some individuals are exposed. In what follows, the set \mathcal{CIR} denotes the set of all distributions $\mathbf{H}, \tilde{\mathbf{H}} \in \mathcal{H}$ for which a correlation increasing risk is applied such that $\Upsilon(\mathbf{H}_{-\mathbf{R}}(p)) = \Upsilon(\tilde{\mathbf{H}}_{-\mathbf{R}}(p)) \neq [\mathbf{0}]$ for all $p \in [0, 1]$ and $\iota(h_{\mathbf{R}}(p_1)) \neq \iota(h_{\mathbf{R}}(p_2))$ – with \mathcal{CIR} being used indifferently as a set or a property label. In other terms, a pure permutation is made between the individuals ranked p_1 and p_2 . Also, the sequence of correlation increasing risk must be non-void, which could occur if $\mathbf{H}(p_1) = \mathbf{H}(p_2)$ since in this case $\Upsilon(\mathbf{H}(p_1)) = \Upsilon(\tilde{\mathbf{H}}(p_1)) = \Upsilon(\mathbf{H}(p_2)) = \Upsilon(\tilde{\mathbf{H}}(p_2))$.

Property 3.2 – Correlation Increasing Risk – \mathcal{CIR} :

For all socio-economic health inequality indices $I_{\nu, \alpha} \in \{I\}_{\nu, \alpha}$, if $(\tilde{\mathbf{H}}, \mathbf{H}) \in \mathcal{CIR}$ such that $\tilde{\mathbf{H}}$ is issued from \mathbf{H} by a non-void correlation increasing risk between two individuals, then:

$$I_{\nu, \alpha}(\tilde{\mathbf{H}}) > I_{\nu, \alpha}(\mathbf{H}).$$

On this basis, it is possible to restrict the values of the α parameter thanks to \mathcal{CIR} .

Lemma 3.1 For all socio-economic health inequality indices $I_{\nu, \alpha} \in \{I\}_{\nu, \alpha}$, if a non-void sequence of correlation increasing risk between two individuals p_i and p_j is applied such that $p_i = p_j$, then the two following statements hold.

(i) Assume that the aggregator $\|\cdot\|_\alpha^\ominus$ is approximated by a twice differentiable function $f : [0, 1]^K \rightarrow [0, 1]$. Then, $I_{\nu, \alpha}$ respects \mathcal{CIR} if and only if $\|\cdot\|_\alpha^\ominus$ is strictly L -superadditive and $\alpha > 1$.

(ii) $I_{\nu, \alpha}$ is invariant to any \mathcal{CIR} if and only if $\alpha \in \{0, 1, \infty\}$. Then,

(ii.a) $\mathcal{CIR} \cap \mathcal{RI} = \emptyset$.

(ii.b) $\mathcal{CIR} \cap \mathcal{RS} = \emptyset$ whenever $\alpha = 1$.

(ii.c) $\mathcal{CIR} \cap \mathcal{ERS} = \emptyset$.

Proof:

(i) Let $I_{\nu, \alpha}(\tilde{\mathbf{H}}) > I_{\nu, \alpha}(\mathbf{H})$ or equivalently $A_{\nu, \alpha}(\tilde{\mathbf{H}}) < A_{\nu, \alpha}(\mathbf{H})$. Taking two individuals ranked p_i and p_j , if their rank after a \mathcal{CIR} are denoted p'_i and p'_j respectively, then:

$$\begin{aligned} & v(p'_i) \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_i))\|_\alpha^\ominus\right) + v(p'_j) \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_j))\|_\alpha^\ominus\right) \\ & < v(p_i) \left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_\alpha^\ominus\right) + v(p_j) \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus\right). \end{aligned} \quad (3.6)$$

Since $v(p) \geq 0$ for all $p \in [0, 1]$ and since the equivalent income y^E is by definition invariant to any \mathcal{CIR} , then $p_i = p_j = p'_i = p'_j$. Thus,

$$\begin{aligned} & \|\Upsilon(\mathbf{H}(p_i)) \vee \Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus + \|\Upsilon(\mathbf{H}(p_i)) \wedge \Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus \\ & > \|\Upsilon(\mathbf{H}(p_i))\|_\alpha^\ominus + \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus. \end{aligned} \quad (3.7)$$

Let $f(x) = \|x\|_\alpha^\ominus$ such that $f : [0, 1]^K \rightarrow [0, 1]$ is twice differentiable. From Eq.(3.7) it follows that f is strictly L -superadditive, see Boland and Proschan (1988). A function $f : [0, 1]^K \rightarrow [0, 1]$ is L -superadditive if, and only if, $\frac{\partial^2 f(x_1, \dots, x_K)}{\partial x_k \partial x_j} \geq 0$. After simple algebraic manipulations, it can be shown that strict L -superadditivity is ensured whenever $\alpha > 1$.

(ii) Consider a correlation increasing risk between two individuals p_i and p_j such that $p_i = p_j$. Since $v(p_i) = v(p_j) = v(p'_i) = v(p'_j)$, then from (3.7):

$$\begin{aligned} & \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_i))\|_\alpha^\ominus\right) + \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_j))\|_\alpha^\ominus\right) \\ & < \left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_\alpha^\ominus\right) + \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus\right). \end{aligned} \quad (3.8)$$

(ii.a) From \mathcal{RI} , there exists $k \in \mathcal{K}_{-\mathbf{r}}$ such that $\iota(h_k(p)) = 0$, then a contradiction arises:

$$\begin{aligned} & \lim_{\alpha \rightarrow 0} \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_i))\|_\alpha^\ominus\right) + \lim_{\alpha \rightarrow 0} \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_j))\|_\alpha^\ominus\right) \\ & = \lim_{\alpha \rightarrow 0} \left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_\alpha^\ominus\right) + \lim_{\alpha \rightarrow 0} \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus\right). \end{aligned} \quad (3.9)$$

(ii.b) Setting $\alpha = 1$ in the \mathcal{RS} case yields also a contradiction since,

$$\begin{aligned} & \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_i))\|_1^\ominus\right) + \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_j))\|_1^\ominus\right) \\ & = \left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_1^\ominus\right) + \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_1^\ominus\right). \end{aligned} \quad (3.10)$$

(ii.c) Finally, following the conditions of \mathcal{ERS} , let $\iota(h_{\mathbf{r}}(p_i)) = 1$ and $\iota(\tilde{h}_{\mathbf{r}}(p_j)) = 0$ such that $\Upsilon(\mathbf{H}_{-\mathbf{r}}) = \Upsilon(\tilde{\mathbf{H}}_{-\mathbf{r}}) = [\mathbf{0}]$. We have $\lim_{\alpha \rightarrow \infty} \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_i))\|_\alpha^\ominus\right) = \lim_{\alpha \rightarrow \infty} \left(1 - \|\Upsilon(\mathbf{H}(p_i))\|_\alpha^\ominus\right) = 0$. Also, $\lim_{\alpha \rightarrow \infty} \left(1 - \|\Upsilon(\tilde{\mathbf{H}}(p_j))\|_\alpha^\ominus\right) = 1$ and $\lim_{\alpha \rightarrow \infty} \left(1 - \|\Upsilon(\mathbf{H}(p_j))\|_\alpha^\ominus\right) = 1$, then the same contradiction occurs. ■

The previous results show that \mathcal{CIR} is only relevant with risk sensibility, provided that $\alpha > 1$ is not in the neighborhood of infinity.⁸ Also, result (i) indicates that the analysis in terms of risk sensibility is general since it can be done with a bounded variable $\iota(h_k(p)) \in [0, 1]$ instead of a Boolean one Eq.(2.1). This means that the use of the threshold τ_k can be relaxed.

3.4 Snowballing risk effect

Let us now investigate the snowballing risk effect by denoting $\phi_\alpha(\mathbf{H}_p(\mathcal{S}))$ the achievement of the individual at rank p being non healthy in some health dimensions represented by the set

⁸Relaxing the condition on the mean would imply $\mu_{\phi(\mathbf{H})} < \mu_{\phi(\tilde{\mathbf{H}})}$.

$\mathcal{S} \subset \mathcal{K}_{-\mathbf{R}}$. Consider that individual p_1 is endowed with $|\mathcal{S}|$ non healthy dimensions whereas individual p_2 is endowed with $|\mathcal{R}|$ non healthy dimensions such that $|\mathcal{R}| = |\mathcal{S}| + 1 < |\mathcal{K}|$. Adding to each individual the same risk factor \mathbf{R} could imply that the increase in the non-healthy situation of individual p_2 is deeper than p_1 . In such a case,

$$\begin{aligned} & \|\Upsilon(\mathbf{H}_{p_1}(\mathcal{S} \cup \{\mathbf{R}\}))\|_{\alpha}^{\ominus} - \|\Upsilon(\mathbf{H}_{p_1}(\mathcal{S}))\|_{\alpha}^{\ominus} \\ & < \|\Upsilon(\mathbf{H}_{p_2}(\mathcal{R} \cup \{\mathbf{R}\}))\|_{\alpha}^{\ominus} - \|\Upsilon(\mathbf{H}_{p_2}(\mathcal{R}))\|_{\alpha}^{\ominus}. \end{aligned} \quad (3.11)$$

In the sequel, we will say that $\tilde{\mathbf{H}}$ is issued from \mathbf{H} by a snowballing risk effect involving two persons p_i and p_j , if their health situations are described as above. In other words, an additional risk factor is added to both p_i and p_j with $\tilde{\mathbf{H}}$ being associated with more non-healthy dimensions than \mathbf{H} .

Property 3.3 – Snowballing Risk Effect – \mathcal{SRE} :

For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, if $(\tilde{\mathbf{H}}, \mathbf{H}) \in \mathcal{SRE}$ such that $\tilde{\mathbf{H}}$ is issued from \mathbf{H} by a snowballing risk effect involving two persons p_i and p_j , then:

$$I_{\nu,\alpha}(\tilde{\mathbf{H}}) > I_{\nu,\alpha}(\mathbf{H}).$$

This property postulates that the risk factor provides an acceleration of the non-healthy state of an individual p_i who is initially less healthy than another individual p_j .

Lemma 3.2 For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, let $(\tilde{\mathbf{H}}, \mathbf{H}) \in \mathcal{SRE}$ for two individuals p_i and p_j such that $p_i = p_j$. Then the two following statements hold.

(i) Assume that the aggregator $\|\cdot\|_{\alpha}^{\ominus}$ is approximated by a twice differentiable function $f : [0, 1]^K \rightarrow [0, 1]$. Then, $I_{\nu,\alpha}$ respects $\mathcal{SRE} \iff \|\cdot\|_{\alpha}^{\ominus}$ is strictly L -superadditive and $\alpha > 1 \iff I_{\nu,\alpha}$ respects \mathcal{CIR} .

(ii) Whenever $\alpha \in \{0, 1, \infty\}$, it results that:

- (a) $\mathcal{SRE} \cap \mathcal{RI} = \emptyset$.
- (b) $\mathcal{SRE} \cap \mathcal{RS} = \emptyset$ whenever $\alpha = 1$.
- (c) $\mathcal{SRE} \cap \mathcal{ERS} = \emptyset$.

Proof:

(i) \mathcal{SRE} implies that:

$$\|\Upsilon(\mathbf{H}(p_i))\|_{\alpha}^{\ominus} + \|\Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\ominus} < \|\Upsilon(\tilde{\mathbf{H}}(p_i))\|_{\alpha}^{\ominus} + \|\Upsilon(\tilde{\mathbf{H}}(p_j))\|_{\alpha}^{\ominus}. \quad (3.12)$$

Let $f(x) = \|x\|_{\alpha}^{\ominus}$ such that $f : [0, 1]^K \rightarrow [0, 1]$ is twice differentiable. The last expression becomes, for $\delta \in [0, 1]$ and setting $|\mathcal{R}| = |\mathcal{S}| + 1$:

$$\begin{aligned} & f(x_1, \dots, x_{|\mathcal{S}|}, 0, \dots, \delta) - f(x_1, \dots, x_{|\mathcal{S}|}, 0, \dots, 0) \\ & < f(x_1, \dots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \dots, \delta) - f(x_1, \dots, x_{|\mathcal{S}|}, x_{|\mathcal{R}|}, 0, \dots, 0). \end{aligned} \quad (3.13)$$

Now let $x_S := (x_1, \dots, x_{|S|}, 0, \dots, 0)$, $x_R := (x_1, \dots, x_{|S|}, x_{|R|}, 0, \dots, 0)$ and let $\delta = x_K \rightarrow 0$, thus dividing both sides of the last expression provides:

$$\frac{\partial f(x_S)}{\partial x_K} < \frac{\partial f(x_R)}{\partial x_R}. \quad (3.14)$$

Since $x_R = x_S + (0, \dots, 0, x_{|R|}, 0, \dots, 0)$, then letting $x_{|R|} \rightarrow 0$ and dividing both sides of the previous expression by $x_{|R|}$ entails:

$$0 < \frac{\partial^2 f(x_R)}{\partial x_R \partial x_{|R|}}. \quad (3.15)$$

As in Lemma 3.1, the last condition is fulfilled whenever $\alpha > 1$. In this respect, we obtain the same implications compared with \mathcal{CIR} , *i.e.*, strict L -superadditivity, so that \mathcal{SRE} and \mathcal{CIR} are equivalent whenever $\alpha > 1$.

(ii) Points (a), (b) and (c) provide contradictions in the same manner as in Lemma 3.1 (ii.a), (ii.b) and (ii.c) since in those cases:

$$\begin{aligned} & f(x_1, \dots, x_{|S|}, 0, \dots, \delta) - f(x_1, \dots, x_{|S|}, 0, \dots, 0) \\ &= f(x_1, \dots, x_{|S|}, x_{|R|}, 0, \dots, \delta) - f(x_1, \dots, x_{|S|}, x_{|R|}, 0, \dots, 0). \end{aligned} \quad (3.16)$$

■

The last result shows that the restriction $\alpha > 1$ (with α that does not tend to ∞) allows for two apparently different properties to be matched, both \mathcal{CIR} and \mathcal{SRE} . In what follows, we characterize extreme risk sensibility (when $\alpha \rightarrow \infty$).

3.5 Critical risk level

Consider a distribution $\mathbf{H} \in \mathcal{H}$, such that $1 = \iota(h_{\mathbf{r}}(p_i)) \neq \iota(h_{\mathbf{r}}(p_j)) = 0$, from which we derive two distributions \mathbf{H}_1 and \mathbf{H}_2 by focusing only on two individuals p_i and p_j . On the one hand, the former is derived from \mathbf{H} such that the worst health dimensions faced by both p_i and p_j are gathered, in particular,

$$\|\Upsilon(\mathbf{H}_1(p_{ij}))\|_{\alpha}^{\ominus} := \|\Upsilon(\mathbf{H}(p_i)) \vee \Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\ominus}. \quad (3.17)$$

More precisely, \mathbf{H}_1 is obtained from \mathbf{H} by replacing individuals p_i and p_j by p_{ij} defined above, with no change for the other individuals. On the other hand, \mathbf{H}_2 is given by the worst situation between individuals p_i and p_j , that is, the maximum (bad health) situation between p_i and p_j :

$$\|\Upsilon(\mathbf{H}_2(p_{ij}))\|_{\alpha}^{\ominus} := \bigvee_{r=i,j} \|\Upsilon(\mathbf{H}(p_r))\|_{\alpha}^{\ominus}. \quad (3.18)$$

Again, \mathbf{H}_2 is obtained from \mathbf{H} by replacing individuals p_i and p_j by p_{ij} , *ceteris paribus*. In the sequel, we will say by Eq.(3.17) and Eq.(3.18) that the distributions \mathbf{H}_1 and \mathbf{H}_2 are derived from \mathbf{H} by a rearrangement of a critical risk level. The property is the following.

Property 3.4 – Critical Risk Level – \mathcal{CRL} :

For all inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, if $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ such that \mathbf{H}_1 and \mathbf{H}_2 are issued from \mathbf{H} by a rearrangement of a critical risk level between two individuals p_i and p_j , then:

$$I_{\nu,\alpha}(\mathbf{H}_1) = I_{\nu,\alpha}(\mathbf{H}_2).$$

This property is actually matched whenever the risk factor \mathbf{R} is associated with a critical risk level, such as it can produce an irreversible situation in at least one of the health dimension. In a given distribution \mathbf{H} , replacing two individuals p_i and p_j by one virtual individual p_{ij} who takes on all their non-healthy dimensions (including the irreversible risk factor), or by one virtual individual who takes on their irreversible risk factor only, yields, *ceteris paribus*, exactly the same socio-economic health inequality index. This is because the dimension associated with critical risk dominates the other ones. Consequently, for an individual ranked p affected by a critical risk factor, his/her bad health situation will be maximum whatever the values of the other dimensions. This is in line with the property of extreme risk sensibility \mathcal{ERS} . The critical risk level \mathcal{CRL} is, however, weaker than the extreme risk sensibility \mathcal{ERS} , since \mathcal{ERS} applies for matrices of zeros $\Upsilon(\mathbf{H}_{-\mathbf{R}})$ and $\Upsilon(\tilde{\mathbf{H}}_{-\mathbf{R}})$, whereas this is not systematically the case for the critical risk level property. The equivalence is given below.

Lemma 3.3 For all socio-economic health inequality indices $I_{\nu,\alpha} \in \{I\}_{\nu,\alpha}$, let $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ with $p_i = p_j$. Then, the two following statements are equivalent.

- (i) $I_{\nu,\alpha}$ satisfies extreme risk sensibility \mathcal{ERS} .
- (ii.a) $I_{\nu,\alpha}$ satisfies critical risk level \mathcal{CRL} and,
- (ii.b) for all $\mathbf{H} \in \mathcal{H}$ and $t \in [0, 1]$, $\|t\Upsilon(\mathbf{H})\|_{\alpha}^{\ominus} = t \|\Upsilon(\mathbf{H})\|_{\alpha}^{\ominus}$ such that $\theta_K = 1$.

Proof:

[(ii) \implies (i)]: For simplicity, set $f(x) := \|x\|_{\alpha}^{\ominus}$ such that $x := \Upsilon(\mathbf{H})$. From (ii.a), equations (3.17) and (3.18) are equivalent *i.e.*:

$$\|\Upsilon(\mathbf{H}_1(p_{ij}))\|_{\alpha}^{\ominus} = \|\Upsilon(\mathbf{H}(p_i)) \vee \Upsilon(\mathbf{H}(p_j))\|_{\alpha}^{\ominus} = \bigvee_{r=i,j} \|\Upsilon(\mathbf{H}(p_r))\|_{\alpha}^{\ominus} = \|\Upsilon(\mathbf{H}_2(p_{ij}))\|_{\alpha}^{\ominus},$$

that is, setting $x_i := \Upsilon(\mathbf{H}(p_i))$ and $x_j := \Upsilon(\mathbf{H}(p_j))$,

$$f(x_i \vee x_j) = \max\{f(x_i), f(x_j)\}. \quad (3.19)$$

From (ii.b), since $\|\mathbf{0}\|_{\alpha}^{\ominus} = 0$, it results that,

$$f(tx) = \max\{tf(x), 0\}. \quad (3.20)$$

From Bricc and Horvath (2004, Proposition 3.0.3.), equations (3.19) and (3.20) hold if, and only if,

$$f(x) = \max_{k=1,\dots,K} \theta_k x_k ; \quad \forall x, \Theta \in \mathbb{R}_+^K. \quad (3.21)$$

Then, setting $\theta_K = 1$ and choosing a distribution $\tilde{\mathbf{H}}$ issued from \mathbf{H} such that $\Upsilon(\mathbf{H}_{-\mathbf{r}}) = [\mathbf{0}]$ and $\iota(h_{\mathbf{r}}(p)) = 1, \iota(\tilde{h}_{\mathbf{r}}(p)) = 0$, then Eq.(3.21) implies that: $\phi_{\infty}(\mathbf{H}(p)) = 0 < 1 = \phi_{\infty}(\tilde{\mathbf{H}}(p))$. [(i) \implies (ii)]: From $\mathcal{ER}\mathcal{S}$, $\|x\|_{\alpha}^{\ominus} = \max_{k=1, \dots, K} \theta_k x_k$. In this case, choosing $(\mathbf{H}_1, \mathbf{H}_2) \in \mathcal{CRL}$ with $p_i = p_j$, implies that $\|\Upsilon(\mathbf{H}_1(p_{ij}))\|_{\infty}^{\ominus} = \|\Upsilon(\mathbf{H}_2(p_{ij}))\|_{\infty}^{\ominus}$, then $I_{\nu, \infty}(\mathbf{H}_1) = I_{\nu, \infty}(\mathbf{H}_2)$, which concludes the proof. ■

Finally, the socioeconomic health inequality indices are endowed with various sensitivities to risk factors. The different properties are summarized in the following proposition.

Proposition 3.1 *For all socioeconomic health inequality indices $I_{\nu, \alpha} \in \{I\}_{\nu, \alpha}$:*

- (i) $I_{\nu, \alpha}$ satisfies risk insensibility \mathcal{RI} , if $\alpha \rightarrow 0$.
- (ii) $I_{\nu, \alpha}$ satisfies risk sensibility \mathcal{RS} , \mathcal{CIR} and \mathcal{SRE} , if $\alpha > 1$.
- (iii) $I_{\nu, \alpha}$ satisfies extreme risk sensibility \mathcal{ERS} , and \mathcal{CRL} , if $\alpha \rightarrow \infty$.

Proof:

See Lemma 3.1 to Lemma 3.3. ■

Though risk sensibility is now captured by the α parameter, nothing has been said about the redistributive preferences of the social planner, which are introduced and discussed in the next section.

4 Boolean Risk Factor and Stochastic Dominance Criteria

In this section, the interplay between parameters ν and α is presented in order to figure out the class of measures $\{I\}_{\nu, \alpha}$. Particularly, the role of ν in determining the willingness of the social planner to perform health redistribution is explained. Lastly, the result in terms of stochastic dominance is set out.

4.1 Distributional sensibility

Distributional sensibility may also be captured by the function $v(p)$, which provides the behavior of the social planner with respect to health transfers between individuals at *different rank levels*, instead of looking for permutations between the situations of individuals. In the same manner, this kind of distributive sensibility principle, however, enables one to choose whether the social planner has to implement transfers towards non healthy people, very non-healthy people, or the most non-healthy ones. Those transfers are actually well-documented in the literature. Aaberge (2009) shows, for Yaari's (1987) dual social welfare function, the conditions needed for respecting the s -th degree positional (income) transfer principle. When $\nu = 1$, an exogenous manna for one individual improves his/her welfare and therefore the overall welfare. The principle of order 2 postulates that the overall welfare increases if a

rich-to-poor transfer occurs between an income donor at rank p_1 and an income recipient at rank p_2 such that $p_1 > p_2$. Those transfers are generalized in such a way that more weights are put on the lower income recipient insofar as s increases.

In the case of socio-economic health inequality indices, the formal property of ν -th degree of the positional transfer principle is captured when the weight function $v(p)$ has derivatives that alternate in signs, see Aaberge (2009). The social planner performs some health transfers between the individuals. The set of socio-economic health inequality indices that respect the ν -th degree positional transfer principle is:

$$\Omega^\nu := \left\{ I_{\nu,\alpha}(\mathbf{H}) \in \{I\}_{\nu,\alpha} \left| \begin{array}{l} v^{(\ell)} \text{ is continuous and } \nu\text{-time differentiable over } [0, 1] \\ (-1)^\ell v^{(\ell)}(p) \geq 0 \quad \forall p \in [0, 1] ; \forall \ell = 1, \dots, \nu - 1 \\ v^{(\ell)}(1) = 0, \quad ; \forall \ell = 1, \dots, \nu. \end{array} \right. \right\}$$

In order to deal with the generalized Gini indices of inequality and to respect the ν -th positional transfer principle for $\nu \geq 2$, the weight function may be restricted to $v(p) = \nu(1-p)^{\nu-1}$. In that case, the social planner is more and more inequality averse to the extent that ν increases. On the contrary, he is inequality loving whenever $\nu \in (1, 2)$, or neutral to inequality if $\nu = 1$. The sensibility to risk and to inequality is summarized in Figure 0.

Figure 0 exhibits a snapshot of two distributional types of sensitivities: inequality aversion (variations of ν) and risk sensibility (variations of α). So, the parametrization of the socio-economic health inequality indices is performed with respect to the different properties inherent to ν and α , that is, according to risk and inequality sensitivities. In the same manner, in Table 1 below, the properties and all possible parametrizations are summarized for the two-parameter family of socio-economic health inequality indices $\{I\}_{\nu,\alpha}$.

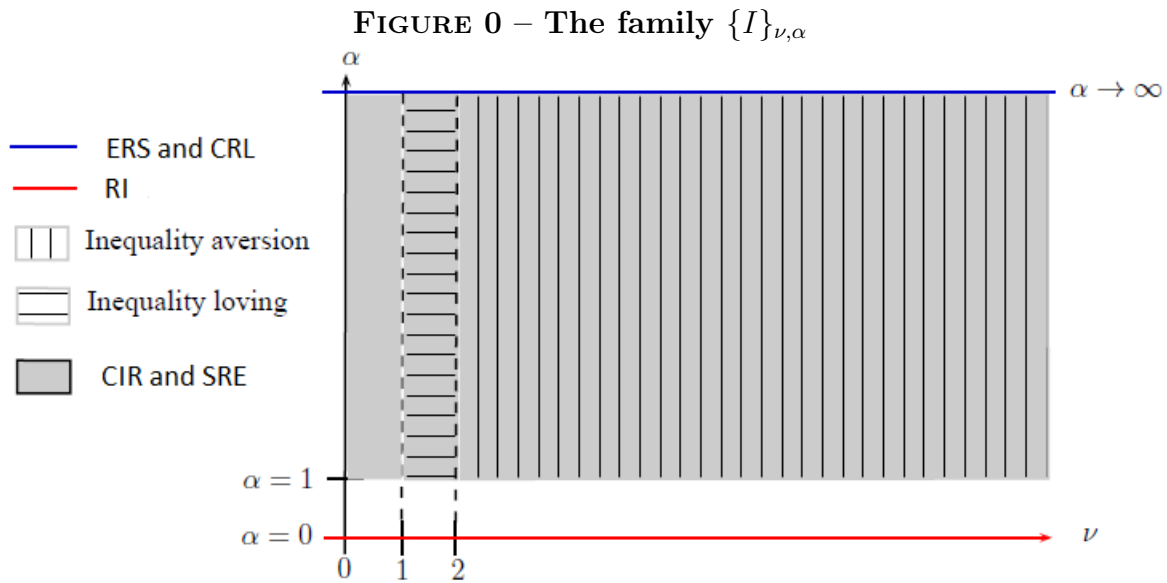


Table 1. Properties of $I_{\nu,\alpha}$

$\nu \downarrow \quad \alpha \rightarrow$	$\alpha = 0 : \mathcal{RI}$	$\alpha > 1 : \mathcal{RS}$	$\alpha \rightarrow \infty : \mathcal{ERS}$
Inequality loving $\nu \in (1, 2)$	\emptyset	$\mathcal{CIR} \cup \mathcal{SRE}$	\mathcal{CRL}
Inequality neutrality $\nu = 1$	\emptyset	$\mathcal{CIR} \cup \mathcal{SRE}$	\mathcal{CRL}
Inequality aversion $\nu \geq 2$	\emptyset	$\mathcal{CIR} \cup \mathcal{SRE}$	\mathcal{CRL}

\emptyset : no particular property related to risk factors (outlined in the paper)

4.2 Stochastic dominance result

In this section, we show that a non-ambiguous ranking between health matrices may be derived, without imposing any functional form on $v(p)$, but taking recourse to a multidimensional socio-economic health inequality index $I_{\nu,\alpha}(\mathbf{H}) \in \Omega^\nu$. Contrary to the previous papers in the literature, see for instance Makdissi and Yazbeck (2014), we propose concentration curves (achievement curves), which involve the attitude of the social planner with respect to the risk factor. For that purpose, let us introduce the achievement curve of order (ν, α) , with $\nu \in \{1, 2, 3, \dots\}$ and $\alpha > 0$.

Definition 4.1 – (ν, α) -Achievement curves – *The achievement curve of order $(1, \alpha)$ is defined as, for all $\alpha > 0$:*

$$A_{\mathbf{H}}^{1,\alpha}(p) := \frac{1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\ominus}{\mu_{\phi(\mathbf{H})}}. \quad (4.1)$$

The (ν, α) -order achievement curve for any given $\nu \in \{2, 3, \dots\}$ and $\alpha > 0$ is:

$$A_{\mathbf{H}}^{\nu,\alpha}(p) := \int_0^p A_{\mathbf{H}}^{\nu-1,\alpha}(u) du. \quad (4.2)$$

Proportionally to the mean, the achievement curve of order $(1, \alpha)$ yields the health achievement of one individual at rank p of the population. The achievement curve of order $(2, \alpha)$ provides the proportion $100p\%$ of the population whose health achievement is no higher than $A_{\mathbf{H}}^{2,\alpha}(p)$. Accordingly, a dominance criterion between achievement curves yields a non-ambiguous ranking between health distributions (say \mathbf{H} and \mathbf{G}). As a result, when the curves do not cross, there is more (or less) inequality for all percentiles $p \in [0, 1]$. Also, this result is in line with the bidimensional parametrization highlighted in the previous section.

Theorem 4.1 *For all socio-economic health inequality indices $I_{\nu,\alpha}(\mathbf{H}) \in \{I\}_{\nu,\alpha}$ such that $I_{\nu,\alpha}(\mathbf{H}) \in \Omega^\nu$ with $\nu \in \{1, 2, 3, \dots\}$, $\alpha > 0$, and for two health distributions \mathbf{H} and \mathbf{G} , the two following statements are equivalent:*

- (i) $I_{\nu,\alpha}(\mathbf{H}) \geq I_{\nu,\alpha}(\mathbf{G})$.
- (ii) $A_{\mathbf{H}}^{\nu,\alpha}(p) \leq A_{\mathbf{G}}^{\nu,\alpha}(p)$, $\forall p \in [0, 1]$.

Proof:

See Appendix A1. ■

Theorem 4.1 provides a non-ambiguous ranking for multidimensional health distributions with respect to the risk factor (insensibility $\alpha \rightarrow 0$, sensibility $\alpha > 1$, and extreme sensibility $\alpha \rightarrow \infty$). However, the non-ambiguous ranking comes at a cost since the ν parametrization becomes discrete only. As a consequence, inequality loving ($\nu \in (1, 2)$) is not available.⁹ The different dominance criteria and their link with the risk properties defined in Section 3 are itemized in Table 2 below.

Table 2. Dominance criteria and properties of $I_{\nu,\alpha}$

$\nu \downarrow$ $\alpha \rightarrow$	$\alpha = 0 : \mathcal{RI}$	$\alpha > 1 : \mathcal{RS}$	$\alpha \rightarrow \infty : \mathcal{ERS}$
Inequality loving $\nu \in (1, 2)$	Impossible	Impossible	Impossible
Inequality neutrality $\nu = 1$	\emptyset	$\mathcal{CIR} \cup \mathcal{SRE}$	\mathcal{CRL}
Inequality aversion $\nu \geq 2$	\emptyset	$\mathcal{CIR} \cup \mathcal{SRE}$	\mathcal{CRL}

\emptyset : no particular property of related to risk factors (outlined in the paper)

5 Data and definitions

In this paper we use data from the Survey of Health, Ageing and Retirement in Europe (SHARE).¹⁰ SHARE is a multidisciplinary and cross-national panel database collecting micro data on health, socio-economic status and social and family networks. Collected every two years since 2004, the objective of the survey is to better understand the ageing process and, in particular, to examine the different ways in which people aged 50 and older live in Europe. The main advantage of using SHARE is that it contains a broad range of variables referring to physical and mental health, which makes it a unique dataset for studying health-related questions, including socio-economic inequalities in health (O’Donnell, 2009).

The analysis is based on data from Wave 5 (collected in 2013) and focused on individuals living in Luxembourg. After excluding all individuals with missing values on any of the variables used in our empirical analysis, our estimation sample includes 1,610 individuals aged 50 and older and their partners. In our sample, 47 percent of individuals are males and 57 percent are over 65 years old.

5.1 Health dimensions

The health status of each individual is measured using the methodology proposed by Pi Alperin (2016). This approach allows to aggregate different dimensions of health (reflecting the mental and physical components of health) in one synthetic indicator. In particular,

⁹Inequality loving may be captured when the derivatives of $v(p)$ alternate in opposite signs $(-1)^\ell v^{(\ell)}(p) \leq 0$.

¹⁰For an overview of SHARE see Börsch-Supan, Brandt, Hunkler, Kneip, Korbmacher, Malter, Schaan, Stuck and Zuber, 2013; Börsch-Supan, 2015, and Malter and Börsch-Supan, 2015.

this methodology permits to relax the threshold τ_k (from Eq. (2.1)), in order to account for different degrees of deprivation for each dimension of health. In other words, for each health dimension there are completely healthy individuals, completely non-healthy individuals and individuals characterized by different intensities of health failure. Specifically, the synthetic scores are calculated as the weighted mean of the K health dimensions.¹¹ In order to determine the weight vector Θ , we use the 'equal weighting' scheme, for which each health dimension has the same importance in the final score, that is, $1/K$.¹²

Table 3. Health dimensions included in the synthetic indicators of health

Global health	Dimensions of health	What is covered by the dimension of health?
Mental Health	Depression Memory	Twelve different symptoms of depression The ability of people to remember things
Physical health	Long term illness Other illnesses Limitation activities 1 Limitation activities 2 Weight problems Eyesight Hearing	Having any long-term health problem, illness or infirmity A list of fourteen health conditions Difficulties with various daily activities because of health problems Difficulties with instrumental activities because of health problems Overweight, obesity and underweight problems Eyesight distance and reading (with or without glasses) Quality of hearing (with or without hearing aid)

5.2 Exogenous risk factors

In order to illustrate how the two-parameters family of health indicators links one risk factor and a set of health dimensions (while accounting for the behavior of the social planner with respect to the risk factor), in this paper, we use childhood circumstances as exogenous risk factors. In other terms, circumstances that are beyond an individual's control and can affect health in adulthood (Case, Lubotsky and Paxson (2002); Case, Fertig, Paxson (2005)). We include four well-known circumstances: parental education, longevity of parents, nationality of parents, and financial situation during childhood. Deutsch *et al.* (2018) show that in Luxembourg, the probability of having good health is higher among natives, the higher the educational level of the father, and for those who did not have financial difficulties when they were young. [In subsection 6.5, we extend the analysis and consider behavioral risk factors.](#)

Consider the migration background of parents as a risk factor is important specially in Luxembourg since the country records the largest share of immigrants in the European Union (European Commission, 2011). In this paper, the variable 'nationality of parents' sets to one for those individuals who have both parents with a migration background and to zero otherwise. Almost 65 percent of individuals have at least one parent with the Luxembourgish nationality.

¹¹See Appendix A2 for a complete description of the construction of each health dimension.

¹²Appendix A3 studies the sensitivity of the results to the choice of the weighting scheme. Note that all the indicators are computed using the MDEPRIV program (see Pi Alperin and Van Kerm, 2020).

The parental education variable is constructed considering the highest educational attainment of both parents according to the 1997 version of the International Standard Classification of Education (ISCED). Parents with lower education are those who have lower-secondary education or any level below it (corresponding to ISCED 1997 categories 0–2). Parents with higher education are those who have any type of upper or post-secondary education (corresponding to ISCED 1997 categories 3–6). In particular, this risk factor is considered to be equal to one if both parents are low educated, and to zero otherwise. In our sample, 34.41 percent of individuals have at least one parent high educated.

The third risk factor included in the analysis is financial situation of the family during childhood. In SHARE, individuals were asked whether their family used to have financial difficulties when they were growing up, from birth to age 15. This factor is equal to one for those individuals with poor families or families whose financial situation varied over time. In contrast, for those individuals with a financial situation during childhood considered as pretty well, or about average, the risk factor is equal to zero. More than 75 percent of the sample is not considered having this risk.

Finally, we consider the longevity of parents as an exogenous risk factor. In particular, we follow Jusot *et al.* (2013) who consider this factor as a proxy for parental’ health. More precisely, individuals in the survey report whether their parents are still alive at the time of the survey and their age at death if applicable. With this information the longevity risk factor was set up to be equal to one if at least one of the parents had short longevity (*i.e.* a parent who died earlier than expected at birth of his/her generation) and equal to zero for those individuals with both parents enjoying longevity. Only 16 percent of the population have one, or both of their parents, in the category short longevity.

6 Empirical application

This section is devoted to the measurement of socio-economic health inequalities in Luxembourg considering childhood circumstances of individuals as exogenous risk factors. In this Section, Figures present different combinations of parameters ν and α . In particular, the parameter ν must be interpreted as the social planner inequality aversion. Specifically, if $\nu \geq 2$ the index displays health inequality aversion, whereas health inequality loving is obtained whenever $\nu \in (1, 2)$.¹³ In other words, more important is the value of parameter ν , more averse to health inequalities is the social planner. Concerning α , this parameter captures the sensibility of the social planner with respect to the association between health dimensions and one risk factor. There are different degrees of risk sensibility: if $\alpha \rightarrow 0$, the sensibility of the social planner is at its minimum level (insensibility) while if $\alpha \rightarrow \infty$, its sensibility is at its maximum level (extreme sensibility).¹⁴

¹³Notice that the value of parameter ν denotes the stochastic dominance order. Hence, par example, $\nu = 2$ represents a two-order stochastic dominance.

¹⁴Section 3.2 explains these notions in detail.

In what follows, the distinction between different sensibilities of the social planner will be made by imposing different values to α : $\alpha = 10, 30, 50, 70$ are related to a moderate risk sensibility, whereas $\alpha = 200$ being the extreme risk sensibility. A code is accessible in the GitHub platform, which allows to select different values for the parameter ν , different levels of risk sensibility of the social planner, and bootstrap confidence intervals.¹⁵

6.1 Risk insensibility

Figures 1a and 1b present the case when the social planner has risk insensibility ($\alpha \rightarrow 0$). In this case, for any given exogenous risk factor associated with the health dimensions, the socio-economic health inequality index remains the same. In other words, the (absolute) achievement curves remain (almost) invariant with respect to any of the considered risk factors. Results are thus in conformity with the theoretical prediction.

Figure 1a further shows that, at the order 2, the achievement curve is close to the 45-degree line. This means that the socio-economic health inequality in the society is the lowest possible, *i.e.* the aggregated health information is equally distributed among the individuals. This is explained by the fact that a social planner with risk insensibility considers an individual being non-healthy if he/she is non-healthy in all dimensions. In addition, the social planner respects the Pigou-Dalton principle of transfers (a Daltonian social planner).¹⁶ Then he/she could operate health transfers even if he/she has risk insensibility. This would be the case for the individuals being non-healthy in all dimensions including in the risk factor.

Figure 1b shows that, at the order 3, the risk factors cannot be distinguished either since the social planner has risk insensibility. In this case, however, the social planner is more inclined to perform redistribution towards non-healthy people, since at the order 3 the Kolm's transfer principle is respected.¹⁷

¹⁵This code, written in Python language, is accessible in the following GitHub platform: <https://github.com/mussard974/Achievement-curves>.

¹⁶The Pigou-Dalton principle of transfers says that a transfer of some defined variable (for example income) from the rich to the poor is desirable, as long as it does not bring the rich to a poorer situation than the poor. Under these conditions, this transfer reduces inequalities.

¹⁷The principle of diminishing transfers of Kolm (1976) requires that a progressive transfer (from a higher to a lower income earner) should have a diminishing effect at higher income levels. Additionally, all curves shift down, showing a higher aversion towards inequality.

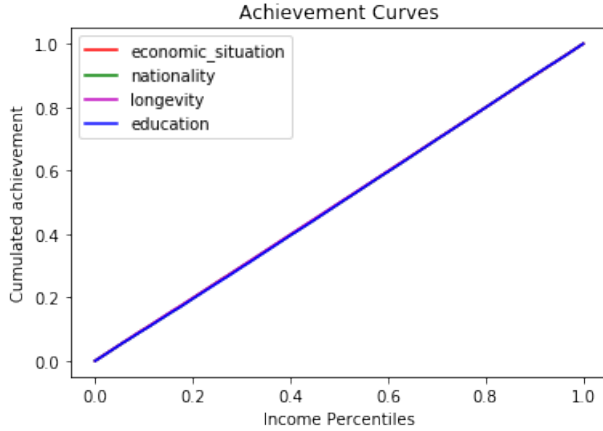


Figure 1a: Order 2, $\alpha = 0$

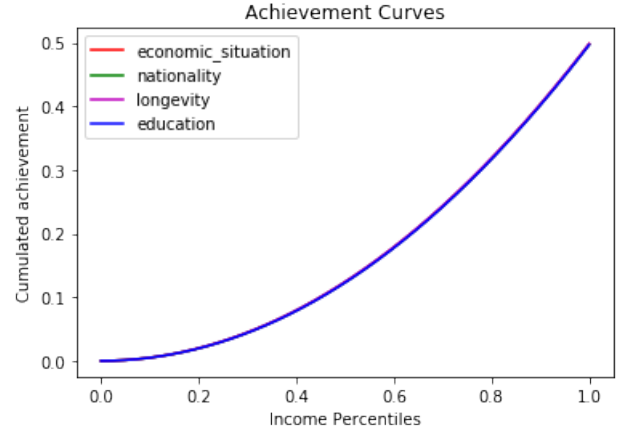


Figure 1b: Order 3, $\alpha = 0$

6.2 Risk sensibility

Figures 2a-2d compare the achievement curves for each exogenous risk factor while imposing different values to the parameter α in order to make a clear distinction between a low risk sensibility and a more important one. When $\alpha = 10$ (Figure 2a), the achievement curves coincide. Then, the risk sensibility of the social planner is close to risk insensibility. If higher values are assigned to α , however, the curves move farther from the 45-degree line showing that the more sensitive to risk is the social planner, the more important is the impact of the risk factors on socio-economic health inequalities. Specifically, if α goes from 30 to 70 (Figures 2b-2d), the risk sensibility increases in such a way that the achievement curve of nationality of parents lies below the others curves. In consequence, the risk inherent to the migration background of parents is more and more valued by the social planner as being the most important risk factor (among the four factors) that increases the socio-economic health inequalities.

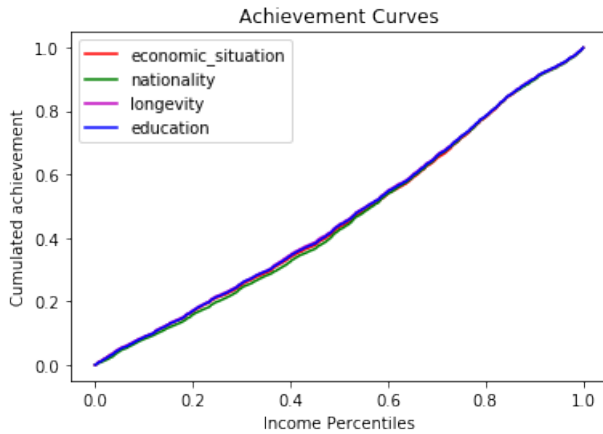


Figure 2a: Order 2, $\alpha = 10$

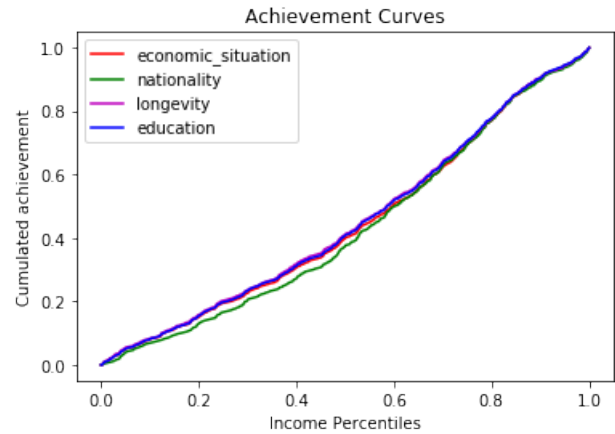


Figure 2b: Order 2, $\alpha = 30$

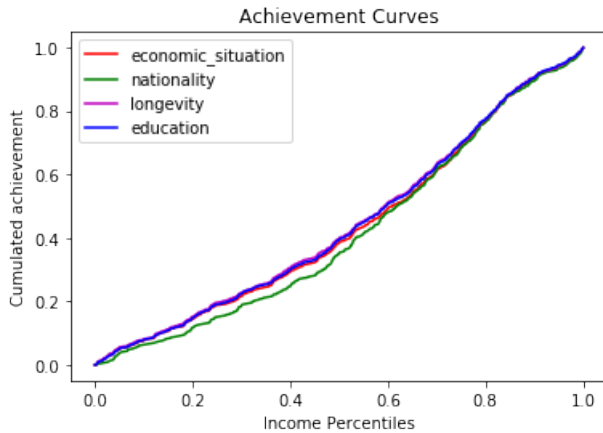


Figure 2c: Order 2, $\alpha = 50$

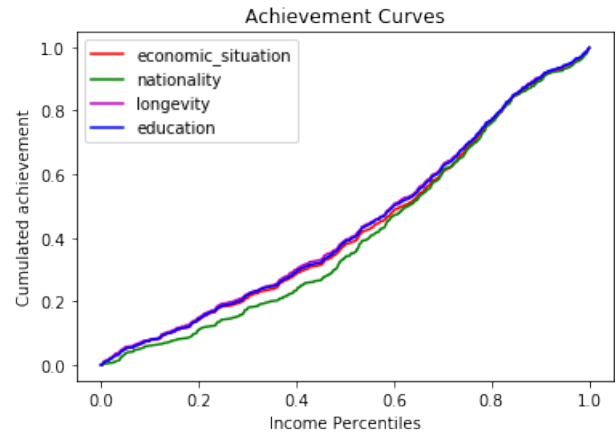


Figure 2d: Order 2, $\alpha = 70$

Figures 2a-2d further suggest that, in this context, the social planner cannot proceed to a non-ambiguous ranking of the health matrices associated with each exogenous risk factors. The reason is that the curves cross. Thus, the Daltonian social planner could not say with certainty whether the nationality of parents, or the parental education for example, is the main factor affecting the health status of individuals and implying a higher socio-economic health inequality in the society.

Figures 3a-3d show that, at the order 3, the curves do not cross when the degree of risk sensibility of the social planner increases. A Kolm social planner with risk sensitive ranks, without ambiguity, the distribution with the risk factor nationality of parents as the most unequal one followed by the distributions related to parental education, financial situation and longevity of parents, respectively. In other words, among the different childhood circumstances of individuals having an impact on the health status in adulthood, having parents with a migration background and low educated constitutes the risk factors that aggravate the most the overall level of inequality in Luxembourg.

It is worth mentioning that it is not necessary to check higher orders of dominance whenever dominance appears for $\alpha = \alpha^*$. All social planners embodied with $\alpha \geq \alpha^*$ agree indeed

about the increase (decrease) of the inequality on the society. In this sense, as socio-economic health inequalities are stronger due to the risk factor nationality of parents when $\alpha^* = 70$ (a moderate risk sensibility), then all socio-economic indices such that $\alpha \geq 70$ are consistent with this ranking (a robust ranking). This reasoning is also valid for the ν parameter. Since the socio-economic health inequalities increase due to the risk factor nationality at the order $\nu = 2$, the same result holds true for all social planners embodied by $\nu \geq 2$. In what follows, we show, however, how the results can be confirmed with a social planner extreme risk averse. Then, all social planners having any combination of parameter $(\alpha, \nu) > (\alpha^*, \nu^*)$, will unanimously select the same risk factors that increase the socio-economic health inequalities.

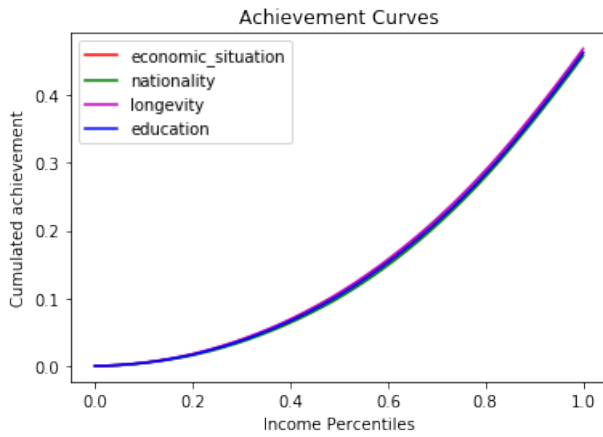


Figure 3a: Order 3, $\alpha = 10$

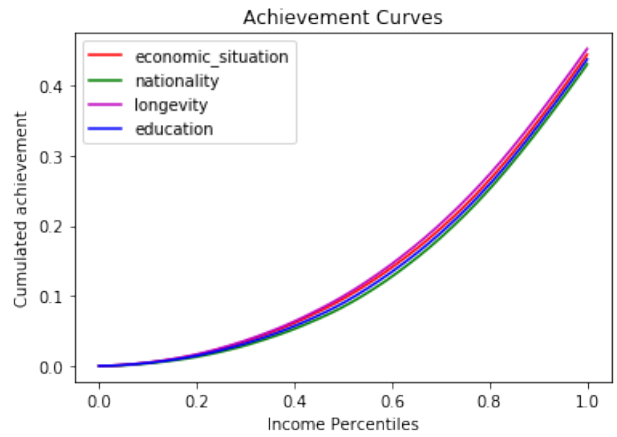


Figure 3b: Order 3, $\alpha = 30$

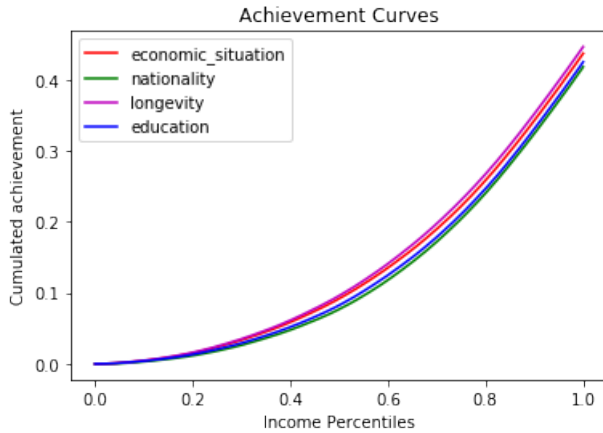


Figure 3c: Order 3, $\alpha = 50$

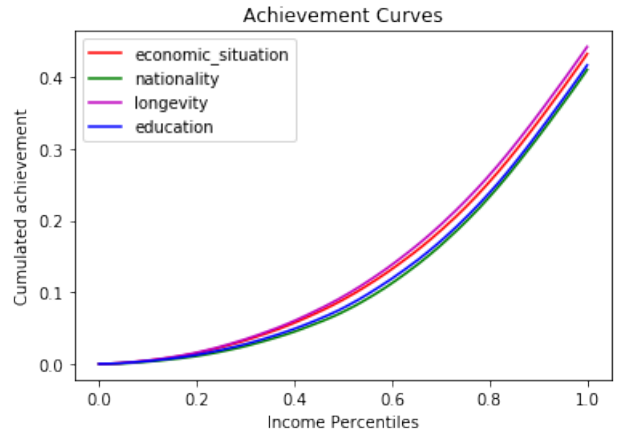


Figure 3d: Order 3, $\alpha = 70$

6.3 Extreme risk sensibility

Figures 4a-4c show three different orders of inequality aversion in the case of extreme risk sensibility ($\alpha = \infty$). With a Daltonian social planner (order 2), the achievement curves cross and it becomes difficult to identify those risk factors that increase socio-economic health

inequality (Figure 4a). With a social planner more inclined to perform transfers to deprived individuals (order 3), it is possible to identify the financial situation during childhood as the less riskiest factor followed by the longevity one. Nationality and education of parents risk factors cannot be distinguished since the achievement curves cross (Figure 4b). Finally, with a social planner that respects composite transfers (order 4), the health information matrix associated with the risk nationality of parents provides an achievement curve that dominates the curve of the educational risk factor. We can then conclude that the nationality of parents is the riskiest factor that contributes to increase overall inequalities.

In order to test the sensitivity of results while comparing two risk factors, it is possible to design achievement curves with bootstrap confidence intervals.¹⁸ Figure 4d shows that the bootstrap confidence interval of longevity of parents lies above the bootstrap confidence interval of nationality of parents. In consequence, taking into account the sampling error, it is possible to affirm that that the migration background of parents yields more socio-economic health inequalities in the society compared to the longevity risk factor (the social planner having an extreme risk sensibility).

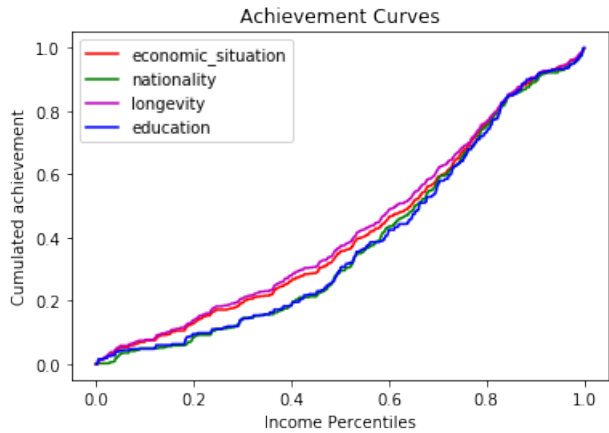


Figure 4a: Order 2, $\alpha = \infty$

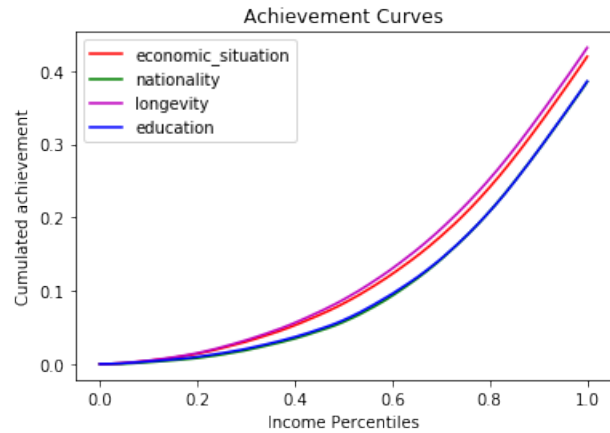


Figure 4b: Order 3, $\alpha = \infty$

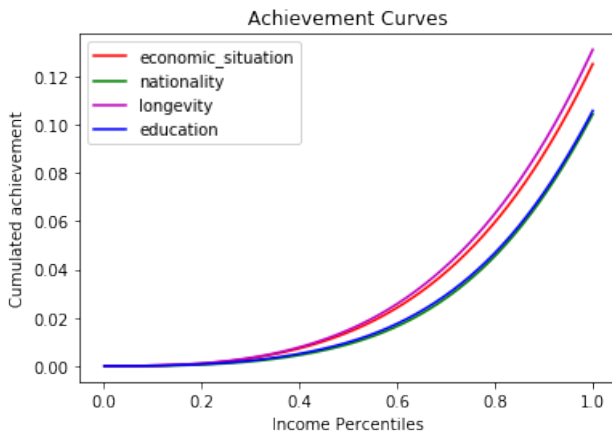


Figure 4c: Order 4, $\alpha = \infty$

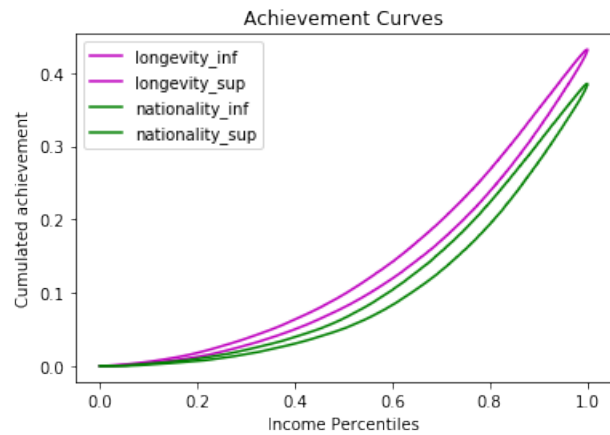


Figure 4d: Order 3, $\alpha = \infty$

¹⁸We use a simple percentile method with $B = 5,000$ replications and a 5 percent risk level.

6.4 Risk mixture

An interesting exercise is to compare each individual risk factor to a factor representing the average of all exogenous risk factors (Risk_mix). This is possible since the socio-economic health inequality indices can be computed with respect to a bounded risk factor $h_{\mathbf{r}}(p) \in [0, 1]$ (see Section 3).

In previous subsections, the risk factor nationality of parents appears as the factor increasing the most the socio-economic health inequalities in Luxembourg. It is then possible to compare the health information matrix associated with this factor to the health information matrix in which an average risk factor is introduced. Figures 5a and 5b show a moderate and an extreme risk sensitive social planner ($\alpha = 50$ and $\alpha = \infty$, respectively) at the order 2. The health information matrix related to the risk factor nationality of parents is the most unequal. Although this result is ambiguous for the moderate risk sensitive social planner (since the curves cross), it is not for the social planner with extreme risk sensibility. The predominance of the migration background of parents as the risk factor increasing the most the socio-economic health inequalities in Luxembourg is unambiguously confirmed by Figures 5c and 5d. At the order 3, the achievement curve associated with the average risk factor dominates the achievement curve with the nationality of parents' risk factor. Additionally, the bootstrap confidence interval of the achievement curve related to the risk mixture lies above the interval conference of nationality of parents along the distribution (see Figure 5d).

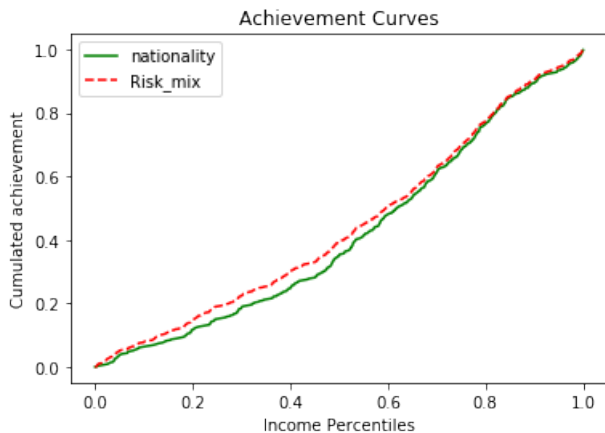


Figure 5a: Order 2, $\alpha = 50$

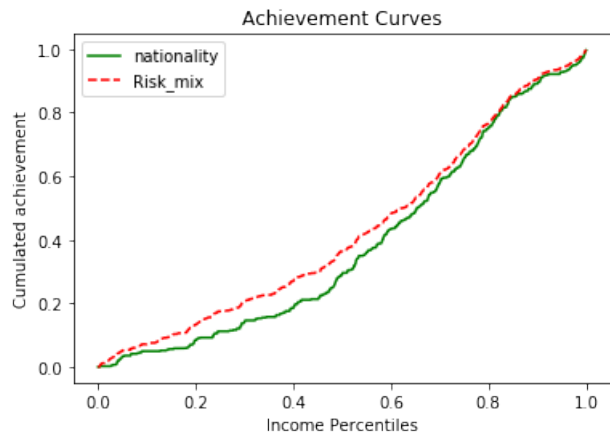


Figure 5b: Order 2, $\alpha = \infty$

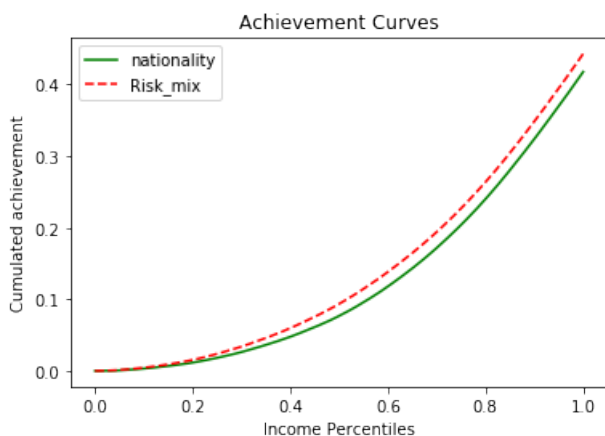


Figure 5c: Order 3, $\alpha = 50$

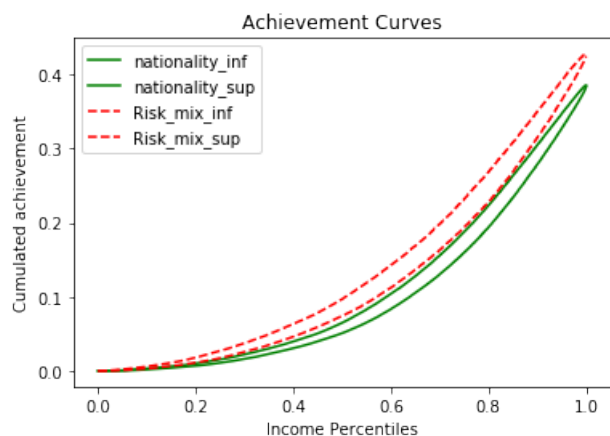


Figure 5d: Order 3, $\alpha = \infty$

6.5 Endogenous risk factors

The determinants of health are factors that have a significant influence, whether positive or negative, on health. Health is thus the result of multiple factors including, among other, those genetics, efforts or lifestyle, environmental, socio-economic and characteristics of the healthcare systems (Laurence, 2014).

The family of indicators proposed in this paper allows social planners and researchers to analyze different risk factors (health determinants) and to identify those, which contribute the most to generate socio-economic health inequalities in the society. From the comparative perspective, this family of indices allows to perform a cross-country analysis in order to establish in which country some specific risk factors generate more inequalities. This information might be important for the elaboration of public policy actions at the international level.¹⁹

¹⁹In 1998, the World Health Organization (WHO) established the Commission on the Social Determinants of Health in charge of recommending policies and interventions (https://www.who.int/social_determinants/thecommission/finalreport/about_csdh/en/). This international organization often initiates projects aiming to reduce socio-economic inequalities in health (WHO, 2005 and 2015).

In previous subsections, we focused our analysis in risk factors that the individual cannot control for (childhood circumstances) as our approach is in line with the literature on equality of opportunity. The analysis, however, can be extended to different categories of risk factors. In this subsection, and following the equality of opportunity theory, we analyze individuals' efforts or lifestyle. These endogenous risk factors, which are controlled by individuals, can influence health and increase (decrease) socio-economic health inequalities.

Three different endogenous risk factors are investigated in what follows: smoking, drinking habits, and physical activities. As before, in order to construct the achievement curves, nine different dimensions of health are considered together with one risk factor.

Figure 6a shows that with a social planner with intermediate values of risk sensibilities ($\alpha = 70$), it is not possible to identify any risk factor as having more impact than other on the overall level of inequality as the achievement curves overlap. Meanwhile, Figure 6b shows that for a social planner more inclined to perform transfers to deprived individuals while having intermediate values of risk sensibility ($\alpha = 70$), the endogenous risk factor drinking habits generates more inequality in the society compared with smoking or physical activities (the achievement curve of drinking habits slightly lies below the others two).

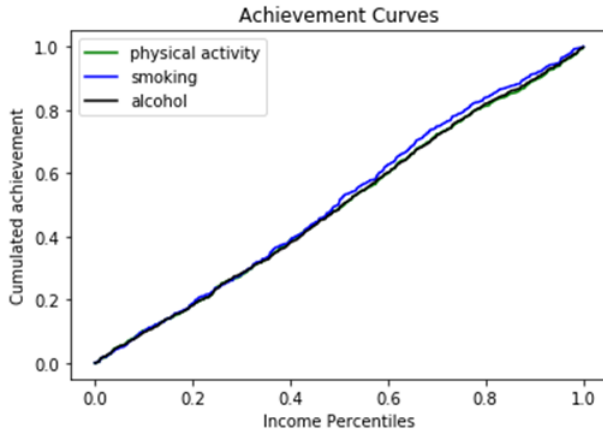


Figure 6a: Order 2, $\alpha = 70$

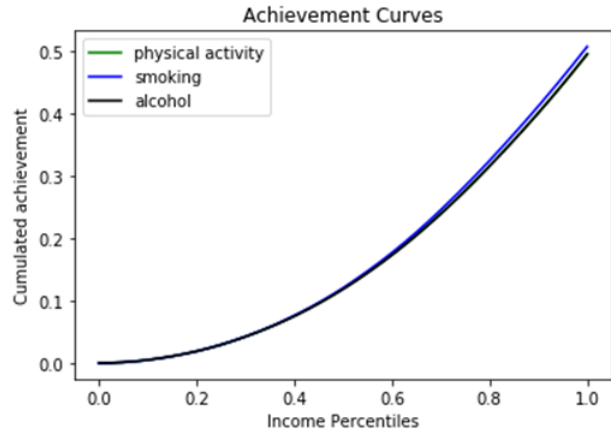


Figure 6b: Order 3, $\alpha = 70$

Increasing the level of risk sensibility to infinity, *i.e.* for social planner with an extreme risk sensibility, confirms that drinking habits is the endogenous risk factor that has the most important impact on increasing socio-economic health inequalities (Figure 7a and 7b at the orders 3 and 4 respectively).

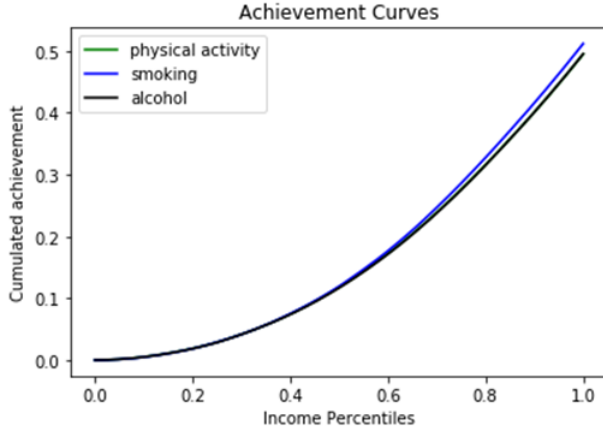


Figure 7a: Order 3, $\alpha = \infty$

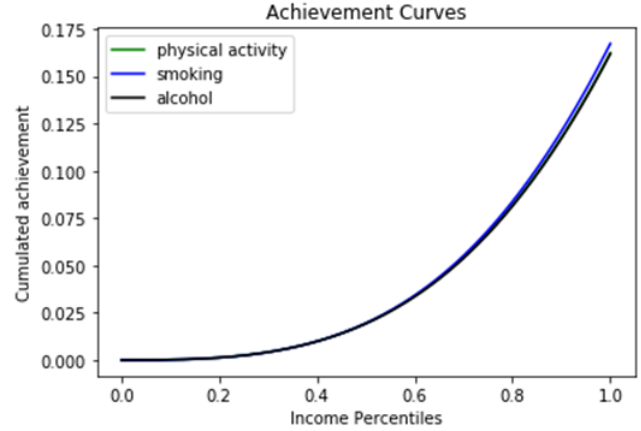


Figure 7b: Order 4, $\alpha = \infty$

Finally, Figure 8 compares the exogenous risk factor nationality of parents and the endogenous risk factor drinking habits, both being the most important factor in their categories. A social planner, with an extreme risk sensibility and more inclined to perform transfers to deprived individuals, can conclude that the migration background of parents has a deeper impact on the overall level of inequality in the society compared with drinking habits of individuals. Indeed, the bootstrap confidence intervals of the achievement curve of nationality lie below those of the achievement curve of alcohol along all the income distribution.

It is interesting to notice that the risk factor that the individuals cannot control for can induce to higher socio-economic health inequalities than those risk factors under the individuals control. In line with the theory of equality of opportunity, in Luxembourg, these unfair inequalities should be compensated.

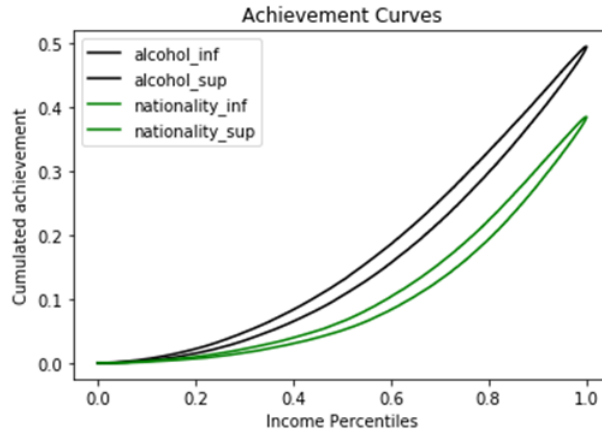


Figure 8: Order 3, $\alpha = \infty$

7 Conclusion

This paper contributes to the existing literature on behavioral OR approaches and proposes a two-parameter family of socio-economic health inequality indices, which depends on different

health dimensions and on one risk factor. These indices allow to capture the risk sensibility and the inequality aversion of the social planner in order to provide a non-ambiguous ranking of multidimensional health distributions. In other words, the social planner behind the veil of ignorance is endowed with a bi-dimensional view, which is necessary to apprehend the impact of different risk factors (beyond the individuals' control) on the level of inequality in a society.

Based on SHARE data for Luxembourg, we investigate four different variables representing childhood circumstances to illustrate how the two-parameters family of health indicators links one risk factor and a set of health dimensions: parental education, longevity of parents, nationality of parents, and financial situation during childhood. In particular, we apply our stochastic dominance rule (compatible with either risk insensibility, risk sensibility, or extreme risk sensibility) by changing the exogenous risk factors and by comparing all possible multidimensional health distributions. We find that the most important exogenous risk factors explaining socio-economic health inequalities in Luxembourg are the migration background and education level of parents. These findings are robust for different levels of risk sensitivity, as well as extremely risk sensitivity social planners, insofar they are inclined to perform health transfers to deprived individuals in order to reduce inequalities in the society.

Although in this paper the selection of risk factors is in line with the circumstances in the equality of opportunity theory, this approach can be extended to different categories of risk factors and different types of social planners (in terms of inequality aversion and risk sensibility). More precisely, the proposed methodology can be adapted to analyze other issues related to multidimensional inequality (with dimensions associated to risk factors). For example, poverty alleviation, lack of access to education, lack of access to the healthcare system, among other. In all these cases, comparing the impact of different risk factors on socio-economic inequalities provides relevant information for the social planner to prioritize redistributive policies on the risk factors that aggravate the level of inequality in the society.

Finally, further research may include dealing with multiple risk factors, for instance. This could determine some priority in the redistribution to be made to the non-healthy individuals affected by exogenous risk factors.

Appendix A1: Proof of Theorem 4.1

Proof:

Sufficiency.

Note that, for all $\alpha > 0$ and $\nu \in \{1, 2, 3, \dots\}$, for all inequality indices $I_{\nu, \alpha}$, we get,

$$I_{\nu, \alpha}(\mathbf{H}) - I_{\nu, \alpha}(\mathbf{G}) = - \int_0^1 v(p) \left[\left(1 - \|\Upsilon(\mathbf{H}(p))\|_\alpha^\ominus\right) - \left(1 - \|\Upsilon(\mathbf{G}(p))\|_\alpha^\ominus\right) \right] dp.$$

Thus, by Definition 4.1, $I_{\nu, \alpha}(\mathbf{H}) - I_{\nu, \alpha}(\mathbf{G}) \geq 0$ is equivalent to:

$$\int_0^1 v(p) [A_{\mathbf{G}}^{1, \alpha}(p) - A_{\mathbf{H}}^{1, \alpha}(p)] dp \geq 0.$$

Integrating by parts $\int_0^1 v(p) A_{\mathbf{H}}^{1, \alpha}(p) dp$, for all $\nu \in \{1, 2, \dots\}$, entails:

$$\int_0^1 v(p) A_{\mathbf{H}}^{1, \alpha}(p) dp = |v(p) A_{\mathbf{G}}^{2, \alpha}(0)|_0^1 - \int_0^1 v^{(1)}(p) A_{\mathbf{H}}^{2, \alpha}(p) dp.$$

Since $v^{(\nu)}(1) = 0$ and by definition $A_{\mathbf{G}}^{1, \alpha}(0) = 0$, then:

$$\int_0^1 v(p) A_{\mathbf{H}}^{1, \alpha}(p) dp = - \int_0^1 v^{(1)}(p) A_{\mathbf{H}}^{2, \alpha}(p) dp.$$

Integrating the previous relation $\nu - 1$ times provides:

$$\int_0^1 v(p) A_{\mathbf{H}}^{\nu, \alpha}(p) dp = (-1)^{\nu-1} \int_0^1 v^{(\nu-1)}(p) A_{\mathbf{H}}^{\nu, \alpha}(p) dp.$$

Hence, for all $\alpha > 0$:

$$I_{\nu, \alpha}(\mathbf{H}) - I_{\nu, \alpha}(\mathbf{G}) = (-1)^{(\nu-1)} \int_0^1 v(p)^{(\nu-1)}(p) [A_{\mathbf{G}}^{\nu, \alpha}(p) - A_{\mathbf{H}}^{\nu, \alpha}(p)] dp. \quad (7.1)$$

Note that $(-1)^{(\nu-1)} \int_0^1 v(p)^{(\nu-1)} dp \geq 0$. If $A_{\mathbf{G}}^{\nu, \alpha}(p) - A_{\mathbf{H}}^{\nu, \alpha}(p) \geq 0$ for all $p \in [0, 1]$, then it results that $I_{\nu, \alpha}(\mathbf{H}) - I_{\nu, \alpha}(\mathbf{G}) \geq 0$.

Necessity.

Consider the following weight function:

$$v^{(\nu-2)}(p) = \begin{cases} (-1)^{\nu-2} \epsilon & p \leq \bar{p} \\ (-1)^{\nu-2} (\bar{p} + \epsilon - p) & \bar{p} < p \leq \bar{p} + \epsilon \\ 0 & p > \bar{p} + \epsilon \end{cases} .$$

It follows that,

$$v^{(\nu-1)}(p) = \begin{cases} 0 & p \leq \bar{p} \\ (-1)^{\nu-1} & \bar{p} < p \leq \bar{p} + \epsilon \\ 0 & p > \bar{p} + \epsilon \end{cases} . \quad (7.2)$$

Assume that $A_{\mathbf{G}}^{\nu, \alpha}(p) - A_{\mathbf{H}}^{\nu, \alpha}(p) < 0$ on an interval $[\bar{p}, \bar{p} + \epsilon]$ for some ϵ close to 0. Substituting (7.2) in (7.1) yields $I_{\nu, \alpha}(\mathbf{H}) - I_{\nu, \alpha}(\mathbf{G}) < 0$, a contradiction. ■

Appendix A2: Description of the health dimensions used for the construction of individuals' health indicators

Table A.2.I: Depression

Depression scale Euro-d*	Degree of membership
Non depressed (0 dimension)	0
Between 1 and 11 dimensions	$1 - (12 - X_i)/12$
Completely depressed (12 dimensions)	1

*Depression, pessimism, suicidal thought, guilty, sleep, interest, irritability, appetite, tiredness, concentration, enjoyment, tearfulness.

Table A.2.II: Memory

Memory and ability to remember things	Degree of membership	
Four questions have been asked regarding the date, the month, the year, and the day of the week	Knows all	0
	Knows 3 of 4	0.3
	Knows 2 of 4	0.6
	Knows 1 of 4	0.9
	None of them	1

Table A.2.III: Chronic illness

Long term illness	Degree of membership	
Do you have any long-term health problems, illness, disability or infirmity?	No	0
	Yes	1

Table A.2.IV: Other illnesses

Other illnesses	Degree of membership	
Did a doctor say you have any of the following conditions?*	No	0
	One of these conditions	0.75
	Two or more of these conditions	1

*A heart problem; High blood pressure or hypertension; High blood cholesterol; A stroke or cerebral vascular disease; Diabetes or high blood sugar; Chronic lung disease such as chronic bronchitis or emphysema; Asthma; Arthritis, including osteoarthritis, or rheumatism; Osteoporosis; Cancer or malignant tumor; Stomach or duodenal ulcer; Parkinson disease; Cataracts; Hip fracture or femoral fracture.

Table A.2.V: Limitation activities 1

Health and daily activities		Degree of membership
Because of a health problem, do you have difficulty doing any of the following daily activities?*	No	0
	One of these activities	0.15
	Two of these activities	0.25
	Three of these activities	0.50
	Four of these activities	0.75
	Five or more of these activities	1

*Walking 100 meters; Sitting for about two hours; Getting up from a chair after sitting for long periods; Climbing several flights of stairs without resting; Climbing one flight of stairs without resting; Stooping, kneeling or crouching; Reaching or extending your arms above shoulder level; Pulling or pushing large objects like a living room chair; Lifting or carrying weight over 5 kilos, like a heavy bag of groceries.

Table A.2.VI: Limitation activities 2

Health and instrumental activities		Degree of membership
Because of a health problem, do you have difficulty doing any of the following instrumental activities?*	No	0
	One of these activities	0.15
	Two of these activities	0.25
	Three of these activities	0.50
	Four of these activities	0.75
	Five or more of these activities	1

*Dressing, including putting on shoes and socks; Walking across a room; Bathing or showering; Eating, such as cutting up for your food; Getting in or out of bed; Using the toilet, including getting up or down; Using a map to figure out how to get around in a strange place; Preparing a hot meal; Shopping for groceries; Making telephone calls; Taking medications; Doing work around the house or garden; Managing money, such as paying bills and keeping track of expenses.

Table A.2.VII: Weight²⁰

Weight problems \leq 65 years old	Degree of membership
BMI < 17.5	1
$17.5 \leq$ BMI < 18.5	$(18.5 - \text{BMI}) / (18.5 - 17.5)$
$18.5 \leq$ BMI < 25	0
$25 \leq$ BMI < 30	$(30 - \text{BMI}) / (30 - 25)$
BMI \geq 30	1
Weight problems \geq 66 years old	Degree of membership
BMI < 21	1
$21 \leq$ BMI < 23	$(23 - \text{BMI}) / (23 - 21)$
$23 \leq$ BMI < 27	0
$27 \leq$ BMI < 30	$(30 - \text{BMI}) / (30 - 27)$
BMI \geq 30	1

Table A.2.VIII: Eyesight

Eyesight distance and reading*	Degree of membership
Both are E or VG	0
One is E or VG, the other is G or F	0.15
One is E or VG, the other is P	0.25
Both are G or F	0.30
One is G or F, the other is P	0.60
Both are P	1

*E: excellent; VG: very good; G: good; F: fair; P: poor

Table A.2.IX: Hearing

Hearing		Degree of membership
Is your hearing*	Excellent or Very good	0
	Good or Fair	0.15
	Poor	1

*We have also considered individuals who are using a hearing aid as usual.

²⁰Body Mass Index (BMI) = weight (in kg)/height² (in meters).

Appendix A3: Robustness check

A.3.1 The weight vector

One important debate in the literature on multidimensional indicators is the role and the intensity of the weight attached to each dimension. The construction of composite indicators relies on the selection of the weighting scheme used to aggregate different health dimensions. In the main part of this paper, we use the equal weighting scheme in order to give the same importance to each single health dimension.

In order to test the sensitivity of the results to the weighting scheme selected to aggregate health dimensions, we recalculated the achievement curves using two alternatives weighting scheme. The first weighting scheme was proposed by Cerioli and Zani (1990) who consider the weight as being an inverse function of the deprivation level of the item in the entire population. The second weighting scheme, proposed by Betti and Verma (1998), not only accounts for the relative frequency of the health dimensions among the population by also limits the influence of those dimensions that are highly correlated.

Specifically, Cerioli and Zani (1990) give a more important weight to relatively rare dimensions:

$$\theta_k^{CZ} = \log\left(\frac{1}{\bar{h}_k}\right), \quad (7.3)$$

where \bar{h}_k is the arithmetic mean of the k -th health dimension (that is of the $\iota(h_k(p))$ over all p). The weight of any dimension of health proposed by Betti and Verma (1998) is defined as follows:

$$\theta_k^{BV} = \theta_k^a \cdot \theta_k^b, \quad (7.4)$$

where θ_k^a depends on the distribution of the k -th dimension and θ_k^b depends on the correlation between k and the other health dimensions. In particular, θ_k^a is determined by the coefficient of variation of $\iota(h_i(p))$,

$$\theta_k^a = \frac{n \int_0^1 [\iota(h_k(p)) - (\bar{h}_k)^2] dp}{(n\bar{h}_k)^{\frac{1}{2}}}.$$

The weights θ_k^b are computed as follows:

$$\theta_k^b = \left[1 + \sum_{\substack{k'=1 \\ k' \neq k}}^K \rho_{k,k'} F(\rho_{k,k'} < \rho_H) \right]^{-1} \left[1 + \sum_{\substack{k'=1 \\ k' \neq k}}^K \rho_{k,k'} F(\rho_{k,k'} \geq \rho_H) \right]^{-1},$$

where $\rho_{k,k'}$ is Pearson's correlation coefficient between dimensions k and k' and $F(\cdot)$ is an indicator function valued to be 1 if the expression in brackets is true and 0 otherwise. The parameter ρ_H is a pre-determined cut-off correlation level between the two dimensions.²¹ In

²¹Betti and Verma (1998) suggest setting ρ_H so as to divide the ordered set of correlations at the point of the largest gap.

other words, it separates high and low correlations. The term θ_k^b is the inverse of a measure of average correlation of dimension k with the others health dimensions. The larger the average correlation with dimension k , the lower the resulting weight for that dimension.

Figures 6a-6c below show the achievement curves with the nationality of parents as risk factor. These curves are computed using the three proposed weighting schemes: the equal weight, Cerioli and Zani (1990) weighting scheme (Eq.(7.3)), and Betti and Verma (1998) weighting scheme(Eq.(7.4)). As it is possible to see, in the three cases analyzed (risk insensibility, risk sensibility and extreme risk sensibility), curves overlap showing that they are not sensitive to the three possible normalized weights θ . In particular, in the extreme sensibility case, the weight vector is totally independent of the health dimensions, so that the achievement curves are exactly the same for any given weight vector. In the three Figures, the smoothed curves lying at the bottom are those of the order 3.

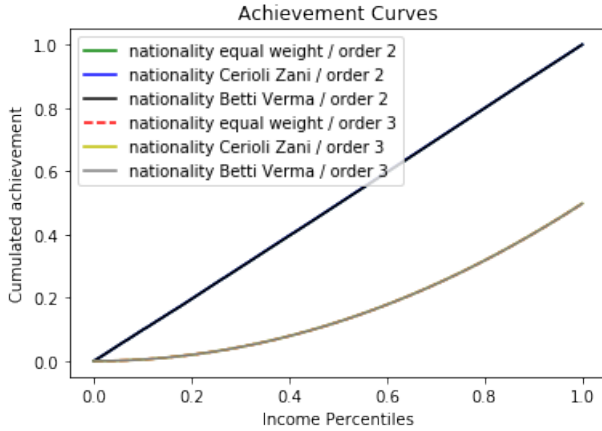


Figure 6a: Risk insensibility

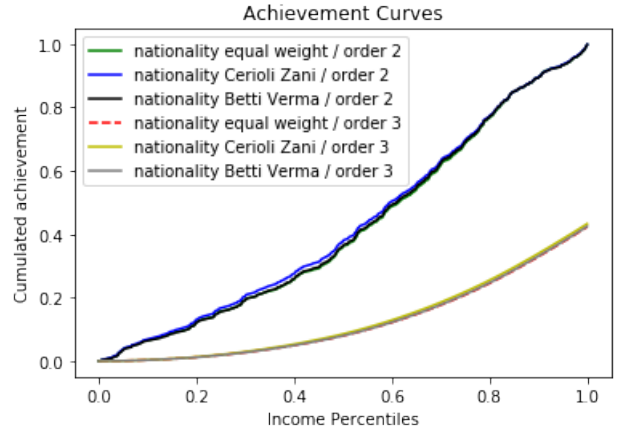


Figure 6b: Risk sensibility $\alpha = 40$

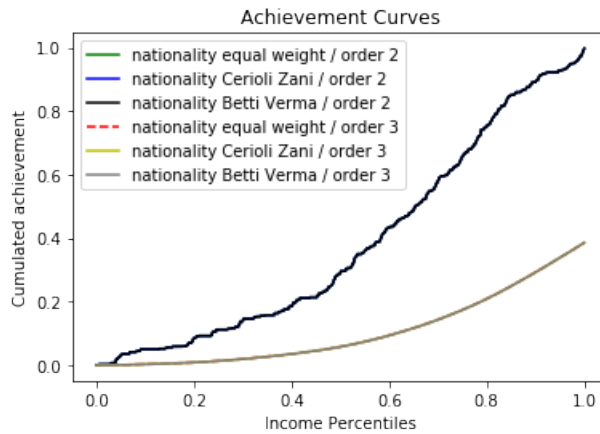


Figure 6c: Extreme risk sensibility

A.3.2 Averaging the risk factors

In order to measure the robustness of our results to the weighting scheme chosen to aggregate the risk factors into one single variable in the case of risk sensibility (Eq. (2.6)), we computed the achievement curves with respect to the mixed risk using the three alternative weighting schemes presented above: equal weight, Cerioli and Zani and Betti and Verma weighting schemes. Figure 7a shows that, in the case of risk sensibility ($\alpha = 40$), the achievement curves are almost the same (and exactly the same for the insensibility and extreme sensibility cases). More precisely, since the risk factors are constructed as Boolean variables, the variations of the achievement curves are very slight, suggesting that the way of averaging has no impact on their curvature.

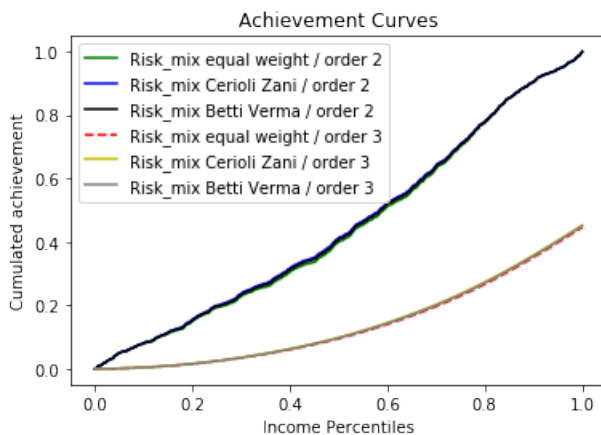


Figure 7a: Risk sensibility $\alpha = 40$

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