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# 1 **EXPLORATORY TESTS ON A BIAXIAL COMPRESSION HOPKINSON BAR SET-UP**

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## 7 **Abstract:**

8 Background: Multiaxial dynamic loadings occur in many industrial cases and multiaxial  
9 dynamic test development is thus a crucial issue. Objective: To meet this challenge, a biaxial  
10 compression Hopkinson bar set-up is designed. Methods: The set-up consists of a striker, an  
11 input bar, an internal output bar and a co-axial external output tube (surrounding the internal  
12 bar). The internal output bar measures the axial loading of the cross sample whereas the external  
13 output bar measures the transverse one via a mechanism. This mechanism uses two intermediate  
14 parts with inclined sliding surfaces. Results: Gauges on the bars enable for force measurements  
15 in the set-up, and the sample displacement field is obtained by digital image correlation. Simple  
16 compression tests on cuboid samples inserted between the input bar and the internal output bar  
17 give the sample material behavior. Then, to determine the friction at the mechanism sliding  
18 surfaces, identical samples are inserted between the input bar and the external output bar, and  
19 are compressed. Conclusions: Finally, the consistency of the measurements obtained during a  
20 biaxial compression test on a cross sample can be checked from the previously measured  
21 parameters and from numerical simulations.

22 **Keywords:** biaxial compression dynamic test, Hopkinson bars

## 1 Introduction

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Multiaxial dynamic loadings usually occur in many industrial cases such as automotive impacts [1], high-speed forming [2] or high-speed machining [3]. Multiaxial dynamic test development is therefore a crucial issue. Unfortunately, most of the dynamic tests are uniaxial. For instance, the very common Hopkinson bar test (which enables for accurate measurements at high strain rates) uses the uniaxial compression loading generated by the impact of a projectile.

In order to perform multiaxial tests, many set-ups have thus been designed to obtain multiaxial loadings from an initial uniaxial loading device. For example, a radial pressure can be applied to a cylindrical sample mounted on Hopkinson bars thanks to a confinement device. One can use a pressure vessel that enables for a controlled quasi-static pressure to be applied from a fluid [4]. Inserting the cylindrical sample inside a rigid tube can generate a dynamic radial loading even though the ratio between the radial pressure and the axial stress strongly depends on the sample material, in particular on its Poisson's ratio [5]. Using a confinement tube made of a perfect plastic behavior material makes it possible to maintain a constant radial pressure [6]. A rigid confinement can also be imposed to a cruciform sample with a pre-loading system [7].

Another idea is to combine shear and compressive loadings. An inclined shear/compression specimen [8] or pressure bars with beveled ends [9] can be used to apply such combined loadings. The combined torsion-compression Hopkinson bar technique using torsional and compressive bars at each side of specimen was also reported [10]. It can also be carried out by blocking a brake on the input bar and by applying both compression/tension and torsion on the input boundary of the bar. Then the sudden fracture of the brake generates both

46 torsion and tension/compression waves [11]. Unfortunately, the difference between the wave  
47 celerities does not enable simultaneous loadings to be obtained [12].

48 In order to apply biaxial loadings, two perpendicular Hopkinson bar devices have been  
49 built [13]. An explosive is used to obtain simultaneous loadings. The system is rather expensive  
50 and difficult to use. Recently, biaxial Hopkinson bar systems using two impactors were reported  
51 to generate biaxial compression states on samples [14], but obtaining two simultaneous impacts  
52 remains difficult. Another simple way to apply an equi-biaxial loading is the bulge-test using  
53 Hopkinson bars described in [15]. In this test, the external boundary of a circular sheet is leant  
54 against the tubular boundary of the output bar while the other side of the sheet is submitted to  
55 the pressure of a fluid compressed by the input bar. A biaxial tensile state is thus obtained at  
56 the center of the sheet. Unfortunately, only sheets can be tested and the displacement field on  
57 the sample cannot be easily measured.

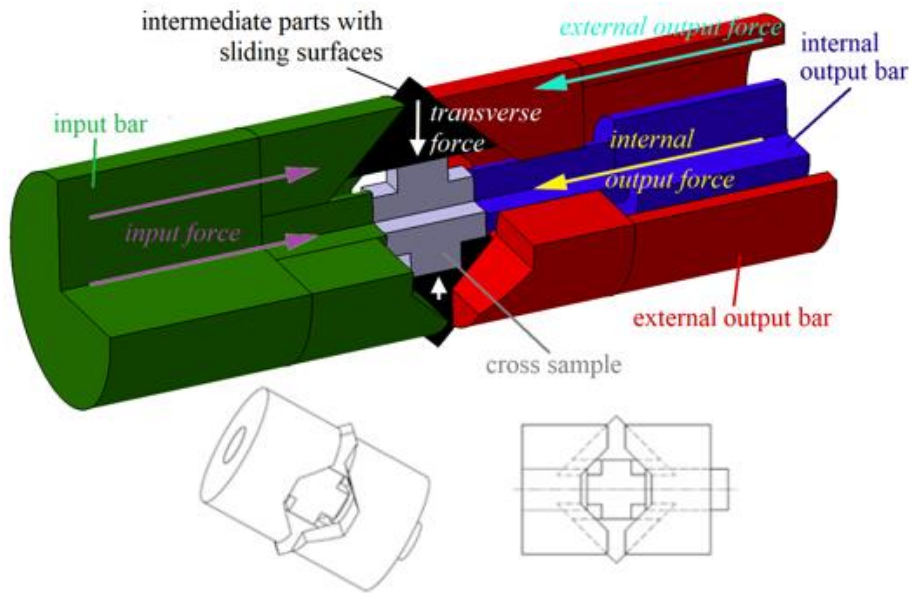
58 From the short review above, it can be seen that the multiaxial testing design is still a  
59 tough issue. There are no commonly admitted testing set-ups and the design of such a test  
60 depend on the aimed loading state and on specimens. This paper is focused on biaxial  
61 compression and a new concept of Hopkinson bar system has been designed and tested. Its  
62 principle and its characteristics are described in Section 2. Then, Sections 3 and 4 present  
63 respectively the raw experimental results obtained from calibration tests and the analysis of a  
64 bi-axial test thanks to numerical simulations.

## 65 **2 Design of the new set-up**

### 66 2.1 Set-up characteristics

67 The designed bi-axial set-up uses a mechanism with intermediate parts with sliding  
68 surfaces and a cruciform specimen. This mechanism with sliding surfaces at  $45^\circ$  with respect

69 to the axial direction was placed between the single input bar, the internal output bar and the  
 70 co-axial external output tube (Figure 1). After the striker impact on the input bar, the internal  
 71 output bar measures the axial loading of the cross sample whereas the external output bar (the  
 72 single tube that surrounds the internal output bar) measures the transversal one via the  
 73 mechanism.



74  
 75 Figure 1: Schematic of the experimental configuration (cut view) with the relative motion of  
 76 each part and the applied forces (up). Three-dimensional (down, left) and uncut (down, right)  
 77 views without the three bars.

78 Using a single loading impulse avoids the difficulty due to non-simultaneous impacts  
 79 occurring with perpendicular Hopkinson bar devices [13], [14]. Another advantage of such a  
 80 design lies in the fact that the ratio between the axial and the transverse loadings is imposed by  
 81 the ratio between the external and the internal output bar impedances and by the sliding surface  
 82 angle of the mechanism.

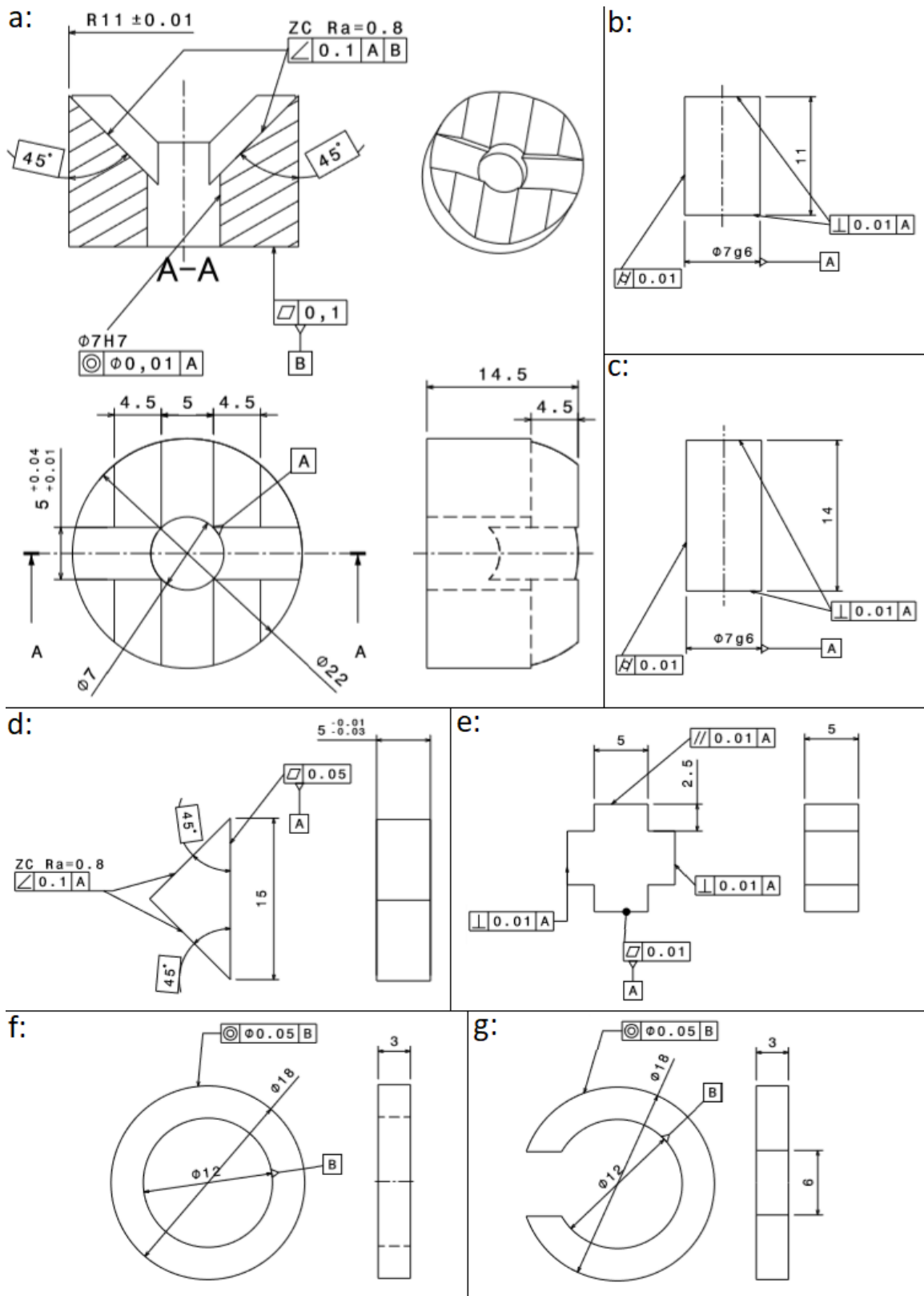
83 The pressure bars are made of steel and their characteristics are given in Table 1. The  
 84 external and the internal output bars have nearly the same impedance in order to ensure, on an

85 isotropic sample, an approximate equality between the internal and the external loadings, and  
86 therefore between the axial and the transverse loadings, thanks to the mechanism. The  
87 difference between the internal radius of the external output bar ( $R_{ieo}$ ) and the internal output  
88 bar radius ( $R_{io}$ ) is 3 mm, which is sufficient to place the gauges glued on the internal bar and  
89 their cables. These electrical cables exit from the external bar at the output end (i.e. at the  
90 opposite of the set-up). Nylon bearings are inserted between the two co-axial output bars and  
91 some of them are opened (Figure 2g) to let a passage to the cables.

bar	density	wave celerity	radius		length
			external	internal	
striker	$\rho_i = 8050 \text{ kg.m}^{-3}$	$C_i = 4600 \text{ m.s}^{-1}$	$R_i = 11 \text{ mm}$		1.25 m
input					4 m
internal output	$\rho_{io} = 7800 \text{ kg.m}^{-3}$	$C_{io} = 5100 \text{ m.s}^{-1}$	$R_{io} = 6 \text{ mm}$		2 m
external output	$\rho_{eo} = 7400 \text{ kg.m}^{-3}$	$C_{eo} = 5200 \text{ m.s}^{-1}$	$R_{eoo} = 11 \text{ mm}$	$R_{ieo} = 9 \text{ mm}$	2 m

92 Table 1: Mass densities, tensile-compressive wave celerities, radii and lengths of the bars.

93 The detailed drawings of the set-up parts are given in Figure 2. The sample size  
94 (boundary to boundary) is  $10 \text{ mm} \times 10 \text{ mm}$  and its thickness is 5 mm (Figure 2e). The shape of  
95 the parts with the two sliding surfaces (noted “a” in Figure 2) has been chosen to maximize the  
96 stiffness. One of this part (the left-hand one in Figure 1, in green) is inserted between the input  
97 bar and the two transverse triangular parts (noted “d”) and the other (the right-hand one in  
98 Figure 1, in red) is inserted between the two transverse triangular parts and the external output  
99 bar.



100

101 Figure 2: Part drawings (general tolerances: 0.1 mm); a: parts with the sliding surfaces, b: left-

102 hand cylinder, c: right-hand cylinder, d: transverse triangular parts, e: sample, f: bearings, g:

103 opened bearings.

104 The cylinder noted “b” in Figure 2 is inserted between the input bar and the sample, and  
105 the cylinder “c” is inserted between the sample and the internal output bar (see Figure 1). Both  
106 cylinders apply the axial loading on the cross sample and the right-hand one in Figure 1 (the  
107 “c”) is free to have an axial motion relatively to the right-hand part with the sliding surfaces  
108 (“a”).

## 109 2.2 Forces and velocities in the set-up

110 Strain gauges are glued at the middle of the input bar and on the two output bars close  
111 to the sample but far enough from the ends (at 0.374 m on the external bar and at 0.612 m on  
112 the internal one) to be within the Saint-Venant conditions.

113 After the striker impact, the input bar gauge measures an incident compressive wave  $\varepsilon_i$   
114 followed by a reflected tensile wave  $\varepsilon_r$ . Moreover, the external output gauge measures a  
115 transmitted wave in the external output bar,  $\varepsilon_{et}$  and the internal output gauge also measures a  
116 transmitted wave in the internal output bar,  $\varepsilon_{it}$ .  $\varepsilon_i$  can be seen as a loading imposed to the biaxial  
117 set-up whereas  $\varepsilon_r$ ,  $\varepsilon_{et}$  and  $\varepsilon_{it}$  can be seen as the set-up response to the imposed loading.

118 These strain waves have to be virtually transported from the gauge positions to the  
119 interfaces between the bars and the set-up presented in Figure 1, down. Thus,  $\varepsilon_i$  has to be  
120 delayed and  $\varepsilon_r$ ,  $\varepsilon_{et}$  and  $\varepsilon_{it}$  have to be shifted forward of the duration necessary for the waves to  
121 propagate from the measurement gauge to the corresponding interface. Then the Hopkinson  
122 formulae enable for the determination of the forces and of the velocities at the interfaces from  
123 these transported waves and from the bar parameters given in Table 1:

$$124 \quad (1) \quad F_i = -\rho_i C_i^2 \pi R_i^2 (\varepsilon_i + \varepsilon_r)$$

$$125 \quad (2) \quad V_i = C_i (\varepsilon_r - \varepsilon_i)$$



126 (3)  $F_{eo} = -\rho_{eo} C_{eo}^2 \pi (R_{eoo}^2 - R_{ieo}^2) \varepsilon_{et}$

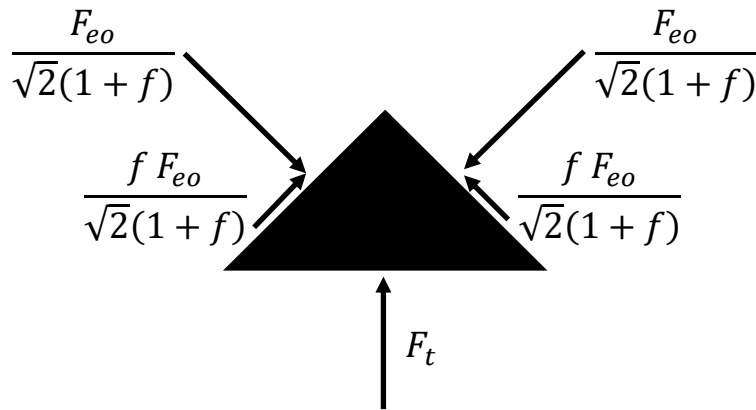
127 (4)  $V_{eo} = -C_{eo} \varepsilon_{et}$

128 (5)  $F_{io} = -\rho_{io} C_{io}^2 \pi R_{io}^2 \varepsilon_{it}$

129 (6)  $V_{io} = -C_{io} \varepsilon_{it}$

130  $F_i$  and  $V_i$  are the force and the velocity at the interface between the input bar and the set-  
 131 up shown in Figure 1,  $F_{eo}$  and  $V_{eo}$  are the force and the velocity at the external output bar  
 132 interface and  $F_{io}$  and  $V_{io}$  are the force and the velocity at the internal output bar interface.

133 Under the assumption of an equilibrium state in the set-up, the axial force in the sample  
 134 can be assumed to be equal to the internal output force  $F_{io}$  and the transverse force  $F_t$  can be  
 135 deduced from the external output force  $F_{eo}$ . Indeed, each transverse triangular part of the  
 136 mechanism is axially submitted to half  $F_{eo}$  and by taking account of friction, the mechanical  
 137 equilibrium of a transverse triangular part leads to Figure 3.



138

139 Figure 3: Equilibrium of a transverse triangular part axially submitted to half  $F_{eo}$ .

140 As the Figure 3 upper triangle moves from top to bottom, the applied friction forces are  
141 oriented from bottom to top. Besides, according to the Coulomb's laws, the friction force over  
142 normal force ratio is imposed to be equal to the friction coefficient at the mechanism sliding  
143 interfaces, noted  $f$ . The axial projection of the force transmitted by a sliding surface corresponds  
144 to half  $F_{eo}$  (the axially applied force) and the transverse projection corresponds to  $F_t$ , which  
145 leads to:

$$146 \quad (7) \quad F_t = \frac{1-f}{1+f} F_{eo}$$

### 147 **3 Experimental results**

#### 148 **3.1 Strain gauge measurement and image processing**

149 Axial strain gauges are glued on each of the three bars and the measurement frequency  
150 is 500 kHz. The Hopkinson formulae (equations (1), (3) and (5)) permit for the calculation of  
151 the forces at the interfaces between the bars and the set-up from these measurements. The  
152 camera trigger signal is measured in the same time-basis as the gauge voltages. The image  
153 which was being recorded in the camera at the arrival of the trigger signal being known, both  
154 measurements can be time shifted.

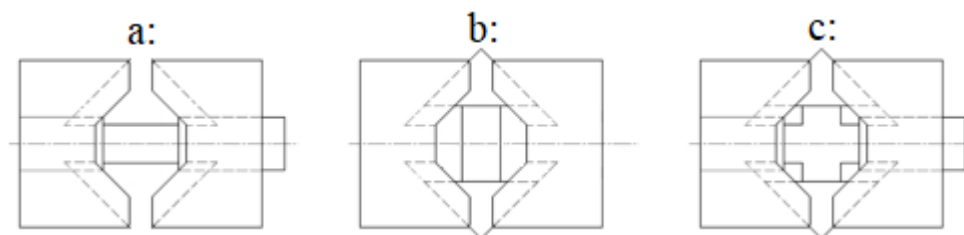
155 The speckled samples are observed during the tests thanks to an SA5 high-speed camera  
156 whose frequency is 50 kHz at a definition of 512 pixels  $\times$  272 pixels. Images of the samples are  
157 acquired during the tests. The first one is called reference image and the displacement between  
158 each image and the reference is calculated thanks to Digital Image Correlation (DIC). DIC is  
159 performed by using the in-house Correli RT3 software [16], [17]. The displacement field is  
160 defined over a finite-element mesh made of triangular 3-noded elements (T3). The chosen  
161 element size is 10 pixels. The mean displacements on the four sample interfaces and the  
162 resulting axial and transverse elongations are determined from DIC. The displacement is given

163 in pixels, which are converted into millimeters knowing the sample dimension. In order to  
164 control the uncertainty of the DIC calculation, an elastic regularization is used [16], [17].

165 The relative weight applied to the reference solution can be seen as the fourth power of  
166 a length called the regularization length. A too high regularization length may lead to erroneous  
167 estimations of the experimental displacement field because this field is thus constrained to be  
168 close to an elastic solution. As a result, the DIC calculations are processed with a regularization  
169 length decreasing from a value corresponding to 3 times the element size (30 pixels) to a value  
170 equal to the element size (10 pixels). DIC gives identical results when the length varies from  
171 30 to 20 pixels but the results obtained with a 10 pixel length are sometimes a bit noisy and  
172 slightly different from the previous ones. To reduce the noise, a 20-pixel length is thus chosen  
173 for the processing.

### 174 3.2 Identification of the sample material behavior

175 The samples are made of an AW-2017A aluminum. The material stress-strain law is  
176 identified from simple compression tests on cuboid samples ( $5\text{ mm} \times 5\text{ mm} \times 10\text{ mm}$ ) inserted  
177 between the input bar and the internal output bar (the external output bar being removed, see  
178 Figure 4a).



179  
180 Figure 4: Schematics of the tests; a: cuboid sample axially compressed, b: cuboid sample  
181 transversally compressed, c: cross sample bi-axially compressed.

182 The compression force in the sample (Figure 4, case a) thus corresponds to the internal  
183 output force  $F_{io}$ . The sample axial and transverse elongations in the image plan are determined  
184 from DIC. The real cross section (which increases because of compression), and then the true  
185 stress, can thus be calculated. Meanwhile, the axial logarithmic strain is deduced from the  
186 corresponding axial elongation. These measurements lead to repeatable axial behaviors.

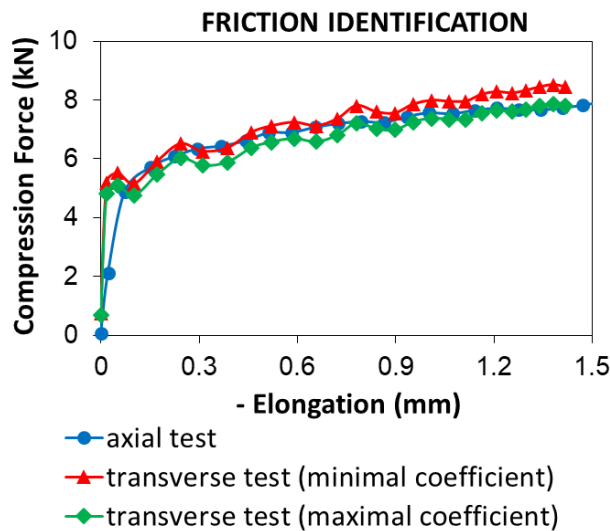
### 187 3.3 Identification of the friction at the mechanism sliding interfaces

188 The friction coefficient in the mechanism of this new bi-axial testing device is a key  
189 parameter which has to be determined under the bi-axial test conditions. In order to reproduce  
190 the reached sliding velocities at the sliding interfaces, identical AW-2017A cuboid samples are  
191 inserted between the input bar and the external output bar, and are compressed via the  
192 mechanism (the internal output bar being removed, see Figure 4b). The ratio between the  
193 transverse compression force  $F_t$  and the external output force  $F_{eo}$  is thus friction dependent  
194 according to relation (7).

195 The set-up being far too small to insert a cell able to measure the compression force in  
196 the sample, this force will be deduced from relation (7) and from the  $F_{eo}$  measurement. The  
197 friction coefficient  $f$  will be estimated knowing that the force-elongation laws identified during  
198 the axial (Figure 4a) and the transverse (Figure 4b) tests must be the same because the tested  
199 cuboid samples are the same too.

200 According to Figure 5, by multiplying the external output force  $F_{eo}$  measured during the  
201 transverse test by a constant ratio, a satisfactory fit can be obtained between the transverse and  
202 the axial tests when the displacements become high enough. However, for low displacements,  
203 the ratio should be a bit lower to obtain a satisfactory fit. It implies that the friction is first higher  
204 (adhesion phase) and then decreases when the displacements become high enough (sliding  
205 phase), which is finally consistent with the Coulomb's law.

206 Figure 5 represents typical curves obtained from an axial test and from a transverse one,  
 207 chosen as average behaviors. The transverse curve shows that the adhesion phase is not  
 208 stationary, unlike the sliding one. The friction will thus be identified only during the sliding  
 209 phase. The noise of the transverse force-elongation curve will lead to an uncertainty on the  
 210 friction. By fitting, in the sliding phase, the transverse curve points with the minimal forces and  
 211 the axial curve, a lower bound of the friction coefficient is obtained. Inversely, by fitting, in the  
 212 sliding phase, the transverse curve points with the maximal forces and the axial curve, an upper  
 213 bound of the coefficient is obtained. One obtains  $0.05 < f < 0.09$ . Such values are consistent  
 214 with Vaseline lubricated interfaces. It finally leads to a 0.87 average  $F_v/F_{eo}$  ratio with a 4 %  
 215 relative uncertainty.



216  
 217 Figure 5: Friction identification by fitting the transverse test and the axial one.

218 The friction coefficient magnitude and the corresponding uncertainty knowledge does  
 219 not matter in itself, but it will enable for the determination of the transverse force and of the  
 220 corresponding uncertainty during the bi-axial test.

221 Without clearances and without intermediate strains in the set-up, the corresponding  
 222 interface sliding velocity should be equal to the difference between the input velocity and the

223 external output one, or to the opposite of the transverse sample elongation rate, both divided by  
224  $\sqrt{2}$  (because of the 45° angle):

$$225 \quad (8) \quad V_{int,bars} = \frac{V_i - V_{eo}}{\sqrt{2}}$$

$$226 \quad (9) \quad V_{int,sample}(t) = \frac{\Delta l(t-\Delta t) - \Delta l(t+\Delta t)}{2\sqrt{2} \Delta t}$$

227  $V_{int,bars}$  is the interface sliding velocity estimated from the bar interface velocities  
228 (relations (2) and (4)) whereas  $V_{int,sample}$  is the same quantity estimated from the sample  
229 elongation  $\Delta l$ . Formula (9) corresponds to a numerical differentiation,  $t$  being a measurement  
230 instant and  $\Delta t$  the elongation acquisition time (given by the camera).

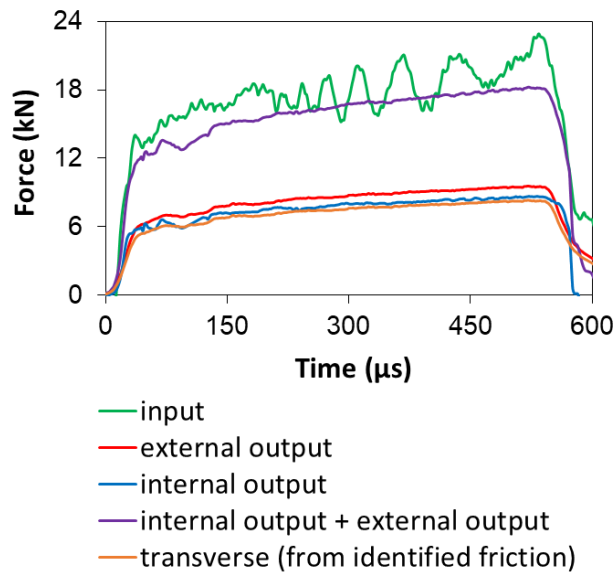
231 These velocity estimations show that a stationary phase with an interface sliding  
232 velocity of around 1 to 3 m/s begins for an elongation of approximately 0.1 mm, which is  
233 consistent with the beginning of the phase with a constant friction identified in Figure 5.

### 234 3.4 Analysis of the bi-axial test measurements

235 An AW-2017A cross sample is tested by using the whole bi-axial mechanism and the  
236 two output bars (see Figure 1 or Figure 4c). The forces applied by the bars on the bi-axial set-  
237 up are determined from the strain gauge measurements and from the Hopkinson formulae and  
238 are shown in Figure 6.

239 Figure 6 shows that the input force and the total output force are in satisfactory  
240 equilibrium, despite of the complex 3-dimensionnal wave propagation phenomena occurring in  
241 the bi-axial mechanism. According to Section 2.2, the axial compression force in the sample  
242 corresponds to  $F_{io}$  whereas the transverse compression force  $F_t$  is lower than  $F_{eo}$  but almost  
243 equal because of the low friction (in Section 3.3, the estimated  $F_t/F_{eo}$  ratio is roughly 0.87). The

244 obtained transverse and axial forces ( $F_t$  and  $F_{io}$ ) being very close, Figure 6 thus displays that  
245 the force loading path in the sample is rather equi-bi-axial.

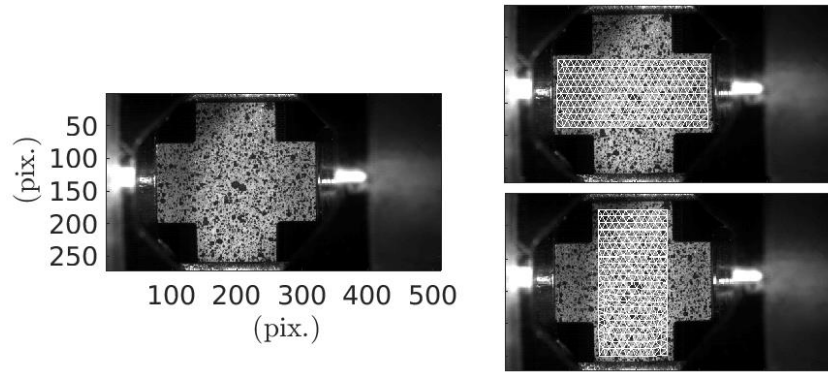


246

247 Figure 6: Time evolutions of the forces at the interfaces.

248 As the external and the internal output bars have nearly the same impedance, the  
249 velocities, and thus the displacements, at the external and the internal output bar boundaries are  
250 also very similar. If there were no clearances and no strains occurring in the mechanism  
251 intermediate parts, the transverse displacement of each of the two triangular parts would be half  
252 the difference between the input bar and the external output bar displacements. The triangular  
253 parts moving symmetrically, the sample elongations, in both directions, would thus be exactly  
254 equal to the difference between the displacements at the input bar and at the output bar  
255 interfaces. It displays that the elongation loading path may be also rather equi-bi-axial.

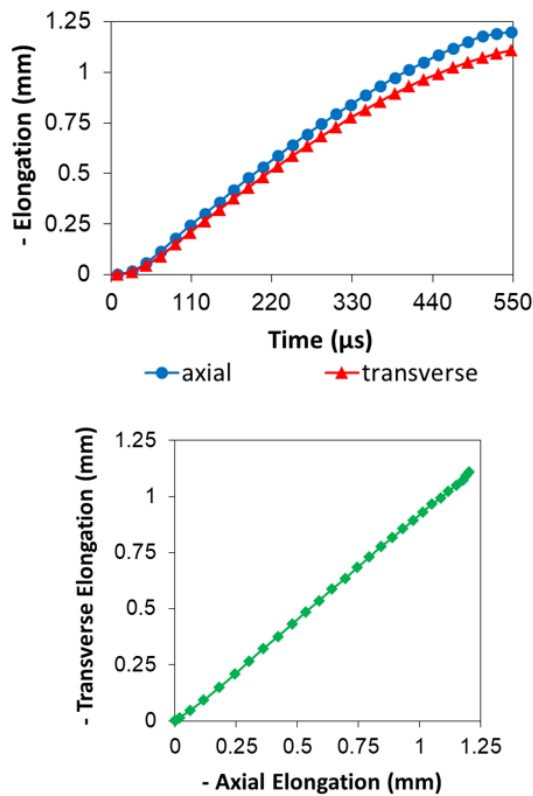
256 The cross sample axial and transverse elongations are determined from the images and  
257 from DIC. The DIC reference image and the corresponding meshes on the sample are shown in  
258 Figure 7.



259

260 Figure 7: Reference image (left), mesh used to determine the axial displacements (right, up)

261 and mesh used to determine the transverse displacements (right, down).



262

263 Figure 8: Time-evolutions of (the opposite of) the elongations in both directions (up) and

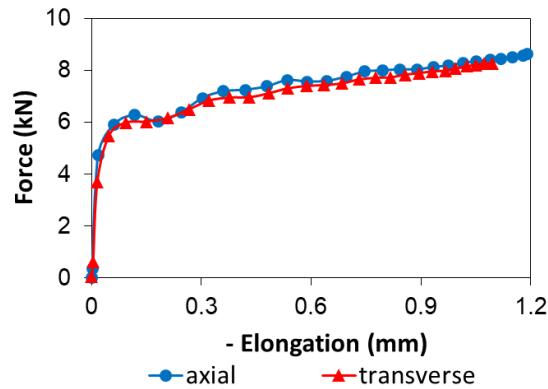
264 corresponding loading path (down).

265 The obtained elongation time-evolutions and the corresponding transverse-axial loading

266 path can be seen in Figure 8, which clearly confirms that an almost equi-bi-axial loading is



267 imposed to the sample. This bi-axial state can also be seen in Figure 9, which also shows the  
268 set-up capacities in both directions.



269

270 Figure 9: Sample force-elongation curves in axial and transverse directions.

271 Formulae (8) and (9) show that the sliding velocity reached at the interface during the  
272 stationary phase is of the order of 1 to 3 m/s, like during the friction identification test. It  
273 confirms the relevance of the friction coefficient value used for data processing.

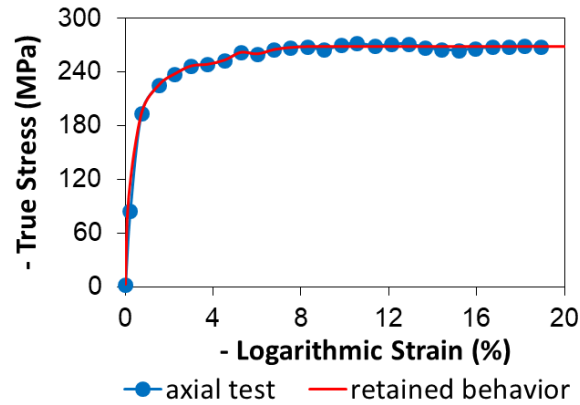
## 274 **4 Check of the measurement consistency thanks to simulations**

### 275 4.1 Material modelling

276 A typical result obtained from an axial compression test on a cuboidal sample (Figure  
277 4a), and chosen as a reference is reported in Figure 10.

278 As there are few measurement points in the elastic phase, a 70 GPa Young modulus and  
279 a 0.3 Poisson's ratio are assumed (AW-2017A characteristics). A yield stress - plastic strain  
280 law with a 100 MPa elastic threshold, and which exactly fits the measurements is then chosen  
281 (see retained behavior in Figure 10). To simplify the modelling, the stress is supposed to  
282 saturate at a 268 MPa threshold value, which also well fits the measurements. In practice, the  
283 axial curves are reproducible for strains lower than 3 %, but a 4 % dispersion is observed for

284 the threshold value. These parameters are implemented in a Von-Mises elastic-plastic model  
285 with an isotropic hardening. The used software is ABAQUS.



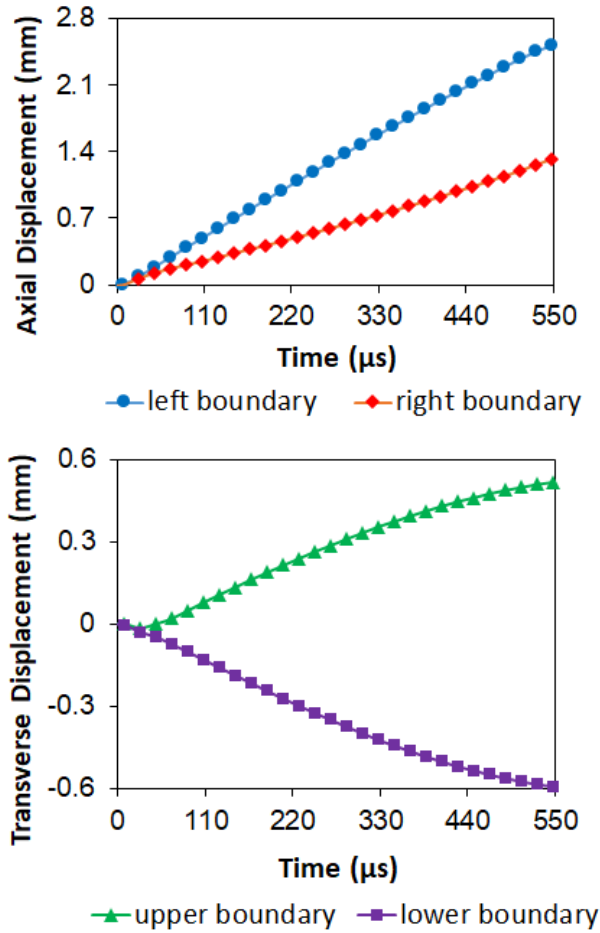
286

287 Figure 10: Stress-strain law identified from a test.

## 288 4.2 Simulation of the sample behavior

289 Because of its cross-shape, the stress and the strain states in this sample are  
290 heterogeneous. So, to check the force and elongation measurement consistency, a finite-element  
291 simulation must be performed. The calculations are performed with the Figure 10 retained  
292 behavior. The fact that the model identification test has been carried out in the same dynamic  
293 conditions as the bi-axial test implies that the method remains valid with a strain rate dependent  
294 behavior. If any contact occurs between the arm free boundaries, it will be supposed frictionless.  
295 According to Figure 6, the bi-axial set-up, and therefore the cross sample, are in a satisfactory  
296 equilibrium. This equilibrium state can be accurately verified by processing a dynamic explicit  
297 calculation. The chosen density is  $2800 \text{ kg.m}^{-3}$ .

298 The displacements estimated thanks to DIC calculations are directly imposed to the  
299 sample interfaces (Figure 11). Because of the image plan symmetry, only half the sample has  
300 to be modelled (Figure 12). 8-node linear brick elements are used. The chosen brick size is  
301 0.25 mm, which leads to the Figure 12 mesh.



302

303

Figure 11: Imposed boundary displacements (axial ones oriented from left to right and

304

transverse ones oriented from top to bottom).

305

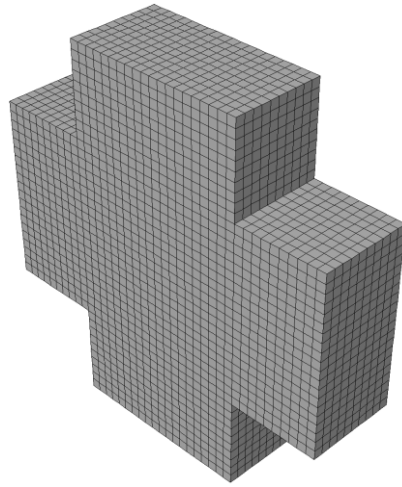
The dynamic explicit calculations display that the forces applied at opposite boundaries are almost equal until 60 μs, and become exactly the same after this duration. The sample equilibrium assumption being checked, the axial force is defined as the left and right boundary force average and the transverse force as the upper and lower boundary force average.

309

The numerical and the experimental forces in both directions can then be compared to check the measurement consistency. Because of the behavior dispersion (see Section 4.1), a 4 % relative uncertainty must be considered for the numerical forces. The friction is taken into account by determining the measured transverse force with the 0.87 estimated  $F_t/F_{eo}$  ratio (see end of section 3.3). The 4 % relative uncertainty of this ratio also leads to a 4 % uncertainty of

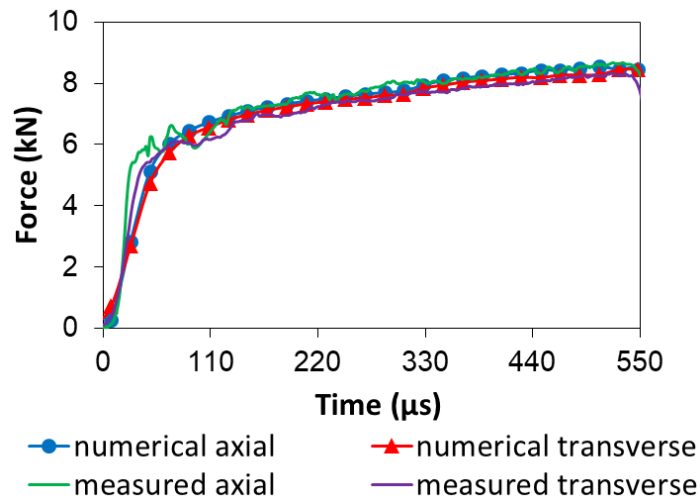
313

314 the measured transverse force. It could be noted that the uncertainty due to friction is not so  
315 high.



316

317 Figure 12: Meshed sample model accounting of the symmetry plan.



318

319 Figure 13: Comparison between the experimental forces and the numerical forces obtained  
320 from the measured displacements and from the identified model.

321 In Figure 13, the numerical forces are obtained at the displacement measurement  
322 frequency (i.e. every 20  $\mu\text{s}$ ). As expected, the fit is not perfect at the very beginning of the test  
323 because of the following reasons:

- 324 - These is a too weak number of acquisition points in the elastic phase to accurately  
325 measure the brutal increase of the forces.
- 326 - The sample equilibrium being not perfect during the first 60  $\mu\text{s}$ , the axial and the  
327 transverse forces in the cross sample do not exactly correspond to measured forces at  
328 the output bar interfaces (i.e.  $F_{io}$  and  $F_{eo}$  multiplied by the friction dependent ratio).
- 329 - The transverse force is determined from the sliding friction coefficient, and as explained  
330 in section 3.3, this method overestimates the force at the very beginning of the test, i.e.  
331 in the adhesion phase.

332 However, during the stationary phase, the measurement consistency is clearly proven.  
333 It validates the identified sample model. Although this processing is not performed to measure  
334 the friction, it also shows that the friction coefficient used to process the data is consistent.

### 335 4.3 Simulation of the whole apparatus behavior

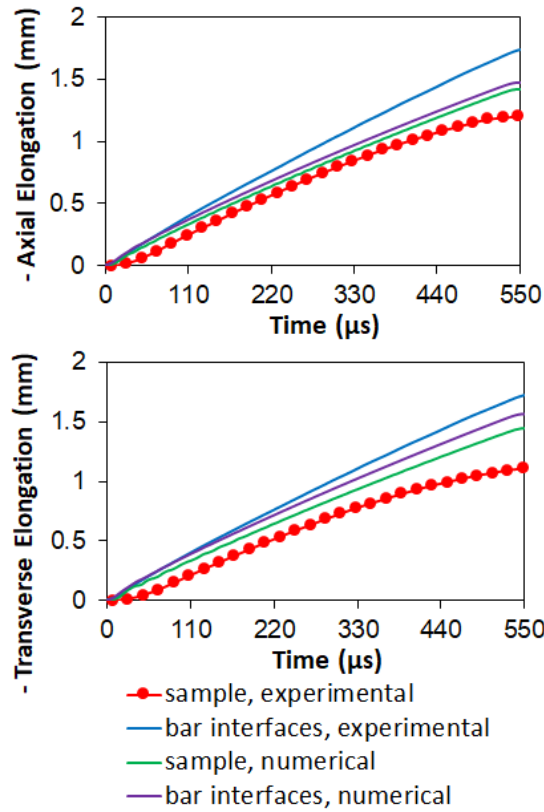
336 The axial and transverse forces in the sample are both determined from the forces at the  
337 interfaces between the output bars and the set-up. This determination is based on the  
338 equilibrium assumption, which actually implies a quick enough transmission of the wave  
339 through the mechanism. A satisfactory, but not perfect, equilibrium is experimentally shown in  
340 Figure 6. However, the incident strain wave  $\varepsilon_i$  and the reflected one  $\varepsilon_r$  being rather opposite, as  
341 shown in Figure 15, the input force  $F_i$  determination from formula (1) is very noise sensitive.  
342 A simulation of the whole bar set-up has thus been performed to study this equilibrium. The  
343 used software is still ABAQUS in its explicit version.

344 The aluminum sample is supposed to have the properties given in Sections 4.1 and 4.2.  
345 The bi-axial mechanism parts are supposed to be made of the input bar steel (it is actually the  
346 case) whose Poisson's ratio is 0.3 (density and wave celerity given in Table 1), and to remain  
347 purely elastic. The same assumptions are used to model the output bars (densities and wave

348 celerities also given in Table 1). The striker initial velocity is  $6 \text{ m.s}^{-1}$ , which is consistent with  
349 experimental measurements (Figure 15). The selected friction coefficient at the contact surfaces  
350 is 0.07, as measured in Section 3.3. The static and the dynamic coefficients are supposed to be  
351 the same.

352         Because of the two symmetry planes (the cutting plane in Figure 1 and the one normal  
353 to the section plane and containing the axis), only a quarter of the system is studied. The bars  
354 and the projectile are modelled by 6-node linear triangular prism elements (ABAQUS  
355 terminology) whose approximate size is 5 mm. The two parts with the sliding surfaces and the  
356 tubes inserted between the sample and the input bar and between the sample and the internal  
357 output bar are modelled by the same elements but their rough sizes are respectively 10 mm and  
358 2.5 mm. The triangular part is composed of two triangular prism elements separated by the  
359 symmetry plane. The sample is merely composed of four 8-node linear brick elements: one  
360 corresponding to the center and three corresponding to the three arms represented in the  
361 simplified model. As shown in Figure 15, element sizes are thin enough to roughly fit the  
362 measurements. The chosen time increment is  $0.1 \mu\text{s}$ .

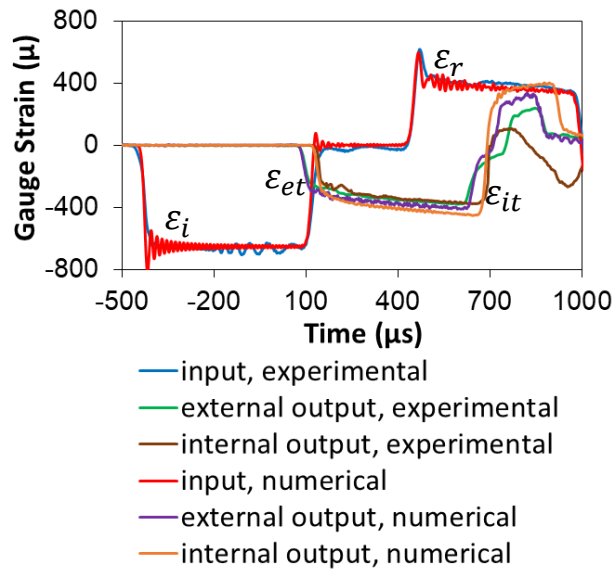
363         The experimental results display that the contacts in the bi-axial mechanism are not  
364 perfect. In Figure 14, a non-negligible gap can be seen between the sample elongations and the  
365 differences between the input and the output bar displacements. As explained in Section 3.4,  
366 these quantities would be identical if there was neither clearances nor intermediate strains. The  
367 numerical equivalent of this gap is lower, and only due to the elastic strains of the mechanism  
368 parts. It implies that the numerical set-up is a little bit stiffer than the real one. However, the  
369 stiffness remaining in the same order, it enables to check the general relevance of the set-up.



370

371 Figure 14: Comparisons between the sample elongations and the differences between the bar  
 372 displacements.

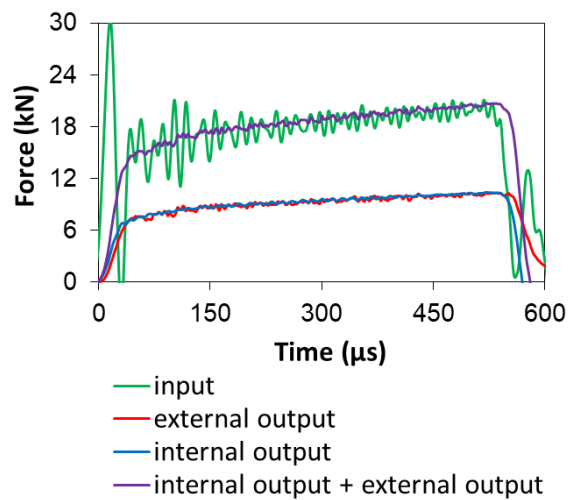
373 The gauges are glued at the middle of the input bar and on the two output bars at 0.374 m  
 374 from the interface with the set-up on the external bar, and at 0.612 m on the internal one. The  
 375 experimental and the numerical strain time-evolutions are compared in Figure 15. The input  
 376 gauge first measures the incident wave  $\varepsilon_i$ , and then the reflected wave  $\varepsilon_r$ . The external output  
 377 gauge and the internal one measure the external transmitted wave  $\varepsilon_{et}$  and the internal transmitted  
 378 wave  $\varepsilon_{it}$ .  $\varepsilon_i$  can be seen as a loading imposed to the bi-axial set-up whereas  $\varepsilon_r$ ,  $\varepsilon_{et}$  and  $\varepsilon_{it}$  can be  
 379 seen as the set-up response to the imposed loading. Figure 15 shows that the simulated  $\varepsilon_i$   
 380 (proportional to the striker initial velocity) rather fits the experimental one, despite some  
 381 spurious oscillations. The numerical transmitted waves are a little bit higher than the  
 382 experimental ones, which is fully consistent with a modelled set-up stiffer than the real one.



383

384 Figure 15: Experimental and numerical strain time evolutions at the gauge positions.

385 Figure 16 finally shows that, in spite of numerical oscillations, the input force and the  
 386 total output force are in mean equilibrium and that both output forces are the same.

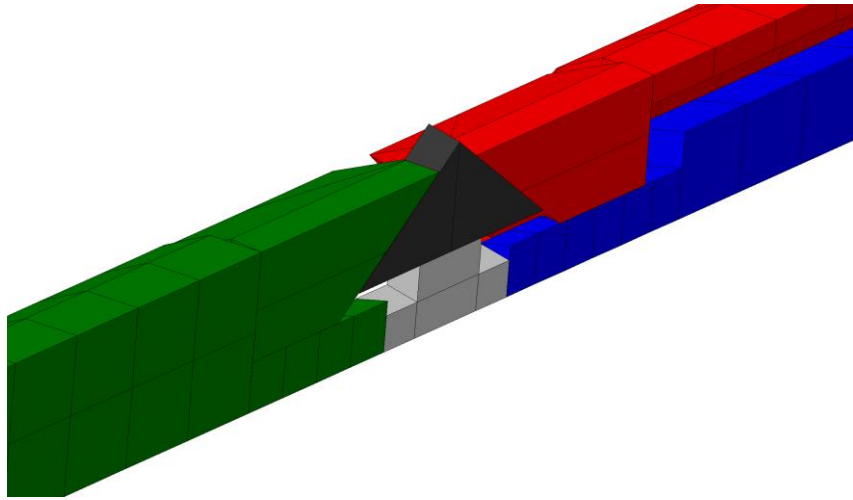


387

388 Figure 16: Time evolutions of the numerical forces at the interfaces.

389 The finite element mesh can be seen in Figure 17. As the contacts are not perfectly  
 390 modeled, using a smaller mesh would be useless to better fit the measurements.





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Figure 17: Finite element mesh.

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## **5 Conclusion**

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The aim of the study was to check the relevance of a newly designed Hopkinson bi-axial compression set-up. The sample axial compression is directly generated by an internal output bar whereas the transverse compression is indirectly generated by an external output bar via a mechanism. The friction in the mechanism is identified in the relevant dynamic conditions from comparisons between axial and transverse compression tests on cuboid samples. To reproduce the sliding velocities reached at the mechanism sliding interfaces during the bi-axial test, these cuboid samples are made of the bi-axial sample material. The friction being identified, any sample can now be tested.

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A calibration bi-axial test has been performed on a cross sample with a simple shape, and therefore easy to model. The measurements show that the device ensures an approximate equality between the axial and the transverse loadings, and the experimental result reliability is proven by a numerical modelling of the bi-axial sample. The set-up can therefore be used in the future for characterization on more complex samples and materials (which was not the aim of this first study).

408 Finally, the design of an easily buildable set-up generating and measuring a rather  
409 isotropic dynamic bi-axial loading has been achieved.

## 410 **Conflict of Interest**

411 The authors declare that they have no conflict of interest.

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