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**Evaluation of visco-elastic sandwich beam damping and natural frequency  
variability  
using a modal stability procedure**

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**SUMMARY**

The modal stability procedure is used along with Monte-Carlo simulation to quantify the variability of frequency and damping ratios of a rectangular three-layered simply supported sandwich visco-elastic beam. The asymptotic numerical method is used to compute the complex eigenfrequencies at nominal in the case of a high damping frequency dependent material (3M ISD112). The results are compared with analytical data.

**Key Words: Variability, Modal Stability Procedure, Visco-elastic sandwich**

**1. INTRODUCTION**

Visco-elastic sandwiches are widely used in the industry as vibration and noise dampers. Since last years, variability of the damping properties of visco-elastic sandwich structures has become a subject of interest. Indeed, visco-elastic materials damping performances can experience high variations due to randomness in material properties [1]. As far as noise/vibration reduction performances are concerned, it is important for several industrial applications to quantify accurately these variations to be able to choose the most suitable application specific damping treatment. A straightforward way to characterize this variability relies on performing Monte-Carlo simulations (MCS) of a numerical model that computes the damping properties of visco-elastic structures. Indeed, since many years, the use of robust finite element methods [2] to predict frequency response curves and/or modal properties of visco-elastic structures in linear and/or non-linear vibrations is commonplace. Combining MCS with a finite element solver can seem promising due to its robustness and simplicity which led to the development of stochastic finite elements (SFEM)[3]. However, it is widely known that performing MCS with finite element solvers involves high computational costs to achieve reasonable levels of accuracy owing to the law of large numbers and the number of random variables considered. A way to circumvent this limitation is to substitute the full numerical model by fast and reliable surrogates that approximate accurately the response while reducing the computational burden [3,4]. In the context of SFEM, many efficient alternatives to reduce computational times have been proposed for visco-elastic structures. Guedri *et al.* [5] used a modal perturbation technique along with asymptotic numerical method within an intrusive SFEM approach with perturbation to compute visco-elastic structures FRF's variability. More recently Sepahvand *et al* [6] used polynomial chaos expansions within a non-intrusive spectral SFEM framework as suggested in Sarsri *et al* [7] to characterize the

variability of damping performances of visco-elastic sandwich structures with spatial random material properties. AMG de Lima *et al.* [8] proposed a promising scheme called component mode synthesis for intrusive SFEM with perturbation along with quasi-Monte-Carlo method to determine FRF's variability. This method belongs to the class of modal re-analysis structural methods [9] that provide efficient numerical procedures for computing the perturbed output responses due to modifying the properties of the structure, without having to solve the equilibrium equation problem several times. Moreover, it is independent of the SFEM formulation that makes use of Karhunen-Loève expansions that are difficult to implement in practical situations and it can be efficiently coupled to MCS given its low computational times. In the same category of methods, one can find the modal stability procedure (MSP) [9] that assumes weak sensitivity of mode shapes to variations in the input parameters of the model. Furthermore, it is a non-intrusive method tailored for high dimensional variability analyses using black-box industrial finite element solvers. In the present work MSP with Monte-Carlo sampling will be used. The stochastic variables, supposed independent, are the Young modulus of the faces and the static shear modulus of the visco-elastic layer. The finite element model (section 2) [2,12] and the asymptotic numerical method with automatic differentiation [10] are employed to obtain the frequencies and damping ratios for nominal values. Then, the MSP (section 3) is applied to derive the first and second order statistical moments. The use case is a simply supported visco-elastic frequency dependent sandwich beam in free linear vibrations whose material properties experience lognormal stochastic variations (section 4). The results will be compared to MCS combined with Rao's analytical formula [11].

## 2. FINITE ELEMENT MODEL

A finite element model of a visco-elastic sandwich beam in free vibrations is built. The kinematics are described by the zig-zag model of Rao [11] along with the convolution visco-elastic modeling to get the governing equations of motion. The interested reader can refer to [2,12] for more details. The vibration problem with frequency dependent visco-elastic material leads to a non-linear eigenvalue problem of the form

$$(K_0(\omega) - E(\omega)K_v(\omega) - \omega^2 M)U = 0 \quad (1)$$

with  $K_0, K_v$  the elastic and visco-elastic rigidity matrixes respectively,  $M$  the mass matrix and  $E(\omega)$  the visco-elastic Young modulus. Eq. (1) is solved using the Diamant approach [10] that implements the asymptotic numerical method with automatic differentiation. The beam is meshed with 200 elements, a truncation order of 20, a maximal number of iterations of 15 and a tolerance of  $10^{-10}$  are considered for solving (1) via Diamant.

## 3. THE MODAL STABILITY PROCEDURE

The MSP assumption considers weak sensitivity of the mode shapes to variations in the input parameters of the model. A single finite element analysis is required, then the MSP formulation is developed. Fast MCS to this formulation provide statistic quantities ( mean, standard deviation and distribution ) leading to evaluate the variability of frequency, damping ratio or FRF. The MSP method is non-intrusive and intended to be used with industrial-size models with a large number of degrees of freedom and a large number of random variables. In the present work, the MSP approach is used as follows :

- Eq (1) is solved exactly using Diamant at nominal values of the faces Young modulus  $E_f$  and visco-elastic layer shear modulus  $G_0$  and  $p$  nominal mode shapes  $(U_0^1, U_0^2, \dots, U_0^p)$  and complex eigenvalues  $(\omega_0^1, \omega_0^2, \dots, \omega_0^p)$  are determined.

- $N^{mac}$  independent trial samples for  $E_f$  and  $G_0$  are generated from the lognormal distribution with prescribed mean  $\mu_{E_f}$  and  $\mu_{G_0}$  and coefficients of variation  $\delta_{E_f}$  and  $\delta_{G_0}$  respectively. The problem (1) is solved exactly  $N^{mac}$  times and perturbed mode shapes  $U_p^i$ , damping ratios  $\eta_p^i$  and frequencies  $f_p^i$  are obtained. Then complex MAC values and error indicators  $\mathcal{E}_f^i$  on the frequency and  $\mathcal{E}_\eta^i$  on the damping are computed to verify the modal stability assumption as follows

$$MAC^i = \frac{(U_0^i, U_p^i)^2}{(U_0^i, U_0^i) * (U_p^i, U_p^i)} \quad (4)$$

$$\mathcal{E}_f^i = \left\| \frac{(f_p^{iDiamant} - f_p^{iMSP})}{f_p^{iDiamant}} \right\|_\infty \quad (5)$$

$$\mathcal{E}_\eta^i = \left\| \frac{(\eta_p^{iDiamant} - \eta_p^{iMSP})}{\eta_p^{iDiamant}} \right\|_\infty \quad (6)$$

with  $(\cdot, \cdot)$  the Hermitian scalar product and  $\|\cdot\|_\infty$  the infinite norm. As the MAC numbers are complex their modulus and phase are compared to 1 and 0 respectively.

- For each nominal mode shape  $U_0^i$ , Eq. (1) is written as follows

$$(\omega^i)^2 = \frac{\sum_{e=1}^{e=N} {}^T \{U_0^i\}^e * \{K_0\}^e * \{U_0^i\}^e}{\sum_{e=1}^{e=N} {}^T \{U_0^i\}^e * \{M\}^e * \{U_0^i\}^e} + E(\omega) * \frac{\sum_{e=1}^{e=N} {}^T \{U_0^i\}^e * \{K_v\}^e * \{U_0^i\}^e}{\sum_{e=1}^{e=N} {}^T \{U_0^i\}^e * \{M\}^e * \{U_0^i\}^e} \quad (2)$$

where  $\{\cdot\}^e$  denotes a quantity at the elemental level and  $N$  the number of finite elements of the model. As suggested in [5] the elemental quantities  ${}^T \{U_0^i\}^e * \{K_0\}^e * \{U_0^i\}^e$  and  ${}^T \{U_0^i\}^e * \{M\}^e * \{U_0^i\}^e$  can be written as  $\sum_{l=1}^{N_k} (\prod_{j=1}^{N_m} \xi_j^{\alpha_j^l}) {}^T \{U_0^i\}^e * k_l^e * \{U_0^i\}^e$  and

$\sum_{l=1}^{N_k} (\prod_{j=1}^{N_m} \xi_j^{\alpha_j^l}) {}^T \{U_0^i\}^e * m_l^e * \{U_0^i\}^e$  respectively. The  $N_k$  matrices  $k_l^e$  and  $m_l^e$  are rigidity and mass matrices respectively that depend only on the finite element shape functions and the  $N_m$  quantities  $\xi_j$  are material properties that can be subjected to random variations. The indices  $\alpha_j^l$  takes value 1 or 0 depending on the presence of the random variable  $\xi_j$  in the product or not. Noting  $c_k^{il} = {}^T \{U_0^i\}^e * k_l^e * \{U_0^i\}^e$  and  $c_m^{il} = {}^T \{U_0^i\}^e * m_l^e * \{U_0^i\}^e$  Eq (2) can be written as

$$(\omega^j)^2 = \frac{\sum_{e=1}^{e=N} \sum_{l=1}^{N_k} \left( \prod_{j=1}^{N_m} \xi_j^{\alpha_j^l} \right) * c_{k_0}^{il}}{\sum_{e=1}^{e=N} \sum_{l=1}^{N_k} \left( \prod_{j=1}^{N_m} \xi_j^{\alpha_j^l} \right) * c_m^{il}} + E(\omega) * \frac{\sum_{e=1}^{e=N} \sum_{l=1}^{N_k} \left( \prod_{j=1}^{N_m} \xi_j^{\alpha_j^l} \right) * c_{k_v}^{il}}{\sum_{e=1}^{e=N} \sum_{l=1}^{N_k} \left( \prod_{j=1}^{N_m} \xi_j^{\alpha_j^l} \right) * c_m^{il}} \quad (3)$$

The quantities  $c_*^{il}$  are computed once and for all for the  $p$  nominal mode shapes. For the beam model in bending (our case), we can write that

$$\{M\}^e = (2\rho_f S_f + \rho_c S_c) m^e$$

$$\{K\}^e = (2(1+\nu_c)) \left( (I_c E^*(\omega) + \frac{E_f S_f h_c^2}{2}) k_1^e - \frac{E_f S_f h_c h_f}{2} k_2^e + (2E_f I_f + \frac{E_f S_f h_f^2}{2}) k_3^e + S_c E^*(\omega) \right)$$

with  $E^*(\omega) = E(\omega) + E(0)$ . All the quantities indexed with  $_c$  are related to the visco-elastic layer and the quantities indexed with  $_f$  are related to the elastic faces.  $h$  is the thickness,  $I$  the quadratic moment,  $S$  the surface of the section,  $\rho$  the density and  $\nu$  the Poisson coefficient

- For each MCS trial, Eq. (3) is solved using a Newton-Raphson procedure where the first derivative is provided by means of automatic differentiation of the function  $E(\omega)$ . The starting point is chosen as the complex frequencies  $\omega_0^j$ . The cost of each trial is then equal to the cost of finding the root of a one variable complex function which is negligible compared to the resolution of (1) using Diamant. In this work,  $P$  is equal to 6 and  $N^{mac}$  is taken equal to 50.

## 4. USE CASE

A three layered simply supported beam with a visco-elastic layer of 1 mm and a total thickness of 5 mm is considered. The length is  $L=300$  mm and the width is  $b=30$  mm. The elastic faces are composed of aluminium whose nominal characteristics are a Young's modulus  $E_f$  of 69 GPa, a Poisson ratio of 0.3 and a density of 2766 kg/m<sup>3</sup>. The visco-elastic frequency dependent material 3M ISD112 is used.

The Young modulus of 3M ISD112 at 27°C is given by  $E(\omega) = 2(1+\nu_c) G_0 \sum_{k=1}^{k=3} \frac{\delta_k \omega}{\omega - i\Omega_k}$  with

$$\nu_c = 0.5, \quad G_0 = 0.5 \times 10^6 \text{ Pa},$$

$$\delta_1 = 0.746, \delta_2 = 3.265, \delta_3 = 43.284, \Omega_1 = 468.7 \text{ Hz}, \Omega_2 = 4742.4 \text{ Hz}, \Omega_3 = 71532.5 \text{ Hz}$$

In the present work, the Young modulus  $E_f$  of the elastic faces and the shear modulus  $G_0$  of the visco-elastic layer are subject to lognormal stochastic variations around their nominal values  $\mu_{E_f}$  and  $\mu_{G_0}$  with coefficients of variation  $\delta_{E_f}$  and  $\delta_{G_0}$  respectively.

Variable	Nominal values	Coefficient of variation	Law
$E_f$	$\mu_{E_f} = 69 \times 10^9$ Pa	$\delta_{E_f} = 10\%$	Lognormal
$G_0$	$\mu_{G_0} = 0.5 \times 10^6$ Pa	$\delta_{G_0} = 10\%$	Lognormal

Table 1. Random variables, nominal values, coefficients of variation and law.

## 5. RESULTS

### 5.1 Modal stability verification

In a first step, the MSP is compared at nominal values with the Diamant approach. The errors  $\mathcal{E}_f^i$  and  $\mathcal{E}_\eta^i$  are given in Table 2 for each mode.

Mode	$\mathcal{E}_f^i$	$\mathcal{E}_\eta^i$
1	6.95e-7	9.9e-7
2	2.58e-6	3.4e-6
3	3.27e-7	4.4e-7
4	4.14e-7	6.0e-7
5	1.87e-7	2.6e-7
6	6.1e-9	8.1e-9

Table 2. Relative maximal errors between Diamant and MSP at nominal.

Then, the modal stability assumption is checked by drawing  $N^{mac} = 50$  values from the lognormal distribution for  $E_f$  and  $G_0$  and performing simulations using Diamant and MSP in order to compute MAC values and error indicators for the sixth first modes. The range of variations of the norm and phase angle of the MAC numbers computed for each mode are given in Table 3. It can be easily noticed that the norm of the MAC is close to one whereas the phase angle is close to 0 for all the modes which shows that the modal stability assumption holds.

Mode	Norm of MAC	Phase of MAC
1	0.99 - 1.0	-1.2e-3 - 2.1e-3
2	0.99 - 1.0	-2.3e-4 - 3.7e-4
3	0.99 - 1.0	-1.0e-4 - 1.7e-4
4	0.99 - 1.0	-6.1e-5 - 1.03e-4
5	0.99 - 1.0	-4.3e-5 - 6.97 e-5
6	0.99 - 1.0	-2.9e-5 - 4.95e-5

Table 3. Norms and phases of MAC numbers

The error indicators are given in Table 4. One can notice that the MSP the maximal relative error of the MSP on the damping is 5.5% and on the frequency 0.36%. These numbers give us confidence in the accuracy of the MSP.

Mode	$\mathcal{E}_f^i$	$\mathcal{E}_\eta^i$
1	3.56e-3	5.47e-2
2	1.66e-3	1.97e-2
3	1.18e-3	1.22e-2
4	7.76e-4	9.54e-3
5	5.31e-4	7.51e-3
6	4.12e-4	6.08e-3

Table 4. Relative maximal errors between Diamant and MSP for perturbed inputs.

Moreover, computational times are given in Table 5 for Diamant and MSP on the basis of 6 modes. One can easily notice that the MSP is 200 hundred times faster than Diamant with acceptable error levels.

Solver	CPU time
Diamant	100 s
MSP	0.5 s

Table 5. CPU time for Diamant and MSP for computing 6 modes.

## 5.2 Monte-Carlo simulation using MSP for 3MISD112

A Monte-Carlo simulation with 100000 trials is performed for 3MISD112 with a variability specified in Table 1. The results are displayed in Table 6. The 95% confidence intervals are computed. One can see that the sampling is accurate since the values of the CIs are very small compared to the mean. For example for mode 6 the mean frequency is 1747 Hz +/- 0.517 which is quite honorable.

Mode	Damping	Frequency
1	Mean =22% , CoV=9.8%,CI=1.4e-4	Mean=58.16 Hz , CoV=3.9%,CI=1.4%
2	Mean=16.7% , CoV=10.9%,CI=1.14e-4	Mean=205.9 Hz , CoV=4.4%,CI=5.6%
3	Mean=14.4% , CoV=11.5%,CI=1.0e-4	Mean=448.5 Hz , CoV=4.64%,CI=13%
4	Mean=11.2% , CoV=11.8%,CI=8.7e-5	Mean=788 Hz , CoV=4.7%,CI=23%
5	Mean=10% , CoV=11.8%,CI=7.4e-5	Mean=1221 Hz , CoV=4.74%,CI=36%
6	Mean=8.9% , CoV=11.7%,CI=6.5e-5	Mean=1747 Hz , CoV=4.78%,CI=51.7%

Table 6. Mean, coefficient of variations and confidence intervals for the sixth first modes using Monte-Carlo with MSP for 3MISD112

Moreover, statistical tests ( $\chi^2$  goodness of fit, Lilliefors test and Jarque-Bera test) are performed to determine whether the outputs are lognormally distributed or not. The results are shown in Table 7.

Mode	Damping	Frequency
1	Chi2=0,Lilli=0,JB=0	Chi2=1,Lilli=1,JB=0
2	Chi2=0,Lilli=0,JB=0	Chi2=1,Lilli=1,JB=0
3	Chi2=1,Lilli=1,JB=1	Chi2=1,Lilli=1,JB=0
4	Chi2=1,Lilli=1,JB=1	Chi2=1,Lilli=1,JB=1
5	Chi2=1,Lilli=1,JB=1	Chi2=1,Lilli=1,JB=1
6	Chi2=1,Lilli=1,JB=1	Chi2=1,Lilli=1,JB=1

Table 6. Results of statistical tests for the sixth first modes (0=fail,1=success)

One can notice that the three tests agree on the fact that the modes 4,5 and 6 have lognormally distributed damping ratios and frequencies, whether the situation is not clear for modes 1, 2 and 3. If mode 3 seems to have log normally distributed damping and eigen-frequency, mode 1 and 2 seem to have non lognormally distributed damping ratios and lognormally distributed frequencies.

## 6. CONCLUSION

The modal stability procedure along with Monte-Carlo sampling have been used to quantify the variability of a three layered sandwich beam for 3M ISD112 a high damping frequency dependent visco-elastic material. In the future, a more complex geometry (plate, laminated plate, etc.) has to be considered and temperature dependence should be included.

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