

# Von Kármán approximation for the vibrations of post-buckled beams: supplementary material

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## Abstract

We plot figures 2, 3, 4, and 5 of the article for different values of  $L/h$ .

## 1 Notations

The beam has length  $L$ , width  $w$ , thickness  $h$ , Young's modulus  $E$  and density  $\rho$ . The bending rigidity is then  $EI = Eh^3w/12$  and the stretching rigidity  $EA = Ehw$ , where  $A$  is the surface area and  $I$  the second moment of area of the cross-section. We consider slender beams, with a small slenderness ratio  $\eta = I/(AL^2) = (1/12)(h/L)^2$ . We use the following scheme to non-dimensionalize the equations

$$y = Y/L \quad , \quad x = X/L \quad (1)$$

$$\mathbf{n} = \mathbf{N}L^2/(EI) \quad , \quad m = ML/(EI) \quad (2)$$

$$t = T/\tau \quad , \quad \text{with } \tau = L^2\sqrt{\rho A/(EI)} \quad (3)$$

and we note that  $\Omega = \omega/\tau$  is the physical angular frequency in radians per second. The (fixed) axial displacement  $D$  is defined as  $D = L - X(L, T)$  and we note  $d = D/L$ ,  $T$  being the physical time in seconds.

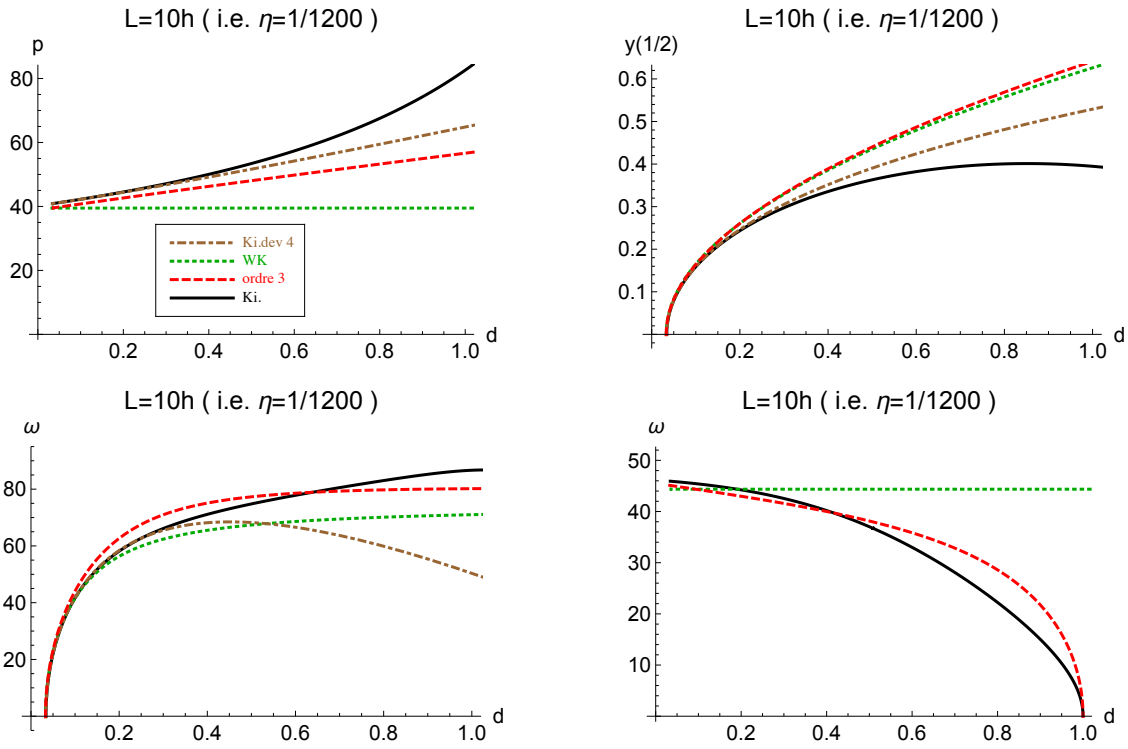


Figure 1: The two upper graphs show post-buckled equilibrium curves for the load  $p_E$  and the transverse deflection at midspan  $y(1/2)$  vs axial displacement  $d$ . The two lower graphs show the frequency  $\omega$  of the 1st and 2nd vibration modes around the buckled equilibrium. The curves for the Kirchhoff model (noted Ki.), the Woinowsky-Krieger model (noted WK), the ordre 3 model (noted ordre 3), and the 4th order development of Kirchhoff model (noted Ki. dev 4) are shown. 'Ki. dev 4' is not plotted for the 2nd vibration mode.

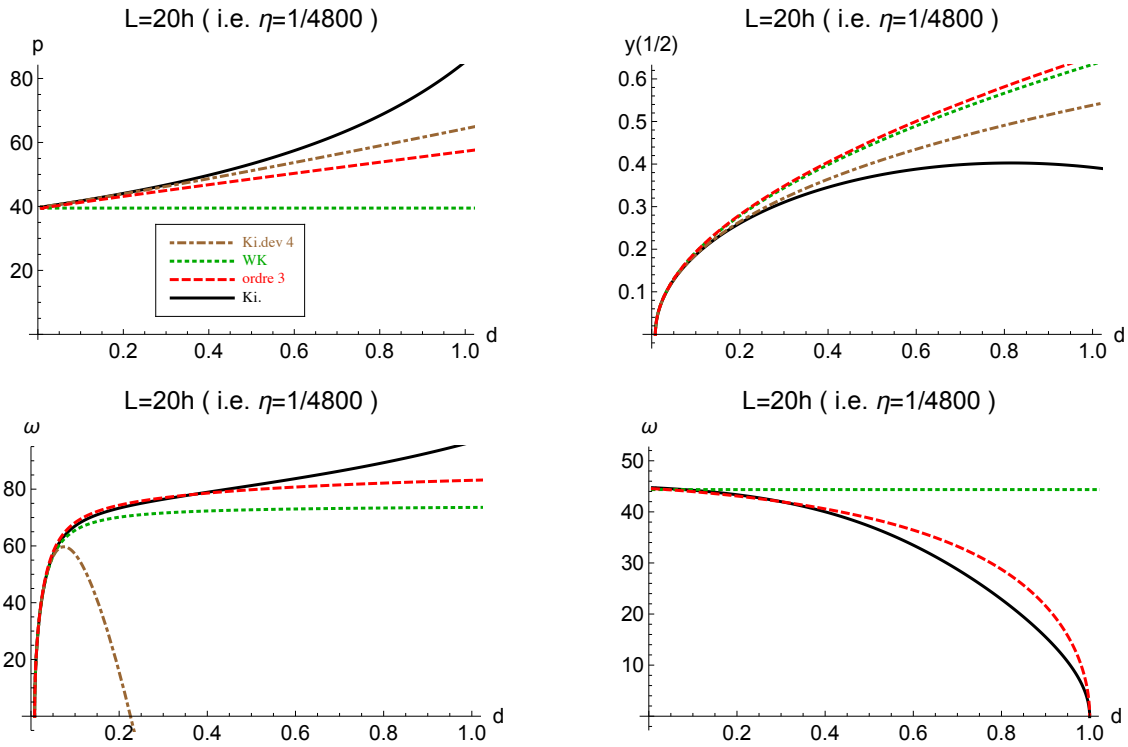


Figure 2: The two upper graphs show post-buckled equilibrium curves for the load  $p_E$  and the transverse deflection at midspan  $y(1/2)$  vs axial displacement  $d$ . The two lower graphs show the frequency  $\omega$  of the 1st and 2nd vibration modes around the buckled equilibrium. The curves for the Kirchhoff model (noted Ki.), the Woinowsky-Krieger model (noted WK), the ordre 3 model (noted ordre 3), and the 4th order development of Kirchhoff model (noted Ki. dev 4) are shown. 'Ki. dev 4' is not plotted for the 2nd vibration mode.

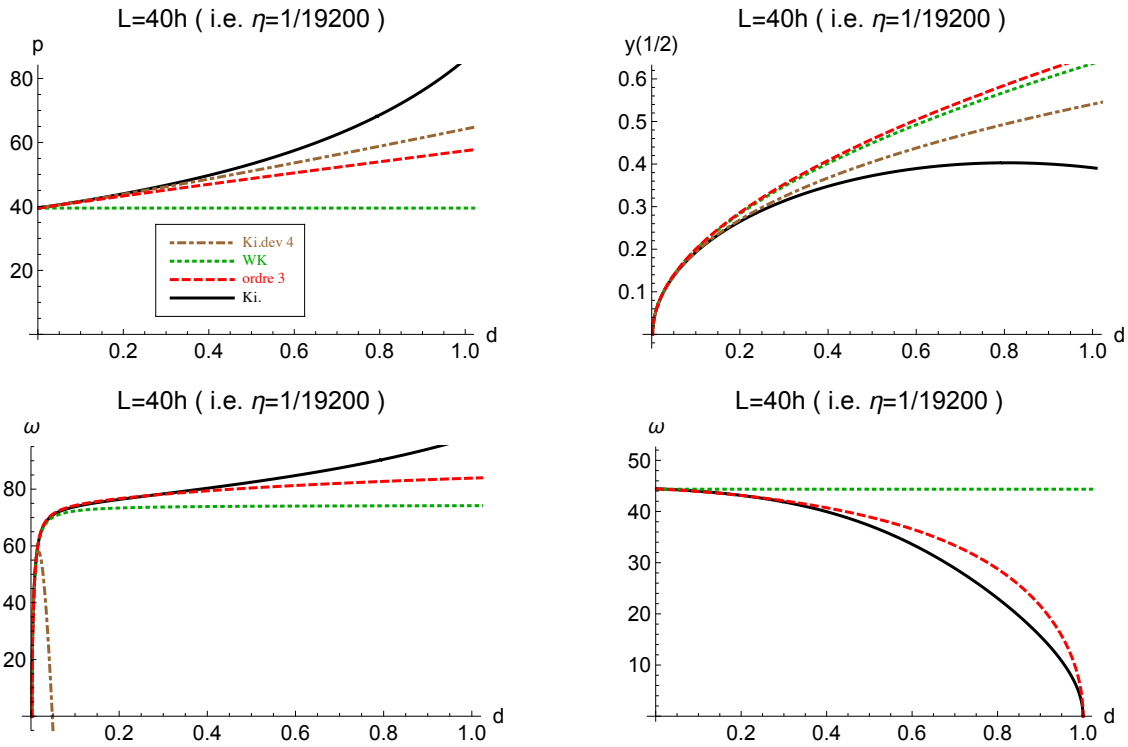


Figure 3: The two upper graphs show post-buckled equilibrium curves for the load  $p_E$  and the transverse deflection at midspan  $y(1/2)$  vs axial displacement  $d$ . The two lower graphs show the frequency  $\omega$  of the 1st and 2nd vibration modes around the buckled equilibrium. The curves for the Kirchhoff model (noted Ki.), the Woinowsky-Krieger model (noted WK), the ordre 3 model (noted ordre 3), and the 4th order development of Kirchhoff model (noted Ki. dev 4) are shown. 'Ki. dev 4' is not plotted for the 2nd vibration mode.

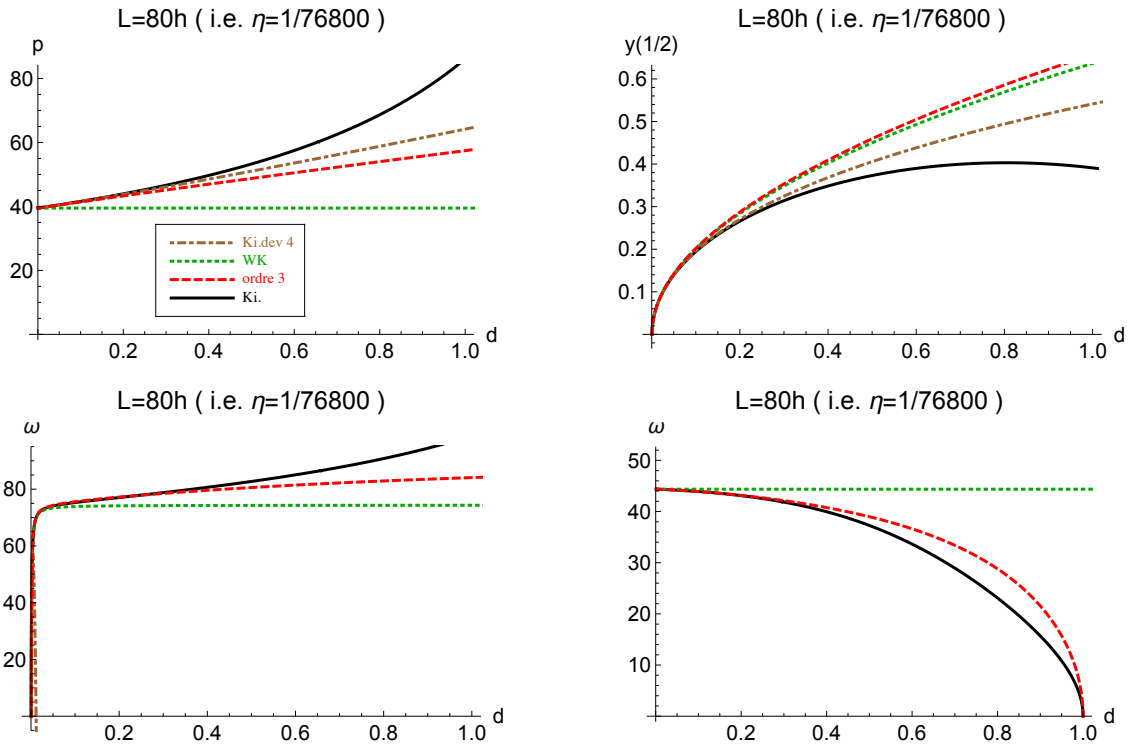


Figure 4: The two upper graphs show post-buckled equilibrium curves for the load  $p_E$  and the transverse deflection at midspan  $y(1/2)$  vs axial displacement  $d$ . The two lower graphs show the frequency  $\omega$  of the 1st and 2nd vibration modes around the buckled equilibrium. The curves for the Kirchhoff model (noted Ki.), the Woinowsky-Krieger model (noted WK), the ordre 3 model (noted ordre 3), and the 4th order development of Kirchhoff model (noted Ki. dev 4) are shown. 'Ki. dev 4' is not plotted for the 2nd vibration mode.