Colloquium on Networks and Evolution - 2020-09-15 Sorbonne Université (Paris)

# Phylogenetic networks: how advanced are the methods? 

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## Outline

- A quick introduction to phylogenetic networks
- Advances in phylogenetic networks:
- simplifying models
- knowing the network space
- finding new techniques
- using powerful tools
- putting everything together!

Phylogenetic networks: generalizing phylogenetic trees

## Phylogenetic networks: generalizing phylogenetic trees

Phylogenetic tree of a set of species:

- Classify them depending on common characters
$\rightarrow$ classification
- Describe their evolution

ORDRE présumé de la formation des Animaux,
offrant 2 séries séparées, subrameuses.


Lamarck: Histoire naturelle des animaux sans vertèbres (1815)

Poissons
Reptiles
Oiseaux
Mammifères.

## Phylogenetic networks: generalizing phylogenetic trees

Phylogenetic tree of a set of species:

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on common characters
- Describe their evolution
$\rightarrow$ modelization


Woese, Kandler, Wheelis: Towards a natural system of organisms: proposal for the domains Archaea, Bacteria, and Eucarya,
Proceedings of the National Academy of Sciences 87(12), 4576-4579 (1990)

## Abstract and explicit phylogenetic networks

Phylogenetic network: network representing evolution data

- abstract / data-display phylogenetic networks: to classify, visualize data




## Abstract and explicit phylogenetic networks

Phylogenetic network: network representing evolution data

- explicit phylogenetic networks: to model evolution



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Phylogenetic network: network representing evolution data

- explicit phylogenetic networks: to model evolution

reticulations: nodes with >1 parent, modeling hybridization, recombination, lateral gene transfer, etc.


## Explicit phylogenetic networks



A gallery of explicit phylogenetic networks : http://phylnet.univ-mlv.fr/recophync/networkDraw.php

## How hard is it to reconstruct a network?

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Quite hard.

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In practice:
« 9.1 Limitations
The biggest limitation of methods to infer introgression and hybridization, including species network methods, is scalability.

Methods which infer a species network directly from multilocus sequences have only been used with a handful of taxa, and less than 200 loci. »

[^0]
## So, what should we do?

# So, what should we do? What has been done? 

> M KEEP CALM SIMPDLIFY YOUR MODEL

## Just put gene trees together

The "hybridization network" problem (ignore duplication, loss, ILS, etc.):
given 2 trees, find the smallest network containing both of them with the minimum number of reticulations

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Easy to find a network containing the two trees!
But $n$ hybrid vertices for trees with $n$ leaves: not optimal!

T1


T2


add a root above the two trees, glue the leaves together


## Just put gene trees together

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NP-hard to minimize the number of reticulations

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NP-hard to minimize the number of reticulations

Even checking a solution (Tree Containment Problem) is hard!
Kanj, Nakhleh, Than \& Xia (2008) Theoretical Computer Science

## The Tree Containment Problem

Input: A binary phylogenetic network $N$ and a tree $T$ over the same set of taxa.

## Question: Does $N$ display $T$ ?

$\rightarrow$ Can we remove one incoming arc, for each vertex with $>1$ parent in $N$, so that the obtained tree is equivalent to $T$ (each $\operatorname{arc}$ in $T$ is a path in $N$ )?


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## The Tree Containment Problem

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tree containment
``` \(-\)


Tushar Agarwal, Philippe Gambette \& David Morrison (2016),

Who is Who in
Phylogenetic
Networks: Articles,
Authors and
Programs, arXiv

\section*{Subclasses of phylogenetic networks}


\section*{Subclasses of phylogenetic networks}


\section*{Understanding tree containment}


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}

\section*{Bounding the size of phylogenetic networks}

How many nodes can a network on \(n\) leaves have?
\(\rightarrow\) unbounded for general networks
\(\rightarrow\) for nearly-stable networks:
- 26n-24

Philippe Gambette, Andreas Gunawan, Anthony Labarre, Stéphane Vialette and Louxin Zhang. Locating a Tree in A Phylogenetic Network in Quadratic Time. RECOMB 2015
- \(8 n-7\)

Andreas Gunawan and Louxin Zhang. Bounding the Size of a Network Defined By Visibility Property. arXiv, 2015.

\section*{Counting phylogenetic networks}

How big is the search space?
\(\rightarrow\) analytic combinatorics techniques to count the number of networks in some subclasses
\begin{tabular}{c||c|c|c|c}
\(n\) & \(g_{n-1}\) & \(r_{n}\) & \(u_{n-1}\) & \(\ell_{n}\) \\
\hline 1 & 0 & 1 & 0 & 1 \\
2 & 1 & 3 & 1 & 18 \\
3 & 2 & 36 & 6 & 143 \\
4 & 15 & 723 & 135 & 120078 \\
5 & 192 & 20280 & 5052 & 17643570 \\
6 & 3450 & 730755 & 264270 & 3332111850 \\
\hline as \(n \rightarrow \infty\) & \(c_{1} \approx 0.20748\) & \(c_{1} \approx 0.1339\) & \(c_{1} \approx 0.07695\) & \(c_{1} \approx 0.02931\) \\
\(x_{n} \sim c_{1} c_{2}^{n} n^{n-1}\) with & \(c_{2} \approx 1.89004\) & \(c_{2} \approx 2.943\) & \(c_{2} \approx 5.4925\) & \(c_{2} \approx 15.4333\) \\
\hline OEIS reference & A 328121 & A 328122 & A 333005 & A 333006
\end{tabular}

Mathilde Bouvel, Philippe Gambette \& Marefatollah Mansouri, Counting Phylogenetic Networks of Level 1 and 2, accepted to JOMB

\section*{Counting phylogenetic networks}

How big is the search space?
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\[
\begin{gathered}
\phi(z)^{n}=\sum_{i \geq 0} \sum_{k=0}^{i} \sum_{p=0}^{k} \sum_{q=0}^{p} \sum_{s=0}^{q} \times\binom{ n+i-1}{i}\binom{i}{k}\binom{k}{p}\binom{p}{q}\binom{q}{s}\left(\frac{12 z}{4(1-z)^{4}}\right)^{i-k}\left(\frac{-30 z^{2}}{4(1-z)^{4}}\right)^{k-p} \\
\left(\frac{32 z^{3}}{4(1-z)^{4}}\right)^{p-q}\left(\frac{-16 z^{4}}{4(1-z)^{4}}\right)^{q-s}\left(\frac{3 z^{5}}{4(1-z)^{4}}\right)^{s} \\
=\sum_{i \geq 0} \sum_{k=0}^{i} \sum_{p=0}^{k} \sum_{q=0}^{p} \sum_{s=0}^{q}\binom{n+i-1}{i}\binom{i}{k}\binom{k}{p}\binom{p}{q}\binom{q}{s} \frac{(3)^{i}\left(\frac{-15}{6}\right)^{k}\left(\frac{-16}{15}\right)^{p}\left(\frac{-1}{2}\right)^{q}\left(\frac{-3}{16}\right)^{s}}{(1-z)^{4 i}} \\
\times z^{i+k+p+q+s .}
\end{gathered}
\]

Mathilde Bouvel, Philippe Gambette \& Marefatollah Mansouri, Counting Phylogenetic Networks of Level 1 and 2, accepted to \(J O M B\)

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How big is the search space?
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counting


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How big is the search space?
\(\rightarrow\) analytic combinatorics techniques to count the number of networks in some subclasses

The next step: random generation of phylogenetic networks?


\section*{Local moves}

\section*{How to explore the search space?}
\(\rightarrow\) NNI moves

Katharina Huber, Vincent Moulton and Taoyang Wu. Transforming phylogenetic networks: Moving
 beyond tree space. JTB 404:30-39, 2016.

Philippe Gambette, Leo van Iersel, Mark Jones, Manuel Lafond, Fabio Pardi and Celine Scornavacca. Rearrangement Moves on Rooted Phylogenetic Networks. PLoS Computational Biology 13(8): e1005611.1-21, 2017.

\section*{Local moves}

\section*{How to explore the search space?}

\section*{\(\rightarrow\) NNI moves, SPR moves}

Katharina Huber, Vincent Moulton and Taoyang Wu. Transforming phylogenetic networks: Moving beyond tree space. JTB 404:30-39, 2016.

Philippe Gambette, Leo van Iersel, Mark Jones, Manuel Lafond, Fabio Pardi and Celine Scornavacca. Rearrangement Moves on Rooted Phylogenetic Networks. PLoS Computational Biology 13(8): e1005611.1-21, 2017.

Magnus Bordewich, Simone Linz and Charles Semple. Lost in space? Generalising subtree prune and regraft to spaces of phylogenetic networks. JTB 423:1-12, 2017

NNI moves


SPR distance \(-\)


\section*{KEEP}

CALM AND FIND NEW TECHNIQUES

\section*{New techniques developed for phylogenetic networks}
- agreement forests: to compute the SPR distance between trees and to solve the hybridization problem between 2 trees


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- agreement forests: to compute the SPR distance between trees and to solve the hybridization problem between 2 trees
- cherry picking: to solve the hybridization problem between > 2 trees
- network decompositions: to solve the tree containment problem on reticulation visible networks


\section*{M KEEP CALM AND USE POWERFUL TOOLS}

\section*{Fixed parameter tractability (FPT algorithms)}
- find an appropriate parameter \(k\) which is small: number of reticulations, level of the network, etc.
- look for an FPT algorithm in \(k\) : computation time in \(O(f(k) \times \operatorname{poly}(n))\)
- computation time may be huge depending on \(k\)
- the problem remains tractable when \(n\) (the number of taxa) increases

Laurent Bulteau \& Mathias Weller, Parameterized Algorithms in Bioinformatics: An Overview, Algorithms 12(12):256, 2019


\section*{Visualization minimizing edge crossings}


FPT algorithm snake-style

polynomial-time solvable!
ear-style


Jonathan Klawitter \& Peter Stumpf. Drawing Tree-Based Phylogenetic Networks with Minimum Number of Crossings. arXiv preprint, 2020


\section*{Use mathematical properties of abstract networks}
explicit rooted network

unrooted network

circular split network


Philippe Gambette, Vincent Berry \& Christophe Paul: Quartets and Unrooted Phylogenetic Networks, JBCB 10(4):1250004, 2012

Philippe Gambette, Katharina Huber \& Guillaume Scholz, Uprooted phylogenetic networks, \(B M B, 79(9): 2022-204,2017\)


\section*{Use solvers}
- SAT, ILP (integer linear programming), CSP (constraint satisfaction problem), maximum clique solvers are available

integer linear programming

- work in progress with Pierre Bourhis and Marion Tommasi:
- an ad hoc algorithm is faster most of the time
- the time taken by the solver does not vary much: more efficient when the ad hoc algorithm takes too long

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\section*{Put everything together}
- requires some good engineering work: use multicore processors, parallel or distributed computing, etc.
- requires easy-to-use software:
- cross-platform software: SplitsTree (1998), Dendroscope (2007), PhyloSketch (2020)
- web applications: T-REX online (2012)
- packages or pipeline bricks: R package Phangorn (2011), Julia package PhyloNetworks (2017)
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[^0]:    R. A. Leo Elworth, Huw A. Ogilvie, Jiafan Zhu and Luay Nakhleh. Advances in Computational Methods for Phylogenetic Networks in the Presence of Hybridization. In Tandy Warnow (editor), Bioinformatics and Phylogenetics. Seminal Contributions of Bernard Moret, Vol. 29 of Computational Biology, 2019

