High electromechanical coupling of a broadband PZT-based vibration energy harvester

David Gibus, Pierre Gasnier, Adrien Morel, Sebastien Boisseau, Adrien Badel

High electromechanical coupling of a broadband PZT-based vibration energy harvester

David GIBUS[12*], Pierre GASNIER[1], Adrien MOREL[12], Sebastien BOISSEAU[1] and Adrien BADEL[2]

[1] *Univ. Grenoble Alpes, CEA, LETI, MINATEC, F-38000 Grenoble, France*

[2] *Univ. Savoie Mont Blanc, SYMME, 74000 Annecy, France*

*david.gibus@cea.fr

Abstract**—This paper reports on the design and experiment of a highly-coupled piezoelectric energy harvester. Such a device is necessary to benefit from non-linear electrical techniques to improve the frequency bandwidth. We improve a previously developed 2-degree of freedom (2-DOF) analytical model with the Rayleigh method to understand 2-dimensional electromechanical interactions and give a preliminary design a PZT-5A cantilever. Then, we use 3D Finite Element Method (FEM) simulations to take into account 3-dimensional effects. The resulting device demonstrates a strong squared electromechanical coupling coefficient (k^2=15.0% for the 1st mode), which represents $k^2/k_{31}^{l\,2} = 99\%$ of the commonly used 31 material coupling coefficient and $k^2/k_{31}^{w\,2} = 43\%$ of the 31 material coupling coefficient computed according to the plane strain assumption.**

I. Introduction

Highly coupled generators are a relevant solution for broadband vibration harvesting. When associated with electrical methods that influence the mechanical resonator dynamics, their harvesting bandwidth can be enlarged thanks to the strong electromechanical coupling of the system [1-2]. Thus, harvesters embedding single crystals have been designed in order to maximize the global coupling. In a previous work [3], we explained the benefits of long proof mass systems to homogenize the strain distribution in the piezoelectric material. We also highlighted the drastic effect of plane strain or plane stress condition on the electromechanical coefficient.

Here, we propose to take benefit of these out-of-plane interactions to maximize the system coupling of a cantilever enrolling a common PZT ceramic material. Indeed, for this type of material, wide cantilevers, operating according to the plane strain assumption, exhibit a much better coupling than narrow beams, operating according to the plane stress assumption. PZT-5A have been chosen for its high Curie temperature (Tc°=360%) and high coupling coefficient.

In section II, we present the Rayleigh method applied on the mode shape computed from the 2 degree of freedom (2-DOF) model presented in [3]. A design methodology is described in section III and the modelling, simulations and measurement results are presented and compared in section IV.

II. Analytical model

The studied system is composed by a bimorph piezoelectric cantilever whose electrodes are connected in parallel as shown in FIGURE 1 and a long proof mass. We firstly neglect the beam mass, as for the 2-DOF model presented in [3]. We consider the proof mass M_t as well as its rotary inertia I_t and the distance D_t of its center of gravity with the end of the bending beam. The two mode shapes of the beam are calculated according to these assumptions.

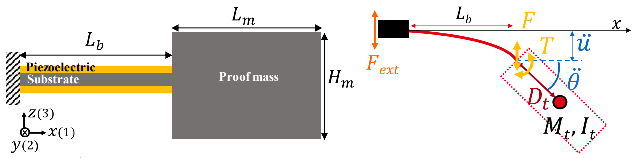

FIGURE 1 : Clamped-free beam system on the left and beam during bending on the right

In a second step, we take the kinetic influence of the beam mass into account thanks to the Rayleigh Method. This method has been extensively detailed for long proof mass systems in [4] where the mode shape were determined with the continuous beam model (contrary to the 2-DOF model used here). By considering energy conversion law, we deduce a single degree of freedom system for each resonant mode:

$$\begin{cases} M_j\ddot{r} + K_j r - \Theta_j v = -B_{f_j}\ddot{w}_B \\ \Theta_j\dot{r} + C_p\dot{v} + i = 0 \end{cases} \qquad (1)$$

Where M_j, K_j, Θ_j, B_{f_j} and C_p are the equivalent inertial mass, stiffness, coupling term, forcing factor and clamped capacitance respectively. $\ddot{w}_b$ is the base acceleration, v is the voltage across the electrodes and i is the current flowing through the electrodes. r denotes the displacement at the beam end, while the subscript $j \in [1, N]$ represents the resonant mode of interest.

By considering the mode shapes given by the 2-DOF model, we can analytically express the system coefficients (1) as well as the resonant frequencies and coupling for both first and second flexural resonant modes. This simplifies both the understanding of the electromechanical interactions and the design.

For instance, we can prove that the proof mass density does not affect the global coupling once the proof mass is much heavier than the beam. Only the proof mass shape has an influence on the strain distribution. Furthermore, to increase the first mode electromechanical coupling, it can be shown that the rotary inertia to mass ratio I_t/M_t and the distance D_t should be maximized.

III. Design proposition

The piezoelectric coefficient of the PZT-5A from Noliac company are listed in Table 1 where the coupling coefficients are computed thanks to [5]. Plane stress assumption is used for narrow beams whereas plane strain assumption is commonly used for wide beams. We noticed that the coupling computed according the plane strain assumption is twice bigger than the one corresponding to plane stress assumption. High width-to-length ratio presents however an issue for low frequency applications when the volume is in concern. Indeed, increasing the beam length is usually done to reduce the resonant frequency. The proposed methodology consists thus in firstly optimizing the cantilever design thanks to the proposed model for a given resonant frequency. Then, we adjust the beam width to tend to correspond to the plane strain assumption by using 3D FEM simulations. We use steel as the substrate and proof mass material

TABLE 1: MATERIAL COEFFICIENTS FROM NOLIAC NCE51 (PZT-5A)

<table>
<tr><td colspan="2">d_{31}</td><td colspan="2">s_{11}^E</td><td colspan="2">ϵ_{33}^T</td></tr>
<tr><td colspan="2">-208 pm.V^{-1}</td><td colspan="2">17×10^{-12} Pa^{-1}</td><td colspan="2">1900 F.m^{-1}</td></tr>
<tr><td colspan="6">Coupling</td></tr>
<tr><td colspan="3">Plane stress $k_{31}^{l\,2}$ (usual k_{31}^2)</td><td colspan="3">Plane strain $k_{31}^{w\,2}$</td></tr>
<tr><td colspan="3">15.1%</td><td colspan="3">34.2%</td></tr>
</table>

The system has been assembled with epoxy glue (FIGURE 2). The substrate and PZT patches thicknesses are equal to 0.4mm and 0.3mm respectively. We performed admittance measurement to experimentally determine the system parameters (FIGURE 3).

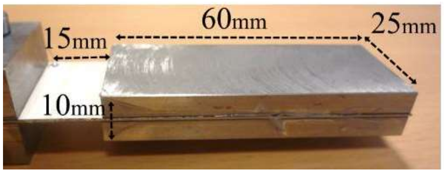

FIGURE 2 : PROTOTYPE WITH PZT-5A

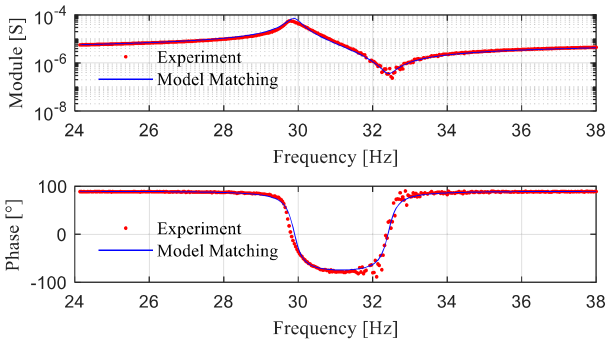

FIGURE 3 : MEASURED ADMITTANCE OF THE PROTOTYPE AND MATCHING WITH THE MODEL PRESENTED IN [1].

IV. DISCUSSIONS

The assembled prototype shows a large squared electromechanical coupling coefficient ($k^2 = 15.0\%$) which is, as far as we know, the best value obtained for a vibration energy harvester made with PZT-5A material. It represents $k^2/k_{31}^{l\,2} = 99\%$ and $k^2/k_{31}^{w\,2} = 43\%$ of the material couplings computed according to the plane stress and the plane strain assumption respectively. The quality factor Q_m and the figure of merit $k_e^2 Q_m = k^2 Q_m/(1-k^2)$ are equal to 85 and 15.2 respectively.

We can notice differences between modelling, 3D FEM simulation and experimental results (TABLE 2). Since the width-to-length ratio of the beam is equal to 1.7, plane strain assumption is not fully respected for our cantilever. The expected coupling is thus reduced between modelling and 3D simulation. Moreover, when we take the imperfectly clamped end and the glue into account in simulations, the resulting global squared electromechanical coupling coefficient k² decreases. This is explained by losses of elastic energy in non-piezoelectric material. While it is impossible to determine accurately the glue thickness and how imperfect is the clamping, we can understand the qualitative influence of each parameter. As an example, we reach a coupling of 17.0% if we consider that the piezoelectric patches are at a distance of 50µm from the clamped-end block and from the proof mass, and if the glue thickness is equal to 5µm (the epoxy Young modulus is estimated to be 4GPa).

TABLE 2: COUPLING AND RESONANT FREQUENCY OBTAINED WITH THE MODEL, 3D FEM AND EXPERIMENT

	f_{sc}	k^2
Proposed plane strain modelling	36.3 Hz	27.8%
3D FEM with perfect clamping	36.0 Hz	23.1%
3D FEM with glue and imperfect clamping	30.8 Hz	17.0%
Experiment	29.9 Hz	15.0%

V. CONCLUSION

In this paper, we propose a design methodology in order to maximize the coupling of linear piezoelectric energy harvesters. The proposed methodology, based on the resolution of a 2DOF model coupled with the Rayleigh method and 3D simulations, is applied to design a PZT-5A-based energy harvester. We have been able to obtain experimentally a squared coupling coefficient of 15.0%. Differences between model, simulations and experiment are explained by the influence of the clamped-end imperfections and the glue thickness.

Future work will focus on improving the fabrication and the understanding of the clamping condition influence.

REFERENCES

[1] A. Badel and E. Lefeuvre, “Wideband Piezoelectric Energy Harvester Tuned Through its Electronic Interface Circuit,” *J. Phys. Conf. Ser.*, vol. 557, p. 012115, Nov. 2014.

[2] A. Morel, G. Pillonnet, P. Gasnier, E. Lefeuvre, and A. Badel, “Frequency tuning of piezoelectric energy harvesters thanks to a short-circuit synchronous electric charge extraction,” *Smart Mater. Struct.*, vol. 28, no. 2, p. 025009, Dec. 2018.

[3] D. Gibus, P. Gasnier, A. Morel, S. Boisseau, and A. Badel, “Modelling and design of highly coupled piezoelectric energy harvesters for broadband applications,” *J. Phys. Conf. Ser.*

[4] M. Kim, M. Hoegen, J. Dugundji, and B. L. Wardle, “Modeling and experimental verification of proof mass effects on vibration energy harvester performance,” *Smart Mater. Struct.*, vol. 19, no. 4, p. 045023, Apr. 2010.

[5] The Institute of Electrical and Electronics Engineers, “IEEE Standard on Piezoelectricity,” *ANSIIEEE Std 176-1987*, 1988.