



**HAL**  
open science

# Operation Modes and Self-motions of a 2-RUU Parallel Manipulator

Latifah Nurahmi, Stéphane Caro, Philippe Wenger

► **To cite this version:**

Latifah Nurahmi, Stéphane Caro, Philippe Wenger. Operation Modes and Self-motions of a 2-RUU Parallel Manipulator. The 3rd IFToMM Symposium on Mechanism Design for Robotics, Jun 2015, Aalborg, Denmark. hal-02947228

**HAL Id: hal-02947228**

**<https://hal.science/hal-02947228>**

Submitted on 23 Sep 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Operation Modes and Self-motions of a 2-RUU Parallel Manipulator

Latifah Nurahmi, Stéphane Caro<sup>†</sup>, Philippe Wenger

*Institut de Recherche en Communications et Cybernétique de Nantes, France,  
e-mails: {latifah.nurahmi,stephane.caro,philippe.wenger}@ircsyn.ec-nantes.fr*

**Abstract.** This paper deals with the characterization of the operation modes of the 2-RUU parallel manipulator with an algebraic approach, namely the Study kinematic mapping of the Euclidean group  $\mathbf{SE}(3)$ . The manipulator is described by a set of eight constraint equations and the primary decomposition reveals that the mechanism has three operation modes. The singularity conditions are obtained by deriving the determinant of the Jacobian matrix of the constraint equations with respect to the Study parameters. It is shown that there exist singular configurations in which the 2-RUU manipulator may switch from one operation mode to another operation mode. All the singular configurations are mapped onto the joint space and are geometrically interpreted. Finally, the mechanism may switch from the 1st Schönflies mode to the 2nd Schönflies mode through the additional mode that contains self-motions.

**Key words:** Schönflies motion, operation mode, Study parameters, 2-RUU manipulator, self-motion.

## 1 Introduction

Lower-mobility parallel manipulators are suitable for wide range of applications that require fewer than six degree-of-freedom end-effector motion (*6-dof*), for example Schönflies Motion Generators (SMGs). The SMGs are manipulators which can exhibit three independent translations and one pure rotation about an axis of fixed direction, for example the 2-RUU parallel manipulator. This mechanism is composed of two RUU limbs in which two joints are actuated in each limb.

By using an algebraic description of the manipulator and the Study parametrization based on [3], [5], the operation modes of the 2-RUU manipulator are discussed in more details in this paper. The constraint equations are initially derived. Then the primary decomposition is computed over the set of constraint equations to reveal the existence of three operations modes. The singularities are examined by deriving the determinant of the Jacobian matrix of the constraint equations with respect to the Study parameters. It is shown that the mechanism is able to change from one mode to another mode by passing through the configurations that belong to both modes.

---

<sup>†</sup> Corresponding author. Email: stephane.caro@ircsyn.ec-nantes.fr. Tel: +33 (02) 40 37 69 68

The singularity conditions are mapped onto the joint space. Eventually, the changes of operation modes for the 2-RUU parallel manipulator are illustrated.

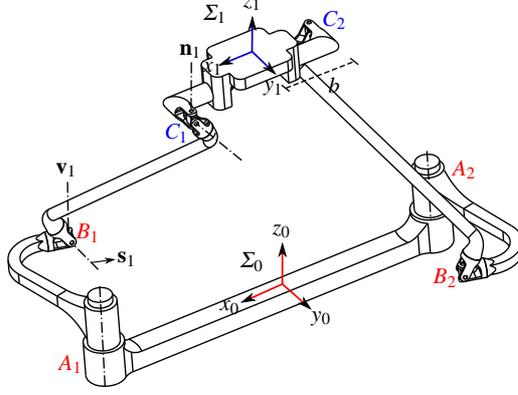


Fig. 1: The 2-RUU parallel manipulator.

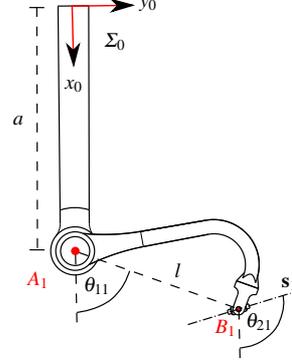


Fig. 2: The axes of R-joints from top view.

## 2 Manipulator architecture

The 2-RUU parallel manipulator shown in Fig.1, is composed of a base, a moving platform, and two identical limbs. Each limb is composed of five revolute joints such that the second and the third ones, as well as the fourth and the fifth ones, are built with intersecting and perpendicular axes. Thus they are assimilated to U-joint.

The first revolute joint is attached to the base and is actuated. Its rotation angle is defined by  $\theta_{1i}$  ( $i = 1, 2$ ). The axes of the first, the second, and the fifth joints are directed along  $z$ -axis. The second axis and the fifth axis are denoted by  $\mathbf{v}_i$  and  $\mathbf{n}_i$  ( $i = 1, 2$ ), respectively. The second revolute joint is also actuated and its rotation angle is defined by  $\theta_{2i}$  ( $i = 1, 2$ ), in Fig. 2.

The axes of the third and the fourth joints are parallel. The axis of the third joint is denoted by  $\mathbf{s}_i$  ( $i = 1, 2$ ) and it changes instantaneously as a function of  $\theta_{2i}$  as shown in Fig. 2, i.e.:  $\mathbf{s}_i = [0, \cos(\theta_{2i}), \sin(\theta_{2i}), 0]^T$  ( $i = 1, 2$ ).

The fixed frame  $\Sigma_0$  is located at the center of the base. The first revolute joint of the  $i$ -th limb is located at point  $A_i$  with distance  $a$  from the origin of  $\Sigma_0$ . The first U-joint is denoted by point  $B_i$  with distance  $l$  from point  $A_i$ . Link  $A_i B_i$  always moves in a plane normal to  $\mathbf{v}_i$ . Hence the coordinates of points  $A_i$  and  $B_i$  expressed in the fixed frame  $\Sigma_0$  are:

$$\begin{aligned} \mathbf{r}_{A_1}^0 &= [1, a, 0, 0]^T & \mathbf{r}_{B_1}^0 &= [1, l \cos(\theta_{11}) + a, l \sin(\theta_{11}), 0]^T \\ \mathbf{r}_{A_2}^0 &= [1, -a, 0, 0]^T & \mathbf{r}_{B_2}^0 &= [1, l \cos(\theta_{12}) - a, l \sin(\theta_{12}), 0]^T \end{aligned} \quad (1)$$

The moving frame  $\Sigma_1$  is located at the center of the moving platform. The moving platform is connected to the limbs by two U-joints, of which the intersection point

of the revolute joint axes is denoted by  $C_i$ . The length of the moving platform from the origin of  $\Sigma_1$  to point  $C_i$  is defined by  $b$ . The length of link  $B_iC_i$  is defined by  $r$ . The coordinates of point  $C_i$  expressed in the moving frame  $\Sigma_1$  are:

$$\mathbf{r}_{C_1}^1 = [1, b, 0, 0]^T \quad \mathbf{r}_{C_2}^1 = [1, -b, 0, 0]^T \quad (2)$$

### 3 Constraint equations

In this section, the constraint equations are derived whose solutions illustrate the possible poses of the moving platform (coordinate frame  $\Sigma_1$ ) with respect to  $\Sigma_0$ . To obtain the coordinates of points  $C_i$  and vectors  $\mathbf{n}_i$  expressed in  $\Sigma_0$ , the Study parametrization of a spatial Euclidean transformation matrix  $\mathbf{M}$  based on [1] is used.

The parameters  $x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3$ , which appear in matrix  $\mathbf{M}$ , are called Study parameters. They are useful in the representation of a spatial Euclidean displacement and they should satisfy [1]  $x_0^2 + x_1^2 + x_2^2 + x_3^2 \neq 0$ . This condition will be used in the following computations to simplify the algebraic expressions. First of all, the half-tangent substitutions for  $\theta_{ij}$  ( $i, j = 1, 2$ ) are performed to remove the trigonometric functions:

$$\cos(\theta_{ij}) = \frac{1 - t_{ij}^2}{1 + t_{ij}^2}, \quad \sin(\theta_{ij}) = \frac{2t_{ij}}{1 + t_{ij}^2}, \quad i, j = 1, 2 \quad (3)$$

where  $t_{ij} = \tan(\frac{\theta_{ij}}{2})$ . The coordinates of points  $C_i$  and vectors  $\mathbf{n}_i$  expressed in  $\Sigma_0$  are obtained by:

$$\mathbf{r}_{C_i}^0 = \mathbf{M} \mathbf{r}_{C_i}^1, \quad \mathbf{n}_i^0 = \mathbf{M} \mathbf{n}_i^1, \quad i = 1, 2 \quad (4)$$

As the coordinates of all points are given in terms of Study parameters and the design parameters, the constraint equations can be obtained by examining the design of the RUU limb. The link connecting points  $B_i$  and  $C_i$  is coplanar to the vectors  $\mathbf{v}_i$  and  $\mathbf{n}_i^0$ . Accordingly, the scalar triple product of vectors  $(\mathbf{r}_{C_i}^0 - \mathbf{r}_{B_i}^0)$ ,  $\mathbf{v}_i$  and  $\mathbf{n}_i^0$  vanishes, namely:

$$(\mathbf{r}_{C_i}^0 - \mathbf{r}_{B_i}^0)^T \cdot (\mathbf{v}_i \times \mathbf{n}_i^0) = 0, \quad i = 1, 2 \quad (5)$$

After computing the corresponding scalar triple product, and removing the common denominators, the following constraint equations come out:

$$\begin{aligned} g_1 : & (at_{11}^2 - bt_{11}^2 - lt_{11}^2 + a - b + l)x_0x_1 + 2lt_{11}x_0x_2 - (2t_{11}^2 + 2)x_0y_0 + 2lt_{11}x_3x_1 + \\ & (-at_{11}^2 - bt_{11}^2 + lt_{11}^2 - a - b - l)x_3x_2 + (-2t_{11}^2 - 2)y_3x_3 = 0 \\ g_2 : & (at_{12}^2 - bt_{12}^2 + lt_{12}^2 + a - b - l)x_0x_1 - 2lt_{12}x_0x_2 + (2t_{12}^2 + 2)x_0y_0 - 2lt_{12}x_1x_3 + \\ & (-at_{12}^2 - bt_{12}^2 - lt_{12}^2 - a - b + l)x_2x_3 + (2t_{12}^2 + 2)x_3y_3 = 0 \end{aligned} \quad (6)$$

To derive the constraint equations corresponding to the link length  $r$  of link  $B_iC_i$ , the distance equation can be formulated as:  $\|(\mathbf{r}_{C_i}^0 - \mathbf{r}_{B_i}^0)\|^2 = r^2$ . As a consequence, the following two equations are obtained:

$$\begin{aligned}
g_3 : & (a^2 t_{11}^2 - 2abt_{11}^2 - 2alt_{11}^2 + b^2 t_{11}^2 + 2blt_{11}^2 + l^2 t_{11}^2 - r^2 t_{11}^2 + a^2 - 2ab + 2al + b^2 - \\
& 2bl + l^2 - r^2)x_0^2 - 8blt_{11}x_0x_3 + (4at_{11}^2 - 4bt_{11}^2 - 4lt_{11}^2 + 4a - 4b + 4l)x_0y_1 + \dots \\
g_4 : & (a^2 t_{12}^2 - 2abt_{12}^2 + 2alt_{12}^2 + b^2 t_{12}^2 - 2blt_{12}^2 + l^2 t_{12}^2 - r^2 t_{12}^2 + a^2 - 2ab - 2al + b^2 + \\
& 2bl + l^2 - r^2)x_0^2 + 8blt_{12}x_0x_3 + (-4at_{12}^2 + 4bt_{12}^2 - 4lt_{12}^2 - 4a + 4b + 4l)x_0y_1 + \dots
\end{aligned} \tag{7}$$

To derive the constraint equations corresponding to the actuation of the second joint of each limb, the scalar product of vector  $(\mathbf{r}_{C_i}^0 - \mathbf{r}_{B_i}^0)$  and vector  $\mathbf{s}_i$  is expressed as:  $(\mathbf{r}_{C_i}^0 - \mathbf{r}_{B_i}^0)^T \mathbf{s}_i = 0$ . Hence, the following constraint equations are obtained:

$$\begin{aligned}
g_5 : & (-at_{11}^2 t_{21}^2 + bt_{11}^2 t_{21}^2 + lt_{11}^2 t_{21}^2 + at_{11}^2 - at_{21}^2 - bt_{11}^2 + bt_{21}^2 - lt_{11}^2 + 4lt_{11}t_{21} - lt_{21}^2 + \\
& a - b + l)x_0^2 - 4bt_{21}(t_{11}^2 + 1)x_0x_3 - 2(t_{21}^2 - 1)(t_{11}^2 + 1)x_0y_1 + 4t_{21}(t_{11}^2 + 1)x_0y_2 \dots \\
g_6 : & (at_{12}^2 t_{22}^2 - bt_{12}^2 t_{22}^2 + lt_{12}^2 t_{22}^2 - at_{12}^2 + at_{22}^2 + bt_{12}^2 - bt_{22}^2 - lt_{12}^2 + 4lt_{12}t_{22} - lt_{22}^2 - \\
& a + b + l)x_0^2 + 4bt_{22}(t_{12}^2 + 1)x_0x_3 - 2(t_{22}^2 - 1)(t_{12}^2 + 1)x_0y_1 + 4t_{22}(t_{12}^2 + 1)x_0y_2 \dots
\end{aligned} \tag{8}$$

For detail expressions of equations  $g_3, g_4, g_5, g_6$ , the reader may refer to [6]. All solutions have to be within the Study quadric, i.e.:  $g_7 : x_0y_0 + x_1y_1 + x_2y_2 + x_3y_3 = 0$ . To exclude the exceptional generators ( $x_0 = x_1 = x_2 = x_3 = 0$ ), the normalization equation is added:  $g_8 : x_0^2 + x_1^2 + x_2^2 + x_3^2 - 1 = 0$ .

## 4 Operation modes

The design parameters are assigned as  $a = 2, b = 1, l = 1, r = 2$ . The set of eight constraint equations is written as a polynomial ideal with variables  $\{x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3\}$  over the coefficient ring  $\mathbb{C}[t_{11}, t_{12}, t_{21}, t_{22}]$ , defined as:  $\mathcal{J} = \langle g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8 \rangle$ . At this point, the following ideal is examined:  $\mathcal{J} = \langle g_1, g_2, g_7 \rangle$ .

The primary decomposition is computed and it turns out that the ideal  $\mathcal{J}$  is decomposed into three components as:  $\mathcal{J} = \bigcap_{i=1}^3 \mathcal{J}_i$ , with the results of primary decomposition:

$$\begin{aligned}
\mathcal{J}_1 &= \langle x_0, x_3, x_1y_1 + x_2y_2 \rangle \\
\mathcal{J}_2 &= \langle x_1, x_2, x_0y_0 + x_3y_3 \rangle \\
\mathcal{J}_3 &= \langle (t_{11}^2 t_{12}^2 + 2t_{12}^2 + 1)x_0x_1 + (-t_{11}^2 t_{12} + t_{11}t_{12}^2 + t_{11} - t_{12})x_0x_2 \dots \rangle
\end{aligned} \tag{9}$$

For complete results of the primary decomposition, the reader may refer to [6]. Accordingly, the 2-RUU manipulator under study has three operation modes. The computation of the Hilbert dimension of ideal  $\mathcal{J}_i$  with  $t_{11}, t_{12}, t_{21}, t_{22}$  treated as variables shows that:  $\dim(\mathcal{J}_i) = 4$  ( $i = 1..3$ ). To complete the analysis, the remaining equations are added by writing:

$$\mathcal{K}_i : \mathcal{J}_i \cup \langle g_3, g_4, g_5, g_6, g_8 \rangle, \quad i = 1..3 \tag{10}$$

It follows that the 2-RUU manipulator has *4-dof*. Each system  $\mathcal{K}_i$  corresponds to a specific operation mode that will be discussed in the following.

#### 4.1 System $\mathcal{K}_1$ : 1st Schönflies mode

In this operation mode, the moving platform is reversed about an axis parallel to the  $\mathbf{xy}$ -plane by 180 degrees from the home position [2]. The condition  $x_0 = 0, x_3 = 0, x_1y_1 + x_2y_2 = 0$  are valid for all poses and are substituted into the transformation matrix  $\mathbf{M}$ , as:

$$\mathbf{M}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2(x_1y_0 - x_2y_3) & x_1^2 - x_2^2 & 2x_1x_2 & 0 \\ 2(x_1y_3 + x_2y_0) & 2x_1x_2 & -x_1^2 + x_2^2 & 0 \\ -\frac{2y_2}{x_1} & 0 & 0 & -1 \end{pmatrix} \quad (11)$$

From the transformation matrix it can be seen that the manipulator has *3-dof* translational motions, which are parametrized by  $y_0, y_2, y_3$  and *1-dof* rotational motion, which is parametrized by  $x_1, x_2$  in connection with  $x_1^2 + x_2^2 - 1 = 0$  [4].

#### 4.2 System $\mathcal{K}_2$ : 2nd Schönflies mode

In this operation mode, the condition  $x_1 = 0, x_2 = 0, x_0y_0 + x_3y_3 = 0$  are valid for all poses. The transformation matrix in this operation mode is:

$$\mathbf{M}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2(x_0y_1 - x_3y_2) & x_0^2 - x_3^2 & -2x_0x_3 & 0 \\ -2(x_0y_2 + x_3y_1) & 2x_0x_3 & x_0^2 - x_3^2 & 0 \\ -\frac{2y_3}{x_0} & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

From this transformation matrix, it can be seen that the manipulator has *3-dof* translational motions, which are parametrized by  $y_1, y_2, y_3$  and *1-dof* rotational motion, which is parametrized by  $x_0, x_3$  in connection with  $x_0^2 + x_3^2 - 1 = 0$  [4].

#### 4.3 System $\mathcal{K}_3$ : Third mode

In this operation mode, the moving platform is no longer parallel to the base. The variables  $x_3, y_0, y_1$  can be solved linearly from ideal  $\mathcal{J}_3$ . Accordingly, the 2-RUU parallel manipulator will perform two translational motions, which are parametrized by variables  $y_2, y_3$  and two rotational motions, which are parametrized by variables  $x_0, x_1, x_2$  in connection with normalization equation  $g_8$ .

Under this operation mode, the joint angles  $t_{21}$  and  $t_{22}$  can be computed from Eq.(8). It turns out that no matter the value of the first actuated joint ( $t_{11}, t_{12}$ ) in each limb, these equations vanish for two real solutions, namely (1.)  $t_{21} = -\frac{1}{t_{22}}$  and (2.)  $t_{21} = t_{22}$ . As a consequence, in this operation mode, the links  $B_iC_i$  ( $i = 1, 2$ ) from both limbs are always parallel to the same plane and the axes  $\mathbf{s}_i$  ( $i = 1, 2$ ) from both limbs are always parallel too.

## 5 Singularity Conditions and Operation Mode Changing

The 2-RUU manipulator reaches a singularity condition when the determinant of its Jacobian matrix vanishes. The Jacobian matrix is the matrix of all first order partial derivatives of the eight constraint equations with respect to the eight Study parameters. Since the 2-RUU manipulator has more than one operation mode, the singular configurations can be classified into two different types, i.e. the configurations that belong to a single operation mode and the singularity configurations that belong to more than one operation mode. The common configurations that belong to more than one operation mode allow the 2-RUU manipulator to switch from one mode to another mode. However, the 1st Schönflies mode and the 2nd Schönflies mode do not have configurations in common, since  $x_0, x_1, x_2, x_3$  can never vanish simultaneously. It means that the 2-RUU manipulator cannot switch from the Schönflies mode into the reversed Schönflies mode directly.

To change from the 1st Schönflies mode into the 2nd reversed Schönflies mode, the mechanism should pass through the third mode, namely system  $\mathcal{K}_3$ , as shown in Fig. 3. There exist some configurations in which the mechanism can change from the 1st Schönflies mode to the third mode and these configurations belongs to both modes. Noticeably, these configurations are also singular configurations since they lie in the intersection of two operation modes.

### 5.1 1st Schönflies Mode ( $\mathcal{K}_1$ ) $\longleftrightarrow$ Third Mode ( $\mathcal{K}_3$ )

For the 1st Schönflies mode, the determinant of Jacobian matrix  $S_1 : \det(\mathbf{J}_1) = 0$  (for complete results of  $\det(\mathbf{J}_1)$ , the reader may refer to [6]) has four factors. The first factor is  $y_2 = 0$ , when the moving platform is coplanar to the base, the mechanism is always in a singular configuration. The second factor shows the singularity configurations that lie in the intersection with  $\mathcal{K}_2$ . However, this factor is neglected since systems  $\mathcal{K}_1$  and  $\mathcal{K}_2$  do not have configurations in common.

The inspection of the third factor yields the singularity configurations that belong to the 1st Schönflies mode  $\mathcal{K}_1$  and the third mode  $\mathcal{K}_3$ . The third factor is added to the system  $\mathcal{K}_1$ . Then, all Study parameters are eliminated. The elimination yields two polynomials of degree eight and degree nine in  $t_{11}, t_{12}, t_{21}, t_{22}$ , respectively. The factorization splits both polynomials into four factors as follows:

$$\begin{aligned} f_1 &: (t_{21}t_{22} + 1)(t_{21} - t_{22})(t_{12} + t_{11})(3t_{11}^2t_{12}^2 - 2t_{11}t_{12} + 8t_{12}^2 + 3) \\ f_2 &: (t_{21}t_{22} + 1)(t_{21} - t_{22})(t_{12}^2 + 1)(3t_{11}^2t_{12}^2 - 2t_{11}t_{12} + 8t_{12}^2 + 3) \end{aligned} \quad (13)$$

Polynomials  $f_1, f_2$  vanish simultaneously when: (1.)  $t_{21} = -\frac{1}{t_{22}}$  and (2.)  $t_{21} = t_{22}$ . It means that when the second link  $B_iC_i$  ( $i = 1, 2$ ) from both limbs are parallel to the same plane and the moving platform is twisted about an axis parallel to the  $\mathbf{xy}$ -plane by 180 degrees, the 2-RUU manipulator is in the intersection of the 1st Schönflies mode and the third mode.

Eventually, the last factor of the determinant of Jacobian matrix  $S_1 : \det(\mathbf{J}_1) = 0$  is analysed. This factor is added to the system  $\mathcal{K}_1$  and all Study parameters are eliminated. Due to the heavy elimination process, the joint angles are assigned as  $t_{11} = 1, t_{12} = 0, t_{21} = -1$  and the result of elimination is a univariate polynomial of degree 12 in  $t_{22}$ .

## 5.2 2nd Schönflies Mode ( $\mathcal{K}_2$ ) $\longleftrightarrow$ Third Mode ( $\mathcal{K}_3$ )

For the 2nd Schönflies mode, the determinant of Jacobian matrix  $S_2 : \det(\mathbf{J}_2) = 0$  (for complete results of  $\det(\mathbf{J}_2)$ , the reader may refer to [6]) has four factors too. The first factor is  $y_3 = 0$  in which the moving platform is coplanar to the base, hence the mechanism is always in a singular configuration. The second factor gives the condition in which the mechanism is in the intersection of systems  $\mathcal{K}_1$  and  $\mathcal{K}_2$ . As explained in Section 5.1, this factor is removed.

The analysis of the third factor yields the singularity configurations that belong to the 2nd Schönflies mode  $\mathcal{K}_2$  and the third mode  $\mathcal{K}_3$ . The third factor is added to the system  $\mathcal{K}_2$ . Then all Study parameters are eliminated and yields:

$$\begin{aligned} f_1 &: (t_{21}t_{22} + 1)(t_{21} - t_{22})(t_{12} + t_{11})(3t_{11}^2t_{12}^2 - 2t_{11}t_{12} + 8t_{12}^2 + 3) \\ f_2 &: (t_{21}t_{22} + 1)(t_{21} - t_{22})(t_{12}^2 + 1)(3t_{11}^2t_{12}^2 - 2t_{11}t_{12} + 8t_{12}^2 + 3) \end{aligned} \quad (14)$$

Notably, the elimination results have the same mathematical expressions as in Eq.(13) and it leads to the same solutions, namely: (1.)  $t_{21} = -\frac{1}{t_{22}}$  and (2.)  $t_{21} = t_{22}$ . It means that when the second link  $B_iC_i$  ( $i = 1, 2$ ) from both limbs are parallel to the same plane and the moving platform is parallel to the base, the 2-RUU manipulator is in the intersection of the 2nd Schönflies mode and the third mode.

Finally, the last factor of the determinant of Jacobian matrix  $S_2 : \det(\mathbf{J}_2) = 0$  is analysed. Due to the heavy elimination process, the joint angles are assigned as  $t_{11} = 1, t_{12} = 0, t_{21} = -1$  and the result of elimination is also a univariate polynomial of degree 12 in  $t_{22}$ .

## 6 Self-motions in System $\mathcal{K}_3$

The determinant of the Jacobian matrix is computed in the system  $\mathcal{K}_3$ , which consists of five constraint equations over five variables. Hence the  $5 \times 5$  Jacobian matrix can be obtained. The determinant of this Jacobian matrix  $S_3 : \det(\mathbf{J}_3) = 0$  (for complete results of  $\det(\mathbf{J}_3)$ , the reader may refer to [6]) consists of 10 factors, two of them are:  $(t_{21}t_{22} + 1)$  and  $(t_{21} - t_{22})$ . It can be seen from Section 4.3 that these factors are the necessary conditions for the manipulator to be in the system  $\mathcal{K}_3$ , namely (1.)  $t_{21} = -\frac{1}{t_{22}}$  and (2.)  $t_{21} = t_{22}$ . It means that every configuration in the system  $\mathcal{K}_3$  amounts to a self-motion. Furthermore, the image of the transition configurations in the joint space from Eq.(13), is contained in  $S_3 : \det(\mathbf{J}_3) = 0$ . As a consequence,

the transition between systems  $\mathcal{H}_1$  and  $\mathcal{H}_2$  occurs through the third system  $\mathcal{H}_3$  that contains self-motions. These transition configurations are illustrated in Figs. 3a-3e.

## 7 Conclusions

In this paper, the method of algebraic geometry was applied to characterize the type of operation modes of the 2-RUU parallel manipulator. The set of eight constraint equations are derived and the primary decomposition is computed. It is shown that the 2-RUU parallel manipulator has three operation modes and the interpretation of each operation mode was given. The singularity conditions were computed and represented in the joint space. It turns out that the mechanism is able to switch from the 1st Schönflies mode to the 2nd Schönflies mode or vice versa, by passing through the third mode that contains self-motions.

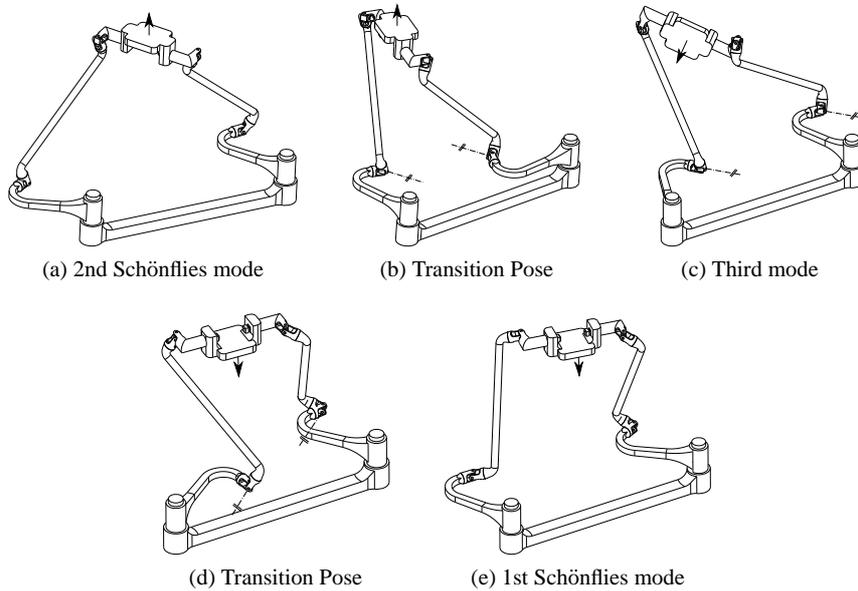


Fig. 3: Transition from the 2nd Schönflies mode to the 1st Schönflies mode via the third mode.

## References

1. Husty, M. L., Pfurner, M., Schröcker, H-P., Brunthaler, K.: Algebraic Methods in Mechanism Analysis and Synthesis. *Robotica*, **25**(6) pp. 661-675 (2007).

2. Xianwen Kong: Reconfiguration Analysis of a 3-DOF Parallel Mechanism Using Euler Parameter Quaternions and Algebraic Geometry Method. *Mechanism and Machine Theory*, **74** pp.188-201 (2014).
3. Nurahmi, L., Schadlbauer, J., Caro, Stephane., Husty, M., Wenger, P.: Kinematic Analysis of the 3-RPS Cube Parallel Manipulator. *Journal of Mechanisms and Robotics*, **7**(1) pp. 011008-1011008-10 (2015).
4. Schadlbauer, J., Nurahmi, L., Husty, M., Wenger, P. and Caro, S.: Operation Modes in Lower-Mobility Parallel Manipulators, Second Conference on Interdisciplinary Applications of Kinematics, Lima, Peru, September 9–11, (2013).
5. Schadlbauer, J., Walter, D.R., Husty, M.: The 3-RPS Parallel Manipulator from an Algebraic Viewpoint. *Mechanism and Machine Theory*, **75** pp.161-176 (2014).
6. <http://www.irccyn.ec-nantes.fr/%7Enurahmi/MEDER2015/Appendix.pdf>