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# Synthesis of 3-RPS Parallel Manipulators based on Prescribed Operation Modes

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**Abstract.** The subject of this paper is about the synthesis of the design parameters by considering the prescribed operation modes at the design stage for a parallel manipulator with three RPS legs. The synthesis is based on the Euler parametrization and the results of primary decomposition. The design parameters and the coordinates of one RPS leg are initially defined to formulate the constraint equation associated with this leg. Seven classes of the RPS leg are identified and the geometric properties of each class are highlighted. By selecting three different or identical classes of the RPS leg, a new 3-RPS parallel manipulator is proposed without specific values of the design parameters. The primary decomposition is computed over a set of three constraint equations. One or more Euler parameters in the results of primary decomposition is constrained to be equal to null, which leads to particular type of operation mode. The methodology also provides new architectures of the 3-RPS parallel manipulators based on a classification of the RPS leg, that satisfy the prescribed operation mode.

**Key words:** Synthesis, design parameters, primary decomposition, operation mode, RPS legs.

## 1 Introduction

The well-known 3-RPS (R, P, S, represent revolute, prismatic, and spherical, respectively) parallel manipulator with different shapes of moving platform and base was extensively studied by many researchers. In 1983 [4], Hunt introduced the 3-RPS manipulator which has an equilateral triangle base and an equilateral triangle platform. Schadlbauer *et al.* in [7] revealed that this manipulator has two distinct operation modes and in [6] the authors characterized the type of motion in both operation modes by using the axodes. Huang *et al.* in 1995 [2] proposed the 3-RPS cube manipulator which is composed of a cube-shaped base and an equilateral triangle platform. By using the Study kinematic mapping, Nurahmi *et al.* in [5] found that this manipulator has only one operation mode in which the 3-*dof* general motion and the Vertical Darboux Motion occur inside the same operation mode.

Accordingly, a general approach to synthesize the design parameters by considering the prescribed operation modes for a parallel manipulator with three RPS legs, is discussed in more details in this paper. The approach is based on the Euler parametrization [1], and the primary decomposition is used to reveal the existence of the number and the type of operation modes [8]. The first essential step is to

characterize the coordinates and to define the design parameters of one RPS leg. Then, the constraint equation of this RPS leg is formulated by means of the Euler parametrization. A classification of seven classes of the RPS legs are found, which gives a general information about the position and the orientation of the RPS legs. By selecting three different or identical classes of the RPS legs, a new 3-RPS manipulator is generated without specific value of the design parameters. The constraint equations of the corresponding new manipulator are derived and the primary decomposition is computed. By restricting one or more Euler parameters to be equal to null, the design parameters can be synthesized.

## 2 Coordinates and design parameters of RPS leg

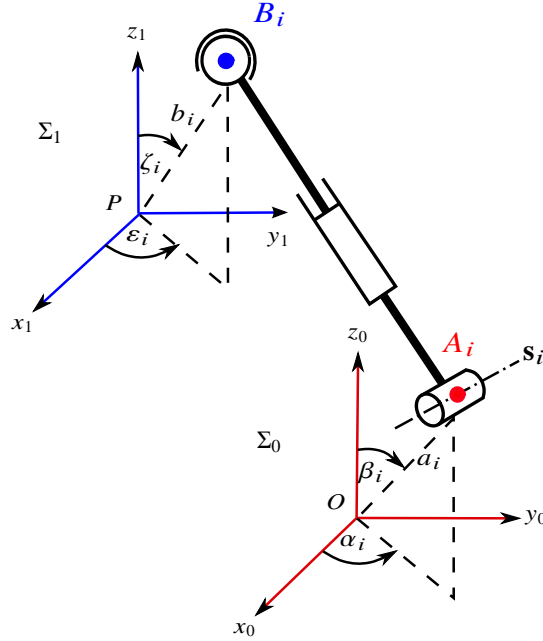


Fig. 1: Coordinates and Design Parameters of RPS Leg.

The RPS leg shown in Fig. 1, is composed of a revolute joint, a prismatic joint, and a spherical joint mounted in series. The revolute joint is attached to the base and denoted by point  $A_i^1$ . This point is located in the three-dimensional space which is specified by the azimuth angle  $\alpha_i$ , the polar angle  $\beta_i$ , and the radial distance  $a_i$  from the origin  $O$  of the fixed frame  $\Sigma_0$ .

The spherical joint is attached to the moving platform and denoted by point  $B_i$ . This point is also located in the three-dimensional space which is specified by the

<sup>1</sup>  $i$  denotes the number of the leg used in the manipulator. In this paper  $i=1,2,3$ .

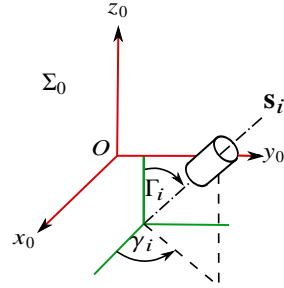


Fig. 2: The axis of revolute joint.

azimuth angle  $\varepsilon_i$ , the polar angle  $\zeta_i$ , and the radial distance  $b_i$  from the origin  $P$  of the moving frame  $\Sigma_1$ . The axis of the revolute joint is along the vector  $\mathbf{s}_i$ , which is specified by the azimuth angle  $\gamma_i$  and the polar angle  $\Gamma_i$  (Fig. 2). The coordinates of points  $A_i$ ,  $B_i$  and unit vector  $\mathbf{s}_i$  are:

$$\begin{aligned} \mathbf{r}_{A_i}^0 &= [1, c_{\alpha_i}c_{\beta_i}a_i, s_{\alpha_i}c_{\beta_i}a_i, s_{\beta_i}a_i]^T, \quad \mathbf{r}_{B_i}^1 = [1, c_{\varepsilon_i}c_{\zeta_i}b_i, s_{\varepsilon_i}c_{\zeta_i}b_i, s_{\zeta_i}b_i]^T \\ \mathbf{s}_i &= [0, c_{\gamma_i}c_{\Gamma_i}, s_{\gamma_i}c_{\Gamma_i}, s_{\Gamma_i}]^T. \end{aligned} \quad (1)$$

where  $c_{\alpha_i} = \cos(\alpha_i)$ ,  $s_{\alpha_i} = \sin(\alpha_i)$ ,  $c_{\beta_i} = \cos(\beta_i)$ ,  $s_{\beta_i} = \sin(\beta_i)$ ,  $c_{\varepsilon_i} = \cos(\varepsilon_i)$ ,  $s_{\varepsilon_i} = \sin(\varepsilon_i)$ ,  $c_{\zeta_i} = \cos(\zeta_i)$ ,  $s_{\zeta_i} = \sin(\zeta_i)$ ,  $c_{\gamma_i} = \cos(\gamma_i)$ ,  $s_{\gamma_i} = \sin(\gamma_i)$ ,  $c_{\Gamma_i} = \cos(\Gamma_i)$ , and  $s_{\Gamma_i} = \sin(\Gamma_i)$ . As a consequence, there are eight design parameters for one RPS leg, namely  $a_i$ ,  $b_i$ ,  $\alpha_i$ ,  $\beta_i$ ,  $\varepsilon_i$ ,  $\zeta_i$ ,  $\gamma_i$ , and  $\Gamma_i$ . Since the manipulator that will be created should have three RPS legs, hence there are 24 design parameters in the 3-RPS manipulator.

### 3 Constraint Equations

In this section, the constraint equation is expressed for one RPS leg shown in Fig. 1. To obtain the coordinates of point  $B_i$  expressed in  $\Sigma_0$ , the transformation matrix  $\mathbf{M}^2$  by means of the Euler parametrization [1] is used.

The parameters  $x_0, x_1, x_2, x_3$ , which appear in matrix  $\mathbf{M}$ , are called Euler parameters of the rotation. They are useful in the representation of a spatial Euclidean displacement and they should satisfy the equation [3]:  $x_0^2 + x_1^2 + x_2^2 + x_3^2 - 1 = 0$ . This condition will be used in the following computations to simplify the algebraic expressions. The coordinate of point  $B_i$  expressed in  $\Sigma_0$  is obtained by:  $\mathbf{r}_{B_i}^0 = \mathbf{M} \mathbf{r}_{B_i}^1$ .

As the coordinates of all points are given in terms of the Euler parameters and the design parameters, the constraint equation can be obtained by examining the design of the RPS leg. The leg connecting points  $A_i$  and  $B_i$  is orthogonal to the axis  $\mathbf{s}_i$  of the revolute joint. Accordingly, the scalar product of vector  $(\mathbf{r}_{B_i}^0 - \mathbf{r}_{A_i}^0)$  and vector  $\mathbf{s}_i$  vanishes, namely:  $(\mathbf{r}_{B_i}^0 - \mathbf{r}_{A_i}^0)^T \mathbf{s}_i = 0$ .

After computing the corresponding scalar products and removing the common denominators the following constraint equation of one RPS leg comes out:

$$\begin{aligned} h_i : & c_{\gamma_i}c_{\Gamma_i}X + c_{\Gamma_i}s_{\gamma_i}Y + (x_0^2 - x_1^2 - x_2^2 + x_3^2)s_{\zeta_i}s_{\Gamma_i}b_i - s_{\beta_i}s_{\Gamma_i}a_i + (2x_1x_2 - 2x_0x_3)b_i \\ & c_{\Gamma_i}c_{\gamma_i}s_{\varepsilon_i}c_{\zeta_i} + (2x_0x_3 + 2x_1x_2)b_is_{\gamma_i}c_{\Gamma_i}c_{\zeta_i}c_{\varepsilon_i} + (x_0^2 + x_1^2 - x_2^2 - x_3^2)c_{\varepsilon_i}c_{\zeta_i}c_{\gamma_i}c_{\Gamma_i}b_i \\ & - c_{\alpha_i}c_{\beta_i}c_{\gamma_i}c_{\Gamma_i}a_i + (x_0^2 - x_1^2 + x_2^2 - x_3^2)c_{\zeta_i}s_{\varepsilon_i}c_{\Gamma_i}s_{\gamma_i}b_i - c_{\beta_i}c_{\Gamma_i}s_{\alpha_i}s_{\gamma_i}a_i + (2x_0x_1 + \\ & 2x_2x_3)b_is_{\Gamma_i}s_{\varepsilon_i}c_{\zeta_i} + (2x_0x_2 + 2x_1x_3)b_ic_{\Gamma_i}c_{\gamma_i}s_{\zeta_i} + (2x_2x_3 - 2x_0x_1)b_is_{\gamma_i}c_{\Gamma_i}s_{\zeta_i} + \\ & (2x_1x_3 - 2x_0x_2)b_is_{\Gamma_i}c_{\zeta_i}c_{\varepsilon_i} + s_{\Gamma_i}Z = 0 \end{aligned} \quad (2)$$

<sup>2</sup> For detail expression of the transformation matrix, the reader may refer to <http://www.irccyn.ec-nantes.fr/%7enurahmi/ISRM2015/Appendix.pdf>

## 4 Classifications of the RPS legs

In this section, the constraint equation associated with the design parameters are solved to synthesize seven classes of the RPS legs. The constraint equation  $h_i$  in Eq. (2) should vanish in any condition, likewise in the identity condition. In the identity condition  $\Sigma_0$  and  $\Sigma_1$  are coincident, and we have the identity transformation  $\mathbf{I}$  in which the parameters become  $x_0 = 1, x_1 = 0, x_2 = 0, x_3 = 0, X = 0, Y = 0, Z = 0$ . By substituting these values into Eq. (2), this yields:

$$h_{\mathbf{I}} : (c_{\varepsilon_i} c_{\zeta_i} c_{\gamma_i} c_{\Gamma_i} + c_{\zeta_i} c_{\Gamma_i} s_{\varepsilon_i} s_{\gamma_i} + s_{\zeta_i} s_{\Gamma_i}) b_i - (c_{\alpha_i} c_{\beta_i} c_{\gamma_i} c_{\Gamma_i} + c_{\beta_i} c_{\Gamma_i} s_{\alpha_i} s_{\gamma_i} + s_{\beta_i} s_{\Gamma_i}) a_i = 0 \quad (3)$$

Eq. (3) can be written as  $h_{\mathbf{I}} : f - g = 0$ . To find the relations between the design parameters for which  $h_{\mathbf{I}}$  vanishes, we compute one particular condition where  $f, g$  vanish simultaneously. Note that it also amounts to the condition to be fulfilled for which  $h_{\mathbf{I}}$  vanishes no matter the values of  $a_i$  and  $b_i$ . One has to discuss the ideal  $\mathcal{I} = \langle f, g \rangle$  and compute the Groebner basis. The relations containing complex terms are discarded and 23 relations remain, and substituted into Eq. (1). Based on their geometric properties, seven classes are identified and each class contains one or more sub-classes<sup>3</sup>. The sub-classes give the location of the RPS legs in the three-dimensional space, in which  $\mathbf{r}_{A_i}^0$  gives the location of the revolute joint with respect to  $\Sigma_0$ ,  $\mathbf{r}_{B_i}^1$  gives the location of the spherical joint with respect to  $\Sigma_1$ , and  $\mathbf{s}_i$  gives the unit vector of the axis of the revolute joint.

By selecting three different or identical classes, a new manipulator with three RPS legs can be created. The user may assign some arbitrary values into the design parameters and assemble the legs accordingly. However, it is interesting to generate various designs of the 3-RPS manipulator that fulfil the prescribed operation modes as presented in the following.

## 5 Synthesis of Design Parameters

In the following, an example of 3-RPS manipulator with three identical classes of the RPS leg will be presented. Then, the design parameters associated with the new manipulator are synthesized by imposing the prescribed operation modes.

### 5.1 Sub-class F.2

In this section, the 3-RPS manipulator is generated by selecting three identical sub-classes, namely sub-class F.2. The RPS leg in this class consists of revolute joint and spherical joint that are located in any position with respect to  $\Sigma_0$  and  $\Sigma_1$ , respectively. The axis of the revolute joint is parallel to the  $\mathbf{xy}$ -plane.

Due to the heavy computations, points  $A_i$  and  $B_i$  are assumed to lie in the  $\mathbf{xy}$ -plane of  $\Sigma_0$  and  $\Sigma_1$ , respectively. Therefore, some values are assigned for  $\beta_1 = \beta_2 =$

<sup>3</sup> For detail expressions of classes and sub-classes of the RPS leg, and complete computation results from Maple; the reader may refer to <http://www.irccyn.ec-nantes.fr/%7enurahmi/ISRM2015/Appendix.pdf>

$\beta_3 = 0$  and  $\zeta_1 = \zeta_2 = \zeta_3 = 0$ . The first RPS leg of the manipulator is fixed by substituting  $\varepsilon_1 = 0$ . To obtain the coordinates of points  $B_1, B_2, B_3$  expressed in  $\Sigma_0$ , the coordinate transformation is performed by means of the Euler parametrization as:  $\mathbf{r}_{B_i}^0 = \mathbf{M} \mathbf{r}_{B_i}^1$  ( $i = 1, 2, 3$ ). The constraint equations are determined by computing the scalar products of the vector  $A_i B_i$  and the unit vector  $\mathbf{s}_i$  which has to vanish as:  $(\mathbf{r}_{B_i}^0 - \mathbf{r}_{A_i}^0)^T \mathbf{s}_i = 0$ . The constraint equations turn out:

$$\begin{aligned} h_1 &: Y + (2x_0x_3 + 2x_1x_2)b_1 = 0 \\ h_2 &: 4c_{\varepsilon_2}^2 b_2 x_1 x_2 - 2(x_1^2 - x_2^2)c_{\varepsilon_2} s_{\varepsilon_2} b_2 + c_{\varepsilon_2} Y - s_{\varepsilon_2} X + (2x_0x_3 - 2x_1x_2)b_2 = 0 \quad (4) \\ h_3 &: 4c_{\varepsilon_3}^2 b_3 x_1 x_2 - 2(x_1^2 - x_2^2)c_{\varepsilon_3} s_{\varepsilon_3} b_3 + c_{\varepsilon_3} Y - s_{\varepsilon_3} X + (2x_0x_3 - 2x_1x_2)b_3 = 0 \end{aligned}$$

For the algebraic computation, the half-tangent substitutions are performed to remove the trigonometric functions in the second and the third legs:  $s_{\varepsilon_i} = (2te_i)/(1 + te_i^2)$ ,  $c_{\varepsilon_i} = (1 - te_i^2)/(1 + te_i^2)$ ,  $i = 2, 3$ . Then the three constraint equations are written as polynomial ideal  $\mathcal{I} = \langle h_1, h_2, h_3 \rangle$  with variables  $\{x_0, x_1, x_2, x_3, X, Y, Z\}$  over the coefficient ring  $\mathbb{C}[b_1, b_2, b_3, te_2, te_3]$ . The primary decomposition is computed and it turns out that  $\mathcal{I}$  does not decompose, i.e. it has general expressions as  $\mathcal{I} = \langle g_1, g_2, g_3 \rangle$ , as follows<sup>4</sup>:

$$\begin{aligned} g_1 &: (2b_2 te_2^3 te_3^3 - 2b_3 te_2^3 te_3^3 + 2b_2 te_2^3 te_3 - 2b_2 te_2 te_3^3 + 2b_3 te_2^3 te_3 - 2b_3 te_2 te_3^3) \dots \\ g_2 &: Y(b_1 te_2^4 te_3^3 - b_1 te_2^3 te_3^4 + b_2 te_2^4 te_3^3 - b_3 te_2^3 te_3^4 + b_1 te_2^4 te_3 - b_1 te_2 te_3^4 - b_3 te_2 \dots \\ g_3 &: X(b_1 te_2^4 te_3^3 - b_1 te_2^3 te_3^4 + b_2 te_2^4 te_3^3 - b_3 te_2^3 te_3^4 + b_1 te_2^4 te_3 - b_1 te_2 te_3^4 + b_2 te_2^4 \dots \end{aligned} \quad (5)$$

It can be seen from Eq. (5) that  $g_1, g_2, g_3$  are free of  $Z$  component. This means that for any value of the design parameters  $(b_1, b_2, b_3, \varepsilon_2, \varepsilon_3)$ , the manipulator can always perform a pure translation along  $\mathbf{z}$  direction. Variable  $x_3$  can be solved linearly from  $g_1$  and  $x_3$  is parametrized by  $x_0, x_1, x_2$ . This means that the manipulator is capable of orientations determined by  $x_0, x_1, x_2$  in which variable  $x_3$  is not null. Equations  $g_2, g_3$  can be solved linearly for variables  $Y$  and  $X$ , respectively. This shows that the manipulator undergoes translational motions along  $\mathbf{x}$  and  $\mathbf{y}$  directions which are coupled to the orientations. In the following, the rotational components  $\{x_0, x_1\}$  from  $g_1, g_2, g_3$  are constrained to be equal to zero, which leads to different operation modes. By fulfilling this condition, the design parameters are synthesized and new architectures are proposed.

### 5.1.1 Case $x_0 = 0$

One variable is constrained to be null, namely  $x_0 = 0$ . Since only the equation  $g_1$  has component  $x_0$ , the computation will be carried out only for  $g_1$ . After substituting  $x_0 = 0$ , equation  $g_1$  becomes:  $g_1 : ax_1^2 + bx_1x_2 + cx_2^2 = 0$ , where  $a, b, c$  are polynomial coefficients in terms of the design parameters  $(b_1, b_2, b_3, te_2, te_3)$ .

To synthesize the design parameters, all polynomial coefficients have to vanish. Hence, one has to discuss the ideal  $\mathcal{J} = \langle a, b, c \rangle$ . The Groebner basis of the ideal  $\mathcal{J}$

<sup>4</sup> For complete results of the primary decomposition, the reader may refer to <http://www.irccyn.ec-nantes.fr/%7enurahmi/ISRM2015/Appendix.pdf>

is computed and 17 solutions of the design parameters are obtained. Not all solutions are possible and hence some assumptions are developed, as follows:

1. The second and the third legs cannot be coincident with the first leg:  
-  $\varepsilon_2 \neq 0$  and  $\varepsilon_3 \neq 0$
2. The second leg cannot be coincident with the third leg:  
-  $\varepsilon_2 \neq \varepsilon_3$
3. The magnitude of  $b_i$  ( $i = 1, 2, 3$ ) should be positive:  
-  $b_i \geq 0, i = 1, 2, 3$
4. The platform cannot be a point:  
-  $b_1 \neq b_2 \neq b_3 \neq 0$
5. No complex solutions:  
-  $\{b_1, b_2, b_3, \varepsilon_2, \varepsilon_3\} \in \mathbb{R}$

After removing the solutions that do not fulfil the assumptions stated above, four solutions of the design parameters are obtained. The solutions are:

$$\begin{aligned}
 L_1 : \quad & b_2 = 0, \quad b_3 = 0, \quad \varepsilon_3 = \pi + \varepsilon_2 \\
 L_2 : \quad & b_2 = \frac{b_1}{\tan(\varepsilon_3)}, \quad b_3 = 0, \quad \varepsilon_2 = \frac{\pi}{2}, \quad \varepsilon_3 \neq 0 \text{ or } \varepsilon_3 \neq \pm\pi \\
 L_3 : \quad & b_2 = -\frac{b_1}{\tan(\varepsilon_3)}, \quad b_3 = 0, \quad \varepsilon_2 = -\frac{\pi}{2}, \quad \varepsilon_3 \neq 0 \text{ or } \varepsilon_3 \neq \pm\pi \\
 L_4 : \quad & b_1 = b_3 \frac{\cos(\varepsilon_2 - \varepsilon_3)}{\cos(\varepsilon_2)}, \quad b_2 = b_3 \frac{\cos(\varepsilon_3)}{\cos(\varepsilon_2)}, \quad \varepsilon_2 \neq \pm\frac{\pi}{2} \text{ or } \varepsilon_2 \neq \pm\frac{3\pi}{2}
 \end{aligned} \tag{6}$$

The 3-RPS manipulator can be generated by selecting one of the solutions ( $L_1, L_2, L_3, L_4$ ). In the following, the application of the solution  $L_2$  is presented.

#### Solution $L_2$

In solution  $L_2$ , some values are assigned as  $b_1 = 1$  and  $\varepsilon_3 = -2\pi/3$ . Other design parameters are obtained as:  $b_2 = \sqrt{3}/3$ ,  $b_3 = 0$ ,  $\varepsilon_2 = \pi/2$ . The new architecture of the 3-RPS manipulator is depicted in Fig. 3. The base and the moving platform have right-angle triangle shape. The unit vectors  $\mathbf{s}_1$  and  $\mathbf{s}_2$  are orthogonal ( $\mathbf{s}_1 \perp \mathbf{s}_2$ ).

The values of the design parameters are substituted into the set of three constraint equations defined in Eq. (4). The primary decomposition is computed and it shows that the mechanism has two operation modes as follows:  $\mathcal{K} = \bigcap_{i=1}^2 \mathcal{K}_i$ , with the results of primary decomposition:

$$\begin{aligned}
 \mathcal{K}_1 &= \langle x_0, 3X - \sqrt{3}Y, 2x_1x_2 + Y \rangle \\
 \mathcal{K}_2 &= \langle x_3, 3X - \sqrt{3}Y, 2x_1x_2 + Y \rangle
 \end{aligned} \tag{7}$$

First operation mode is shown by the first sub-ideal  $\mathcal{K}_1$ , in which  $x_0 = 0$ . All possible poses of the mechanism in this operation mode are obtained by rotating the platform from the identity condition about a transformation axis by  $\pi$  and translating along the same direction. Second operation mode is shown by the sub-ideal  $\mathcal{K}_2$  with  $x_3 = 0$ . In this operation mode, the transformation axis is parallel to the  $\mathbf{xy}$ -plane. The investigation of these two operation modes are discussed in more details in [7].

### 5.1.2 Case $x_1 = 0$

In this section, the variable  $x_1$  in Eq. (5) is constrained to be null. After computing the Groebner basis, 11 solutions of the design parameters are obtained. Not all solutions are possible and hence by following the aforementioned assumptions in Section 5.1.1, three solutions are obtained as:

$$\begin{aligned} L_1 : \quad & b_2 = -\frac{b_1}{\tan(\varepsilon_3)}, \quad b_3 = 0, \quad \varepsilon_2 = \frac{\pi}{2}, \quad \varepsilon_3 \neq 0 \text{ or } \varepsilon_3 \neq \pm\pi \\ L_2 : \quad & b_2 = \frac{b_1}{\tan(\varepsilon_3)}, \quad b_3 = 0, \quad \varepsilon_2 = -\frac{\pi}{2}, \quad \varepsilon_3 \neq 0 \text{ or } \varepsilon_3 \neq \pm\pi \\ L_3 : \quad & b_1 = -b_3 \frac{\cos(\varepsilon_2 - \varepsilon_3)}{\cos(\varepsilon_2)}, \quad b_2 = b_3 \frac{\cos(\varepsilon_3)}{\cos(\varepsilon_2)}, \quad \varepsilon_2 \neq \pm\frac{\pi}{2} \text{ or } \varepsilon_2 \neq \pm\frac{3\pi}{2} \end{aligned} \quad (8)$$

By choosing one of the solutions ( $L_1, L_2, L_3$ ), a new 3-RPS manipulator can be built. The application of the solution  $L_3$  is presented in the following.

#### Solution $L_3$

Solution  $L_3$  is selected to generate the 3-RPS manipulator whose operation modes contain  $x_1 = 0$ . The design parameters  $b_3 = 1$ ,  $\varepsilon_2 = \pi/4$ , and  $\varepsilon_3 = -\pi/4$  are assigned, hence  $b_1 = \sqrt{2}$  and  $b_2 = 1$  are determined. The 3-RPS manipulator with these design parameters is depicted in Fig. 4, in which the base and the moving platform have right-angle triangle shape. The axes of the second and the third revolute joints are orthogonal and meet at point  $A_1$ .

The values of the design parameters are substituted into the set of three constraint equations defined in Eq. (4). The primary decomposition is computed and it shows that the mechanism has two operation modes as follows:  $\mathcal{K} = \bigcap_{i=1}^2 \mathcal{K}_i$ , with the results of primary decomposition:

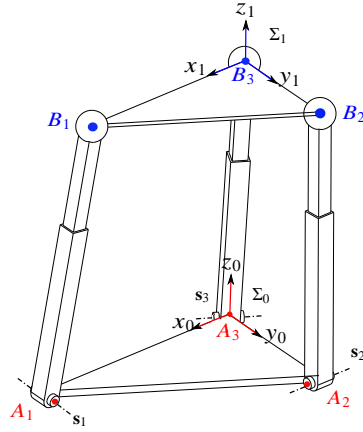
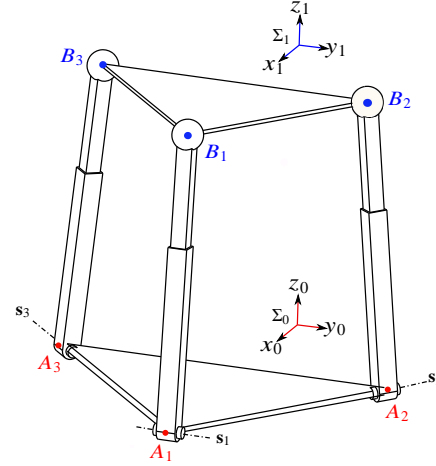
$$\begin{aligned} \mathcal{K}_1 &= \langle x_1, 4x_0x_3 + \sqrt{2}Y, 2x_2^2 - \sqrt{2}X \rangle \\ \mathcal{K}_2 &= \langle x_2, 4x_0x_3 + \sqrt{2}Y, 2x_1^2 + \sqrt{2}X \rangle \end{aligned} \quad (9)$$

The sub-ideal  $\mathcal{K}_1$  shows the first operation mode of this manipulator, in which  $x_1 = 0$ . In this operation mode, the moving platform is transformed from the identity condition about an axis parallel to the  $\mathbf{yz}$ -plane. The second operation mode of this manipulator is shown by sub-ideal  $\mathcal{K}_2$  with  $x_2 = 0$ . The transformation axis of this operation mode is parallel to the  $\mathbf{xz}$ -plane.

## 6 Conclusions

In this paper, the synthesis of the design parameters corresponding to the prescribed operation modes for a parallel manipulator with three RPS legs was addressed. The Euler parametrization and the results of primary decomposition were used to define the synthesis procedure by considering the type of operation modes at the design stage. The design parameters and the coordinates of one RPS leg were initially defined. Then, the constraint equation corresponding to the RPS leg was derived. Seven classes of the RPS legs were developed, in which each class contains several sub-classes that corresponds to the specific position and orientation of the RPS legs.



Fig. 3: Design with solution  $L_2$ .Fig. 4: Design with solution  $L_3$ .

As a result, it is possible to generate new architectures of the 3-RPS manipulator by selecting three different or identical classes of the RPS legs. Some constraints were applied to the Euler parameters in the results of primary decomposition that leads to particular types of operation modes, and then the design parameters were synthesized. Several architectures of the 3-RPS manipulators corresponding to the prescribed operation modes were presented. The applications of the proposed approach for parallel manipulators with different type of legs, will be the subject of future research.

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