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# Dimensionally Homogeneous Extended Jacobian and Condition Number

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**Keywords:** screw theory, condition number, dimensionally homogeneous extended Jacobian matrix, singularities, and parallel manipulators.

**Abstract.** This paper deals with the formulation of the dimensionally homogeneous extended Jacobian matrix, which is an important issue for the performance analysis of  $f$  degrees-of-freedom ( $f \leq 6$ ) parallel manipulator having coupled rotational and translational motions. By using the  $f$  independent coordinates to define the permitted motions and  $(6-f)$  independent coordinates to define the restricted motions of the moving platform, the  $6 \times 6$  dimensionally homogeneous extended Jacobian matrix is derived for non-redundant parallel manipulators. The condition number is computed over the homogeneous extended Jacobian matrix, the homogeneous actuation wrench matrix, and the homogeneous constraint wrench matrix to evaluate the performance of the parallel manipulators. By using these indices, the closeness of a pose to different singularities can be detected. An illustrative example with the 3-RPS parallel manipulator is provided to highlight the effectiveness of the approach and the proposed indices.

## Introduction

The input-output relationship of the manipulators is represented by the Jacobian matrix that maps the actuators rates onto the velocity twist of the moving platform. By applying the theory of reciprocal screw, Joshi and Tsai in [1] proposed a systematic procedure to express a  $6 \times 6$  overall Jacobian matrix that comprises both  $f$  linearly independent actuation wrenches and  $(6-f)$  linearly independent constraint wrenches. However, the overall Jacobian matrix considers only the instantaneous motions accessible by the moving platform. In [2], Huang *et al.* developed a methodology to formulate the Jacobian matrix that takes into account the instantaneous motions inaccessible by the moving platform, named the generalized Jacobian matrix.

The Jacobian matrix of parallel manipulators with coupled rotational and translational motions, possess nonhomogeneous physical units. It leads to significant problem in which the computation of the condition number will vary with the scaling of dimensions. Dated back to 1990 [3], Gosselin used the velocities of two points to describe the velocity of the end effector hence the entries of the Jacobian matrix would bear the same units. Stocco *et al.* in [4] presented the diagonal scaling matrices by separating the task-space forces and the joint-space torques. The characteristic/natural length was introduced in [5-7] to overcome the issue of nonhomogeneous physical units. Kim and Ryu in [8] used three non collinear points on the moving platform and linked the linear velocities of these points with the actuators rates. In [9], Liu *et al.* introduced a tetrahedron containing five points and specified relevant axes at these points. A set of independent axes are selected which represents the permitted motions of the moving platform. Then, the dimensionally homogeneous Jacobian matrix is derived by combining the linear map between the twist of moving platform with the selected independent coordinates, and the generalized Jacobian matrix defined in [2]. However, this method does not take into account the restricted motions due to the existence of the constraint wrenches on the moving platform, that also affect the performance of the lower-mobility parallel manipulators.

Accordingly, this paper focuses on the formulation of the dimensionally homogeneous extended Jacobian matrix for non-redundant lower-mobility parallel manipulators with coupled rotational and translational motions. By considering a set of linear independent axes which depicts the restricted motions of the moving platform, the  $6 \times 6$  dimensionally homogeneous extended Jacobian matrix is derived. The method is applied to the 3-RPS (R, P, S denote prismatic, revolute, and spherical joints, respectively) manipulator. The condition number is computed over the homogeneous extended

Jacobian matrix, the homogeneous actuation wrench matrix, and the homogeneous constraint wrench matrix. These indices can measure the closeness of a pose to different singularities, namely the actuation singularity, the constraint singularity, and the compound singularity.

### Formulation of the Jacobian Matrix

This section briefly reviews the derivation of the Jacobian matrix for non-redundant parallel manipulators having  $f$ -*dof*. The instantaneous motions of a body must form a six dimensional vector, known as twist space  $\mathcal{T}$ , that comprises a permitted twist subspace  $\mathcal{T}_a$  and a restricted twist subspace  $\mathcal{T}_c$ . The subspaces  $\mathcal{T}_a$  and  $\mathcal{T}_c$  have  $\$_{tai}$  and  $\$_{tcj}$  being its elements, respectively.  $\$_{tai}$  is the  $i$ -th permitted twist screw of the moving platform and  $\$_{tcj}$  is the  $j$ -th virtual restricted twist screw that is produced by releasing the correspondence  $j$ -th constraint wrench.

On the other hand, the entire set of wrenches exerted in the moving platform forms six dimensional vector space, known as wrench space  $\mathcal{W}$ , that comprises an actuation wrench subspace  $\mathcal{W}_a$  and a constraint wrench subspace  $\mathcal{W}_c$ . The subspaces  $\mathcal{W}_a$  and  $\mathcal{W}_c$  have  $\hat{\$}_{wai}$  and  $\hat{\$}_{wcj}$  being its elements, respectively. Therefore, the relationships between all four subspaces are:  $\mathcal{W}_a = \mathcal{T}_c^\perp$  and  $\mathcal{W}_c = \mathcal{T}_a^\perp$ .

The twist of the moving platform at each instant  $\$t \in \mathcal{T}$  can be expressed as linear combination of the elements of  $\mathcal{T}_a$  and  $\mathcal{T}_c$ , as:

$$\$t = \$_{tai} + \$_{tcj} = \sum_{i=1}^f \dot{q}_{ai} \hat{\$}_{tai} + \sum_{j=1}^{(6-f)} \dot{q}_{cj} \hat{\$}_{tcj} \quad (1)$$

where  $\dot{q}_{ai}$  and  $\dot{q}_{cj}$  are the actuated joints rates and the virtual constrained joints rates, respectively. The twist  $\$t$  contains the linear velocity  $\mathbf{v}_o$  and the angular velocity  $\boldsymbol{\omega}_o$  of the moving platform origin. By performing the reciprocal product on both sides of Eq. (1) with  $\hat{\$}_{wai}$  and  $\hat{\$}_{wcj}$ , we obtain:

$$\mathbf{J}_t \$t = \mathbf{J}_q \dot{\mathbf{q}} \quad (2)$$

$$\mathbf{J}_t = \begin{bmatrix} \mathbf{J}_{ta} \\ \mathbf{J}_{tc} \end{bmatrix}, \mathbf{J}_{ta} = \begin{bmatrix} \hat{\$}_{wa1} \\ \vdots \\ \hat{\$}_{waf} \end{bmatrix}, \mathbf{J}_{tc} = \begin{bmatrix} \hat{\$}_{wc1} \\ \vdots \\ \hat{\$}_{wc(6-f)} \end{bmatrix}, \mathbf{J}_q = \begin{bmatrix} \mathbf{J}_{qa} \\ \mathbf{J}_{qc} \end{bmatrix}, \mathbf{J}_{qa} = \begin{bmatrix} \hat{\$}_{wa1}^T \hat{\$}_{ta1} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & \hat{\$}_{waf}^T \hat{\$}_{taf} \end{bmatrix},$$

$$\mathbf{J}_{qc} = \begin{bmatrix} \hat{\$}_{wc1}^T \hat{\$}_{tc1} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & \hat{\$}_{wc(6-f)}^T \hat{\$}_{tc(6-f)} \end{bmatrix}, \dot{\mathbf{q}} = \begin{bmatrix} \dot{\mathbf{q}}_a \\ \dot{\mathbf{q}}_c \end{bmatrix}, \dot{\mathbf{q}}_a = \begin{bmatrix} \dot{q}_{a1} \\ \vdots \\ \dot{q}_{af} \end{bmatrix}, \dot{\mathbf{q}}_c = \begin{bmatrix} \dot{q}_{c1} \\ \vdots \\ \dot{q}_{c(6-f)} \end{bmatrix}, \$t = \begin{bmatrix} \boldsymbol{\omega}_o \\ \mathbf{v}_o \end{bmatrix}.$$

where  $\mathbf{J}_t$  and  $\mathbf{J}_q$  are  $6 \times 6$  Jacobian matrices for non-redundant parallel manipulators. The derivation of the Jacobian matrices defined in Eq. (2) will be employed to formulate the  $6 \times 6$  dimensionally homogeneous extended Jacobian matrix discussed hereafter.

### Formulation of the $6 \times 6$ Dimensionally Homogeneous Extended Jacobian Matrix

Theoretically, the rotations and the translations of a body can be sufficiently described by the translations of three non collinear points embedded on it. Let us consider a moving platform as shown in Fig. 1 with the origin  $O$  and the moving frame  $\Sigma_1$  attached to it. It rotates with angular velocity  $\boldsymbol{\omega}_o$  and translates with linear velocity  $\mathbf{v}_o$  with respect to the fixed frame  $\Sigma_0$  of the base. If point  $P$  is embedded at the moving platform with the position vector  $\mathbf{p}^1$  (with respect to the moving frame  $\Sigma_1$ ), its velocity with respect to the fixed frame  $\Sigma_0$  can be defined as:

$$\mathbf{v}_p^0 = \mathbf{v}_o + \boldsymbol{\omega}_o \times \mathbf{p}^0 \quad (3)$$

where  $\mathbf{p}^0 = \mathbf{M} \mathbf{p}^1$ .  $\mathbf{M}$  is a spatial Euclidean transformation matrix  $\mathbf{M} \in SE(3)$  defined in [10]. Place a set of axes at point  $P$  defined as  $\mathbf{e}_{i,j}^1$ . After transforming into  $\mathbf{e}_{i,j}^0$ , take the dot product on both sides of Eq. (3) with  $\mathbf{e}_{i,j}^0$ , it yields:

$$\mathbf{v}_{p(i,j)}^0 = \begin{bmatrix} \mathbf{p}^0 \times \mathbf{e}_{i,j}^0 \\ \mathbf{e}_{i,j}^0 \end{bmatrix} \$_t \quad (4)$$

To describe the velocity of the moving platform, a tetrahedron adopted from [9] is defined in Fig. 2. It contains five points ( $P_i, i = 0..4$ ) with an equilateral base and three identical isosceles faces. The orthogonal axes are attached at each point i.e.  $\mathbf{e}_{i,1}^1 \perp \mathbf{e}_{i,2}^1$ ,  $\mathbf{e}_{i,1}^1 \perp \mathbf{p}^1$ , and  $\mathbf{e}_{i,2}^1 \perp \mathbf{p}^1$ , ( $i = 1..4$ ).

$$\begin{aligned} \mathbf{e}_{1,1}^1 &= [0 \ 1 \ 0]^T, \mathbf{e}_{2,1}^1 = [0 \ -\frac{\sqrt{3}}{2} \ -\frac{1}{2}]^T, \mathbf{e}_{3,1}^1 = [0 \ \frac{\sqrt{3}}{2} \ -\frac{1}{2}]^T, \mathbf{e}_{1,2}^1 = [0 \ 0 \ 1]^T, \\ \mathbf{e}_{2,2}^1 &= [0 \ 0 \ 1]^T, \mathbf{e}_{3,2}^1 = [0 \ 0 \ 1]^T, \mathbf{e}_{4,1}^1 = [1 \ 0 \ 0]^T, \mathbf{e}_{4,2}^1 = [0 \ 1 \ 0]^T, \\ \mathbf{e}_{0,1}^1 &= [1 \ 0 \ 0]^T, \mathbf{e}_{0,2}^1 = [0 \ 1 \ 0]^T, \mathbf{e}_{0,3}^1 = [0 \ 0 \ 1]^T. \end{aligned} \quad (5)$$

Transform all the position vectors and the unit vectors in Eq. (5) with respect to the fixed frame  $\Sigma_0$  and substitute into Eq. (4), we can obtain in matrix form:

$$\mathbf{v}_p^0 = \mathbf{T} \$_t \quad (6)$$

where

$$\mathbf{v}_p^0 = [\mathbf{v}_{p0,1}^0 \ \mathbf{v}_{p0,2}^0 \ \cdots \ \mathbf{v}_{p4,2}^0]^T \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} (\mathbf{p}^0 \times \mathbf{e}_{0,1}^0) & \mathbf{e}_{0,1}^0 \\ \vdots & \vdots \\ (\mathbf{p}^0 \times \mathbf{e}_{4,2}^0) & \mathbf{e}_{4,2}^0 \end{bmatrix}$$

and  $\mathbf{T}$  is a  $11 \times 6$  matrix. For any  $f$ -dof non-redundant parallel manipulators with certain motion type, any  $f$  linearly independent row vectors of  $\mathbf{T}$  can be selected to represent the permitted motions of the moving platform. Furthermore,  $(6-f)$  linearly independent row vectors of  $\mathbf{T}$  are selected such that it depicts the restricted motions of the moving platform. By substituting Eq. (2) into Eq. (6), we can obtain the linear map from the joint rates  $\dot{\mathbf{q}}$  onto the linear and angular velocities of the moving platform, represented by the selected points ( $P_i$ ).

$$\mathbf{J}_p \mathbf{v}_p^0 = \mathbf{J}_q \dot{\mathbf{q}}, \quad \mathbf{J}_p = \mathbf{J}_t \mathbf{T}^{-1}, \quad \mathbf{J}_p = \begin{bmatrix} \mathbf{J}_{pa} \\ \mathbf{J}_{pc} \end{bmatrix}. \quad (7)$$

where  $\mathbf{J}_p$  is a  $6 \times 6$  dimensionally homogeneous extended Jacobian matrix for non-redundant parallel manipulators and it contains a  $f \times 6$  homogeneous actuation wrench matrix ( $\mathbf{J}_{pa}$ ), and a  $(6-f) \times 6$  homogeneous constraint wrench matrix ( $\mathbf{J}_{pc}$ ).

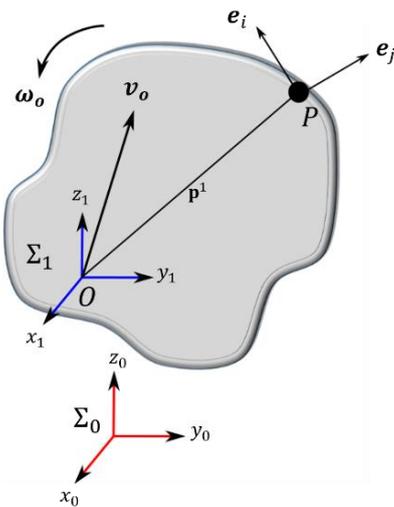


Figure 1: Rigid body displacement

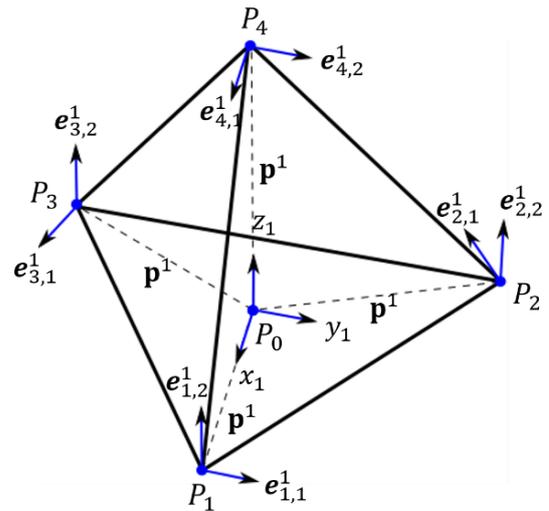


Figure 2: Points and axes in tetrahedron

### Condition Number

The condition number is quite often used as an index to explain the accuracy/dexterity of a parallel manipulator. It is also used to measure the closeness of a pose to a singularity. For a parallel manipulator with mix rotations and translations, the computation of the condition number is in general

not possible due to the inconsistent units. However, after formulating the  $6 \times 6$  dimensionally homogeneous extended Jacobian matrix, the condition number can be evaluated throughout the workspace, defined as:  $\kappa(\mathbf{J}_p^{-1})$ . The condition number can also be computed over the  $f \times 6$  homogeneous actuation wrench matrix ( $\mathbf{J}_{pa}$ ) and the  $(6-f) \times 6$  homogeneous constraint wrench matrix ( $\mathbf{J}_{pc}$ ), defined as  $\kappa((\mathbf{J}_{pa} \mathbf{J}_{pa}^T)^{-1})$  and  $\kappa((\mathbf{J}_{pc} \mathbf{J}_{pc}^T)^{-1})$ , respectively.

Consequently, the parallel manipulators are said to be at an **actuation singularity** if  $\kappa((\mathbf{J}_{pa} \mathbf{J}_{pa}^T)^{-1}) = 0$  and at a **constraint singularity** if  $\kappa((\mathbf{J}_{pc} \mathbf{J}_{pc}^T)^{-1}) = 0$ . It also indicates that the actuation wrench matrix and the constraint wrench matrix degenerate, respectively. On the other hand, there is a case in which the parallel manipulators are neither at the actuation singularity nor at the constraint singularity while the system is subjected to a singularity. In this case, the Jacobian matrix degenerates due to the dependency between the actuation wrench matrix and the constraint wrench matrix, hence  $\kappa(\mathbf{J}_p^{-1}) = 0$ , while  $\kappa((\mathbf{J}_{pa} \mathbf{J}_{pa}^T)^{-1}) \neq 0$  and  $\kappa((\mathbf{J}_{pc} \mathbf{J}_{pc}^T)^{-1}) \neq 0$ . This type of singularity is called a **compound singularity** and the moving platform gains one or more uncontrolled motion. Therefore, larger values of the condition number signify larger distances of a pose to different types of singularities.

### Application: 3-RPS Parallel Manipulator

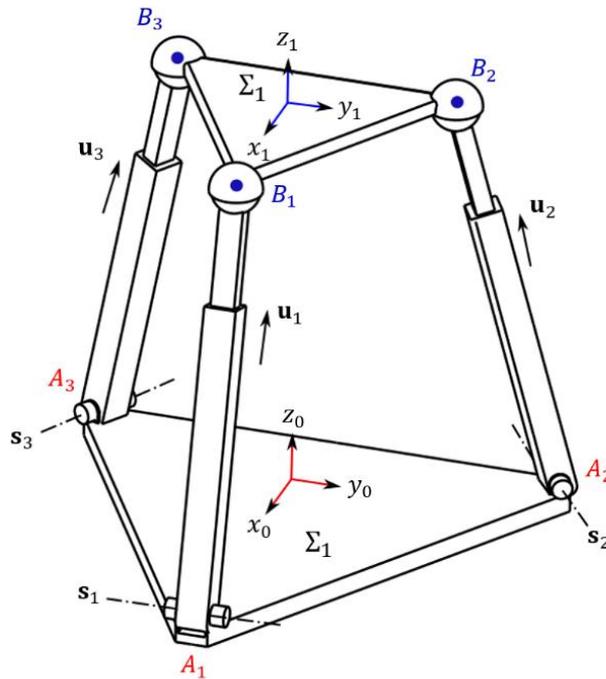


Figure 3: 3-RPS Parallel Manipulator

The 3-RPS parallel manipulator shown in Fig. 3 is a  $3\text{-dof}$  parallel manipulator. It is composed of an equilateral triangle base (circumradius  $h_0$ ), an equilateral triangle platform (circumradius  $h_1$ ), and three RPS limbs. The fixed frame  $\Sigma_0$  and the moving frame  $\Sigma_1$  are located at the origin of the base and at the origin of the moving platform, respectively. The revolute joint in the  $i$ -th limb is located at point  $A_i$ , its axis being along vector  $\mathbf{s}_i$ , while the spherical joint is located at point  $B_i$  ( $i = 1..3$ ). Each pair of vertices ( $A_i B_i$ ) is connected by a prismatic joint along the direction  $\mathbf{u}_i$ .

Due to the manipulator architecture, each limb applies one constraint force, which is perpendicular to the actuated prismatic joint and parallel to the axis  $\mathbf{s}_i$  of the revolute joint. By considering that the prismatic joints are actuated, each leg applies one actuation force whose axis is along the direction of the corresponding actuated joint  $\mathbf{u}_i$ . By collecting all the actuation forces and the constraint forces, the Jacobian matrix  $\mathbf{J}_t$  can be obtained.

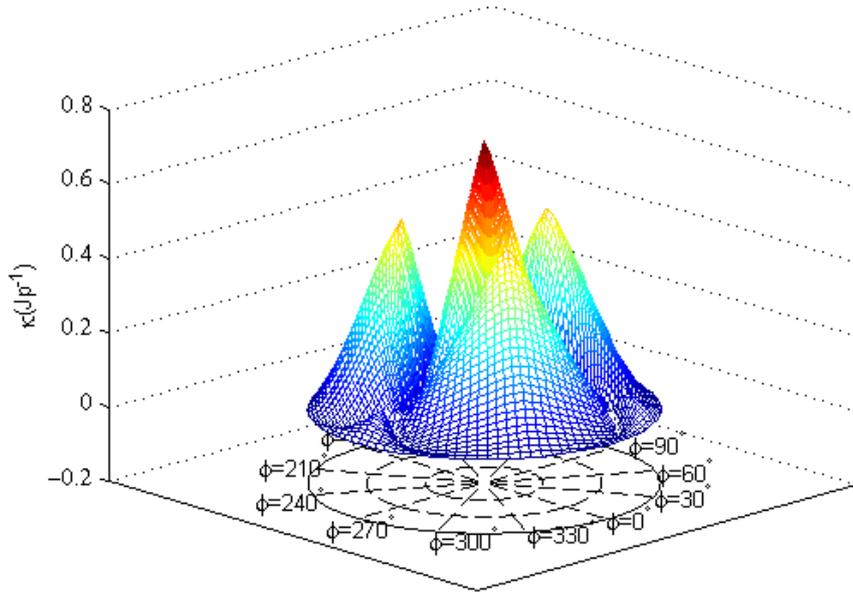
$$\mathbf{J}_t = \begin{bmatrix} (\mathbf{r}_{B1}^0 \times \mathbf{u}_1) & (\mathbf{r}_{B2}^0 \times \mathbf{u}_2) & (\mathbf{r}_{B3}^0 \times \mathbf{u}_3) & (\mathbf{r}_{B1}^0 \times \mathbf{s}_1) & (\mathbf{r}_{B1}^0 \times \mathbf{s}_2) & (\mathbf{r}_{B3}^0 \times \mathbf{s}_3) \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 \end{bmatrix}^T$$

$$\mathbf{J}_q = \mathbf{I} \text{ (6} \times \text{6 Identity matrix)}$$

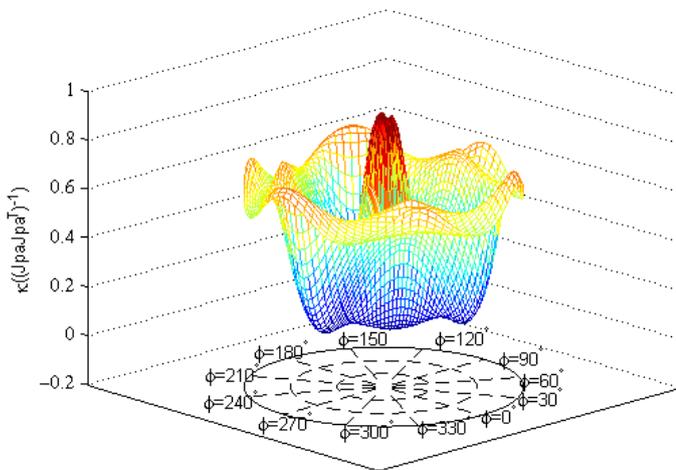
Since the 3-RPS parallel manipulator has one pure translation along vertical direction and two rotations, three linearly independent axes can be selected from the tetrahedron to represent 3-dof permitted motions i.e.  $\mathbf{e}_{i,2}^1, i = 1..3$ . Furthermore, three other independent axes are selected to represent the restricted motions of the manipulators i.e.  $\mathbf{e}_{i,1}^1, i = 1..3$ . Hence:

$$\mathbf{T} = \begin{bmatrix} (\mathbf{r}_{B1}^0 \times \mathbf{e}_{1,2}^0) & (\mathbf{r}_{B2}^0 \times \mathbf{e}_{2,2}^0) & (\mathbf{r}_{B3}^0 \times \mathbf{e}_{3,2}^0) & (\mathbf{r}_{B1}^0 \times \mathbf{e}_{1,1}^0) & (\mathbf{r}_{B1}^0 \times \mathbf{e}_{2,1}^0) & (\mathbf{r}_{B3}^0 \times \mathbf{e}_{3,1}^0) \\ \mathbf{e}_{1,2}^0 & \mathbf{e}_{2,2}^0 & \mathbf{e}_{3,2}^0 & \mathbf{e}_{1,1}^0 & \mathbf{e}_{2,1}^0 & \mathbf{e}_{3,1}^0 \end{bmatrix}^T$$

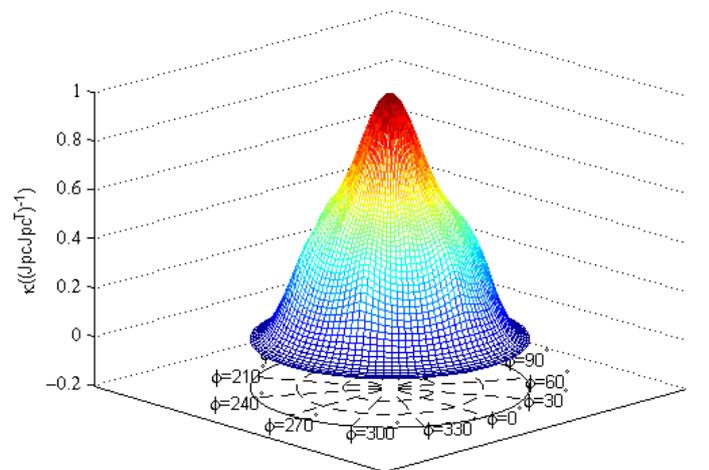
Eventually, the 6x6 dimensionally homogeneous extended Jacobian matrix of the 3-RPS parallel manipulator is formulated i.e.  $\mathbf{J}_p \mathbf{v}_p^0 = \dot{\mathbf{q}}$ . The Euler angles (azimuth-tilt-torsion) is used to represent



(a) Distribution of  $\kappa(\mathbf{J}_p^{-1})$



(b) Distribution of  $\kappa(\mathbf{J}_{pa} \mathbf{J}_{pa}^T)^{-1}$



(c) Distribution of  $\kappa(\mathbf{J}_{pc} \mathbf{J}_{pc}^T)^{-1}$

Figure 4: Distribution of the Condition number as a function of the azimuth angle ( $\phi$ ) and tilt angle ( $\theta$ )

the orientation workspace of the moving platform. It is well known that the orientation capability of the 3-RPS parallel manipulator is defined as the maximum tilt angle  $\theta$  for a given azimuth angle  $\phi$ . Its maximum orientation capability depends mostly on the singularity loci.

The design parameters are assigned as  $h_0 = 2m$  and  $h_1 = 1m$ . Let us consider the moving platform works at altitude  $Z = 3m$ , then the condition number of  $\mathbf{J}_p, \mathbf{J}_{pa}, \mathbf{J}_{pc}$  is computed throughout the entire workspace and its distributions are plotted in Fig. 4(a)-4(c). For a given value of azimuth angle  $\phi$ , the performance of the 3-RPS manipulator reaches its maximum at  $\theta = 0^\circ$ . Figure 4(b) shows that the manipulator never reaches the actuation singularity. In [11-12], it reveals that the 3-RPS manipulator has two operation modes and the constraint singularity occurs at the boundary of each operation mode. This condition is explained in Fig. 4(c) when the moving platform is tilted by  $\theta = 180^\circ$  for any value of azimuth angle  $\phi$ .

Figure 5 represents the singularity loci of the 3-RPS manipulator and the distribution of the condition number  $\kappa(\mathbf{J}_p^{-1})$  over the entire orientation workspace. The performance of the manipulator decreases when the tilt angle is no longer zero and eventually reaches the singularity loci. It reveals that the singularity loci in Fig. 5 corresponds to the compound singularity since neither the actuation wrench matrix nor the constraint wrench matrix degenerates. Likewise  $(\mathbf{J}_p^{-1}) = 0$ , while  $\kappa((\mathbf{J}_{pa} \mathbf{J}_{pa}^T)^{-1}) \neq 0$  and  $\kappa((\mathbf{J}_{pc} \mathbf{J}_{pc}^T)^{-1}) \neq 0$ . It means that when the 3-RPS manipulator is in the compound singularity, there is the dependency between the actuation wrench matrix and the constraint wrench matrix.

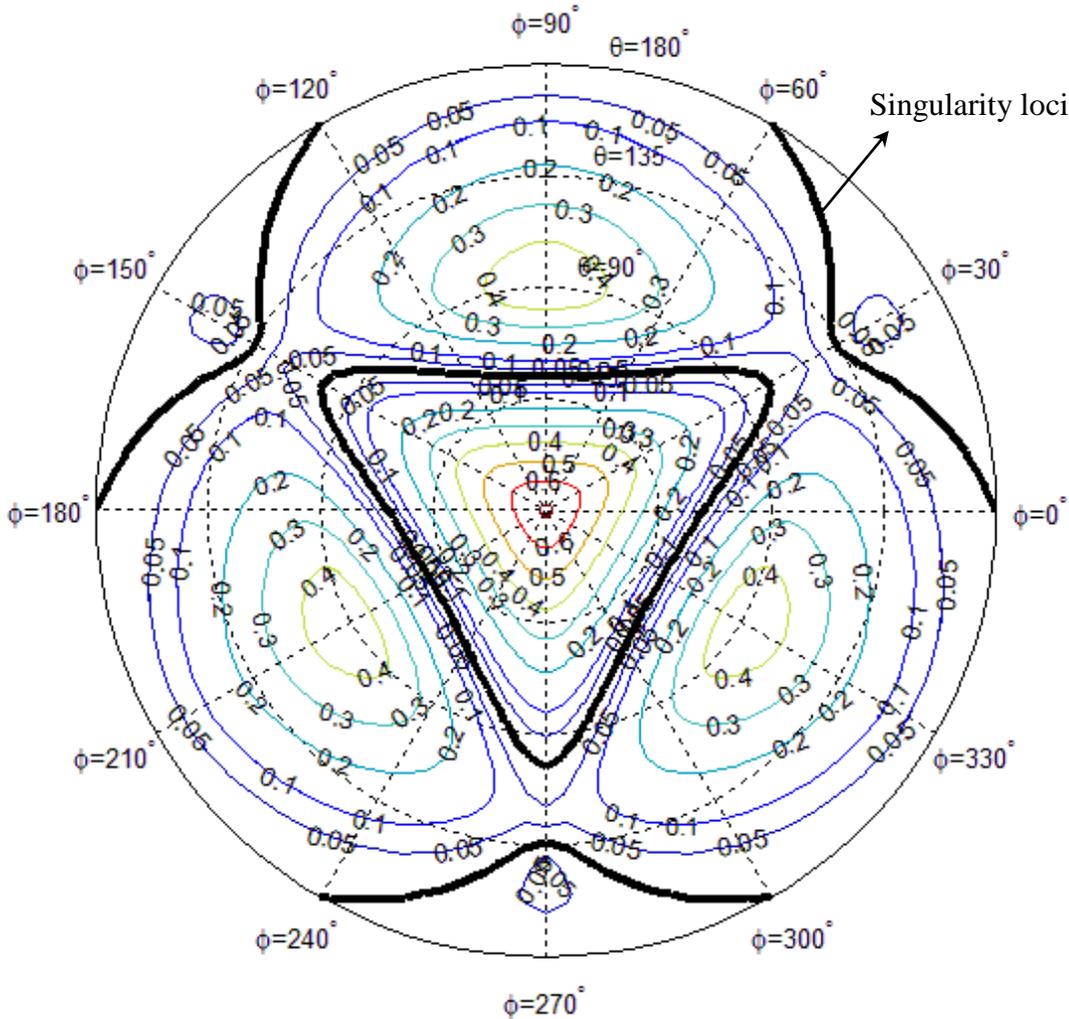


Figure 5: Atlas of singularity loci and condition number  $\kappa(\mathbf{J}_p^{-1})$

## Conclusions

In this paper, the  $6 \times 6$  dimensionally homogeneous extended Jacobian matrix is derived for non-redundant parallel manipulators. The method can be used for the parallel manipulators with coupled rotations and translations by considering a set of linear independent axes at the points of a tetrahedron, which represents the permitted motions and the restricted motions of the moving platform. The proposed approach is applied to the 3-RPS manipulator and the condition number is computed over the homogeneous extended Jacobian matrix ( $\mathbf{J}_p$ ), the homogeneous actuation wrench matrix ( $\mathbf{J}_{pa}$ ), and the homogeneous constraint wrench matrix ( $\mathbf{J}_{pc}$ ). It reveals that at altitude  $Z = 3m$ , the singularity loci of the 3-RPS manipulator corresponds to the compound singularity, in which the 3-RPS manipulator is neither at the actuation singularity nor at the constraint singularity. The 3-RPS manipulator reaches the constraint singularity when the moving platform is tilted by  $\theta = 180^\circ$  for any value of azimuth angle  $\phi$ . Accordingly, these indices can measure the closeness of a pose to different types of singularities, namely the actuation singularity, the constraint singularity, and the compound singularity.

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