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Rotating shallow water flow under location uncertainty

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Rotating shallow water flow under location uncertainty

Part II: some numerical results

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Governing equations

- Conservation of momentum

$$D_t \mathbf{u} + \mathbf{f} \times \mathbf{u} dt = -g \nabla h dt \quad (1)$$

Stochastic transport operator: $D_t[\bullet] = \left(d_t + (\mathbf{u} - \nabla \cdot \frac{1}{2} \mathbf{a}) dt + \boldsymbol{\sigma} d\mathbf{B}_t \right) \cdot \nabla [\bullet] - \nabla \cdot \left(\frac{1}{2} \mathbf{a} \nabla [\bullet] \right)$

Governing equations

- Conservation of momentum

$$D_t \mathbf{u} + \mathbf{f} \times \mathbf{u} dt = -g \nabla h dt \quad (1)$$

- Conservation of mass

$$D_t h + h \nabla \cdot \mathbf{u} dt = 0 \quad (2)$$

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- Incompressible constraints

$$\nabla \cdot \boldsymbol{\sigma} d\mathbf{B}_t = 0, \quad \nabla \cdot \nabla \cdot \mathbf{a} = 0 \quad (3)$$

Stochastic transport operator: $D_t[\bullet] = \left(dt + (\mathbf{u} - \nabla \cdot \frac{1}{2} \mathbf{a}) dt + \boldsymbol{\sigma} d\mathbf{B}_t \right) \cdot \nabla [\bullet] - \nabla \cdot \left(\frac{1}{2} \mathbf{a} \nabla [\bullet] \right)$

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- Incompressible constraints

$$\nabla \cdot \boldsymbol{\sigma} d\mathbf{B}_t = 0, \quad \nabla \cdot \nabla \cdot \mathbf{a} = 0 \quad (3)$$

- Conservation of energy

$$d_t \int_{\Omega} \frac{\rho}{2} (h |\mathbf{u}|^2 + gh^2) d\mathbf{x} = 0 \quad (4)$$

Stochastic transport operator: $D_t[\bullet] = \left(d_t + (\mathbf{u} - \nabla \cdot \frac{1}{2} \mathbf{a}) dt + \boldsymbol{\sigma} d\mathbf{B}_t \right) \cdot \nabla [\bullet] - \nabla \cdot \left(\frac{1}{2} \mathbf{a} \nabla [\bullet] \right)$

Energy diagnosis

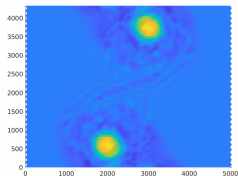
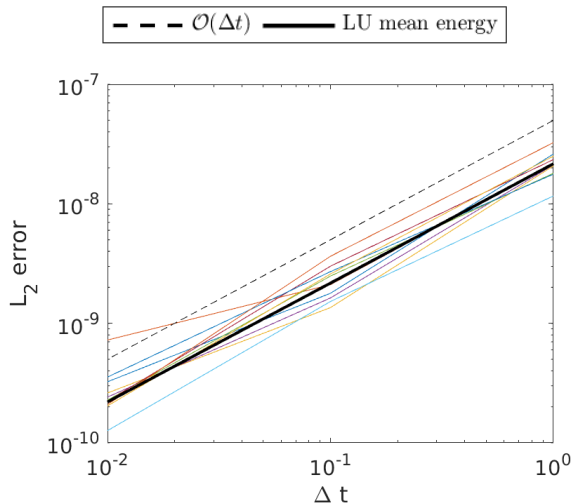


Figure: Convergence in temporal resolution of LU ensemble energy to the reference (at spatial resolution 128^2).

Energy diagnosis

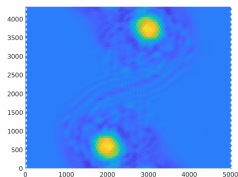
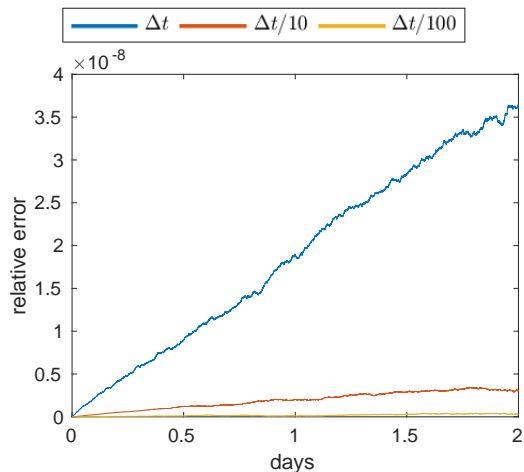


Figure: Relative errors of LU ensemble mean energy compared to the reference (at spatial resolution 128^2).

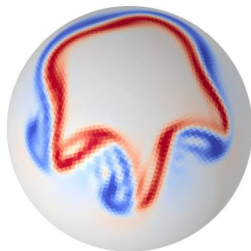
Barotropic instability

Day 6

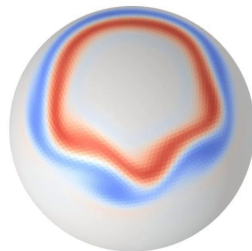
Reference



LU



LES



Video: Evolution of vorticity fields.

Barotropic instability

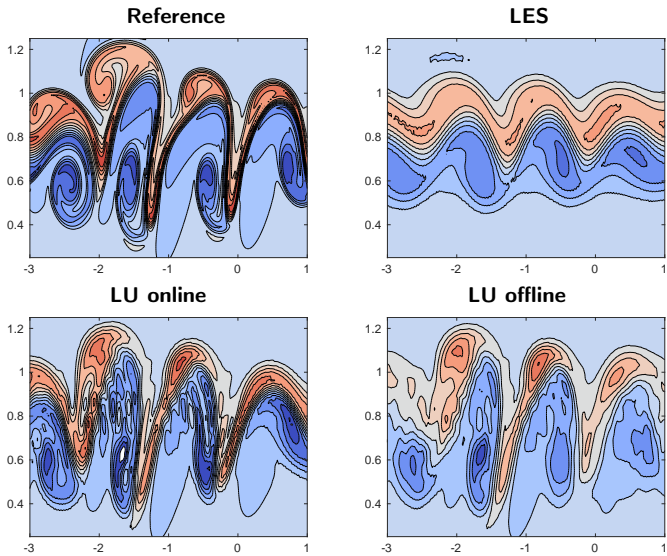


Figure: Contour snapshots of vorticity field at day 6 with $CI = 10^{-5} \text{ s}^{-1}$.

Barotropic instability

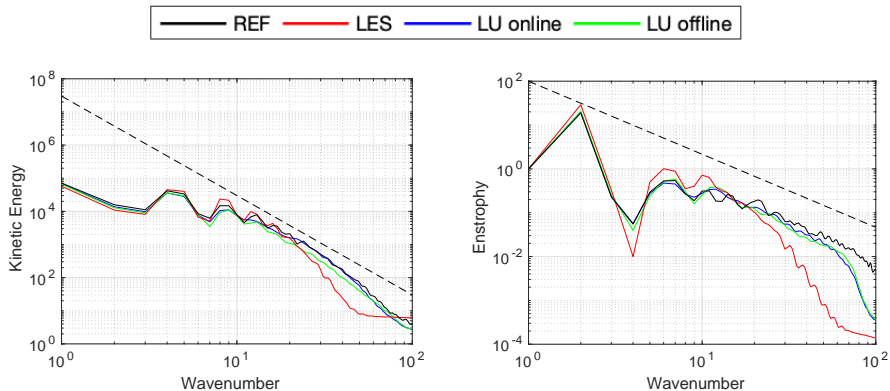
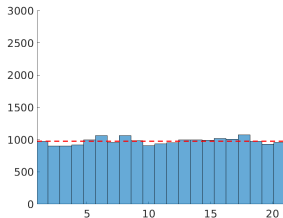


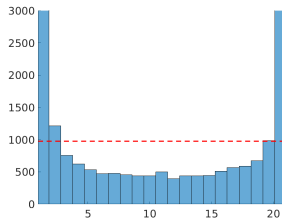
Figure: Spectrums of kinetic energy (left) and enstrophy (right) at day 6. The dashed lines are power laws of slope -3 (left) and $-5/3$ (right).

Ensemble forecast

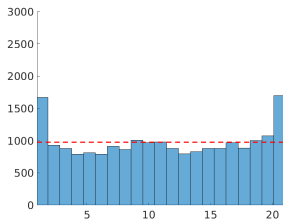
LU offline



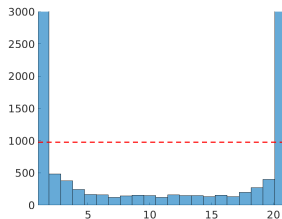
PIC



LU online



PIC



Video: Evolution of rank histograms from day 1 to day 20.

Thank for Your Attention!