Stochastic modeling of the oceanic mesoscale eddies

Long Li, Etienne Mémin, Deremble Bruno

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Motivations

- To better represent the mesoscale eddies effect on the large–scale circulations.

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- To improve the prediction of ocean variability in very coarse–grid simulations.

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- To improve the prediction of ocean variability in very coarse–grid simulations.
- To provide a more reliable ensemble forecasting system and more efficient ensemble spread for data assimilation system.

Outline

1. Location Uncertainty (LU) model
2. Numerical results: LU coarse–grid simulations

LU model

Stochastic flow (Mémin, 2014)

$$\mathrm{d}\boldsymbol{X}_t = \boldsymbol{u}(\boldsymbol{X}_t, t)\mathrm{d}t + \underbrace{\boldsymbol{\sigma}(\boldsymbol{X}_t, t)\mathrm{d}\boldsymbol{B}_t}_{\text{Uncertainty or Noise}} \quad (1)$$

LU model

Spatial structure of noise

- Correlation operator

$$\boldsymbol{\sigma}[\boldsymbol{f}](\boldsymbol{x},t) = \int_{\Omega} \breve{\boldsymbol{\sigma}}(\boldsymbol{x},\boldsymbol{y},t)\boldsymbol{f}(\boldsymbol{y})\mathrm{d}\boldsymbol{y} \tag{2}$$

- $\breve{\boldsymbol{\sigma}}$ is assumed to be bounded

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- Variance tensor

$$\boldsymbol{a} = \mathbb{E}\left[(\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t)(\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t)^T\right]/\mathrm{d}t = \boldsymbol{\sigma}\boldsymbol{\sigma}^T \tag{3}$$

- $\breve{\boldsymbol{\sigma}}$ is assumed to be bounded

LU model

Spectral representation

- Noise

$$\boldsymbol{\sigma}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{B}_t = \sum_{n\in\mathbb{N}} \phi_n(\boldsymbol{x},t)\mathrm{d}\beta_t^n \tag{4}$$

- ϕ_n: orthogonal eigenfunctions of the covariance
- β^n: 1D standard Brownian motions

LU model

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- Variance

$$\boldsymbol{a}(\boldsymbol{x},t) = \sum_{n\in\mathbb{N}} \boldsymbol{\phi}_n(\boldsymbol{x},t)\boldsymbol{\phi}_n^T(\boldsymbol{x},t) \tag{5}$$

- ϕ_n: orthogonal eigenfunctions of the covariance
- β^n: 1D standard Brownian motions

LU model

Transport of extensive tracers (Resseguier et al., 2017)

- Stochastic transport operator

$$\mathbb{D}_t\theta \triangleq \mathrm{d}_t\theta + (\boldsymbol{u} - \boldsymbol{u}_s)\cdot\boldsymbol{\nabla}\,\theta\mathrm{d}t + \underbrace{\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t\cdot\boldsymbol{\nabla}\,\theta}_{\text{forcing}} - \underbrace{\frac{1}{2}\,\boldsymbol{\nabla}\cdot(\boldsymbol{a}\boldsymbol{\nabla}\theta)\mathrm{d}t}_{\text{diffusion}} = 0 \quad (6)$$

- Incompressible noise: $\boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = 0$
- Itô-Stokes drift (Bauer et al., 2020a): $\boldsymbol{u}_s \triangleq \frac{1}{2}\,\boldsymbol{\nabla}\cdot\,\boldsymbol{a}$

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- Conservation of energy

$$\boldsymbol{\nabla}\cdot(\boldsymbol{u} - \boldsymbol{u}_s) = 0 \quad \Longrightarrow \quad \mathrm{d}_t \int_\Omega \frac{1}{2}\theta^2 \mathrm{d}\boldsymbol{x} = 0 \quad (7)$$

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LU model

Connection to physical parameterizations

- Isopycnal projector

$$\mathbf{P} = \mathbf{I} - \frac{\boldsymbol{\nabla}\rho\,(\boldsymbol{\nabla}\rho)^T}{|\boldsymbol{\nabla}\rho|^2} \quad \Longrightarrow \quad \mathbf{P}\boldsymbol{\nabla}\rho = 0 \tag{8}$$

■ Projector: $\mathbf{P}^T = \mathbf{P}, \quad \mathbf{P}^2 = \mathbf{P}$

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- Isopycnal noise

$$\boldsymbol{\sigma} = \mathbf{P}\boldsymbol{\sigma}_0 \quad \Longrightarrow \quad \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t \perp \boldsymbol{\nabla}\rho \text{ and } \boldsymbol{a}\boldsymbol{\nabla}\rho = 0 \tag{9}$$

$$\Longrightarrow \quad \partial_t \rho + \boldsymbol{\nabla}\cdot\Big(\rho(\boldsymbol{u} - \boldsymbol{u}_s)\Big) = 0 \tag{10}$$

- Projector: $\mathbf{P}^T = \mathbf{P}, \quad \mathbf{P}^2 = \mathbf{P}$

Connection to physical parameterizations

- Transport of passive tracer

$$\mathrm{d}_t\theta + (\boldsymbol{u} - \boldsymbol{u}_s) \cdot\boldsymbol{\nabla}\, \theta \mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t \cdot\boldsymbol{\nabla}\, \theta - \frac{1}{2}\, \boldsymbol{\nabla}\cdot (\boldsymbol{a}\boldsymbol{\nabla}\theta)\mathrm{d}t = 0 \qquad (11)$$

LU model

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- Isopycnal diffusion (Gent and McWilliams, 1990; Redi, 1982)

$$\boldsymbol{a}_0 = \kappa\mathbf{I} \quad \Longrightarrow \quad \boldsymbol{a} = \mathbf{P}\boldsymbol{a}_0\mathbf{P}^T = \kappa\mathbf{P} \tag{12}$$

LU model

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$$|\boldsymbol{s}| \ll 1 \quad\Longrightarrow\quad \boldsymbol{a} = \kappa\begin{pmatrix} 1 & 0 & s_x \\ 0 & 1 & s_y \\ s_x & s_y & |\boldsymbol{s}|^2 \end{pmatrix} \tag{13}$$

with $\boldsymbol{s} = [s_x, s_y]^T,\ s_x = -\partial_x\rho/\partial_z\rho,\ s_y = -\partial_y\rho/\partial_z\rho$

LU model

Quasi-geostrophic (QG) flow under LU

- Potential vorticity (PV)

$$q = \nabla^2 \psi + f + f_0^2 \partial_z \big(\partial_z \psi / N^2 \big) \tag{14}$$

- ψ is stream function ($\boldsymbol{u} = \boldsymbol{\nabla}^{\perp} \psi$), f is Coriolis and N is buoyancy frequency

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$$q = \nabla^2\psi + f + f_0^2\partial_z\left(\partial_z\psi/N^2\right) \tag{14}$$

- Evolution of PV under LU

$$\mathbb{D}_t q = \boldsymbol{\nabla}\cdot\boldsymbol{F} \tag{15}$$

$$\boldsymbol{F} \triangleq \underbrace{-\boldsymbol{u}\cdot\boldsymbol{\nabla}^{\perp}(\boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t - \boldsymbol{u}_s\mathrm{d}t) - \boldsymbol{a}\boldsymbol{\nabla}f\mathrm{d}t}_{\text{flux of PV source}} + \underbrace{\frac{1}{2}(\partial_x\boldsymbol{a}\boldsymbol{\nabla}v - \partial_y\boldsymbol{a}\boldsymbol{\nabla}u)\mathrm{d}t}_{\text{flux of PV sink}} \tag{16}$$

$\boldsymbol{F}$ ensures conservation of total energy (Bauer et al., 2020a)

- ψ is stream function ($\boldsymbol{u} = \boldsymbol{\nabla}^{\perp}\psi$), f is Coriolis and N is buoyancy frequency

Outline

1. Location Uncertainty (LU) model

2. Numerical results: LU coarse–grid simulations

Eddy–resolving simulations (Hogg et al., 2003)

Exp.1: $L_d = [30, 17]km, \ \Delta = 5km$ **Exp.2:** $L_d = [51, 32]km, \ \Delta = 10km$

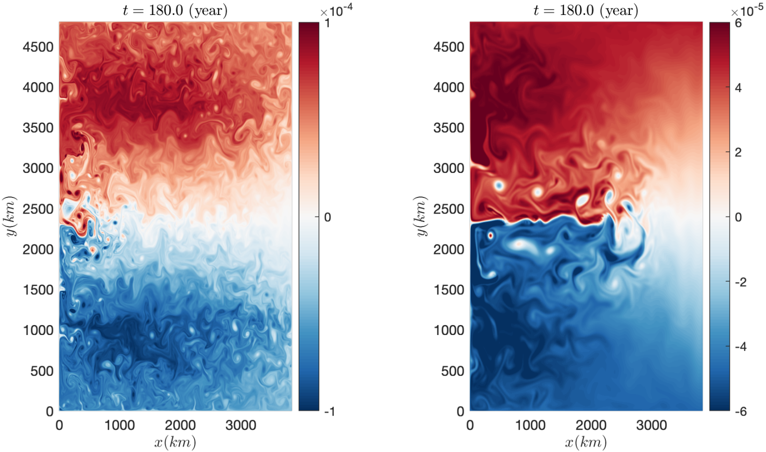


Figure: Eddy–resolving snapshots of surface layer PV (`http://www.q-gcm.org`).

LU–POD noise

Learning of noise from velocity data (Bauer et al., 2020b):

$$\frac{1}{\Delta t}\boldsymbol{\sigma}(\boldsymbol{x})\mathrm{d}\boldsymbol{B}_t \approx \sum_{n=M_0}^{M_1} \boldsymbol{\phi}_n(\boldsymbol{x})\xi_n, \qquad \xi_n \sim \mathcal{N}(0,1) \tag{17}$$

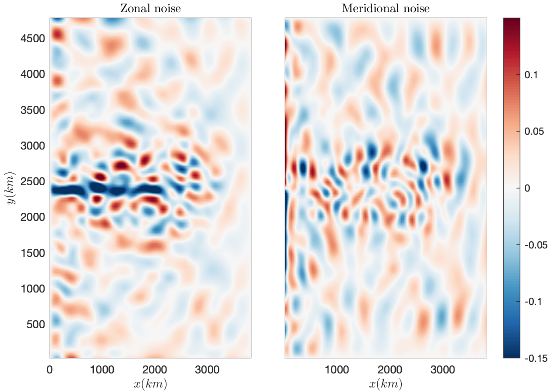


Figure: Zonal (left) and meridional (right) noise velocity at resolution $80km$ learned from eddy–resolving data of Exp.2.

LU–POD–P noise

Update of noise along isopycnal:

$$\mathbf{P} = \mathbf{I} - \frac{\boldsymbol{\nabla}\eta\ (\boldsymbol{\nabla}\eta)^T}{|\boldsymbol{\nabla}\eta|^2}, \qquad \eta \propto \partial_z(\partial_z \psi / N^2) \tag{18}$$

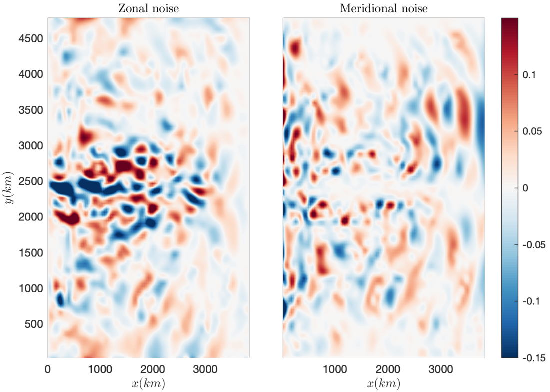
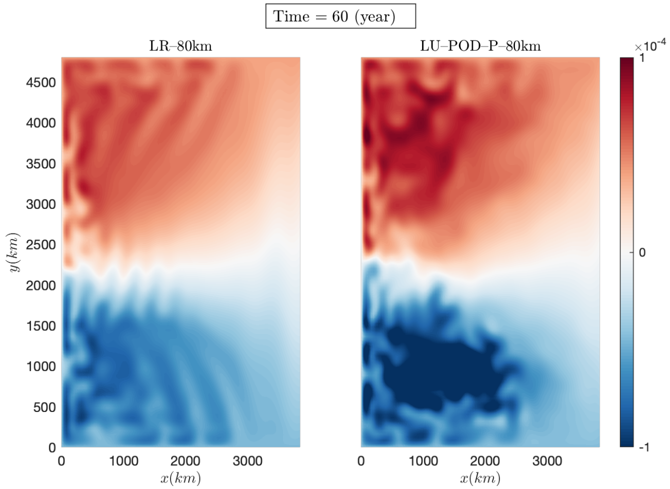


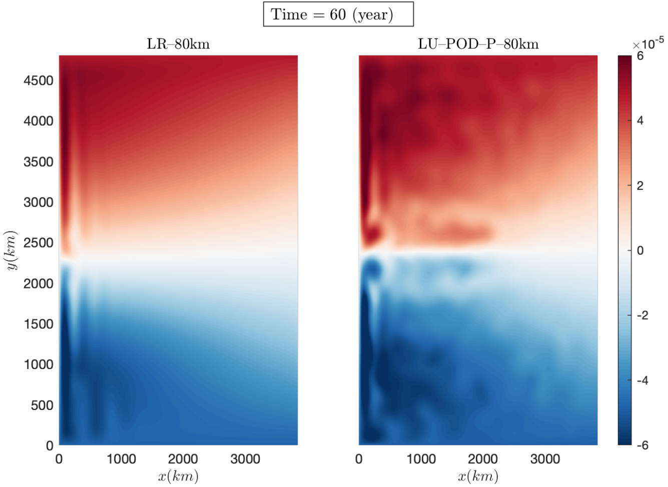
Figure: Zonal (left) and meridional (right) noise velocity at resolution $80km$ by adapting POD noise along isopycnal.

LU coarse simulation (Exp.1)



Video: Evolution of surface layer PV at resolution $80km$. Left: Deterministic model (LR); Right: LU model under POD–P noise.

LU coarse simulation (Exp.2)



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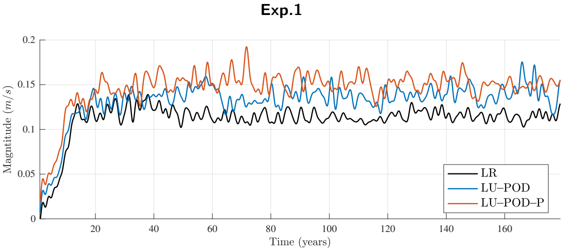
LU coarse simulation



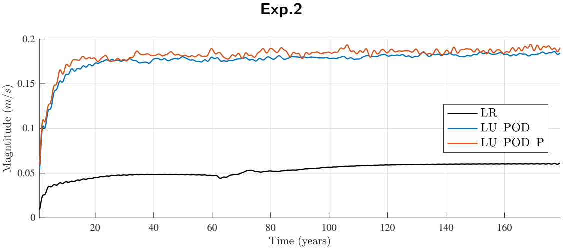


Figure: Maximal values of zonal jet velocity (sum over all layers) at resolution $80km$.

Low–frequency variability diagnosis

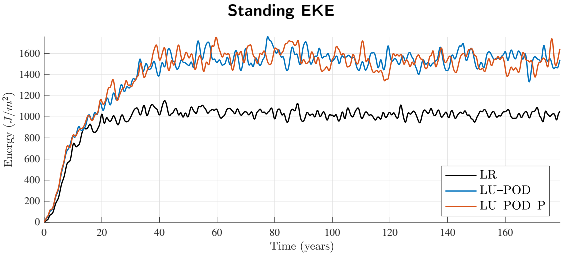
Eddy kinetic energy (EKE) decomposition

For $k = 1, \ldots, N$ (with N total number of ocean layers):

$$\mathsf{KE}_k = \frac{\rho H_k}{2} \Big(\underbrace{\big(\overline{u_k}^2 + \overline{v_k}^2\big)}_{\text{Standing EKE}} + \underbrace{\big(u_k'^2 + v_k'^2\big)}_{\text{Transient EKE}} \Big), \tag{19}$$

where $\overline{\cdot}$ is approximated by a 2–years–low–pass Fourier filter and $\cdot\,'$ is the residual.

Low–frequency variability diagnosis (Exp.1)



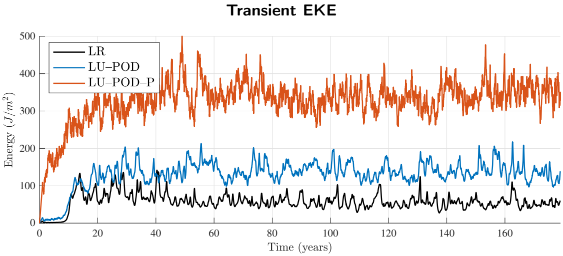


Figure: Standing EKE and Transient EKE (sum over all layers) at resolution $80km$.

Low–frequency variability diagnosis (Exp.2)

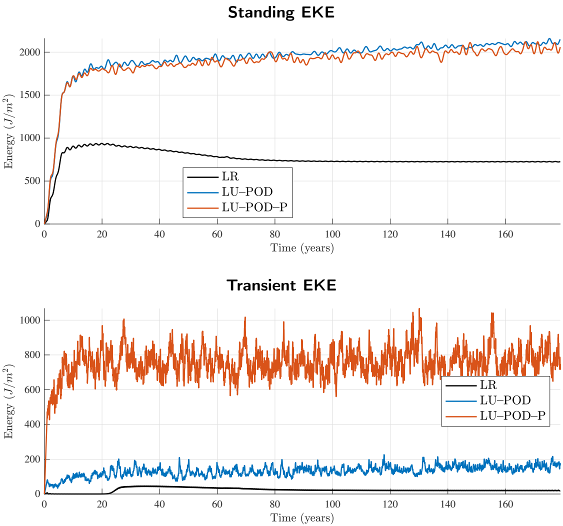


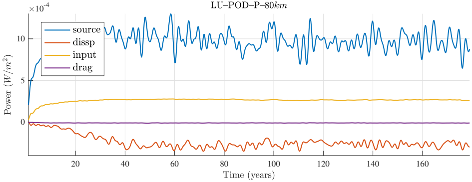
Figure: Standing EKE and Transient EKE (sum over all layers) at resolution $80km$.

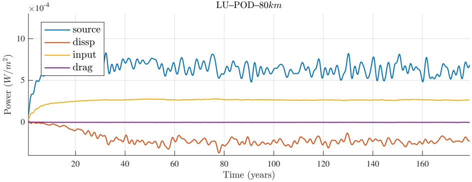
Low–frequency variability diagnosis

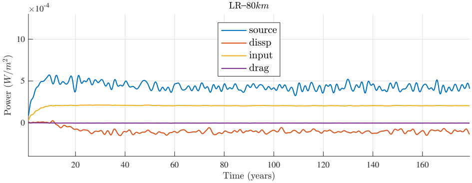
Energy transfert (Hogg and Blundell, 2006)

$$
\begin{aligned}
\partial_t \mathsf{KE} = & \underbrace{-\partial_t \mathsf{PE}}_{\text{source}} \; \underbrace{+\, \rho \int_{\Omega} u_1 \tau^x \mathrm{d}\boldsymbol{x}}_{\text{input wind}} \\
& \underbrace{-\frac{1}{2}\rho \delta_{ek} f_0 \int_{\Omega} (u_N^2 + v_N^2) \mathrm{d}\boldsymbol{x}}_{\text{linear drag}} \\
& \underbrace{-\sum_{k=1}^{N} A_4 \rho H_k \int_{\Omega} (u_k \nabla^4 u_k + v_k \nabla^4 v_k) \mathrm{d}\boldsymbol{x} + \cdots}_{\text{dissipation}}
\end{aligned}
\tag{20}
$$

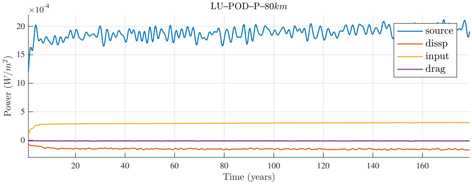
Low–frequency variability diagnosis (Exp.1)

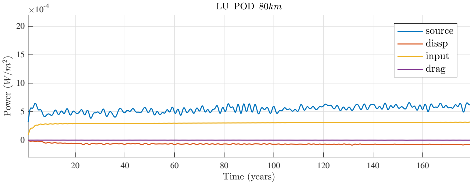


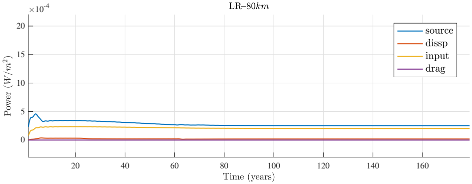




Low–frequency variability diagnosis (Exp.2)







Low–frequency variability diagnosis

Measures of statistics (Grooms et al., 2015)

- Pattern correlation (PC)

$$\mathsf{PC} = \frac{\int_{\Omega} \sigma_f^2 \sigma_{f_{\textsf{ref}}}^2 \mathrm{d}\boldsymbol{x}}{(\int_{\Omega} \sigma_f^4 \mathrm{d}\boldsymbol{x})(\int_{\Omega} \sigma_{f_{\textsf{ref}}}^4 \mathrm{d}\boldsymbol{x})} \qquad \nearrow \tag{21}$$

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- Gaussian approximation of relative entropy

$$\mathsf{Dispersion} = \frac{1}{|\Omega|} \int_\Omega \left(\frac{\sigma_{f_{\mathsf{ref}}}^2}{\sigma_f^2} - 1 - \log\left(\frac{\sigma_{f_{\mathsf{ref}}}^2}{\sigma_f^2} \right) \right) \mathrm{d}\boldsymbol{x} \quad \searrow \tag{22}$$

$$\mathsf{Entropy} = \frac{1}{2} \left(\frac{1}{|\Omega|} \int_\Omega \frac{(\overline{f}^t - \overline{f}_{\mathsf{ref}}^t)^2}{\sigma_f^2} \mathrm{d}\boldsymbol{x} + \mathsf{Dispersion} \right) \quad \searrow \tag{23}$$

Low–frequency variability diagnosis (Exp.1)

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.44	0.71	0.44	851	26
LU–POD	0.44	0.69	0.46	36	18
LU–POD–P	0.41	0.66	0.50	15	8

Table: Measures of skill for surface layer pressure ($80km$).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.26	0.60	0.58	188	94
LU–POD	0.26	0.59	0.61	67	34
LU–POD–P	0.25	0.55	0.70	18	9

Table: Measures of skill for middle layer pressure ($80km$).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.12	0.67	0.56	1701	851
LU–POD	0.12	0.65	0.65	248	124
LU–POD–P	0.12	0.59	0.80	40	20

Table: Measures of skill for bottom layer pressure ($80km$).

Low–frequency variability diagnosis (Exp.2)

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	1.89	1.51	0.11	1e4	5e3
LU–POD	1.16	1.41	0.29	156	82
LU–POD–P	1.17	1.25	0.54	19	12

Table: Measures of skill for surface layer pressure ($80km$).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.47	1.16	0.26	5e4	2e4
LU–POD	0.41	1.07	0.78	143	72
LU–POD–P	0.41	0.90	0.90	15	8

Table: Measures of skill for middle layer pressure ($80km$).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.17	1.15	0.23	2e5	1e5
LU–POD	0.16	1.06	0.93	169	85
LU–POD–P	0.16	0.89	0.92	17	9

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Future works

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3. Implement the Atmosphere–ocean coupled model (Hogg et al., 2003).

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1. Run LU ensemble simulations and verify ensemble spread by Rank histogram, CRPS, etc.
2. Add noise into steady wind and study the response of ocean variability.
3. Implement the Atmosphere–ocean coupled model (Hogg et al., 2003).
4. Data assimilation with particle filter (Cotter et al., 2020).

Thank for Your Attention!

References

W. Bauer, P. Chandramouli, B. Chapron, L. Li, and E. Mémin. Deciphering the role of small-scale inhomogeneity on geophysical flow structuration: a stochastic approach. *Journal of Physical Oceanography*, 50(4):983–1003, 2020a.

W. Bauer, P. Chandramouli, L. Li, and E. Mémin. Stochastic representation of mesoscale eddy effects in coarse-resolution barotropic models. *Ocean Modelling*, 151:101646, 2020b.

C. Cotter, D. Crisan, D. D. Holm, W. Pan, and I. Shevchenko. Data assimilation for a quasi-geostrophic model with circulation-preserving stochastic transport noise. *Journal of Statiscal Physics*, 2020.

P. R. Gent and J. C. McWilliams. Isopycnal mixing in ocean circulation models. *Journal of Physical Oceanography*, 20(1):150–155, 1990.

I. Grooms, A. J. Majda, and K. S. Smith. Stochastic superparameterization in a quasigeostrophic model of the Antarctic Circumpolar Current. *Ocean Modelling*, 85:1–15, 2015.

A. McC. Hogg and J. R. Blundell. Interdecadal variability of the southern ocean. *Journal of Physical Oceanography*, 36:1626–1645, 2006.

A. McC. Hogg, W. K. Dewar, P. D. Killworth, and J. R. Blundell. A quasi-geostrophic coupled model (Q-GCM). *Monthly Weather Review*, 131(10):2261–2278, 2003.

E. Mémin. Fluid flow dynamics under location uncertainty. *Geophysical & Astrophysical Fluid Dynamics*, 108(2): 119–146, 2014.

M. H. Redi. Oceanic isopycnal mixing by coordinate rotation. *Journal of Physical Oceanography*, 12(10):1154–1158, 1982.

V. Resseguier, E. Mémin, and B. Chapron. Geophysical flows under location uncertainty, part I: Random transport and general models. *Geophysical & Astrophysical Fluid Dynamics*, 111(3):149–176, 2017.