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► **To cite this version:**

Long Li, Etienne Mémin, Deremble Bruno. Stochastic modeling of the oceanic mesoscale eddies. Stochastic Transport in Upper Ocean Dynamics Workshop, Sep 2020, London, United Kingdom. hal-02946702

HAL Id: hal-02946702

<https://hal.science/hal-02946702>

Submitted on 23 Sep 2020

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Stochastic modeling of the oceanic mesoscale eddies

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- To better represent the mesoscale eddies effect on the large-scale circulations.

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- To improve the prediction of ocean variability in very coarse-grid simulations.

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- To improve the prediction of ocean variability in very coarse-grid simulations.
- To provide a more reliable ensemble forecasting system and more efficient ensemble spread for data assimilation system.

- 1 Location Uncertainty (LU) model
- 2 Numerical results: LU coarse-grid simulations

Stochastic flow (Mémin, 2014)

$$d\mathbf{X}_t = \mathbf{u}(\mathbf{X}_t, t)dt + \underbrace{\boldsymbol{\sigma}(\mathbf{X}_t, t)d\mathbf{B}_t}_{\text{Uncertainty or Noise}} \quad (1)$$

Spatial structure of noise

- Correlation operator

$$\sigma[f](\mathbf{x}, t) = \int_{\Omega} \check{\sigma}(\mathbf{x}, \mathbf{y}, t) f(\mathbf{y}) d\mathbf{y} \quad (2)$$

- $\check{\sigma}$ is assumed to be **bounded**

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- Correlation operator

$$\sigma[f](\mathbf{x}, t) = \int_{\Omega} \check{\sigma}(\mathbf{x}, \mathbf{y}, t) f(\mathbf{y}) d\mathbf{y} \quad (2)$$

- Variance tensor

$$\mathbf{a} = \mathbb{E}[(\sigma d\mathbf{B}_t)(\sigma d\mathbf{B}_t)^T] / dt = \sigma \sigma^T \quad (3)$$

- $\check{\sigma}$ is assumed to be **bounded**

Spectral representation

- Noise

$$\sigma(\mathbf{x}, t) d\mathbf{B}_t = \sum_{n \in \mathbb{N}} \phi_n(\mathbf{x}, t) d\beta_t^n \quad (4)$$

- ϕ_n : orthogonal eigenfunctions of the covariance
- β^n : 1D standard Brownian motions

Spectral representation

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- Variance

$$\mathbf{a}(\mathbf{x}, t) = \sum_{n \in \mathbb{N}} \phi_n(\mathbf{x}, t) \phi_n^T(\mathbf{x}, t) \quad (5)$$

- ϕ_n : orthogonal eigenfunctions of the covariance
- β^n : 1D standard Brownian motions

Transport of extensive tracers (Resseguier et al., 2017)

- Stochastic transport operator

$$\mathbb{D}_t \theta \triangleq d_t \theta + (\mathbf{u} - \mathbf{u}_s) \cdot \nabla \theta dt + \underbrace{\boldsymbol{\sigma} dB_t \cdot \nabla \theta}_{\text{forcing}} - \underbrace{\frac{1}{2} \nabla \cdot (\mathbf{a} \nabla \theta) dt}_{\text{diffusion}} = 0 \quad (6)$$

- Incompressible noise: $\nabla \cdot \boldsymbol{\sigma} = 0$
- Itô-Stokes drift (Bauer et al., 2020a): $\mathbf{u}_s \triangleq \frac{1}{2} \nabla \cdot \mathbf{a}$

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- Conservation of energy

$$\nabla \cdot (\mathbf{u} - \mathbf{u}_s) = 0 \quad \implies \quad d_t \int_{\Omega} \frac{1}{2} \theta^2 d\mathbf{x} = 0 \quad (7)$$

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Connection to physical parameterizations

- Isopycnal projector

$$\mathbf{P} = \mathbf{I} - \frac{\nabla\rho (\nabla\rho)^T}{|\nabla\rho|^2} \implies \mathbf{P}\nabla\rho = 0 \quad (8)$$

- Projector: $\mathbf{P}^T = \mathbf{P}$, $\mathbf{P}^2 = \mathbf{P}$

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- Isopycnal noise

$$\boldsymbol{\sigma} = \mathbf{P} \boldsymbol{\sigma}_0 \quad \Longrightarrow \quad \boldsymbol{\sigma} d\mathbf{B}_t \perp \nabla \rho \text{ and } \mathbf{a} \nabla \rho = 0 \quad (9)$$

$$\Longrightarrow \quad \partial_t \rho + \nabla \cdot (\rho (\mathbf{u} - \mathbf{u}_s)) = 0 \quad (10)$$

- Projector: $\mathbf{P}^T = \mathbf{P}$, $\mathbf{P}^2 = \mathbf{P}$

Connection to physical parameterizations

- Transport of passive tracer

$$d_t\theta + (\mathbf{u} - \mathbf{u}_s) \cdot \nabla \theta dt + \sigma d\mathbf{B}_t \cdot \nabla \theta - \frac{1}{2} \nabla \cdot (\mathbf{a} \nabla \theta) dt = 0 \quad (11)$$

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- Isopycnal diffusion (Gent and McWilliams, 1990; Redi, 1982)

$$\mathbf{a}_0 = \kappa \mathbf{I} \quad \implies \quad \mathbf{a} = \mathbf{P} \mathbf{a}_0 \mathbf{P}^T = \kappa \mathbf{P} \quad (12)$$

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$$|\mathbf{s}| \ll 1 \quad \Longrightarrow \quad \mathbf{a} = \kappa \begin{pmatrix} 1 & 0 & s_x \\ 0 & 1 & s_y \\ s_x & s_y & |\mathbf{s}|^2 \end{pmatrix} \quad (13)$$

with $\mathbf{s} = [s_x, s_y]^T$, $s_x = -\partial_x \rho / \partial_z \rho$, $s_y = -\partial_y \rho / \partial_z \rho$

Quasi-geostrophic (QG) flow under LU

- Potential vorticity (PV)

$$q = \nabla^2 \psi + f + f_0^2 \partial_z (\partial_z \psi / N^2) \quad (14)$$

- ψ is stream function ($\mathbf{u} = \nabla^\perp \psi$), f is Coriolis and N is buoyancy frequency

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$$q = \nabla^2 \psi + f + f_0^2 \partial_z (\partial_z \psi / N^2) \quad (14)$$

- Evolution of PV under LU

$$\mathbb{D}_t q = \nabla \cdot \mathbf{F} \quad (15)$$

$$\mathbf{F} \triangleq \underbrace{-\mathbf{u} \cdot \nabla^\perp (\sigma d\mathbf{B}_t - \mathbf{u}_s dt)}_{\text{flux of PV source}} - \underbrace{\mathbf{a} \nabla f dt + \frac{1}{2} (\partial_x \mathbf{a} \nabla v - \partial_y \mathbf{a} \nabla u) dt}_{\text{flux of PV sink}} \quad (16)$$

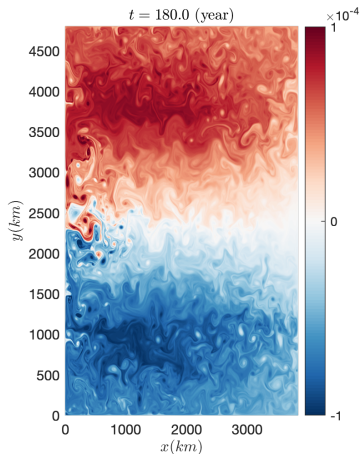
\mathbf{F} ensures **conservation of total energy** (Bauer et al., 2020a)

- ψ is stream function ($\mathbf{u} = \nabla^\perp \psi$), f is Coriolis and N is buoyancy frequency

- 1 Location Uncertainty (LU) model
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Eddy-resolving simulations (Hogg et al., 2003)

Exp.1: $L_d = [30, 17]km$, $\Delta = 5km$



Exp.2: $L_d = [51, 32]km$, $\Delta = 10km$

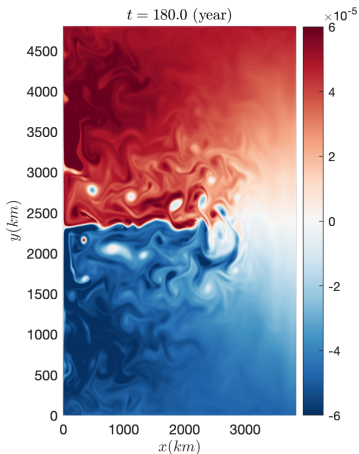


Figure: Eddy-resolving snapshots of surface layer PV (<http://www.q-gcm.org>).

Learning of noise from velocity data (Bauer et al., 2020b):

$$\frac{1}{\Delta t} \sigma(\mathbf{x}) d\mathbf{B}_t \approx \sum_{n=M_0}^{M_1} \phi_n(\mathbf{x}) \xi_n, \quad \xi_n \sim \mathcal{N}(0, 1) \quad (17)$$

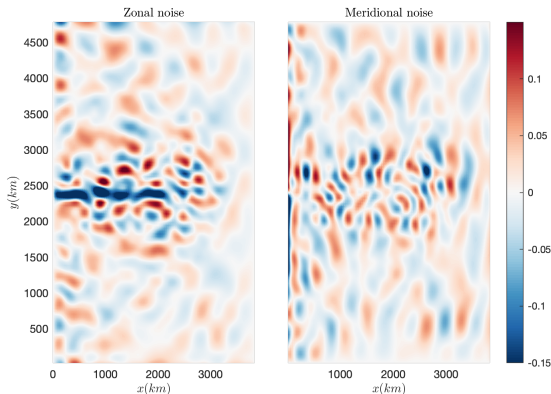


Figure: Zonal (left) and meridional (right) noise velocity at resolution 80km learned from eddy-resolving data of Exp.2.

Update of noise along isopycnal:

$$\mathbf{P} = \mathbf{I} - \frac{\nabla\eta (\nabla\eta)^T}{|\nabla\eta|^2}, \quad \eta \propto \partial_z(\partial_z\psi/N^2) \quad (18)$$

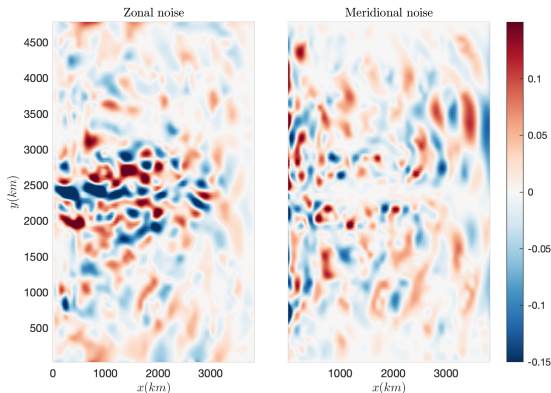
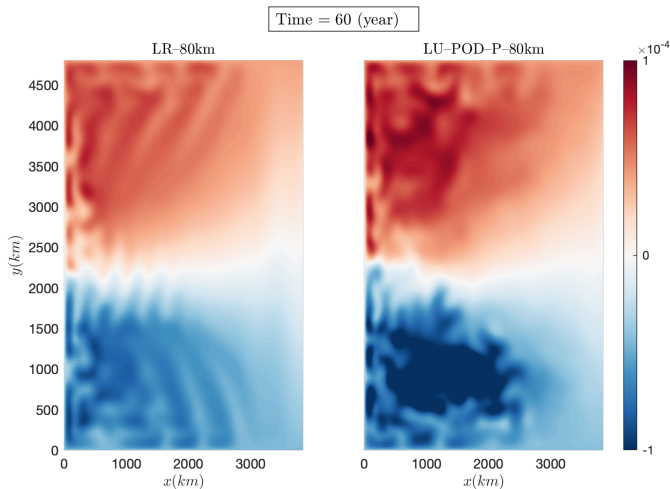


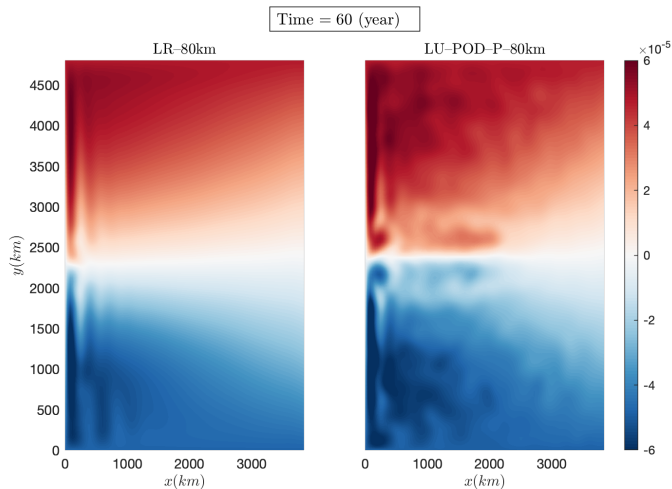
Figure: Zonal (left) and meridional (right) noise velocity at resolution $80km$ by adapting POD noise along isopycnal.

LU coarse simulation (Exp.1)



Video: Evolution of surface layer PV at resolution $80km$. Left: Deterministic model (LR);
Right: LU model under POD-P noise.

LU coarse simulation (Exp.2)



Video: Evolution of surface layer PV at resolution $80km$. Left: Deterministic model (LR);
Right: LU model under POD-P noise.

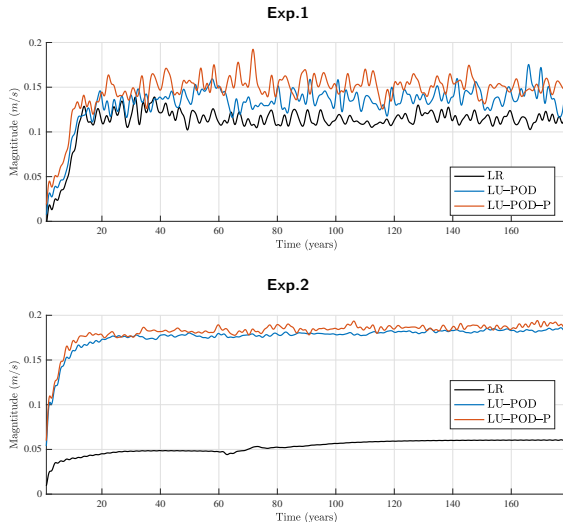


Figure: Maximal values of zonal jet velocity (sum over all layers) at resolution $80km$.

Eddy kinetic energy (EKE) decomposition

For $k = 1, \dots, N$ (with N total number of ocean layers):

$$\text{KE}_k = \frac{\rho H_k}{2} \left(\underbrace{(\overline{u_k^2} + \overline{v_k^2})}_{\text{Standing EKE}} + \underbrace{(u_k'^2 + v_k'^2)}_{\text{Transient EKE}} \right), \quad (19)$$

where $\overline{\cdot}$ is approximated by a 2-years-low-pass Fourier filter and \cdot' is the residual.

Low-frequency variability diagnosis (Exp.1)

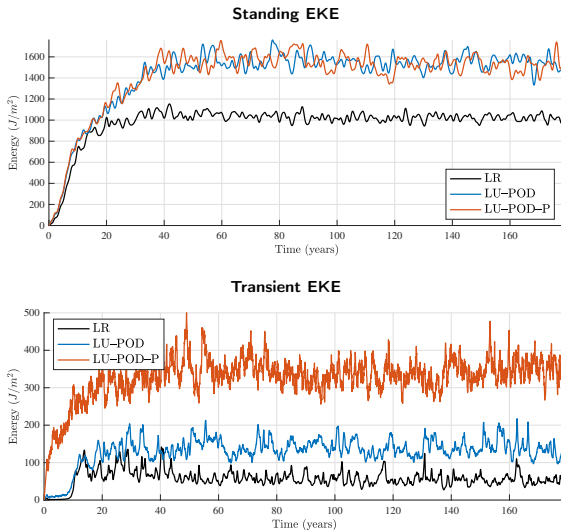


Figure: Standing EKE and Transient EKE (sum over all layers) at resolution $80km$.

Low-frequency variability diagnosis (Exp.2)

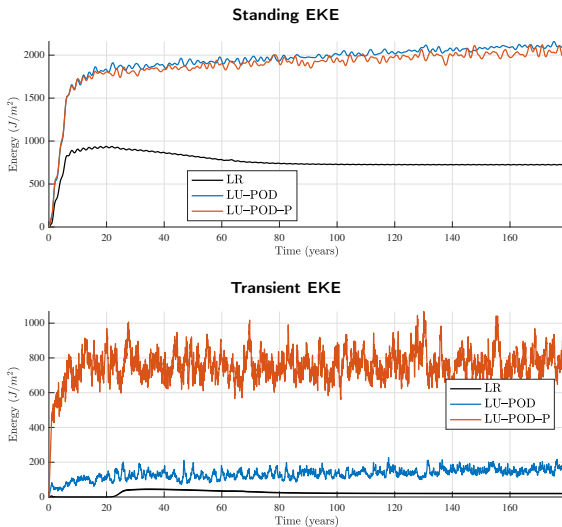
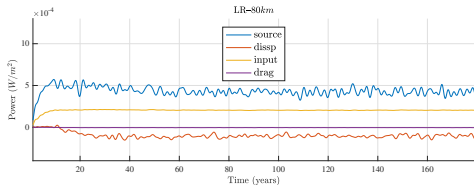
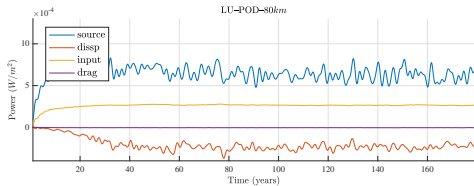
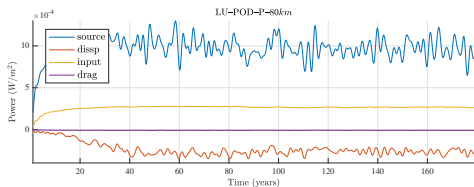


Figure: Standing EKE and Transient EKE (sum over all layers) at resolution $80km$.

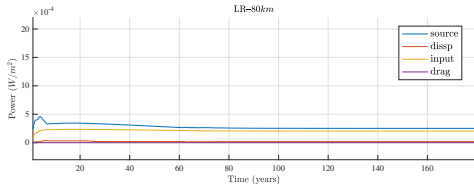
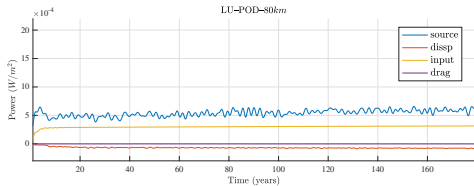
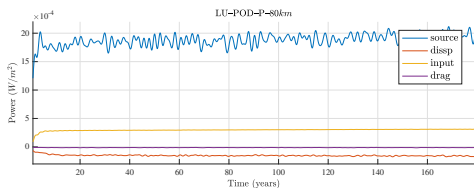
Energy transfert (Hogg and Blundell, 2006)

$$\begin{aligned} \partial_t \text{KE} = & \underbrace{-\partial_t \text{PE}}_{\text{source}} + \underbrace{\rho \int_{\Omega} u_1 \tau^x d\mathbf{x}}_{\text{input wind}} \\ & - \underbrace{\frac{1}{2} \rho \delta_{ek} f_0 \int_{\Omega} (u_N^2 + v_N^2) d\mathbf{x}}_{\text{linear drag}} \\ & - \underbrace{\sum_{k=1}^N A_4 \rho H_k \int_{\Omega} (u_k \nabla^4 u_k + v_k \nabla^4 v_k) d\mathbf{x} + \dots}_{\text{dissipation}} \end{aligned} \quad (20)$$

Low-frequency variability diagnosis (Exp.1)



Low-frequency variability diagnosis (Exp.2)



Measures of statistics (Grooms et al., 2015)

- Pattern correlation (PC)

$$\text{PC} = \frac{\int_{\Omega} \sigma_f^2 \sigma_{f_{\text{ref}}}^2 d\mathbf{x}}{(\int_{\Omega} \sigma_f^4 d\mathbf{x})(\int_{\Omega} \sigma_{f_{\text{ref}}}^4 d\mathbf{x})} \quad \nearrow \quad (21)$$

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- Gaussian approximation of relative entropy

$$\text{Dispersion} = \frac{1}{|\Omega|} \int_{\Omega} \left(\frac{\sigma_{f_{\text{ref}}}^2}{\sigma_f^2} - 1 - \log \left(\frac{\sigma_{f_{\text{ref}}}^2}{\sigma_f^2} \right) \right) d\mathbf{x} \quad \searrow \quad (22)$$

$$\text{Entropy} = \frac{1}{2} \left(\frac{1}{|\Omega|} \int_{\Omega} \frac{(\bar{f}^t - \bar{f}_{\text{ref}}^t)^2}{\sigma_f^2} d\mathbf{x} + \text{Dispersion} \right) \quad \searrow \quad (23)$$

Low-frequency variability diagnosis (Exp.1)

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.44	0.71	0.44	851	26
LU-POD	0.44	0.69	0.46	36	18
LU-POD-P	0.41	0.66	0.50	15	8

Table: Measures of skill for surface layer pressure (80km).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.26	0.60	0.58	188	94
LU-POD	0.26	0.59	0.61	67	34
LU-POD-P	0.25	0.55	0.70	18	9

Table: Measures of skill for middle layer pressure (80km).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.12	0.67	0.56	1701	851
LU-POD	0.12	0.65	0.65	248	124
LU-POD-P	0.12	0.59	0.80	40	20

Table: Measures of skill for bottom layer pressure (80km).

Low-frequency variability diagnosis (Exp.2)

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	1.89	1.51	0.11	1e4	5e3
LU-POD	1.16	1.41	0.29	156	82
LU-POD-P	1.17	1.25	0.54	19	12

Table: Measures of skill for surface layer pressure (80km).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.47	1.16	0.26	5e4	2e4
LU-POD	0.41	1.07	0.78	143	72
LU-POD-P	0.41	0.90	0.90	15	8

Table: Measures of skill for middle layer pressure (80km).

Model	RMSE of mean	RMSE of std	PC of std	Dispersion	Entropy
LR	0.17	1.15	0.23	2e5	1e5
LU-POD	0.16	1.06	0.93	169	85
LU-POD-P	0.16	0.89	0.92	17	9

Table: Measures of skill for bottom layer pressure (80km).

- 1 Run LU ensemble simulations and verify ensemble spread by Rank histogram, CRPS, etc.

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- 4 Data assimilation with particle filter (Cotter et al., 2020).

Thank for Your Attention!

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