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***Abstract*—The purpose of this paper is to show a new methodology for solving inverse problems of design. It is based on the use of a deterministic global optimization algorithm. In front of limitations induced by the use of analytical models, the authors present a way to associate Interval Branch and Bounds techniques and numerical computations based on finite element method. Thanks to this methodology, we are able to solve exactly the associated inverse problem of a magnetic coupling design.**

I. INTRODUCTION

Magnetic couplings are very useful devices for many applications as seal-less pumps in the chemical and petrochemical industries, or the aeronautical and maritime ones for instance. Their functionalities are to transmit a motion between two separated zones without mechanical contacts. The studied structure is a co-axial synchronous magnetic torque coupling. It consists of two rings of permanent magnets separated by an insulating partition. On each side of it, we find an airgap and a binding band.

A. Previous Methodology

In [1], the authors solve the problem of such devices design by applying a rational methodology. The problem is understood and defined as an *inverse problem*, i.e. from some characteristic values given by the schedule of conditions (for example the torque), get the dimensions as well as the structure and the composition of a co-axial magnetic torque coupling. These inverse problems are formulated as a *mixed constrained global optimization problem*.

In order to solve exactly the so-formulated global optimization problems, a particular algorithm (called IBBA) based on a Branch and Bound technique where the bounds are computed using interval analysis has been developed and extended. See [1], [2] and [3] for details.

The advantages of these kinds of methods are :

- they use mixed variables (real, integer, boolean, ...);
- they do not need a starting point (or a set of ones);
- they guarantee to obtain the global minimum of the problem (they are *deterministic* and *global*).

Their main drawbacks are their computation times and the fact that they need (until nowadays) explicit analytic expressions of criteria and constraints.

The inverse problem of couplings design problem is formulated as a mixed constrained global optimization problem defined as follows :

$$\left\{\begin{array}{l} \min\limits_{\substack{x\in\mathbb{R}^{n_r},\ z\in\mathbb{N}^{n_e},\\ b\in B^{n_b},\ \sigma\in\prod_{i=1}^{n_c}K_i}} f(x,z,\sigma,b) \\ g_i(x,z,\sigma,b)\leq 0 \quad \forall i\in\{1,\ldots,n_g\} \\ \Gamma(x,z,\sigma,b)=\Gamma_{fixed} \end{array}\right. \quad (1)$$

where f is a real function (for example the volume), K_i represents an enumerated set of categorical variables (for instance, the kind of magnet), and $B=\{0,1\}$ is the boolean set (for the fact that there are binding bands or not). g_i are some geometrical constraints. Moreover, an equality constraint upon the maximum electromagnetic torque Γ is added, traducing the fact that it must be equal to a fixed value (Γ_{fixed}).

The torque expression comes from an analytical model based on the resolution of Poisson's equations using the separation of variables by keeping only the term due to the first non-zero harmonic (the fundamental) [1].

Several techniques of constraints propagation, and others adaptations were applied in order to reduce the time convergence. Indeed, it was very difficult to compute efficient bounds with interval arithmetic, because of the complexity of our torque formula. Thanks to these techniques, the authors obtained encouraging results dealing with the minimization of the global volume or with the volume of magnets. In our knowledge, it was the first time that such a problem was solved.

B. Need of a More Accurate Model

If a finite element software is used to compute the electromagnetic torque and the mean value of flux density in yokes (the physical constraints of our problem), we notice a non-negligible difference between the values given by our model and the numerical ones (around 20%). However, the kind of analytical models used is known to provide results close to those obtained by a numerical method [4]. Actually, the observed gap is due to the fact that we do not have considered the harmonics greater than the fundamental, and moreover the optimization process is inclined to maximize the error due to these assumptions.

II. New Approach : Use a Hybrid Model

The idea is to associate the advantages of two kinds of models : the swiftness of analytics and the accuracy of finite elements based techniques.

The analytic expression of the electromagnetic torque is given by the resolution of Poisson's equations using the separation of variables. Instead of taking into account only the fundamental (Γ_{o1}) as in [1], the used expression (Γ_{o3}) is the sum of the two first non-zero harmonics (1 and 3). The thicknesses of iron yokes are deduced from the Gauss's law of magnetism and the fact that the mean value of flux density in the yokes B_y must be less or equal to the maximum value $B_M(\sigma_y)$ above which the iron is definitively saturated.

A specially dedicated finite element code has been written in order to automatically draw the geometry, create the mesh, and perform a magneto-static resolution of a magnetic coupling. In output, the torque Γ_{FE} (computed with the Maxwell Stress Tensor) and the mean values of flux density in the inner and outer yokes (B_{yi}^{FE} and B_{yo}^{FE}) are given in less than 1 second. This tool is in the form of a black-box (named NUMTFD) which can be easily called by another program.

III. Formalization of The New Design Problem

Now the question is to know how to associate such a model with an Interval Branch and Bound Algorithm.

A. Inverse Problem Formulation

The design problem must be re-formulated. In fact it is not possible to directly include our black-box as a constraint. Indeed, when the optimization process begins, the lengths of the intervals are too large to perform a valid finite element computation [3].

The idea is to define several zones corresponding to different expressions. If we are far from the wanted value (Γ_{fixed}), we use Γ_{o1}. If we get close, but not close enough to call Γ_{FE}, Γ_{o3} is used. The associate problem is then :

$$\left\{\begin{array}{l} \displaystyle\min_{\substack{x\in\mathbb{R}^{n_r},\ z\in\mathbb{N}^{n_e},\\ b\in B^{n_b},\ \sigma\in\prod_{i=1}^{n_c}K_i}} f(x,z,\sigma,b) \\ g_j(x,z,\sigma,b)\leq 0 \quad \forall j\in\{1,\ldots,n_g\} \\ \left\{\begin{array}{l} (1-\lambda_{ana})\Gamma_{fixed}\leq\Gamma_{o1}\leq(1+\lambda_{ana})\Gamma_{fixed} \\ (1-\lambda_{FE})\Gamma_{fixed}\leq\Gamma_{o3}\leq(1+\lambda_{FE})\Gamma_{fixed} \\ \mathbf{\Gamma_{FE}}(x,z,\sigma,b)=\Gamma_{fixed} \end{array}\right. \\ \mathbf{B_{yi}^{FE}}(x,z,\sigma,b)\leq B_M(\sigma_{yi}) \\ \mathbf{B_{yo}^{FE}}(x,z,\sigma,b)\leq B_M(\sigma_{yo}) \end{array}\right.$$

where λ_{ana} and λ_{FE} are used to define the different zones (for the tests, we have chosen 40% and 20%). Two inequality constraints are added to ensure that the yokes are not saturated.

B. New Interval Branch & Bound Algorithm

The corresponding Interval Branch & Bound Algorithm (IBBA+NUMTFD) has been coded. Its principle is to bisect the initial domain into smaller and smaller boxes and then to eliminate the boxes where the global optimum cannot occurs:

- by proving, using interval bounds, that no point in a box can produce a better solution than the current best one;
- by proving, using interval arithmetic, that at least one constraint cannot be satisfied by any point in such a box.

Some techniques of constraint propagation and limitations have been included to improve the convergence as in [1].

IV. Results and Conclusion

The problem corresponding to the minimization of the global volume of a studied magnetic coupling is solved and results are given in Table I (strike-through text represents non-satisfied constraints). We have chosen Γ_{fixed} = 10 N·m, the real parameters represent geometric quantities, the integer p is the number of poles pairs, and the different σ values are used to choose the kinds of materials. To compare with the previous methodology, the problem is solved using only Γ_{o1} (IBBA), next using Γ_{o3} (IBBAo3), and finally using our hybrid model (IBBA+NUMTFD).

TABLE I
Minimization of the magnetic coupling's global volume V_g

Param.	Bounds	Unit	IBBA	IBBAo3	IBBA+ NUMTFD
θ_{int}; θ_{ext}	[30; 70]	%	48.1; 30.0	55.6; 30.0	52.5; 37.5
R_1; R_2	[1; 5]	cm	2.20; 2.50	2.20; 2.50	2.87; 3.17
R_3; R_4	[1; 5]	cm	2.70; 3.00	2.70; 3.00	3.39; 3.69
p	$[\![4;9]\!]$	-	6	6	5
σ_{mi}; σ_{me}	{1 , 2}	-	2; 2	2; 2	1; 1
σ_{yi}; σ_{ye}	{1 , 2}	-	1; 1	2; 1	2; 1
V_g		cm^3	**84.37**	**84.78**	**116.27**
Γ_{o1}		N·m	**9.80**	10.96	13.61
Γ_{o3}		N·m	~~8.02~~	**9.80**	11.05
Γ_{FE}		N·m	~~**7.39**~~	~~**8.98**~~	**9.92**
B_{yi}^{FE}; B_{yo}^{FE}		T	~~1.71~~; ~~2.12~~	~~1.85~~; ~~2.13~~	**1.53**; **1.54**
CPU Time			0'17"	0'21"	181'37"
Iterations			32124	38991	80514
Numerical Computations			-	-	19951

Only the results corresponding to the new methodology answer perfectly to our *non-homogeneous mixed constrained global optimization problem.*

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