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# Mathematical Development of the Interplexing 6 Signals for the Galileo E1 Band

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The general expression of the Interplexing 6 signals is:

$$\begin{aligned} s(t) &= \sqrt{2P} \cos\left(2\pi f_c t - \frac{\pi}{2} s_1(t) + \beta_2 s_1(t) s_2(t) + \beta_3 s_1(t) s_3(t) + \beta_4 s_1(t) s_4(t) + \beta_5 s_1(t) s_5(t) + \beta_6 s_1(t) s_6(t) + \phi\right) \\ &= \sqrt{2P} \cos(2\pi f_c t + A + B + C + D + E + \phi) \end{aligned} \quad (1)$$

where:

- $A = \beta_2 s_1(t) s_2(t)$
- $B = \beta_3 s_1(t) s_3(t)$
- $C = \beta_4 s_1(t) s_4(t)$
- $D = \beta_5 s_1(t) s_5(t)$
- $E = \beta_6 s_1(t) s_6(t)$

then:

$$s(t) = \sqrt{2P} \begin{bmatrix} \cos(2\pi f_c t + \phi) \left[ \cos\left(-\frac{\pi}{2} s_1(t)\right) \cos(A + B + C + D + E) - \right. \\ \left. \sin\left(-\frac{\pi}{2} s_1(t)\right) \sin(A + B + C + D + E) \right] \\ + \sin(2\pi f_c t + \phi) \left[ \sin\left(-\frac{\pi}{2} s_1(t)\right) \cos(A + B + C + D + E) + \right. \\ \left. \cos\left(-\frac{\pi}{2} s_1(t)\right) \sin(A + B + C + D + E) \right] \end{bmatrix} \quad (2)$$

where  $\cos\left(-\frac{\pi}{2} s_1(t)\right) = 0$ , yielding  $s(t)$  to:

$$s(t) = \sqrt{2P} \begin{bmatrix} \cos(2\pi f_c t + \phi) [s_1(t) \sin(A + B + C + D + E)] \\ + \sin(2\pi f_c t + \phi) [s_1(t) \cos(A + B + C + D + E)] \end{bmatrix} \quad (3)$$

where  $\sin(A + B + C + D + E)$  is:

$$\sin(A + B + C + D + E) = \begin{bmatrix} \sin(A) \cos(B) \cos(C) \cos(D) \cos(E) - \sin(A) \cos(B) \cos(C) \sin(D) \sin(E) - \\ \sin(A) \cos(B) \sin(C) \sin(D) \cos(E) - \sin(A) \cos(B) \sin(C) \cos(D) \sin(E) - \\ \sin(A) \sin(B) \sin(C) \cos(D) \cos(E) + \sin(A) \sin(B) \sin(C) \sin(D) \sin(E) - \\ \sin(A) \sin(B) \cos(C) \sin(D) \cos(E) - \sin(A) \sin(B) \cos(C) \cos(D) \sin(E) + \\ \cos(A) \sin(B) \cos(C) \cos(D) \cos(E) - \cos(A) \sin(B) \cos(C) \sin(D) \sin(E) - \\ \cos(A) \sin(B) \sin(C) \sin(D) \cos(E) - \cos(A) \sin(B) \sin(C) \cos(D) \sin(E) + \\ \cos(A) \cos(B) \sin(C) \cos(D) \cos(E) - \cos(A) \cos(B) \sin(C) \sin(D) \sin(E) + \\ \cos(A) \cos(B) \cos(C) \sin(D) \cos(E) + \cos(A) \cos(B) \cos(C) \cos(D) \sin(E) \end{bmatrix} \quad (4)$$

and

$\cos(A + B + C + D + E)$  is:

$$\cos(A + B + C + D + E) = \begin{bmatrix} \cos(A) \cos(B) \cos(C) \cos(D) \cos(E) - \cos(A) \cos(B) \cos(C) \sin(D) \sin(E) - \\ \cos(A) \cos(B) \sin(C) \sin(D) \cos(E) - \cos(A) \cos(B) \sin(C) \cos(D) \sin(E) - \\ \cos(A) \sin(B) \sin(C) \cos(D) \cos(E) + \cos(A) \sin(B) \sin(C) \sin(D) \sin(E) - \\ \cos(A) \sin(B) \cos(C) \sin(D) \cos(E) - \cos(A) \sin(B) \cos(C) \cos(D) \sin(E) - \\ \sin(A) \sin(B) \cos(C) \cos(D) \cos(E) + \sin(A) \sin(B) \cos(C) \sin(D) \sin(E) + \\ \sin(A) \sin(B) \sin(C) \sin(D) \cos(E) + \sin(A) \sin(B) \sin(C) \cos(D) \sin(E) - \\ \sin(A) \cos(B) \sin(C) \cos(D) \cos(E) + \sin(A) \cos(B) \sin(C) \sin(D) \sin(E) - \\ \sin(A) \cos(B) \cos(C) \sin(D) \cos(E) - \sin(A) \cos(B) \cos(C) \cos(D) \sin(E) \end{bmatrix} \quad (5)$$

Then, from equation (3) and considering the definition of  $A, B, C, D, E$ , we derive:

$$s_1(t) \sin(A + B + C + D + E) = \begin{bmatrix} s_2(t) \sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \\ s_2(t) s_5(t) s_6(t) \sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6) - \\ s_2(t) s_4(t) s_5(t) \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6) - \\ s_2(t) s_4(t) s_6(t) \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6) - \\ s_2(t) s_3(t) s_4(t) \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) + \\ s_2(t) s_3(t) s_4(t) s_5(t) s_6(t) \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6) - \\ s_2(t) s_3(t) s_5(t) \sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) - \\ s_2(t) s_3(t) s_6(t) \sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) + \\ s_3(t) \cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \\ s_3(t) s_5(t) s_6(t) \cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6) - \\ s_3(t) s_4(t) s_5(t) \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6) - \\ s_3(t) s_4(t) s_6(t) \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6) + \\ s_4(t) \cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) - \\ s_4(t) s_5(t) s_6(t) \cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6) + \\ s_5(t) \cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) + \\ s_6(t) \cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) \end{bmatrix} \quad (6)$$

Let us now redefine the Galileo E1 OS equations in [1] as a function of  $s_2(t)$ ,  $s_3(t)$ ,  $s_4(t)$  and  $s_5(t)$ :

$$\begin{aligned} s_{E1-OS_{data}}(t) &= d_{E1-B}(t) c_{E1-B}(t) (\alpha BOC(1, 1) + \beta BOC(6, 1)) = \alpha s_2(t) + \beta s_4(t). \\ s_{E1-OS_{pilot}}(t) &= c_{E1-C}(t) (\alpha BOC(1, 1) - \beta BOC(6, 1)) = \alpha s_3(t) - \beta s_5(t). \end{aligned} \quad (7)$$

considering equation (7), equation (4) can be simplified to :

$$\begin{bmatrix} s_2(t) [\sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6)] \\ + s_2(t) s_5(t) s_6(t) [-\sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6) - \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6)] \\ + s_3(t) [\cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6)] \\ + s_2(t) s_4(t) s_6(t) [-\sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6) - \cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6)] \\ + s_5(t) [\cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) - \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6)] \\ + s_6(t) [\cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) + \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6)] \\ + s_4(t) [\cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) - \sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6)] \\ + s_2(t) s_3(t) s_6(t) [-\sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) - \cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6)] \end{bmatrix} \quad (8)$$

Consequently:

$$\begin{aligned} (\sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6)) &= \alpha \\ (\cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6)) &= \alpha \\ (\cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) - \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6)) &= -\beta \\ (\cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) - \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6)) &= \beta \end{aligned} \quad (9)$$

This linear system lead to  $\beta_2 = \beta_3$  and  $\beta_4 = -\beta_5$ , leaving equation (6) to:

$$s_1(t) \sin(A + B + C + D + E) = \begin{bmatrix} (s_2(t) + s_3(t)) \cos(\beta_6) \frac{\sin(2\beta_2)}{2} + \\ (s_4(t) - s_5(t)) \cos(\beta_6) \frac{\sin(2\beta_4)}{2} + \\ s_6(t) \sin(\beta_6) \frac{[\cos(2\beta_2) + \cos(2\beta_4)]}{2} + \\ s_2(t)s_3(t)s_6(t) \sin(\beta_6) [\cos^2(\beta_2) \sin^2(\beta_4) - \sin^2(\beta_2) \cos^2(\beta_4)] \end{bmatrix} \quad (10)$$

Now, from equation (3) and considering the definition of  $A, B, C, D, E$ , we derive:

$$s_1(t) \cos(A + B + C + D + E) = \begin{bmatrix} s_1(t) \cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \\ s_1(t)s_5(t)s_6(t) \cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6) - \\ s_1(t)s_4(t)s_5(t) \cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6) - \\ s_1(t)s_4(t)s_6(t) \cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6) - \\ s_1(t)s_3(t)s_4(t) \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) + \\ s_1(t)s_3(t)s_4(t)s_5(t)s_6(t) \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6) - \\ s_1(t)s_3(t)s_5(t) \cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) - \\ s_1(t)s_3(t)s_6(t) \cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) - \\ s_1(t)s_2(t)s_3(t) \sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) + \\ s_1(t)s_2(t)s_3(t)s_5(t)s_6(t) \sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6) + \\ s_1(t)s_2(t)s_3(t)s_4(t)s_5(t) \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6) + \\ s_1(t)s_2(t)s_3(t)s_4(t)s_6(t) \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6) - \\ s_1(t)s_2(t)s_4(t) \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) + \\ s_1(t)s_2(t)s_4(t)s_5(t)s_6(t) \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6) - \\ s_1(t)s_2(t)s_5(t) \sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) - \\ s_1(t)s_2(t)s_6(t) \sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) \end{bmatrix} \quad (11)$$

Considering equation (7), equation (11) can be simplified to :

$$\begin{bmatrix} s_1(t) [\cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6)] \\ + s_1(t)s_5(t)s_6(t) [-\cos(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6) + \sin(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6)] \\ + s_1(t)s_2(t)s_3(t) [-\sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \cos(\beta_6) - \cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \cos(\beta_6)] \\ + s_1(t)s_4(t)s_6(t) [-\cos(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \sin(\beta_6) + \sin(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \sin(\beta_6)] \\ + s_1(t)s_3(t)s_4(t) [\sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6) - \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6)] \\ + s_1(t)s_2(t)s_6(t) [-\sin(\beta_2) \cos(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) + \cos(\beta_2) \sin(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6)] \\ + s_1(t)s_2(t)s_4(t) [-\sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \cos(\beta_5) \cos(\beta_6) - \cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \sin(\beta_5) \cos(\beta_6)] \\ + s_1(t)s_3(t)s_6(t) [-\cos(\beta_2) \sin(\beta_3) \cos(\beta_4) \cos(\beta_5) \sin(\beta_6) + \sin(\beta_2) \cos(\beta_3) \sin(\beta_4) \sin(\beta_5) \sin(\beta_6)] \end{bmatrix} \quad (12)$$

with  $\beta_2 = \beta_3$  and  $\beta_4 = -\beta_5$ , equation (12) yields to:

$$s_1(t) \cos(A + B + C + D + E) = \begin{bmatrix} s_1(t) \cos(\beta_6) \frac{[\cos(2\beta_2) + \cos(2\beta_4)]}{2} + \\ s_1(t)s_2(t)s_3(t) \cos(\beta_6) [\cos^2(\beta_2) \sin^2(\beta_4) - \sin^2(\beta_2) \cos^2(\beta_4)] - \\ (s_4(t) - s_5(t))s_1(t)s_6(t) \sin(\beta_6) \frac{\sin(2\beta_4)}{2} - \\ (s_2(t) + s_3(t))s_1(t)s_6(t) \sin(\beta_6) \frac{\sin(2\beta_2)}{2} \end{bmatrix} \quad (13)$$

Considering equations (10) and (13), equation (3) yields to

$$s(t) = \sqrt{2P} \begin{bmatrix} \cos(2\pi f_c t + \phi) \begin{bmatrix} (s_2(t) + s_3(t)) \cos(\beta_6) \frac{\sin(2\beta_2)}{2} + \\ (s_4(t) - s_5(t)) \cos(\beta_6) \frac{\sin(2\beta_4)}{2} + \\ s_6(t) \sin(\beta_6) \frac{[\cos(2\beta_2) + \cos(2\beta_4)]}{2} + \\ s_2(t)s_3(t)s_6(t) \sin(\beta_6) [\cos^2(\beta_2) \sin^2(\beta_4) - \sin^2(\beta_2) \cos^2(\beta_4)] \end{bmatrix} \\ + \sin(2\pi f_c t + \phi) \begin{bmatrix} s_1(t) \cos(\beta_6) \frac{[\cos(2\beta_2) + \cos(2\beta_4)]}{2} + \\ s_1(t)s_2(t)s_3(t) \cos(\beta_6) [\cos^2(\beta_2) \sin^2(\beta_4) - \sin^2(\beta_2) \cos^2(\beta_4)] - \\ (s_4(t) - s_5(t))s_1(t)s_6(t) \sin(\beta_6) \frac{\sin(2\beta_4)}{2} - \\ (s_2(t) + s_3(t))s_1(t)s_6(t) \sin(\beta_6) \frac{\sin(2\beta_2)}{2} \end{bmatrix} \end{bmatrix} \quad (14)$$

## References

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