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Measures and LMIs for Validation of an Aircraft with MRAC and Uncertain Actuator Dynamics

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It is well known that actuator dynamics with degraded performance can lead to poor stability and tracking for adaptive control. In this paper, we use occupation measures and LMI relaxations (called the moment sums of squares or Lasserre hierarchy) for verification and validation of a longitudinal F-16 aircraft model with MRAC and higher order actuator dynamics with uncertain parameters and deflection saturation. These uncertain parameters are represented explicitly in the space of occupation measures. To improve numerical scalability, we exploit sparsity for ordinary differential equations (ODEs) using parsimony to partition the dynamics. We finally compare these main results to those obtained using Monte-Carlo methods.

I. Nomenclature

$\alpha(t)$	=	angle of attack [rad]
$q(t)$	=	pitchrate [rad/s]
$v(t)$	=	elevator actuator output [deg]
$\mathbf{x}_p(t)$	=	short-period longitudinal dynamics
$\mathbf{x}_c(t)$	=	third order actuator dynamics

II. Introduction

A challenge in the design of adaptive control laws is addressing system stability and acceptable command following in the presence of uncertain, unmodeled actuator dynamics. In particular, degraded actuator performance can lead to poor tracking and even instability [1]. In real world applications, actuators seldom follow a first order model and their dynamics cannot be safely neglected as they can limit achievable stability of model reference adaptive control (MRAC). Therefore, it remains imperative that there exists a way to ensure safe interaction between the higher order actuator and the MRAC in the design phase.

Current literature attempts to address this by using MRAC control modifications. See [2, 3] where the authors consider a pseudo control hedging modification to include actuator dynamics in the design phase of MRAC. The authors of [4, 5] revisit pseudo-control hedging with a novel Lyapunov linear matrix inequality (LMI) method to ensure ideal bounded reference trajectories for a range of admissible bandwidths, natural frequency, and damping inherent in the actuator dynamics.

As discussed mainly in [5], the relationship between the damping and natural frequency can be bilinear. In other words, any variation in the damping can adversely affect the range of admissible natural frequencies required to maintain

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closed-loop stability for higher order actuators and vice-versa. There is also a relationship between the MRAC learning rate and the closed-loop stability. Consequently, traditional Monte-Carlo methods can become costly for systems with these parameter uncertainties inherent in their dynamics. Every combination of these parameters must be simulated, and there remains risk of state-space explosion or unexplored, unsafe trajectories for medium and large scale systems.

Moment sum of squares (SOS) is a state-of-the-art method for verification and validation (VV) in aerospace. See [6, 7] where the authors focus on polynomial dynamical models and polynomial SOS Lyapunov candidate function. Occupation measures and LMI relaxations (called the moment sums of squares or Lasserre hierarchy) have also been used in VV of aircraft models with MRAC [8]. See also more recent development in using this framework where the authors [9, 10] exploit sparsity for ODEs using parsimony. Specifically, scaling issues brought about by optimizing directly over medium to large scale systems can be addressed by partitioning the dynamics.

The procedure remains similar to [10–12]. First, our control law validation problem can be written as a polynomial dynamical optimization problem similar to that in optimal control. This problem is then written as its equivalent infinite dimensional linear programming (LP) problem by introducing occupation measures supported over admissible trajectories. Lastly, we relax the infinite dimensional LP problem of measures to a finite dimensional moment LMIs of truncated sequences.

The main contributions are the following:

- We validate an F-16 longitudinal model with MRAC and uncertain third order elevator dynamics with deflection saturation. The elevator contains bounded uncertainties inherent in its damping and natural frequency. Since uncertain parameters have an explicit representation in the space of occupation measures, we can identify numerically the worst case behavior caused by inherent uncertainties in damping and natural frequencies coming from the actuator. To avoid scaling problems, we exploit sparsity for ODEs to partition out the dynamics into a class of smaller systems that serve as control inputs for each other. In total, we are able to validate medium and large scale systems (up to 11 states) using this approach.
- Two examples are considered where the MRAC tolerates uncertainty in the natural frequency and damping, and one example where it obviously fails. Then, we compare our main results to those achieved using Monte-Carlo simulations and a search algorithm. The search algorithm locates the worst case damping and natural frequency behavior from the actuator.

Unlike the results of [2, 4, 5], we do not use the pseudo control hedging modification to address actuator bandwidth uncertainty. Furthermore, none of these results take into account deflection saturation of the actuator. The results in [6, 7] provide only ultimate stability for aircraft models with linear feedback. Our previous work [8, 9] do not take into account uncertainties with a polynomial structure. In other words, our framework guarantees finite-time convergence for non-autonomous systems with piecewise dynamics. This is solution better than the solution provided by Barbalat’s lemma [13] provided by the readily available Lyapunov certificate.

The organization of this work is as follows: Section III contains some necessary mathematical preliminaries, section IV discusses the short period of the F-16 with third order actuator dynamics, section V discusses our main results, and section VI gives our main conclusions and suggestions for future research.

III. Mathematical Preliminaries

We begin by very briefly stating some standard notation used for occupations measures. If X is a compact subset of \mathbb{R}^n , $\mathcal{C}(X)$ denotes the space of continuous functions on X and $\mathcal{M}(X)$ (resp., $\mathcal{M}_+(X)$) denotes the cone of (resp., non-negative) measures. We also assume that there is no relaxation gap ($J = J_\infty$) between the polynomial dynamical optimization problem and its representation in the infinite dimensional space of measures. For the proceeding sections, we rely on the main theoretical results discussed extensively in [8, 9]. In particular, consider the polynomial dynamical optimization with piecewise polynomial different constraints

$$\begin{aligned}
 J &= \inf_{h_T, h} h_T(T, \mathbf{x}(T)) + \int_0^T h(t, \mathbf{x}(t)) dt \\
 \text{s.t.} \quad &\dot{\mathbf{x}}(t) = \mathbf{f}_j(t, \mathbf{x}(t)), \quad \mathbf{x}(t) \in X_j, \quad j = 1, \dots, N \\
 &\mathbf{x}(0) \in X_0, \quad \mathbf{x}(t) \in X_T, \quad t \in [0, T],
 \end{aligned} \tag{1}$$

with given costs $h, h_T \in \mathbb{R}[t, x]$, polynomial dynamics $f_j \in \mathbb{R}[t, x]$, $j = 1, \dots, N$, state trajectory $\mathbf{x}(t)$ constrained in compact basic semialgebraic sets X_j , and initial (resp. terminal) given compact basic semialgebraic sets X_0 (resp., X_T). We also assume that the state space partitioning sets, or cells X_j , are such that all of their respective intersections have zero Lebesgue measure and all belong to a given compact semialgebraic set X .

The nonconvex optimization problem eq. (1) can be approximated as a generic convex infinite dimensional LP problem of measures using parsimony

$$\begin{aligned}
J_\infty = \inf & \int h_T(T, \mathbf{x}(t))d\mu_T + \int h(t, \mathbf{x}(t))d\mu \\
\text{s.t.} & \left(\frac{\partial \mu}{\partial t} + \text{div} \mathbf{f}_j \mu_j \right) + \mu_T = \mu_0 \\
& \left(\frac{\partial \nu}{\partial t} + \text{div} \mathbf{f}_k \nu_k \right) + \nu_T = \nu_0 \\
& \sum_{j=1}^N \pi_{t, \mathbf{y} \#} \mu_j = \sum_{k=1}^M \pi_{t, \mathbf{y} \#} \nu_k \\
& \int \mu_0 = 1, \quad \int \nu_0 = 1,
\end{aligned} \tag{2}$$

where div is the divergence operator and the infimum is with respect to the occupation measure $\mu, \nu \in \mathcal{M}_+([0, T] \times X)$, initial measure $\mu_0, \nu_0 \in \mathcal{M}_+(\{0\} \times X_0)$, terminal measure $\mu_T, \nu_T \in \mathcal{M}_+(\{T\} \times X_T)$, terminal time $T > 0$. Each measure μ_j (resp. ν_k) supported on their respective cell X_j (resp., V_k) so that the global occupation measure becomes

$$\mu = \sum_{j=1}^N \mu_j, \quad \nu = \sum_{k=1}^M \nu_k,$$

with marginal $\pi_{t, \mathbf{y} \#} \mu$ (resp., $\pi_{t, \mathbf{y} \#} \nu$) of measure μ (resp., ν) with respect to variables t, \mathbf{y} . As noted in [10], this approach allows some flexibility with the computational limit caused by the largest moment SDP block. With appropriate partitioning strategies, we can solve problems with medium to large number of states.

IV. F-16 Short Period with Actuator Dynamics

The short-period mode of an F-16 aircraft can be expressed by the dynamics

$$\dot{\mathbf{x}}_p(t) = A \mathbf{x}_p(t) + B v(t), \quad \mathbf{x}_p(0) = \mathbf{x}_{p0} \tag{3}$$

where $\mathbf{x}_p(t) = \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix}$ are the short period dynamics, $\alpha(t) \in \mathbb{R}$ is the angle of attack, $q(t) \in \mathbb{R}$ is the pitchrate,

$$A = \begin{bmatrix} -1.0189 & +0.9051 \\ +0.8223 & -1.0774 \end{bmatrix}, \quad B = \begin{bmatrix} -0.0022 \\ -0.1756 \end{bmatrix}$$

are the open loop short period dynamics. Elevator actuator output $v(t) \in \mathbb{R}$ is described the third order unmodeled actuator dynamics

$$\dot{\mathbf{x}}_c(t) = (F + L(k_1, k_2)) \mathbf{x}_c(t) + G u(t) \tag{4}$$

$$v(t) = H \mathbf{x}_c(t), \quad v(0) = v_0 \tag{5}$$

where $u(t) \in \mathbb{R}$ is a measurable control input,

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a\omega_n^2 & \omega_n^2 + 2\zeta\omega_n a & 2\zeta\omega_n + a \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad H = \begin{bmatrix} a\omega_n^2 \\ 0 \\ 0 \end{bmatrix},$$

$\zeta, \omega_n, a \in \mathbb{R}_+$ are the damping, natural frequency, and gain respectively,. Matrix

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -ak_1^2 & k_1^2 + 2k_2k_1a & 2k_2k_1 + a \end{bmatrix}$$

contains inherent uncertainties within damping and natural frequency of the actuator dynamics described by bounded parameters $k_1, k_2 \in [-k_{max}, k_{max}]$, $k_{max} \in \mathbb{R}_+$. We assume that resulting matrix $(F + L)$ is Hurwitz, the DC gain between the control input $u(t)$ and actuator output $v(t)$ is unity, and pair (A, B) is controllable.

A. Model Reference Adaptive Control

Given a measurable command signal $c(t)$, we want to design a control law

$$u(t) = u_n(t) + u_a(t) \quad (6)$$

such that the combined nominal $u_n(t)$ and adaptive control $u_a(t)$ laws allow eq. (3) to asymptotically track reference model

$$\dot{\mathbf{x}}_r(t) = A_r \mathbf{x}_r(t) + B_r c(t), \quad \mathbf{x}_r(0) = \mathbf{x}_{r0} \quad (7)$$

where $\mathbf{x}_r(t) = \begin{bmatrix} \alpha_r \\ q_r \end{bmatrix}$ and the error dynamics

$$\mathbf{e}(t) = \mathbf{x}_p(t) - \mathbf{x}_r(t) \quad (8)$$

satisfies $\lim_{t \rightarrow \infty} \mathbf{e}(t) = 0$. Matrices $A_r = (A - BK_1)$ and $B_r = BK_2$ are from the nominal controller

$$u(t) = K_1 \mathbf{x}_p(t) + K_2 c(t) \quad (9)$$

with feedback/feedforward gains

$$K_1 = \begin{bmatrix} +4.7432 & -2.3163 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -4.3396 \end{bmatrix}$$

obtained using the LQR method [14, 15]. We assume that there exists a feedback matrix K_1 such that matrix A_r is Hurwitz.

The adaptive controller

$$u_a(t) = -\hat{\mathbf{W}}^T(t) \Phi(\mathbf{x}_p(t)), \quad (10)$$

with the given basis function $\Phi_i(\mathbf{x}_p(t)) = (1 + e^{x_{pi}})^{-1}$, $i = 1, 2$ and weight estimate $\hat{\mathbf{W}}(t)$, satisfies the update law

$$\dot{\hat{\mathbf{W}}}(t) = \Gamma \Phi(\mathbf{x}_p(t)) \mathbf{e}^T(t) P B_p, \quad \hat{\mathbf{W}}(0) = \hat{\mathbf{W}}_0 \quad (11)$$

where $\Gamma = \text{diag} \left(\left[\epsilon \ 10 \right] \right)$, $\epsilon \ll 1$ is the learning rate and positive definite matrix P is generated by the Lyapunov equality

$$0 = A_r^T P + P A_r + I_{2 \times 2}. \quad (12)$$

Theorems that highlight the boundedness and long range stability of MRAC are discussed extensively in [13, 16]. For some discussion on control modifications, refer to [17]. For papers that discuss using these control modifications, refer to [8, 9].

V. Main Results

We wish to validate our existing closed loop polynomial aircraft model eq. (3) with actuator dynamics eq. (5) by finding the initial state that maximizes of the norm of the concave cost function $J = -\|\hat{\mathbf{e}}(T)\|_2^2$ where

$$\hat{\mathbf{e}}(t) = \begin{bmatrix} \alpha(t) - \alpha_r(t) \\ q(t) - q_{ss}(t) \end{bmatrix} \approx \mathbf{e}(t)$$

approximates the error dynamics and $q_{ss} = \lim_{t \rightarrow \infty} q_r(t)$. We also have given terminal time $T = 25$ s, $c(t) = 0.1$ and actuator constants

$$a = 5, \quad \omega_n = 20, \quad \zeta = 0.1,$$

which are all guaranteed feasible solutions from [5]. If we can show that for every chosen initial state

$$\underbrace{\begin{bmatrix} \mathbf{x}_p(0) & \mathbf{x}_c(0) & \mathbf{x}_r(0) & \hat{\mathbf{W}}(0) \end{bmatrix}}_{\mathbf{x}(0)} \in X_0 \triangleq \begin{bmatrix} -0.2 & 0.2 \end{bmatrix}^2 \times \begin{bmatrix} -1 \times 10^{-6} & 1 \times 10^{-6} \end{bmatrix}^6$$

that all trajectories remain bounded in the box

$$\underbrace{\begin{bmatrix} \mathbf{x}_p(t) & \mathbf{x}_c(t) & \mathbf{x}_r(t) & \hat{\mathbf{W}}(t) \end{bmatrix}}_{\mathbf{x}(t)} \in X \triangleq \begin{bmatrix} -2 & 2 \end{bmatrix}^7 \times \begin{bmatrix} -3 & 3 \end{bmatrix}$$

until they reach the terminal cost target $\hat{\mathbf{e}}(T) \in \{J \leq 4 \times 10^{-3}\}$, then our control law is validated. For our main results, we consider a 1% and 10% increase in the uncertainty the elevator dynamics by increasing k_{max} . We also include an example where the MRAC *fails* due to severe degraded actuator performance invoked by a 5000% increase in the dynamic uncertainty.

To include saturation in the elevator deflection, we partition the dynamics using locally affine functions in three cells $X_j, j = 1, \dots, 3$ corresponding to linear, upper saturation, lower regimes

$$X_1 \triangleq \{\mathbf{x}(t) \in \mathbb{R}^8 : |v(t)| \leq \theta_{max}\}, \quad v(t) = H\mathbf{x}_c(t) \quad (13)$$

$$X_2 \triangleq \{\mathbf{x}(t) \in \mathbb{R}^8 : v(t) \geq \theta_{max}\}, \quad v(t) = \theta_{max} \quad (14)$$

$$X_3 \triangleq \{\mathbf{x}(t) \in \mathbb{R}^8 : v(t) \leq -\theta_{max}\}, \quad v(t) = -\theta_{max} \quad (15)$$

where $\theta_{max} = \frac{25\pi}{180}$.

With the *combined* short period dynamics eq. (3), actuator dynamics, in-state actuator uncertainties k_1 and k_2 , and controller dynamics all partitioned into the saturation regimes, we can write eq. (1) as

$$\begin{aligned} J &= \inf_{\hat{\mathbf{e}}(T)} - \|\hat{\mathbf{e}}(T)\|_2^2 \\ \text{s.t. } \dot{\mathbf{x}}_p(t) &= \mathbf{f}_p(t, \mathbf{x}_p(t), v(t)), \\ \dot{\mathbf{x}}_r(t) &= \mathbf{f}_r(t, \mathbf{x}_r(t)), \\ \begin{bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\hat{\mathbf{W}}}(t) \end{bmatrix} &= \mathbf{f}_j(t, \mathbf{x}_c(t), \mathbf{x}_p(t), \hat{\mathbf{W}}(t), \alpha_r, k_1, k_2), \\ \mathbf{x}(t) &\in X_j, \quad k_1, k_2 \in \begin{bmatrix} -k_{max} & k_{max} \end{bmatrix}, \quad j = 1, \dots, 3 \\ \mathbf{x}(0) &\in X_0, \quad \mathbf{x}(t) \in X_T, \quad t \in [0, T], \end{aligned} \quad (16)$$

and the measure LP of parsimony eq. (2) as

$$\begin{aligned} J_\infty &= \inf_{\mu_T} - \int \|\hat{\mathbf{e}}(T)\|_2^2 d\mu_T \\ \text{s.t. } \frac{\partial \mu}{\partial t} + \text{div} \mathbf{f}_p \mu(t, \mathbf{x}_p, v) + \mu_T &= \mu_0 \\ \frac{\partial v}{\partial t} + \text{div} \mathbf{f}_r v(t, \mathbf{x}_r) + v_T &= v_0 \\ \frac{\partial \xi}{\partial t} + \text{div} \mathbf{f}_j \xi(t, \mathbf{x}_c, \hat{\mathbf{W}}, \mathbf{x}_p, \alpha_r, k_1, k_2) + \xi_T &= \xi_0 \\ \pi_{t, \mathbf{x}_p, v} \mu &= \pi_{t, \mathbf{x}_p, v} v \\ \pi_{t, \mathbf{x}_r} v &= \sum_{j=1}^3 \pi_{t, \mathbf{x}_r} \xi \\ \int \mu_0 &= 1, \quad \int v_0 = 1, \quad \int \xi_0 = 1 \end{aligned} \quad (17)$$

with respect to each occupation measure μ, v, ξ . Marginal $\pi_{t, \mathbf{x}_p, v} \mu$ (respectively, $\pi_{t, \mathbf{x}_p, v} v$) of measure μ (respectively, v) with respect to variables t, \mathbf{x}_p , and v . Marginal $\pi_{t, \mathbf{x}_r} v$ (respectively, $\pi_{t, \mathbf{x}_r} \xi$) of measure μ (respectively, ξ) with respect to variables t, \mathbf{x}_r . As discussed in section III we use this strategy to address scaling issues inherited from the problem. With this approach, we can reduce the overall size of the problem by 3 variables. As discussed in [8, 9], we also employ some basic scaling strategies so that all dynamics are normalized on the interval $[-1 \quad 1]$.

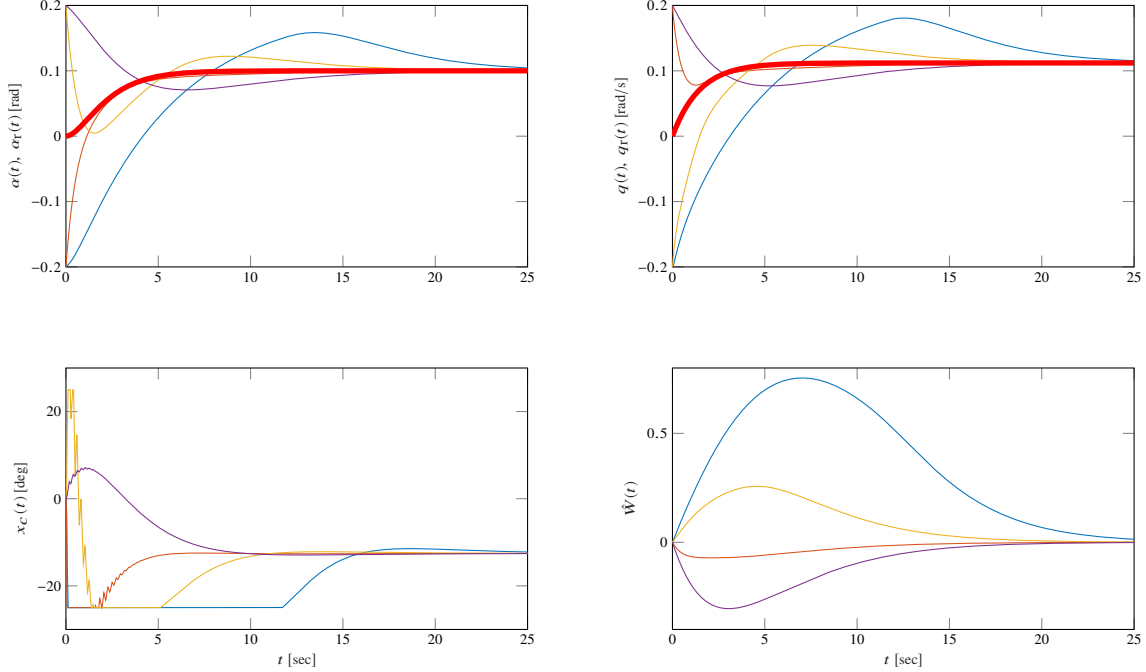


Fig. 1 F-16 Monte-Carlo ($k_{max} = 0.01$, $k_1 = -0.01$, $k_2 = 0.01$)

Rel Ord	$k_{max} = 0.01$		$k_{max} = 0.1$		$k_{max} = 50$	
	Upper Bnd J	CPU [s]	Upper Bnd J	CPU [s]	Upper Bnd J	CPU [s]
1	3.00×10^{-2}	2.81	3.10×10^{-2}	3.59	1.13	3.82
2	9.78×10^{-3}	3.17×10^2	9.66×10^{-3}	3.36×10^2	1.03×10^{-1}	4.34×10^2
3	1.98×10^{-3}	1.38×10^5	2.91×10^{-3}	1.15×10^5	2.86×10^{-2}	1.20×10^5

Table 1 Gloptipoly 3 + MOSEK Upper Bounds for section V

The Monte-Carlo use a fixed time step and evenly distributed initial conditions to simulate the trajectories. For every simulation, we also use a search algorithm to find the matrix $L(k_1, k_2)$ that maximizes the terminal cost J . The worst case Monte-Carlo simulations and upper bounds are given in Figs. 1 to 3 and Table 2, respectively,. Worst case parameters for natural frequency and damping are also shown in the captions.

As shown in Table 1, the upper bounds obtained using our framework (Gloptipoly 3 + MOSEK) show the MRAC can tolerate a 0.1% and 10% uncertainty in damping and natural frequency. At $k_{max} = 50$, the actuator performance degrades significantly and tracking is lost. We can also extract the time spent in the saturation regime, by extracting the mass of each occupation measure [11].

VI. Conclusion

We validated a short period F-16 model with MRAC and a higher order elevator dynamics. The elevator contained parameter uncertainties and deflection saturation. We compared the results using our framework with those obtained using Monte-Carlo and a search algorithm. We also showed an example where the MRAC obviously fails due to poor

k_{max}	Upper Bound J (LQR + MRAC)	CPU Time [s]
0.01	3.13×10^{-5}	7.74×10^1
0.1	3.13×10^{-5}	7.96×10^1
50	2.27×10^{-2}	7.43×10^1

Table 2 Monte-Carlo Upper Bounds for section V

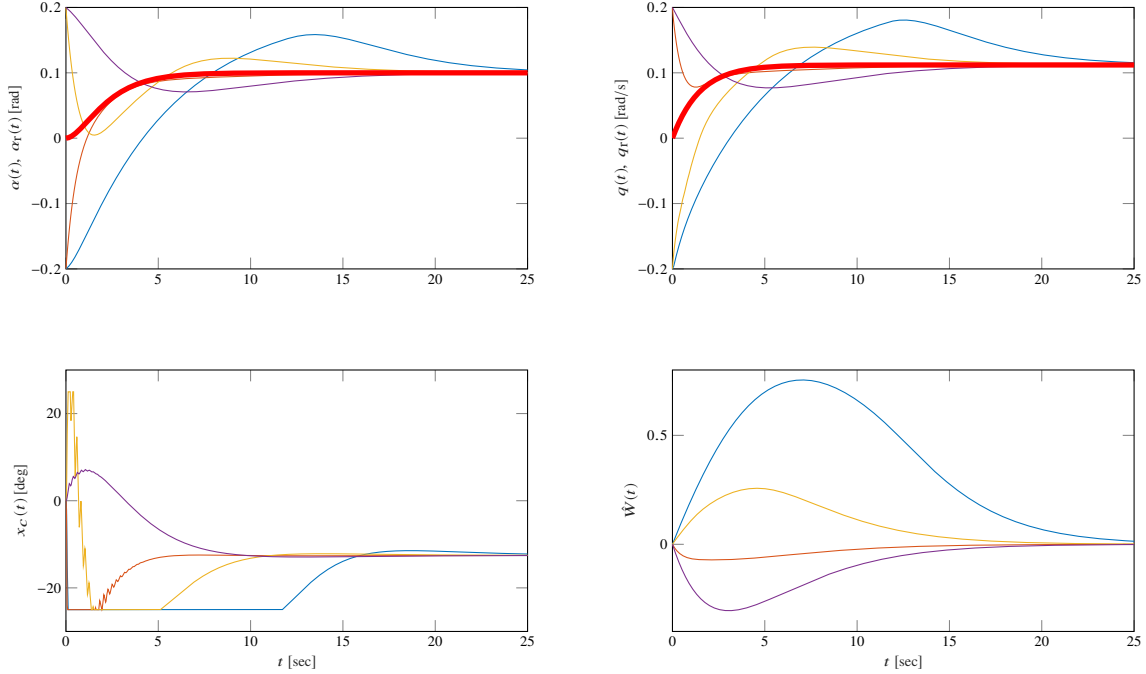


Fig. 2 F-16 Monte-Carlo ($k_{max} = 0.1, k_1 = -0.1, k_2 = 0.1$)

performance from the elevator. For future results, we wish to find new ways to partition the dynamical problem to solve even larger scale systems and use region of attraction framework to find the numerical upper bounds for the uncertain parameters.

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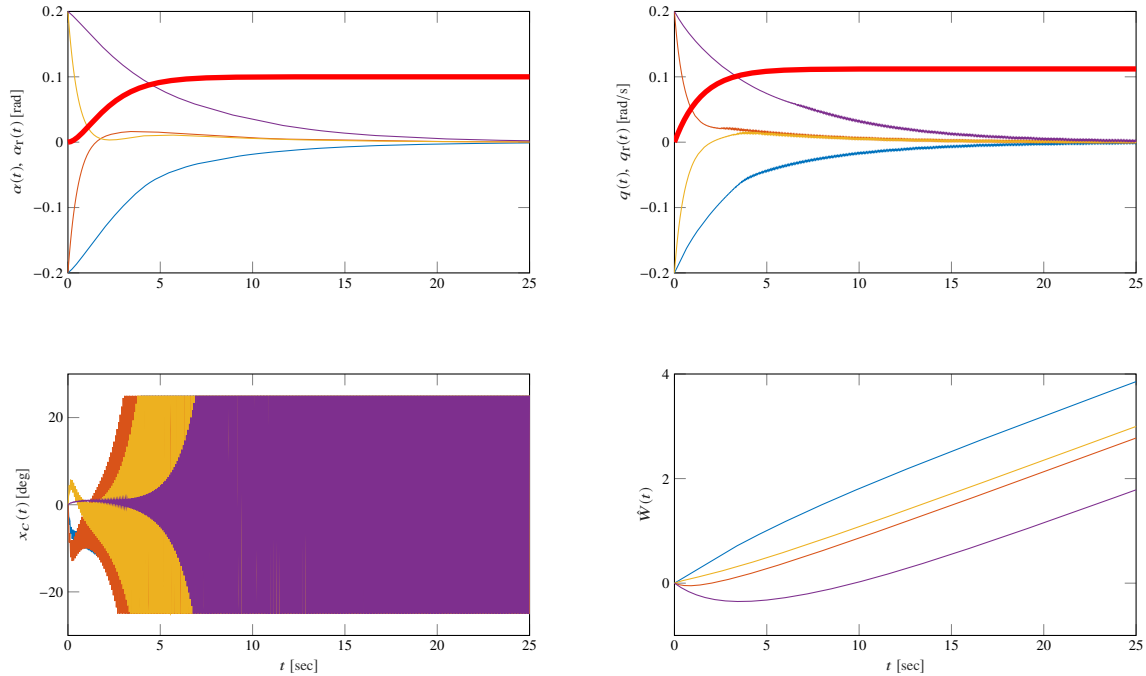


Fig. 3 F-16 Monte-Carlo ($k_{max} = 50, k_1 = -50, k_2 = 0$)

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