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JOINT FREQUENCY DOMAIN CHANNEL ESTIMATION AND EQUALIZATION BASED ON EXPECTATION PROPAGATION FOR SINGLE CARRIER TRANSMISSIONS

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ABSTRACT

In this paper, a novel category of expectation propagation (EP) based frequency domain (FD) semi-blind receivers are proposed for single-carrier block transmissions. A recently proposed EP-based framework for deriving double-loop turbo detectors is extended to handle joint data-aided channel estimation along with EP-based soft interference cancellation (IC). When addressing this problem in a message passing framework, an unconventional probability density function prevent us from establishing analytical update functions for estimating data and channel estimates. This is solved with variational inference methods, such as mean-field (MF), expectation maximization (EM) or EP, using a three-loop receiver structure with flexible performance-complexity trade-off, thanks to fast Fourier transform (FFT) based processing.

Index Terms—turbo equalization, joint channel estimation and detection, expectation propagation, message passing.

1. INTRODUCTION

Interference mitigation is a key technology for wireless receivers that need to cope with increasing throughput requirements. Channel estimation is among the foremost aspects of interference mitigation techniques, a poorly-estimated channel state information (CSI) limits these algorithms' potential. While the availability of a high number of pilot symbols for channel estimation improves the CSI quality, it also severely degrades the system's spectral efficiency. Hence, semi-blind channel estimation algorithms which also exploit data offer more attractive performance - spectral efficiency trade-off.

Conventional turbo-iterative equalizers [1], based on Gaussian-approximated belief propagation (GaBP) message-passing on factor graphs [2] use soft data estimates from the decoder feedback, to improve detection for systems with bit-interleaved coded-modulation (BICM). These soft estimates are also usable for data-aided channel estimation [3], as demonstrated for multi-carrier [4] or single-carrier systems with frequency domain equalization (SC-FDE) [5].

In recent years, more advanced approximate Bayesian inference algorithms, such as expectation propagation (EP) or mean field (MF) [6, 7], have gained significant interest. Such techniques, when formulated as message passing algorithms,

have proven themselves to be practical for addressing complex communications systems [8]. In particular, they have been used for channel impulse response (CIR) estimation in multi-carrier systems with hybrid BP-(EP)-MF frameworks [9, 10], and with an EP-only frameworks in [11, 12]. In these works, FD data symbols are discrete variables (for instance, due to the use of orthogonal frequency domain multiplexing, i.e. OFDM), which allows for low-complexity message computations, however this is not the case for SC-FDE systems, for which alternative joint estimation techniques have been investigated in [5, 13].

In this paper, the doubly-iterative low-complexity frequency domain (FD) receivers with perfect CSI in [14], are extended by applying the EP framework with temporally white message statistics for channel frequency response (CFR) estimation for SC-FDE. The technical contribution of this paper is CSI estimation with non-discrete data in the FD with EP, which requires resolving factor nodes where Gaussian variables are multiplied. This problem is addressed by resolving the multiplier node with three approaches: (a) with expectation-maximization (EM) [2], (b) hybrid EP-MF [9, 10] and (c) EP extended with the quadratic-approximation (QA) [12]. Moreover, semi-blind receivers need to handle the correlations caused by the CFR interpolation of CIR over the data block through a truncated discrete-Fourier transform (DFT). To this end, we propose thrice-iterated joint channel estimator and equalizer, using a factor graph approach.

Notations Bold lowercase letters are used for vectors: let \mathbf{u} be a $N \times 1$ vector, then $u_n, n = 1, \dots, N$ are its entries. Capital bold letters denote matrices: for a $N \times M$ matrix \mathbf{A} , $[\mathbf{A}]_{n,:}$ and $[\mathbf{A}]_{:,m}$ respectively denote its n^{th} row and m^{th} column, and $a_{n,m} = [\mathbf{A}]_{n,m}$ is the entry (n, m) . \mathbf{I}_N is the $N \times N$ identity matrix, $\mathbf{0}_{N,M}$ and $\mathbf{1}_{N,M}$ are respectively all zeros and all ones $N \times M$ matrices. $\text{Diag}(\mathbf{u})$ denotes the diagonal matrix whose diagonal is defined by \mathbf{u} . \mathcal{F}_K is the normalized K -point DFT matrix with $[\mathcal{F}_K]_{k,l} = \exp(-2j\pi(k-1)(l-1)/K)/\sqrt{K}$, and such that $\mathcal{F}_K \mathcal{F}_K^H = \mathbf{I}_K$. For random vectors \mathbf{x} and \mathbf{y} , $\boldsymbol{\mu}_{\mathbf{x}} = \mathbb{E}[\mathbf{x}]$ is the expected value, and $\boldsymbol{\Sigma}_{\mathbf{x},\mathbf{y}} = \text{Cov}[\mathbf{x}, \mathbf{y}]$ is the covariance matrix and $\boldsymbol{\Sigma}_{\mathbf{x}} = \text{Cov}[\mathbf{x}, \mathbf{x}]$. The circularly-symmetric complex Gaussian probability density function (PDF) of mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$ is $\mathcal{CN}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$. Bernoulli distribution of success probability p is $\mathcal{B}(b; p)$ and $\delta(\cdot)$ is the Dirac delta function.

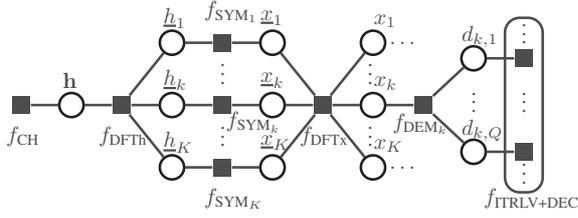


Fig. 1. Joint channel estimation and detection factor graph.

2. SYSTEM MODEL

Single-carrier transmission of a block of K data symbols is carried out with a BICM scheme. A K_b -bit information block \mathbf{b} is encoded by a rate- R_c forward-error-correction code \mathcal{C} to provide a K_d -bit codeword \mathbf{c} , which is then interleaved to the coded block \mathbf{d} with an interleaver Π . A memoryless modulator φ maps \mathbf{d} into the data block $\mathbf{x} \in \mathcal{X}^K$, with $\mathcal{X} \subset \mathbb{C}$, $|\mathcal{X}| = M$ and $Q = \log_2 M$. This symbolwise operation maps each Q -bit vector $\mathbf{d}_k \triangleq [d_{Q(k-1)+1}, \dots, d_{Qk}]$ to the symbol x_k , and we use $\varphi_q^{-1}(x_k)$ or $d_{k,q}$ to refer to $d_{(k-1)Q+q}$. \mathcal{X} is such that independently and identically distributed (IID) data symbols have a zero-mean and unit variance, i.e. $\sigma_x^2 = 1$.

The end-to-end baseband channel between the transmitter and the receiver is assumed to be a quasi-static multipath fading channel, with the CIR $\mathbf{h} = [h_1; \dots; h_L]$, and $L < K$. This model assumes a symbol-level synchronization of the receiver to the emitter and the receiver is affected by noise and extra-system interference, jointly modelled as a complex additive white Gaussian noise (AWGN) \mathbf{w} of variance σ_w^2 .

The data blocks are received with circular transmissions through schemes such as cyclic-prefixing. Baseband data observations are $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$, with \mathbf{H} being the circulant matrix with column $\mathbf{h}_D = [\mathbf{h}; \mathbf{0}_{K-L,1}]$. Using a K -point DFT, FD observations are

$$\underline{\mathbf{y}} = \underline{\mathbf{H}}\underline{\mathbf{x}} + \underline{\mathbf{w}}, \quad (1)$$

where $\underline{\mathbf{y}} = \mathcal{F}_K \mathbf{y}$, $\underline{\mathbf{H}} = \mathcal{F}_K \mathbf{H} \mathcal{F}_K^H = \text{Diag}(\sqrt{K}\underline{\mathbf{h}})$ with $\underline{\mathbf{h}} = \mathcal{F}_K \mathbf{h}_D$, $\underline{\mathbf{x}} = \mathcal{F}_K \mathbf{x}$, and $\underline{\mathbf{w}}$ remains AWGN with variance σ_w^2 .

In this system, the CIR is interpolated to K -point CFR on data observations. To model this more succinctly, we denote the truncated DFT matrix $\mathcal{F}'_K = \mathcal{F}_K[\mathbf{I}_L; \mathbf{0}_{K-L,L}]$, of size $K \times L$ such that $\underline{\mathbf{h}} = \mathcal{F}'_K \mathbf{h}$. Note that while $\mathcal{F}'_K{}^H \mathcal{F}'_K = \mathbf{I}_L$, $\mathcal{F}'_K \mathcal{F}'_K{}^H$ is a non-diagonal and a non-invertible matrix. Then, by denoting $\tilde{\mathbf{y}} = \underline{\mathbf{y}}/\sqrt{K}$ observations are rewritten as

$$\tilde{y}_k = \underline{h}_k \underline{x}_k + \tilde{w}_k, \quad k = 1, \dots, K, \quad (2)$$

where $\tilde{\mathbf{w}} \sim \mathcal{CN}(\mathbf{0}_K, \sigma_w^2)$ and $\sigma_w^2 = \sigma_w^2/K$. In the remainder of this paper, we will consider that σ_w^2 is perfectly known, and that there is a prior knowledge on the CIR, with the PDF

$$p(\mathbf{h}) = \mathcal{CN}(\mathbf{h}; \mathbf{h}_0, \sigma_{h,0}^2 \mathbf{I}_L), \quad (3)$$

which, for instance, might be provided by a pilot-aided (K_P pilots) least-squares (LS) channel estimator [3].

Table 1. Posteriors and messages of variables nodes.

Description	Notation	PDF
Posterior at SYM _k	$q_{\text{SYM}_k}(\underline{h}_k)$	$\mathcal{CN}(\underline{h}_k; \mu_{\underline{h},k}^s, \gamma_{\underline{h},k}^s)$
	$q_{\text{SYM}_k}(\underline{x}_k)$	$\mathcal{CN}(\underline{x}_k; \mu_{\underline{x},k}^s, \gamma_{\underline{x},k}^s)$
Extrinsic from SYM _k	$m_{\text{SYM}_k \rightarrow \underline{h}_k}(\underline{h}_k)$	$\mathcal{CN}(\underline{h}_k; \underline{h}_k^s, v_{\underline{h},k}^s)$
	$m_{\text{SYM}_k \rightarrow \underline{x}_k}(\underline{x}_k)$	$\mathcal{CN}(\underline{x}_k; \underline{x}_k^s, v_{\underline{x},k}^s)$
DFT _h post.	$q_{\text{DFT}_h}(\underline{\mathbf{h}})$	$\mathcal{CN}(\underline{\mathbf{h}}; \mu_{\underline{\mathbf{h}}}^f, \Gamma_{\underline{\mathbf{h}}}^f)$
DFT _h extr.	$m_{\text{DFT}_h \rightarrow \underline{\mathbf{h}}}(\underline{\mathbf{h}})$	$\mathcal{CN}(\underline{\mathbf{h}}; \underline{\mathbf{h}}^f, \mathbf{V}_{\underline{\mathbf{h}}}^f)$
Post. and ext. at CH	$q_{\text{CH}}(\mathbf{h})$	$\mathcal{CN}(\mathbf{h}; \mu_{\mathbf{h}}^c, \Gamma_{\mathbf{h}}^c)$
	$m_{\text{CH} \rightarrow \mathbf{h}}(\mathbf{h})$	$\mathcal{CN}(\mathbf{h}; \mathbf{h}^c, \mathbf{V}_{\mathbf{h}}^c)$
Posterior at DFT _x	$q_{\text{DFT}_x}(\mathbf{x})$	$\mathcal{CN}(\mathbf{x}; \mu_{\mathbf{x}}^f, \gamma_{\mathbf{x}}^f \mathbf{I}_K)$
	$q_{\text{DFT}_x}(\underline{\mathbf{x}})$	$\mathcal{CN}(\underline{\mathbf{x}}; \mu_{\underline{\mathbf{x}}}^f, \Gamma_{\underline{\mathbf{x}}}^f)$
Extrinsic from DFT _x	$m_{\text{DFT}_x \rightarrow \mathbf{x}}(\mathbf{x})$	$\mathcal{CN}(\mathbf{x}; \mathbf{x}^f, v_{\mathbf{x}}^f \mathbf{I}_K)$
	$m_{\text{DFT}_x \rightarrow \underline{\mathbf{x}}}(\underline{\mathbf{x}})$	$\mathcal{CN}(\underline{\mathbf{x}}; \underline{\mathbf{x}}^f, \mathbf{V}_{\underline{\mathbf{x}}}^f)$
Posterior at DEM _k	$q_{\text{DEM}_k}(x_k)$	$\mathcal{CN}(x_k; \mu_{x,k}^d, \gamma_{x,k}^d)$
	$q_{\text{DEM}_k}(\mathbf{d}_k)$	$\prod_q \mathcal{B}(d_{k,q}; \pi_{k,q})$
Extrinsic from DEM _k	$m_{\text{DEM}_k \rightarrow x_k}(x_k)$	$\mathcal{CN}(x_k; x_k^d, v_{x,k}^d)$
	$m_{\text{DEM}_k \rightarrow \mathbf{d}_k}(\mathbf{d}_k)$	$\prod_q \mathcal{B}(d_{k,q}; p_{k,q}^e)$
Prior to DEM _k	$m_{\mathbf{d}_k \rightarrow \text{DEM}_k}(\mathbf{d}_k)$	$\prod_q \mathcal{B}(d_{k,q}; p_{k,q}^a)$

3. TURBO RECEIVER DESIGN WITH SCALAR EP

The receiver which can optimize the packet error rate performance is given by the MAP criterion

$$\hat{\mathbf{b}} = \arg \max_{\mathbf{b}} p(\mathbf{b} | \tilde{\mathbf{y}}, \sigma_w^2). \quad (4)$$

whose resolution is often intractable or too complex, and variational Bayes methods are used to maximize this PDF.

3.1. Factor Graph Modelling with Imperfect CSI

Assuming IID equiprobable information bits, the Bayes rule yields $p(\mathbf{b} | \tilde{\mathbf{y}}, \sigma_w^2) \propto p(\tilde{\mathbf{y}} | \mathbf{b}, \sigma_w^2)$, and $p(\tilde{\mathbf{y}} | \mathbf{b}, \sigma_w^2)$ is

$$\int p(\tilde{\mathbf{y}}, \mathbf{x}, \underline{\mathbf{x}}, \mathbf{h}, \underline{\mathbf{h}}, \mathbf{d}, \mathbf{c} | \mathbf{b}, \sigma_w^2) d\mathbf{x} d\underline{\mathbf{x}} d\mathbf{h} d\underline{\mathbf{h}} d\mathbf{d} d\mathbf{c}. \quad (5)$$

The argument of this marginalization is factorized as

$$p(\tilde{\mathbf{y}}, \mathbf{x}, \underline{\mathbf{x}}, \mathbf{h}, \underline{\mathbf{h}}, \mathbf{d}, \mathbf{c} | \mathbf{b}, \sigma_w^2) = p(\mathbf{h})p(\underline{\mathbf{h}} | \mathbf{h})p(\underline{\mathbf{x}} | \mathbf{x}) p(\mathbf{d} | \mathbf{c})p(\mathbf{c} | \mathbf{b}) \prod_k p(\tilde{y}_k | \underline{x}_k, \underline{h}_k, \sigma_w^2) p(x_k | \mathbf{d}_k) \quad (6)$$

where the distributions from the system model are denominated as factor nodes (FN), with $f_{\text{CH}}(\mathbf{h}) \triangleq p(\mathbf{h})$ and $f_{\text{SYM}}(\underline{x}_k, \underline{h}_k) \triangleq p(\tilde{y}_k | \underline{x}_k, \underline{h}_k, \sigma_w^2) = \mathcal{CN}(\tilde{y}_k; \underline{h}_k \underline{x}_k, \sigma_w^2)$, where \tilde{y}_k and σ_w^2 are omitted in the FN, as they are known.

DFT and mapping constraints are modeled with $f_{\text{DFT}_x}(\underline{\mathbf{x}}, \mathbf{x}) \triangleq p(\underline{\mathbf{x}} | \mathbf{x}) = \delta(\underline{\mathbf{x}} - \mathcal{F}_K \mathbf{x})$, $f_{\text{DFT}_h}(\underline{\mathbf{h}}, \mathbf{h}) \triangleq p(\underline{\mathbf{h}} | \mathbf{h}) = \delta(\underline{\mathbf{h}} - \mathcal{F}'_K \mathbf{h})$ and $f_{\text{DEM}_k}(x_k, \mathbf{d}_k) \triangleq p(x_k | \mathbf{d}_k) = \delta(x_k - \varphi(\mathbf{d}_k))$. For interleaving and decoding we denote $f_{\text{ITRLV}}(\mathbf{d}, \mathbf{c}) \triangleq p(\mathbf{d} | \mathbf{c}) = \delta(\mathbf{d} - \Pi(\mathbf{c}))$ and $f_{\text{DEC}}(\mathbf{c}, \mathbf{b}) \triangleq p(\mathbf{c} | \mathbf{b}) = \delta(\mathbf{c} - \mathcal{C}(\mathbf{b}))$. The factor graph of this system is given in Figure 1.

3.2. EP, Variable Node Assumptions and Scheduling

For brevity, the reader is referred to [6, 14] for EP-based message passing rules. We denote *prior* and *extrinsic* messages between a variable node (VN) v_i , and a FN F respectively as $m_{v_i \rightarrow F}(v_i)$ and $m_{F \rightarrow v_i}(v_i)$. Moreover, $q_F(v_i)$ is the *approximate posterior* on v_i at F , which is obtained through the Kullback-Leibler projection of the *belief* (or the pre-projection posterior) on v_i , denoted as $\tilde{q}_F(v_i)$.

The posteriors and extrinsic messages of the considered variables nodes for the factor graph are listed in Table 1. Coded and interleaved bit $d_{k,q}$ is a Bernoulli variable, with $\mathbb{P}[d_{k,q} = 1]$ measured as a priori $p_{d_{k,q}}^a$, extrinsic $p_{d_{k,q}}^e$ and a posteriori $\pi_{d_{k,q}}$ estimates from DEM's point of view. These are respectively characterized by the log-likelihood ratios (LLRs) $L_a(d_{k,q})$, $L_e(d_{k,q})$ and $L(d_{k,q})$, such that the LLR for a success probability p is $L = \log[(1-p)/p]$. Time-domain variable nodes \mathbf{x} and \mathbf{h} lie in white Gaussian distribution, following the scalar EP framework in [14]. FD quantities $\underline{\mathbf{x}}$ and $\underline{\mathbf{h}}$ are un-correlated, but coloured Gaussians.

Furthermore, the extended framework uses a three-loop schedule, where the decoding loop (*turbo*-iterations between DEM and ITRLV+DEC) consists of a channel estimation loop (*estimation*-iterations between CH and DFTh), which in its turn includes an inner-detection loop (*self*-iteration between DFTx and DEM). The number of iterations of each loop are respectively denoted by \mathcal{T} , \mathcal{E} and \mathcal{S} . Self-iterations use the damping heuristic with exponential smoothing on moments, with $\beta = 0.6$, as detailed in [14], for the Equation (7).

3.3. Exact Message Computations

3.3.1. At factor node DEC+ITRLV

Derivation of messages between DEC and ITRLV is omitted, as these are well-known for any BICM scheme, with $L_e(d_{k,q})$ being used to update $L_a(d_{k,q})$ [15, 16].

3.3.2. At factor node DEM

DEM receives $m_{\mathbf{d} \rightarrow \text{DEM}}(\mathbf{d}_k)$ from the nodes DEC+ITRLV and $m_{\mathbf{x} \rightarrow \text{DEM}}(x_k)$ from DFTx. Its belief on VN $d_{k,q}$ follows

$$\tilde{q}_{\text{DEM}}(d_{k,q}) \propto \sum_{\beta=0}^1 \sum_{\alpha \in \mathcal{X}_q^\beta} \mathcal{D}_k(\alpha) \delta(d_{k,q} - \beta),$$

where $\mathcal{X}_q^\beta = \{\alpha \in \mathcal{X}, \varphi_q^{-1}(\alpha) = \beta\}$, $\beta \in \mathbb{F}_2$, and $\mathcal{D}_k(\alpha) \propto \mathcal{CN}(x_k^f; \alpha, v_x^f) \prod_{q=1}^Q e^{-\varphi_q^{-1}(\alpha) L_a(d_{k,q})}$. The extrinsic LLR on $d_{k,q}$ is given by $L_e(d_{k,q}) = \ln \sum_{\alpha \in \mathcal{X}_q^0} \mathcal{D}_k(\alpha) - \ln \sum_{\alpha \in \mathcal{X}_q^1} \mathcal{D}_k(\alpha) - L_a(d_{k,q})$. The belief on VN x_k is

$$\tilde{q}_{\text{DEM}}(x_k) \propto \sum_{\alpha \in \mathcal{X}} \mathcal{D}_k(\alpha) \delta(x_k - \alpha),$$

and the parameters of $q_{\text{DEM}}(x_k)$ are obtained through moment matching, with $\mu_{x,k}^d = \mathbb{E}_{\mathcal{D}_k}[x_k]$, $\gamma_x^d = K^{-1} \sum_k \text{Var}_{\mathcal{D}_k}[x_k]$. Hence, the parameters of the extrinsic message are

$$x_k^d = v_x^d (\mu_{x,k}^d / \gamma_x^d - x_k^f / v_x^f), \quad v_x^d = 1 / (1 / \gamma_x^d - 1 / v_x^f). \quad (7)$$

3.3.3. At factor node DFTx

Prior messages at DFTx are $m_{\mathbf{x} \rightarrow \text{DFTx}}(\mathbf{x})$ from DEM and $m_{\underline{\mathbf{x}} \rightarrow \text{DFTx}}(\underline{\mathbf{x}})$ from SYM. This FN's belief on VN \mathbf{x} is

$$\tilde{q}_{\text{DFTx}}(\mathbf{x}) = \mathcal{CN}(\mathcal{F}_K \mathbf{x}; \underline{\mathbf{x}}^s, \mathbf{V}_{\underline{\mathbf{x}}}^s) \mathcal{CN}(\mathbf{x}; \mathbf{x}^d, v_x^d \mathbf{I}_K),$$

i.e. a correlated Gaussian PDF. Following projection on to a white Gaussian PDF, the parameters of $q_{\text{DFTx}}(\mathbf{x})$ are

$$\begin{aligned} \boldsymbol{\mu}_{\mathbf{x}}^f &= \mathcal{F}_K^H (\mathbf{V}_{\underline{\mathbf{x}}}^s + v_x^d \mathbf{I}_K)^{-1} (\mathbf{V}_{\underline{\mathbf{x}}}^s \underline{\mathbf{x}}^d + v_x^d \underline{\mathbf{x}}^s), \\ \gamma_x^f &= v_x^d (1 - \xi_x v_x^d), \quad \xi_x \triangleq K^{-1} \sum_k (v_{\underline{\mathbf{x}},k}^s + v_x^d)^{-1}, \end{aligned} \quad (8)$$

where $\underline{\mathbf{x}}^d = \mathcal{F}_K \mathbf{x}^d$. Extrinsic message parameters are

$$\begin{aligned} \mathbf{x}^f &= \mathcal{F}_K^H \left[\underline{\mathbf{x}}^d + \xi_x^{-1} (\mathbf{V}_{\underline{\mathbf{x}}}^s + v_x^d \mathbf{I}_K)^{-1} (\underline{\mathbf{x}}^s - \underline{\mathbf{x}}^d) \right], \\ v_x^f &= \xi_x^{-1} - v_x^d. \end{aligned} \quad (9)$$

Moreover, DFTx's belief on VN $\underline{\mathbf{x}}$ is

$$\tilde{q}_{\text{DFTx}}(\underline{\mathbf{x}}) = \mathcal{CN}(\underline{\mathbf{x}}; \underline{\mathbf{x}}^s, \mathbf{V}_{\underline{\mathbf{x}}}^s) \mathcal{CN}(\mathcal{F}_K^H \underline{\mathbf{x}}; \mathbf{x}^d, v_x^d \mathbf{I}_K),$$

and thus, the approximate posterior parameters are

$$\begin{aligned} \boldsymbol{\Gamma}_{\underline{\mathbf{x}}}^f &= v_x^d \mathbf{V}_{\underline{\mathbf{x}}}^s (v_x^d \mathbf{I}_K + \mathbf{V}_{\underline{\mathbf{x}}}^s)^{-1}, \\ \boldsymbol{\mu}_{\underline{\mathbf{x}}}^f &= \mathbf{V}_{\underline{\mathbf{x}}}^s (v_x^d \mathbf{I}_K + \mathbf{V}_{\underline{\mathbf{x}}}^s)^{-1} (\underline{\mathbf{x}}^d + v_x^d \mathbf{V}_{\underline{\mathbf{x}}}^{s-1} \underline{\mathbf{x}}^s), \end{aligned} \quad (10)$$

and the resulting extrinsic message parameters are

$$\mathbf{V}_{\underline{\mathbf{x}}}^f = v_x^d \mathbf{I}_K, \quad \underline{\mathbf{x}}^f = \underline{\mathbf{x}}^d. \quad (11)$$

3.3.4. At factor node DFTh

The computations in this FN are more tedious, as $\mathcal{F}_K' \mathcal{F}_K'^H$ is non-invertible. Hence the equivalent factor graph with $\mathbf{h}_{\mathbf{D}}$ is solved, assuming that priors $p(h_{\mathbf{D},k})$, $k > l$, are Gaussians with zero means and covariances being infinitesimals.

The parameters of $q_{\text{DFTh}}(\underline{\mathbf{h}})$ are then given by

$$\begin{aligned} \boldsymbol{\mu}_{\underline{\mathbf{h}}}^f &= \underline{\mathbf{h}}^c + v_h^c \mathcal{F}_K' \mathcal{F}_K'^H \boldsymbol{\Xi}_{\mathbf{h}} (\underline{\mathbf{h}}^s - \underline{\mathbf{h}}^c), \\ \gamma_{\underline{\mathbf{h}},k}^f &= v_{\underline{\mathbf{h}},k}^s (1 - v_{\underline{\mathbf{h}},k}^s [\boldsymbol{\Xi}_{\mathbf{h}}]_{k,k}), \end{aligned} \quad (12)$$

where $\boldsymbol{\Xi}_{\mathbf{h}} = (\mathbf{V}_{\underline{\mathbf{h}}}^s + v_h^c \mathcal{F}_K' \mathcal{F}_K'^H)^{-1}$ and $\underline{\mathbf{h}}^c = \mathcal{F}_K' \mathbf{h}^c$. The corresponding extrinsic message is characterized by

$$\begin{aligned} \underline{\mathbf{h}}_k^f &= \underline{\mathbf{h}}_k^s + [\boldsymbol{\Xi}_{\mathbf{h}}]_{k,k}^{-1} \mathbf{e}_{k,k}^H \boldsymbol{\Xi}_{\mathbf{h}} (\underline{\mathbf{h}}_k^c - \underline{\mathbf{h}}_k^s), \\ v_{\underline{\mathbf{h}},k}^f &= [\boldsymbol{\Xi}_{\mathbf{h}}]_{k,k}^{-1} - v_{\underline{\mathbf{h}},k}^s. \end{aligned} \quad (13)$$

The computation of $\boldsymbol{\Xi}_{\mathbf{h}}$ is obtained with L iterations of Sherman-Morrison formula on matrix inversion [17].

3.3.5. At factor node SYM

The belief of SYM on joint variables $\underline{\mathbf{x}}_k$ and $\underline{\mathbf{h}}_k$ is

$$\begin{aligned} \tilde{q}_{\text{SYM}}(\underline{\mathbf{x}}_k, \underline{\mathbf{h}}_k) &= \mathcal{CN}(\tilde{\mathbf{y}}_k; \underline{\mathbf{h}}_k \underline{\mathbf{x}}_k, \sigma_w^2) \\ &\quad \mathcal{CN}(\underline{\mathbf{x}}_k; \underline{\mathbf{x}}_k^f, v_{\underline{\mathbf{x}},k}^f) \mathcal{CN}(\underline{\mathbf{h}}_k; \underline{\mathbf{h}}_k^f, v_{\underline{\mathbf{h}},k}^f), \end{aligned} \quad (14)$$

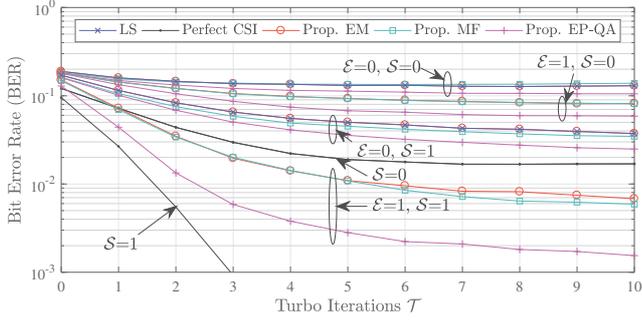


Fig. 2. Bit error rate (BER) performance versus turbo-iterations for $E_s/N_0 = \sigma_x^2/\sigma_w^2 = 16$ dB.

which involves a multiplier node [2], whose resolution is non-trivial. Indeed, the belief of SYM on \underline{x}_k is

$$\tilde{q}_{\text{SYM}}(\underline{x}_k) = \frac{\mathcal{CN}(\underline{x}_k; \underline{y}_k^f, v_{\underline{x},k}^f)}{\pi(\sigma_w^2 + v_{\underline{h},k}^f |\underline{x}_k|^2)} \exp\left(-\frac{|\tilde{y}_k - \underline{h}_k^f \underline{x}_k|^2}{\sigma_w^2 + v_{\underline{h},k}^f |\underline{x}_k|^2}\right), \quad (15)$$

and the belief on \underline{h}_k is obtained by symmetry. Moments of $\tilde{q}_{\text{SYM}}(\underline{x}_k)$ and $\tilde{q}_{\text{SYM}}(\underline{h}_k)$ cannot be analytically computed and approximations are needed to solve the message passing.

4. PROPOSED PRACTICAL RECEIVERS

4.1. Joint Estimation and Detection with MF and EM

The MF approach [10] estimates both \underline{x}_k and \underline{h}_k with a posteriori point-estimates, as follows

$$\underline{x}_k^s = \frac{\mu_{\underline{h},k}^s * \tilde{y}_k}{|\mu_{\underline{h},k}^s|^2 + \gamma_{\underline{h},k}^s}, \quad v_{\underline{x},k}^s = \frac{\sigma_w^2}{|\mu_{\underline{h},k}^s|^2 + \gamma_{\underline{h},k}^s}, \quad (16)$$

where $\mu_{\underline{h},k}^s$ and $\gamma_{\underline{h},k}^s$ are posterior statistics of \underline{h}_k , with

$$\mu_{\underline{h},k}^s = \frac{v_{\underline{h},k}^s \underline{h}_k^f + v_{\underline{h},k}^f \underline{h}_k^s}{v_{\underline{h},k}^s + v_{\underline{h},k}^f}, \quad \gamma_{\underline{h},k}^s = \frac{v_{\underline{h},k}^s v_{\underline{h},k}^f}{v_{\underline{h},k}^s + v_{\underline{h},k}^f}, \quad (17)$$

\underline{h}_k^s and $v_{\underline{h},k}^s$ are obtained by symmetry, with $\mu_{\underline{x},k}^s$ and $\gamma_{\underline{x},k}^s$.

The EM approach [2] simplifies the data estimates, by neglecting CSI estimates' reliability, i.e. $\gamma_{\underline{h},k}^s = 0$.

4.2. Joint Estimation and Detection with EP-QA

The final alternative we consider for computing messages at the FN SYM is the QA method proposed in [12], which consists in computing the second-order local approximation of the argument of the exponential in Eq. (15), around a point-estimate $m_{\underline{x},k}$ of the mean of \underline{x}_k . As a result, we have

$$\tilde{q}_{\text{SYM}}(\underline{x}_k) \approx \mathcal{CN}(\underline{x}_k; \underline{y}_k^f, v_{\underline{x},k}^f) \mathcal{CN}(\underline{x}_k; \underline{x}_k^s, v_{\underline{x},k}^s), \quad (18)$$

where the approximated Gaussian component's statistics are

$$\underline{x}_k^s = \frac{\underline{h}_k^f * \tilde{y}_k}{|\underline{h}_k^f|^2 + v_{\underline{h},k}^f \chi_{\underline{x},k}}, \quad v_{\underline{x},k}^s = \frac{\sigma_w^2 + v_{\underline{h},k}^f |m_{\underline{x},k}|^2}{|\underline{h}_k^f|^2 + v_{\underline{h},k}^f \chi_{\underline{x},k}}, \quad (19)$$

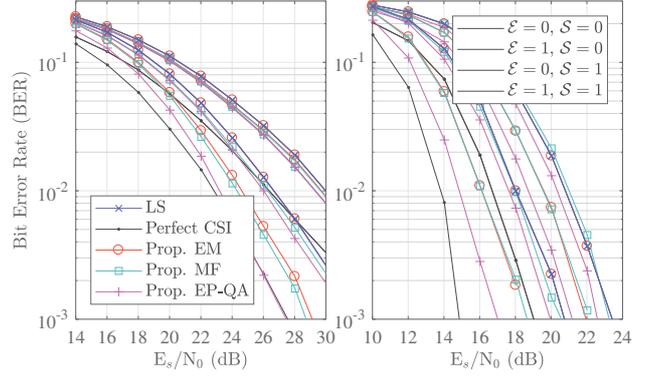


Fig. 3. BER vs SNR for $\mathcal{T} = 0$ (left) and $\mathcal{T} = 5$ (right).

where $\chi_{\underline{x},k} = 1 - |\tilde{y}_k - \underline{h}_k^f m_{\underline{x},k}|^2 / (\sigma_w^2 + v_{\underline{h},k}^f |m_{\underline{x},k}|^2)$. The selection of the hyper-parameter $m_{\underline{x},k}$ has an impact on the convergence speed of the algorithm. The message parameters \underline{h}_k^s and $v_{\underline{h},k}^s$ in SYM node are obtained by symmetry.

The selection of $m_{\underline{x},k}$ and $m_{\underline{h},k}$ for local quadratic approximations, is handled by focusing on their impact on the extrinsic messages of SYM. We have selected $m_{\underline{x},k} = \underline{x}_k^f$ to ensure statistical consistency in the computation of ξ_x (and similarly $m_{\underline{h},k} = \underline{h}_k^f$ for stabilizing Ξ_h). Moreover, to avoid degrading the extrinsics when the priors are poorly known, $\chi_{\underline{x},k}$ is clipped to be positive with $\chi_{\underline{x},k} = \max(0, \chi_{\underline{x},k})$.

4.3. Numerical Results

We consider a SC-FDE transmissions with 8-PSK coded by a convolutional code of polynomials [5, 7]₈, $K = 128$, over the Proakis C channel ($\mathbf{h} = [1; 2; 3; 2; 1]/\sqrt{19}$). Prior channel estimates are obtained through a pilot-aided LS estimation, with $K_P = 10$, and we compare the performance of the three variants of the EP-based receiver (EM, MF, EP-QA), with mismatched CSI ($p(\mathbf{h}) = \delta(\mathbf{h} - \mathbf{h}_0)$) and with a perfect CSI receiver. In the Fig. 2 the impact of turbo-iterations \mathcal{T} is given, and while all three proposed algorithms significantly benefit from self-iterations, EP-QA achieves lower BER, and faster convergence. In the Fig. 3 the decoding capabilities are given. Among joint receivers ($\mathcal{E} = 1$), proposed EP-QA approach has over 1 dB gain over the proposed ones based on EM and MF, for $\mathcal{S} = 1$. Moreover, joint estimation brings up to 3 dB gain over pilot-only CSI ($\mathcal{E} = 0$, i.e. [14]), when comparing receivers with the same \mathcal{T} and \mathcal{S} .

5. CONCLUSIONS AND PERSPECTIVES

In conclusion, self-iterations significantly improve data-based channel estimation, and EP-based inference yields receivers with flexible options for scheduling. EP-QA approach for joint channel-estimation also appears to be promising for SC-FDE systems. In future works, the complexity - spectral-efficiency trade-off of these structures has to be explored. Moreover it needs to be extended to multi-block CSI estimation for comparison with alternative approaches [5, 13].

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