



**HAL**  
open science

# Adaptive Approximation Control of Robotic Manipulators: Centralized and Decentralized Control Algorithms

Hayder Al-Shuka, Yibin Li, Rui Song

► **To cite this version:**

Hayder Al-Shuka, Yibin Li, Rui Song. Adaptive Approximation Control of Robotic Manipulators: Centralized and Decentralized Control Algorithms. [Research Report] School of Control Science and Engineering, Shandong University. 2020. <hal-02942621>

**HAL Id: hal-02942621**

**<https://hal.science/hal-02942621v1>**

Submitted on 18 Sep 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Shandong University  
School of Control Science and Engineering

# **Adaptive Approximation Control of Robotic Manipulators: Centralized and Decentralized Control Algorithms**

A Postdoctoral report

Hayder Al-Shuka  
Jan. 2019

Supervisors:

Prof. Dr. Yibin Li  
Ass. Prof. Dr. Rui Song

## Abstract

The regressor-based adaptive control is useful for controlling robotic systems with uncertain parameters but with known structure of robot dynamics. Unmodeled dynamics could lead to instability problems unless modification of control law is used. In addition, exact calculation of regressor for robots with more than 6 degrees of freedom is hard to be calculated, and the task could be more complex for robots. Whereas the adaptive approximation control is a powerful tool for controlling robotic systems with unmodeled dynamics. The local (partitioned) approximation-based adaptive control includes representation of the uncertain matrices and vectors in the robot model as finite combinations of basis functions. Update laws for the weighting matrices are obtained by the Lyapunov-like design. Therefore, this work is focused function approximation-based control algorithms considering centralized and decentralized approaches.

In this work, the following control algorithms are designed:

- (1) Adaptive hybrid regressor-approximation control. This work attempts to combine the features of both the regressor and the approximation techniques in adaptive control. The regressor technique is a powerful tool for adaptive control of the known structure of modeling while the approximation is useful for estimation of time-varying uncertainty. Therefore, this work proposes adaptive hybrid regressor and approximation control for robots in both free and constrained spaces. The control law consists of three terms: (i) regressor term for initial estimation of the known structure of the robot dynamics, e.g. inertia matrix, Coriolis and centripetal matrix and gravity vector, and (ii) approximation term for estimation of internal and external disturbances resulted from the inexact calculation of regressor matrix and unknown modeling of friction, etc, and (iii) robust term consists of switching  $sgn(\cdot)$  function. The control law is designed based on updating the uncertain parameters and the weighting coefficients corresponding to regressor and approximation respectively with position/force tracking purposes. The proposed controller is stable in the sense of Lyapunov stability.
- (2) Decentralized adaptive partitioned approximation control. Partitioned approximation control is avoided in most decentralized control algorithms; however, it is essential to design feedforward control with improved tracking accuracy. As a result, this work is focused on decentralized adaptive partitioned approximation control for complex robotic systems using the orthogonal basis functions as strong approximators. In essence, the partitioned approximation technique is intrinsically decentralized with some modifications. The proposed decentralized control law consists of three terms: the partitioned approximation-based feedforward term that is necessary for precise tracking, the high gain-based feedback term, and the adaptive sliding gain-based term for compensation of modeling error. The passivity property is essential to prove the stability of local stability of the individual subsystem with guaranteed global stability.

Simulation experiments on 2-link robot and 6-link biped robot are performed to prove the effectiveness of the proposed algorithms.

## Chapter 1: Introduction

In effect, there are two essential control techniques dealing with uncertain plants: (1) robust control, and (2) adaptive control [1]. The robust control deals with bounded uncertainty and is designed to stabilize the target system for any uncertainty within the assumed bounds. In contrast, adaptive control attempts to estimate the uncertain parameters, and to design a control law based on the estimated parameters [1, 2]. In general, adaptive control can be viewed as being composed of two parts [3]:

- An identification portion, which identifies parameters of the plant.
- A control law portion, which implements a control law that is in some way a function of the parameters identified.

Most researchers adopt the regressor approach as a basis to design their adaptive control law [2-6]. In regressor technique, the equation of motion is linearly parameterized in terms of the regressor matrix (which is a function of the state variables of the robot) and the uncertain parameter vector. It should be pointed out that the regressor approach may pose difficulty in practical implementation due to the computational complexity associated with regressor calculation [7]. In general, it has the following characteristics:

- Derivation of regressor matrix is not unique.
- It embodies the full dynamics of the target system. It is complex for high DoF (more than six) robotic system. The derivation of regressor matrix is boring and it is subjected to calculation errors.
- In real time implementation, the regressor matrix must be recomputed in every control cycle because it depends on state variables of the investigated robot.
- The property (linear-in-the-parameter) means that each individual robot has its own regression matrix.
- The uncertain parameters are time invariant.

In light of above, there are some inherent limitations associated with regressor implementation. Some modifications of regressor-based control law can be implemented to deal with uncertain modeling such as  $\sigma$ -modification, etc. However, these robust adaptive laws do not guarantee zero position errors and some bounded error could be produced [1,2]. Therefore, much attention has been focused on evading the regressor computations with miscellaneous techniques [7-14].

The function approximation technique (FAT) is an essential tool for approximating the uncertain parameters of the dynamic system without using regressor matrix. It has been used successfully for low dimensional robotic systems without considering the computational complexity of the proposed algorithms for larger dimension systems [15-17]. In general, the adaptive approximation control has the following characteristics:

- It can carefully deal with time-variant parameters and unmodeled dynamics of the investigated dynamic system, e.g. hysteresis and friction phenomena [19].
- Due to approximation technique, the accumulated approximation errors should be treated carefully [16,20], e.g. Ahmadi and Nourisola [31] have proposed  $H_\infty$  controller for compensation of approximation errors and the produced disturbances.
- No optimal approximator is suggested for now. Beuschel [18] has compared the approximation performance of radial basis functions, generalized basis functions, and fuzzy control. The author proposed that the generalized basis function may rather give better approximation than the others. Whereas some researchers may prefer Support Vector Machines (SVM) as powerful approximators due to their strong approximation performance [20].
- It could be difficult to apply the local approximation technique for high DoF robots due to the curse of dimensionality associated with the approximation of dynamic matrices. The researchers tried to use neural network (NN) approximation which has parallel property to solve this problem, or they resort to global approximation-based control. In effect, the aim of this work is to treat the problem of the curse of dimensionality associated with local approximation; the details will be shown later.

Therefore, this work is focused on adaptive approximation techniques considering centralized and decentralized control algorithms.

The report is organized as follows. Chapters 2 and 3 propose adaptive hybrid regressor-approximation control algorithms for robots in free and constrained spaces respectively. Whereas, chapter 4 introduces the decentralized adaptive partitioned approximation control without considering actuator dynamics. Chapter 5 presents the decentralized adaptive partitioned approximation control considering three actuator control modes.

The following publications have been made during my work in Shandong University:

- [1] Hayder FN Al-Shuka, Rui Song. Decentralized Adaptive Partitioned Approximation Control of Robotic Manipulators. Chapter in book: ROMANSY 22–Robot Design, Dynamics and Control, pp. 10-17 (2019). [https://link.springer.com/chapter/10.1007/978-3-319-78963-7\\_3](https://link.springer.com/chapter/10.1007/978-3-319-78963-7_3)
- [2] Hayder FN Al-Shuka, R Song. Decentralized adaptive partitioned approximation control of high degrees-of-freedom robotic manipulators considering three actuator control modes. International Journal of Dynamics and Control, Springer, pp. 1-14 (2018). <https://link.springer.com/article/10.1007%2Fs40435-018-0482-3>

- [3] Hayder FN Al-Shuka, Steffen Leonhardt, Wen-Hong Zhu, Rui Song, Chao Ding, Yibin Li. Active impedance control of bioinspired-motion robotic manipulators: an overview. *Applied Bionics and Biomechanics*, pp. 1-19 (2018).<https://www.hindawi.com/journals/abb/2018/8203054/>
- [4] Hayder FN Al-Shuka, Rui Song, Chao Ding. On high-level control of power-augmentation lower extremity exoskeletons: Human walking intention. 2018 Tenth International Conference on Advanced Computational Intelligence (ICACI), pp. 169-174 (2018).<https://ieeexplore.ieee.org/abstract/document/8377601/>
- [5] Hayder FN Al-Shuka, Rui Song. On low-level control strategies of lower extremity exoskeletons with power augmentation. 2018 Tenth International Conference on Advanced Computational Intelligence (ICACI), pp. 63-68 (2018).<https://ieeexplore.ieee.org/abstract/document/8377581>
- [6] Hayder Al-Shuka, Rui Song. Hybrid Regressor and Approximation-Based Adaptive Control of Robotic Manipulators with Contact-Free Motion. 2018 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference(IMCEC 2018), pp. 325-329 (2018)<https://ieeexplore.ieee.org/abstract/document/8469628>
- [7] Hayder Al-Shuka, Rui Song. Hybrid Regressor and Approximation-Based Adaptive Control of Piezoelectric Flexible Beams. 2018 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference(IMCEC 2018), pp. 330-334 (2018).<https://ieeexplore.ieee.org/abstract/document/8469279>
- [8] Hayder F.N. Al-Shuka, R. Song. Adaptive Hybrid Regressor and Approximation Control of Robotic Manipulators in Constrained Space. *International Journal of Mechanical and Mechatronics Engineering*, vol. 17, No. 3, pp. 11-21 (2017).<https://pdfs.semanticscholar.org/6183/52ed1ffa4ed57bb0ec760c538d164cb7ad0a.pdf>

## References

1. Farrell JA, Polycarpou MM (2006) Adaptive approximation based control: Unifying neural, fuzzy and traditional adaptive approximation approach. USA: John Wiley & Sons Inc
2. Loannou P, Fidan B (2006) Adaptive control tutorial. Philadelphia: SIAM
3. Craig JJ (1988) Adaptive control of mechanical manipulators. USA: Addison-Wesley Publishing company Inc
4. Spong MW, Vidyasagar M (1989) Robot dynamics and control. USA: John Wiley & Sons
5. Slotine JJE, Li W (1991) Applied nonlinear control. USA: Prentice-Hall, Inc
6. Zhu W-H (2010) Virtual Decomposition control: toward hyper degrees of freedom robots. Berlin; Springer
7. Song YD (1994) Adaptive motion tracking control of robot manipulators-Non-regressor based approach. Proceeding of IEEE International Conference on Robotics and Automation 4:3008-3013
8. Lu WS, Meng QH (1991a) Recursive computation of manipulator regressor and its application to adaptive motion control of robot. IEEE Conference on Communication, Computation and Signal Processing: 170-173
9. Kawasaki H, Bito T, Kanzaki K (1996) An efficient algorithm for model-based adaptive controller of robot manipulators. IEEE Transactions on Robotics and Automation 12(3): 496-501
10. Yang JH (1999) Adaptive tracking control for manipulators with only position feedback. IEEE Canadian Conference on Electrical and Computer Engineering: 1740-1745
11. Qu Z, Dorsey J (1991) Robust tracking control of robots by a linear feedback law. IEEE transactions on Automatic Control 36(9): 1081-1084
12. Park JS, Jiang YA, Hesketh T, Clements DJ (1994) Trajectory control of manipulators using adaptive sliding mode control. Proceedings of IEEE, Southeastcon: 142-146
13. Yuan J, Stepanenko Y (1993) Adaptive PD control of flexible joint robots without using the high-order regressor. Proceedings of the 36<sup>th</sup> Midwest Symposium on Circuits and Systems: 389-393
14. Su C -Y, Stepanenko Y (1996) Adaptive control for constrained robots without using regressor. IEEE International Conference on Robotics and Automation: 264-269
15. Huang A -C, Chien M -C (2010) Adaptive control of robot manipulators: A unified Regressor-Free approach. Singapore: World Scientific Publishing Co. Pte. Ltd
16. Cong S, Liang Y, Shang W (2008) Function approximation-based sliding mode adaptive control for time-varying uncertain nonlinear systems. *Nonlinear Dynamics* 54(5): 223-230
17. Liang W, Chen H -Y, Tsu Y -T (2009) FAT-based adaptive sliding mode control for a piezoelectric-actuated system. IEEE International Conference on Control and Automation: 848-853
18. Beuschel M (2000) Nonlinear function approximators. In: Intelligent observer and control design for nonlinear systems (D. Schroeder ed.). Berlin: Springer-Verlag
19. Ci C, Zhi L, Yun Z (2015) Fuzzy approximation based adaptive control for multiple robotic arms with input hysteresis nonlinearities. Proceedings of the 34th Chinese Control Conference, China: 5948-5952
20. Ahmadi B, Nourisola H (2015) Robust adaptive  $H_\infty$  controller based on hybrid genetic wavelet kernel principal component or nonlinear uncertain systems. *Majlesi Journal of Electrical Engineering* 9(4):13-21

## Chapter 2: Adaptive hybrid regressor-approximation control of robots in free space: centralized control algorithm

This chapter proposes a hybrid adaptive controller based on both regressor and approximation techniques. The equation of motion of the target manipulator is separated into two terms: (i) linear parameterization of the known modeling of the robot dynamics (inertia, Coriolis and centrifugal matrices and gravity vector) by using regressor technique, and (ii) approximation of disturbance terms (e.g. unmodeled dynamics of friction) in terms of weighting and orthogonal basis function matrices. The idea behind the work is an estimation of unmodeled dynamics, which could be hard modeled with conventional regressor unless some modifications of control law are used. Regressor works well only when it is correct. In contrary, approximation approaches provide additional robustness. The proposed Lyapunov's stability-based control law consists of four terms: a regressor term, an approximation term, a high-gain feedback term, and a robust sliding term for compensation of modeling errors. The efficacy of the proposed controller is proved via simulation of 2-R robots.

### 2.1 Introduction

In general, there are two well-known techniques that deal with uncertain plants: robust control and adaptive control. The former attempts to stabilize the uncertain plants within assumed bounds, whereas adaptive control improves the stabilization performance by updating the estimated parameters [1-3].

Since most robotic manipulators are highly nonlinear dynamic systems with strong couplings, fixed-parameter control for precise tracking is hard to be guaranteed. Cancellation of unknown nonlinear terms from the robot dynamics can result in instability with a fixed-parameter controller. Literature proves that there are two common techniques for adaptive control of robotic manipulators: regressor-based control and approximation-based control. The former includes linear parameterization of an equation of motion in terms of regressor matrix and uncertain parameters vector. The regressor matrix is highly nonlinear because it is a function of joint position, velocity, and acceleration. There are some limits inherent in the regressor matrix such as:

- Derivation of regressor matrix is not unique, though the process is standardized.
- It represents the full dynamics of the investigated robotic systems; it is complex for high DoF (more than six) robotic system; however, Virtual Decomposition Control (VDC) simplifies this problem [4].
- In practice, the regressor matrix must be recalculated in every sampling cycle.
- Each individual robot has its own regression matrix [5-7].

Therefore, many researchers have attempted to avoid regressor computations with miscellaneous techniques. For more detail, the reader is referred to [8-15].

On the other hand, the function approximation technique is an alternative tool for adaptive control. The idea behind it is also linear parameterization of the target matrix/vector (decomposition of the target matrix/vector into two matrices): the weighting matrix and the (orthogonal) basis function matrix [16-19]. In general, there are two techniques for approximation-based adaptive control: local (partitioned) approximation and global (augmented) approximation. The former attempts to approximate each dynamic matrix/vector (mass, Coriolis matrices, gravity vector etc.) separately, whereas the global approximation includes approximation of the whole robot dynamics (the left-hand side of the equation of motion) by using one term of weighting and basis function matrices. In general, features and limitations of approximation-based adaptive control are described as follows.

- It can be used as a unified module for different robotic systems. There is no specific criterion for selection of the best approximator.
- If we apply local approximation-based adaptive control for high DoF robotic manipulators, difficulty can be encountered associated with the computational complexity of weighting and orthogonal matrices. In general, the computational complexity grows exponentially with a dimension of the target dynamic system.
- The adaptive law includes updating the weighting matrix of the target matrix. Estimation of initial values of the weighting matrix may not be easy, whereas, this is not the case for regressor approach.
- Due to the approximation inherent in this method, an approximation error can be produced. As a result, a special strategy is required for compensation of the modeling error, see [16] for more details.
- For local approximation-based adaptive control, more adaptation gains can be produced while there is only one adaptation gain for regressor matrix. There is no specific strategy for tuning these adaptation and feedback gains; instead, a try-and-error strategy is used.

One of the important disturbances that should carefully be dealt is friction. Although friction is useful for the motion of mobile robots and control of pneumatic/hydraulic systems, it has hard disadvantages for instability of dynamic systems with low velocity. For low velocity, friction can lead to undesirable steady-state error [20]. Using regressor-based adaptive control for friction compensation requires exact modeling of friction; however, there is no exact mathematical formula for friction terms, for more details on friction phenomenon see [20,21] and the references therein. The idea behind the regressor is to estimate some unknown coefficients of friction terms (e.g. Coulomb coefficient, viscous damping coefficient etc.) based on the known

structure of the system with unknown parameters [4]. Canceling some terms of friction may lead to deviations of desired tracks. Sure, the strategy of bounded control can be integrated with regressor for compensation of friction disturbances [1,4]; most robust adaptive control strategies can be used for control purpose in the presence of unknown modeling. However, these strategies can be accompanied by saturation problems or some problems for asymptotical stability [1].

On the other hand, approximation-based adaptive control can estimate the disturbance term integrated with motion equation easily without the complication of regressor strategy [25]; therefore, this work proposes a new controller based on a combination of regressor matrix and approximation strategies. The idea is to estimate online the dynamic coefficients (mass and Coriolis matrices and gravity vector) using the regressor matrix strategy, while the unknown friction terms are estimated using the approximation technique. An adaptive control law based on both regressor and approximation techniques is proposed with updating adaptive laws for both uncertain vector and weighting matrix corresponding to dynamic matrices/vector and unknown friction terms respectively. The modeling error is compensated using a robust sliding mode gain. Two-R manipulator in free contact motion is simulated for validation of results.

The remainder of this work can be organized as follows. Section 2.2 describes the formulation of the investigated problem. Section 2.3 introduces the proposed controller. Section 2.4 shows simulation results and discussion. Section 5 concludes.

## 2.2 Problem Formulation

To illustrate the core of the work, consider the following second-order scalar system for one robot joint (actuator with load)

$$I\ddot{q} + \tau_f = \tau \quad (2.1)$$

$$\tau_f = k_c \text{sign}(\dot{q}) \quad (2.2)$$

where  $I$  is the effective moment of inertia,  $\ddot{q}$  is the angular acceleration of the joint,  $\tau_f$  represents the friction torque at the joint,  $k_c$  is the coefficient of viscous Coulomb friction, and  $\tau$  is the control torque.

If some or all parameters of (2.1) and (2.2) are unknown, the adaptive control can be used for tracking of the desired trajectory of the joint motion. Classical adaptive control uses the regressor matrix (or vector for scalar systems) to estimate the unknown parameters. Therefore, depending on the regressor-based adaptive control, the following control law can be suggested

$$\tau = \mathbf{y}\hat{\mathbf{p}} - Ks \quad (2.3a)$$

with

$$\mathbf{y} = [\dot{r} \quad \text{sgn}(r)]$$

$$\hat{\mathbf{p}} = \begin{bmatrix} \hat{I} \\ \hat{k}_c \end{bmatrix} \quad (2.3b)$$

$$r = \dot{q}_d - \Lambda e \quad (2.3c)$$

$$s = \dot{q} - r = \dot{e} + \Lambda e \quad (2.3d)$$

$$e = q - q_d \quad (2.3e)$$

where  $\mathbf{y}$  is the regressor vector,  $\hat{\mathbf{p}}$  is the uncertain parameter vector, the symbol  $(\hat{\cdot})$  refers to the uncertainty of parameters,  $r$  is the reference velocity,  $e$  is the tracking error of the angular joint displacement,  $\dot{q}_d$  represents the reference angular velocity of the joint,  $s$  refers to virtual error velocity,  $K$  and  $\Lambda$  are feedback gains.

As we see, the friction term  $\tau_f$  should correctly be modeled for estimation of the unknown coefficients of friction terms and for precise tracking. Sure, there are some solutions for this problem with some modifications of the updating adaptive law, but the approximation technique is more flexible and easier than these techniques. Therefore, the core of this work is to use regressor approach to estimate the unknown parameters which are connected to the inertia, Coriolis matrices, and gravity vector ((2.1) includes only inertia term), while the friction term is estimated based on approximation technique, i.e.

$$\tau = \underbrace{\mathbf{y}(q, \dot{q}, r, \dot{r})}_{\text{regressor term}} \hat{\mathbf{p}} + \underbrace{\hat{\mathbf{w}}^T \boldsymbol{\phi}}_{\text{approximation term}} - Ks \quad (2.4a)$$

$$\tau_f = \hat{\mathbf{w}}^T \boldsymbol{\phi} + \varepsilon \quad (2.4b)$$

where  $\mathbf{y} = \dot{r}$ ,  $\hat{\mathbf{p}} = \hat{I}$ ,  $\mathbf{w} \in R^{\beta \times 1}$  is the weighting vector,  $\boldsymbol{\phi} \in R^{\beta \times 1}$  refers to the orthogonal polynomial function vector,  $\varepsilon$  denotes to the modeling error, and  $\beta$  is the number of approximation terms.

## 2.3 Methodology

### 2.3.1 Dynamics

The Euler-Lagrange equation of motion for a robotic manipulator in free space can be expressed as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f + \boldsymbol{\tau}_d = \boldsymbol{\tau} \quad (2.5)$$

where  $\mathbf{M} \in R^{n \times n}$  is the inertia matrix,  $n$  refers to the number of degrees of freedom (DoFs),  $\mathbf{q} \in R^n$  is the angular displacement vector of the robot joints,  $\mathbf{C} \in R^{n \times n}$  represents the Coriolis and centripetal robot matrix,  $\mathbf{g} \in R^n$  is the gravity vector,  $\boldsymbol{\tau} \in R^n$  is the actuating torque vector,  $\boldsymbol{\tau}_f \in R^n$  represents the dissipative torques resulted from joint friction, and  $\boldsymbol{\tau}_d \in R^n$  represents any additional disturbance.

As stated previously, the first three terms on the left hand side of (2.5) can be rewritten in a linearly parameterized form of the regressor matrix  $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in R^{n \times l}$  ( $l$  refers to the number of uncertain parameters) and the uncertain parameter vector  $\hat{\mathbf{p}} \in R^l$ , while both the friction and disturbance terms are approximated in terms of weighting matrix  $\mathbf{W} \in R^{n \times \beta}$  and basis function vector  $\boldsymbol{\varphi} \in R^\beta$ .

The following properties assumptions are necessary assuming robotic manipulators with revolute joints [23].

**Property 1.** The inertia matrix, Coriolis and centrifugal matrix, and the gravity vectors are uniformly bounded.

**Property 2.** The matrix  $\dot{\mathbf{M}} - 2\mathbf{C}$  is a skew-symmetric matrix, i.e.,  $\mathbf{s}^T(\dot{\mathbf{M}} - 2\mathbf{C})\mathbf{s} = 0$ , if  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is defined using the Christoffel symbols.

**Property 3.** The left-hand side of (2.5) can linearly be parameterized in terms of state variables-based matrix and the constant physical parameters vector.

**Assumption 1.** The robotic manipulator is actuated with direct drive (the gear ratio is equal to one).

**Assumption 2.** The states of the robotic manipulator are bounded and measurable.

### 2.3.2 The Proposed Control Law

There are different controller structures for dealing with electro-mechanical systems with uncertain parameters. In this work, we will exploit the control strategy coined by [22]. Accordingly, the suggested controller law can be expressed as

$$\boldsymbol{\tau} = \underbrace{\hat{\mathbf{M}}(\mathbf{q})\dot{\mathbf{r}} + \hat{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{r} + \hat{\mathbf{g}}(\mathbf{q})}_{\text{regressor term}} + \underbrace{\hat{\boldsymbol{\tau}}_f + \hat{\boldsymbol{\tau}}_d}_{\text{approximation term}} - \mathbf{K}\mathbf{s} - \boldsymbol{\tau}_r \quad (2.6)$$

where  $\mathbf{K} \in R^{n \times n}$  is a high gain feedback diagonal matrix with positive definite properties,  $\boldsymbol{\tau}_r = \delta[\text{sgn}(s_1) \ \cdots \ \text{sgn}(s_n)]^T$ , with  $\delta$  representing a positive number that should be greater than the modeling error  $\|\boldsymbol{\varepsilon}\|$ , and  $\mathbf{r} \in R^n$  is a reference velocity computed as

$$\mathbf{r} = \dot{\mathbf{q}}_d - \boldsymbol{\Lambda}\mathbf{e} \quad (2.7a)$$

$$\mathbf{s} = \dot{\mathbf{q}} - \mathbf{r} = \dot{\mathbf{e}} + \boldsymbol{\Lambda}\mathbf{e} \quad (2.7b)$$

$$\mathbf{e} = \mathbf{q} - \mathbf{q}_d \quad (2.7c)$$

where  $\boldsymbol{\Lambda} \in R^{n \times n}$  is a diagonal positive definite matrix.

Equation (2.6) can be rewritten as

$$\boldsymbol{\tau} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}})\hat{\mathbf{p}} + \hat{\mathbf{W}}^T \boldsymbol{\varphi} - \mathbf{K}\mathbf{s} - \boldsymbol{\tau}_r \quad (2.8)$$

Subtracting (2.6) from (2.5) and using (2.7) to get the following closed loop control system

$$\mathbf{M}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}\mathbf{s} + \boldsymbol{\tau}_r = -(\tilde{\mathbf{M}}\dot{\mathbf{r}} + \tilde{\mathbf{C}}\mathbf{r} + \tilde{\mathbf{g}} + \tilde{\boldsymbol{\tau}}_f + \tilde{\boldsymbol{\tau}}_d) + \boldsymbol{\varepsilon} \quad (2.9)$$

where  $(\tilde{\cdot})$  refers to the difference between the actual and estimated values. Exploiting the linear parameterization form for the right hand side, (2.9) can be rewritten as

$$\mathbf{M}\dot{\mathbf{s}} + \mathbf{C}\mathbf{s} + \mathbf{K}\mathbf{s} + \boldsymbol{\tau}_r = -(\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}})\tilde{\mathbf{p}} + \tilde{\mathbf{W}}^T \boldsymbol{\varphi}) + \boldsymbol{\varepsilon} \quad (2.10)$$

The next step is to design updating adaptive laws to estimate  $\hat{\mathbf{p}}$  and  $\hat{\mathbf{W}}$  separately such that (2.10) is asymptotically stable.

### 2.3.3 The Updating Adaptive Law

To determine feasible updating adaptive laws, let us consider the following Lyapunov-like function for the target robotic system.

$$v = \frac{1}{2} \mathbf{s}^T \mathbf{M} \mathbf{s} + \frac{1}{2} \tilde{\mathbf{p}}^T \boldsymbol{\Gamma} \tilde{\mathbf{p}} + \frac{1}{2} \text{Tr}(\tilde{\mathbf{W}}^T \boldsymbol{\Omega} \tilde{\mathbf{W}}) \quad (2.11)$$

where  $\Gamma \in R^{l \times l}$  and  $Q \in R^{n \times n}$  are the adaptation gains corresponding to the regressor and approximation terms respectively.

Taking derivative of the above equation to get

$$\dot{v} = s^T \dot{M}s + \frac{1}{2} s^T \dot{M}s - \tilde{p}^T \Gamma \dot{p} - Tr(\tilde{W}^T Q \dot{W}) \quad (2.12)$$

Substituting (2.10) into (2.12) to obtain

$$\dot{v} = s^T (-Y\tilde{p} - \tilde{W}^T \phi - Cs - Ks - \tau_r + \varepsilon) + \frac{1}{2} s^T \dot{M}s - \tilde{p}^T \Gamma \dot{p} - Tr(\tilde{W}^T Q \dot{W}) \quad (2.13)$$

Simplification of above equation to get

$$\dot{v} = -s^T Ks + s^T (-\tau_r + \varepsilon) + \frac{1}{2} s^T (\dot{M} - 2C) - \tilde{p}^T (\Gamma \dot{p} + Y^T s) - Tr(\tilde{W}^T (\phi s^T + Q \dot{W})) \quad (2.14)$$

Hence, the updating adaptive law can be suggested as

$$\dot{p} = -\Gamma^{-1} Y^T s \quad (2.15a)$$

$$\dot{W} = -Q^{-1} \phi s^T \quad (2.15b)$$

Exploiting the passivity property [23], assuming  $\|\varepsilon\| \leq \delta$ , and substituting (2.15) into (2.14) to get

$$\dot{v} \leq -s^T Ks - s^T \tau_r + \delta \|s\| \leq -s^T Ks \leq 0 \quad (2.16)$$

which is stable according to Barbalat's lemma [23].

**Remark 1.** In order to avoid drift problems of the estimated parameters and weighting coefficients (2.15), the dead zone robust technique is a powerful technique to resolve this problem. The reader is referred to [16] for more details.

## 2.4 Simulation Results and Discussions

A two-R robot with two actuated joints and physical parameters borrowed from [16] was used for proving the validity of the proposed controller, please see Fig. 2.1. In this section, a contact-free motion robot was simulated using MATLAB/SIMULINK with the desired reference trajectories cited from [24].

In this section, the target robot is controlled by using (2.8) and (2.15). As stated previously, the proposed adaptive controller consists of four terms: a regressor term for estimation of physical parameters of the robot, an approximation term for estimation of unknown disturbances (e.g. friction terms), a high-gain feedback term, and a robust sliding mode term for compensation for modeling errors. Chebyshev polynomials are used for approximation purposes with 11 terms ( $\beta=11$ ). In effect, increasing the number of terms may not have an effect on the accuracy of results, please see [19,24-26]. The values of feedback and adaptive gains used in the simulation are as follows:  $K = 70I_2$ ,  $A = 70I_2$ ,  $Q^{-1} = 100I_{22}$ , and  $\Gamma^{-1} = \text{diag}(35, 25, 20, 10, 5, 3)$ , where  $I_{(.)}$  refers to square diagonal matrix with  $(.)$  elements.

From Fig. 2.2, it should be noted that the position errors are acceptable and well precise tracking could be got with the proposed controller. The input controls of robot motion are shown on the right-hand side of Fig. 2.2. Some important points should be noted: (i) these gains should carefully be selected to avoid instability of dynamic response of the target robot, (ii) a try-and-error procedure is used for tuning the adaptation and feedback gains, (iii) increasing the feedback gain  $K$  is necessary to get accurate tracking but with possible oscillations. Therefore, a compromise should be selected in order to get low oscillations with low position errors. In addition, simulation proves that the adapting gain  $Q^{-1}$  should be high in order to get good tracking; in effect, low adapting gain can result in low tracking. Although there are different approximators (e.g. neural networks, fuzzy approximation, orthogonal polynomials etc.), Chebyshev's polynomials are used for approximation of friction terms. Selecting the best approximator is the task of future work. The objective of the proposed adaptive controller is to track the desired trajectory rather than convergence of physically uncertain parameters. As a result, Figs 2.2 and 2.3 show that the robot can track the desired references but without exact convergence for uncertain parameters, please see [2] for more details.

## 2.5 Conclusions

This work proposes a hybrid regressor and approximation-based adaptive controller for robots in free space. The strength of the proposed control is its ability to compensate for external/internal disturbances by using one approximation term combined

with regressor technique. Its performance seems better than robust regressor-based control which could have inherently bounded position error. The modification of the control law and the associated updating adaptive laws can be further investigated considering the effect of the switching functions.

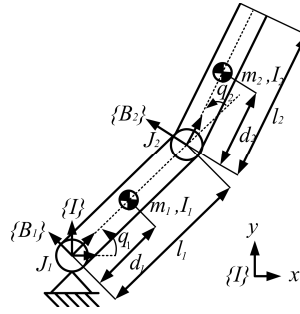


Fig. 1. Two-R robot

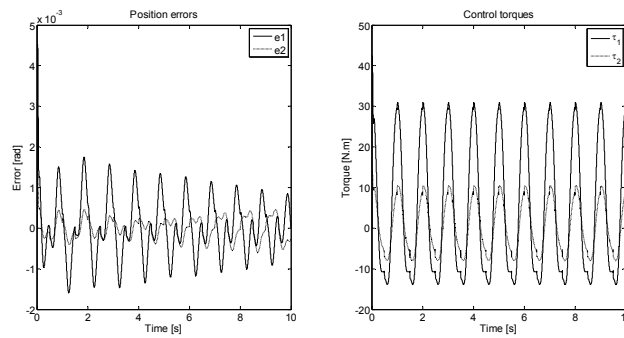


Fig. 2. Dynamic response of 2-R robots

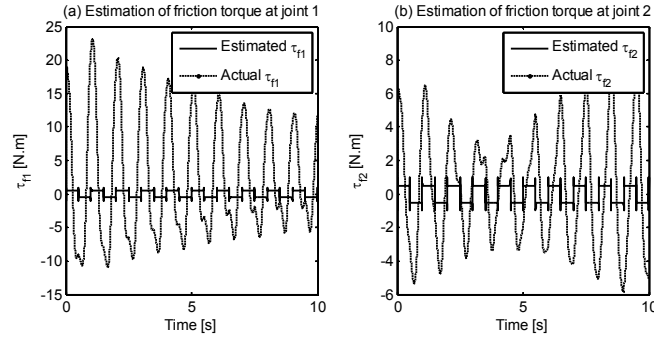


Fig. 3. Estimation of friction torques based on approximation term

### Acknowledgements

This work was funded by the Integration Fund Project of China NSF and Zhejiang province, China, under Award No.U1509212.

### References

- [1] J.A. Farrell and M.M.Polycarpou, Adaptive approximation based control: Unifying neural, fuzzy and traditional adaptive approximation approach, John Wiley & Sons Inc, USA, 2006.
- [2] P. Loannou and B.Fidan, Adaptive control tutorial, SIAM, Philadelphia, 2006.
- [3] J.J.Craig, Adaptive control of mechanical manipulators, Addison-Wesley Publishing company Inc, USA, 1988.
- [4] W.-H.Zhu, Virtual Decomposition control: toward hyper degrees of freedom robots, Springer-Verlag, Berlin, Heidelberg, 2010.
- [5] F. L.Lewis, A.Yesildirek, and K.Liu, "Multilayer neural net robot controller with guaranteed tracking performance," IEEE Trans Neural Netw. vol. 7, pp. 1–12, 1996.
- [6] F.L.Lewis, K.Liu, and A.Yesildirek, "Neural net robot controller with guaranteed tracking performance," IEEE Trans Neural Netw.vol. 6, pp. 703–716, 1995b.
- [7] F.L.Lewis, A.Yesildirek, and K. Liu, "Neural net robot controller: Structure and stability proofs," J. Intell Robot Syst. vol. 13, pp. 1–23, 1995a.
- [8] N. Sadegh and R.Horowitz, "Stability and robustness analysis of a class of adaptive controller for robotic manipulators," International Journal of Robotics Research, Vol. 9, pp. 74-92, 1990.

- [9] Y.D.Song, "Adaptive motion tracking control of robot manipulators-Non-regressor based approach," Proceeding of IEEE International Conference on Robotics and Automation vol. 4, pp.3008-3013, 1994.
- [10] W.S. Lu and Q.H.Meng, "Recursive computation of manipulator regressor and its application to adaptive motion control of robot," IEEE Conference on Communication, Computation and Signal Processing, pp. 170-173, 1991.
- [11] H.Kawasaki, T.Bito, and K.Kanzaki, "An efficient algorithm for model-based adaptive controller of robot manipulators,"IEEE Transactions on Robotics and Automation, vol.12, no. 3, pp. 496-501, 1996.
- [12] Z.Qu and J.Dorsey, "Robust tracking control of robots by a linear feedback law," IEEE transactions on Automatic Control, Vol.36, no. 9, pp. 1081-1084, 1991.
- [13] J.S.Park, Y.A.Jiang, T.Hesketh, and D.J.Clements, "Trajectory control of manipulators using adaptive sliding mode control," Proceedings of IEEE, Southeastcon, pp. 142-146, 1994.
- [14] J. Yuan and Y.Stepanenko, "Adaptive PD control of flexible joint robots without using the high-order regressor," Proceedings of the 36<sup>th</sup> Midwest Symposium on Circuits and Systems, pp. 389-393, 1993.
- [15] C.-Y. Su and Y.Stepanenko, "Adaptive control for constrained robots without using regressor," IEEE International Conference on Robotics and Automation, 264-269 (1996)
- [16] A.-C.Huang and M.-C Chien, Adaptive control of robot manipulators: A unified regressor-free approach, World Scientific Publishing Co. Pte. Ltd. Singapore, 2010.
- [17] S.Cong, Y.Liang, and W.Shang,"Function approximation-based sliding mode adaptive control for time-varying uncertain nonlinear systems,"Nonlinear Dynamics, vol. 54, no.5, pp. 223-230, 2008.
- [18] W.Liang, H.-Y.Chen, and Y.-T.Tsu,"FAT-based adaptive sliding mode control for a piezoelectric-actuated system," IEEE International Conference on Control and Automation, pp. 848-853, 2009.
- [19] Hayder F.N. Al-Shuka, B.Corves, and W.-H.Zhu,"Function approximation technique-based adaptive virtual decomposition control of a seria-chain manipulator,"Robotica, vol. 32, no. 3, pp. 375-399, 2014.
- [20] J.Wang, S.S.Ge, and T.H.Lee,"Adaptive friction compensation for sevo mechanisms,"G. Tao et al. (Eds.), Adaptive control of nonsmooth dynamic systems, pp. 211-248, 2001.
- [21] H.Olsson, K.J.Aström, C.Canudasde Wit, M.Gäfvert, and P.Lischinsky," Friction models and fricton compensation,"European Journal of Control, vol. 4, no. 3, pp. 176-195, 1998.
- [22] J.-J.E. Slotine and W.Li,Applied nonlinear control, Prentice-Hall Inc, USA, 1991.
- [23] M.W. Spong and M.Vidyasagar, Robot dynamics and control, John Wiley & Sons, USA, 1989.
- [24] Hayder F.N. Al-Shuka,"On local approximation-based adaptive control with applications to robotic manipulators and biped robots,"Springer, International Journal of Dynamics and Control, pp. 1-15, 2017.
- [25] Hayder F.N. Al-Shuka and R. Song,"Adaptive hybrid regressor and approximation control of robotic manipulators in constrained space,"International Journal of Mechanical & Mechatronics Engineering IJMME –IJENS, vol:17, no:03, pp. 11-21, 2017.
- [26] Hayder F.N. Al-Shuka, Modelingwalking pattern generators and adaptive control of biped robot, PhD Dissertation, RWTH Aachen University, Germany, 2014.

## Chapter 3: Adaptive hybrid regressor-approximation control of robots in constrained space: centralized control algorithm

This work attempts to combine the features of both the regressor and the approximation techniques in adaptive control. The regressor technique is a powerful tool for adaptive control of the known structure of modeling while the approximation is useful for estimation of time-varying uncertainty. Therefore, this work proposes adaptive hybrid regressor and approximation control for robots in constrained space. The control law consists of three terms: (i) regressor term for initial estimation of the known structure of the robot dynamics, e.g. inertia matrix, Coriolis and centripetal matrix and gravity vector, and (ii) approximation term for estimation of internal and external disturbances resulted from the inexact calculation of regressor matrix and unknown modeling of friction, etc, and (iii) robust term consists of switching  $sgn(\cdot)$  function. The control law is designed based on updating the uncertain parameters and the weighting coefficients corresponding to regressor and approximation respectively with position/force tracking purposes. The proposed controller is stable in the sense of Lyapunov stability and applied to 2-link manipulator moving along a circular arc. The coordinates partition is performed for model reduction of the constraint dynamics of the investigated robot. The results show the effectiveness of the proposed controller for dealing with miscellaneous disturbances.

### 3.1 Introduction

When a robot contacts with the environment via its end-effector, some important points should be noted:

- Position control strategies are not sufficient for precise tracking of desired references; both the position and interaction force should be controlled carefully. For example, if the task of target robot is to write something, neglecting control of interaction force may lead to either loss of contact or press so hard on the target environment [1].
- Besides, the robot loses some degrees of freedom (DoFs) during the contact phase. Consequently, the generalized coordinates of the target robot could be larger than its DoFs due to its constrained motion; it constitutes closed chain mechanism with redundant coordinates.
- The robot may change its configuration during the transition from open chain mechanism to the closed chain. In effect, three motion phases could be produced: free motion phase, contact motion phase (impact phase), and constrained motion phase. Every phase may have its own features and control law.

In literature, there are different techniques for force control of constrained robots; however, they can be classified as follows:

- impedance control, and admittance control [2-4]
- hybrid techniques such as hybrid position/force control [5-7], and hybrid impedance control [8]
- stiffness control [9-11]
- explicit force control [12,13]. For more details on principles of robot force control, please see [14-16].

The above-mentioned studies have assumed full knowledge of robot/environment dynamics; however, neglecting uncertainty of robot/environment dynamics may lead to instability problems and large deviations of desired position and force references. Therefore, several adaptive force strategies have been proposed [15-18]. Most of these approaches are based on regressor technique which embodies full dynamics of the investigated robot. In other words, the left-hand side of the equation of robot motion can be linearly parameterized in terms of regressor matrix (it is a function of the state variables of the robot) and a vector of uncertain parameters (e.g. link length, mass, inertia etc.). The regressor-based adaptive control may not be suitable for control of dynamic systems with unmodeled disturbance (e.g. friction) unless the modification of control law is used. One of its limitations is its dependence on the available structure of the target dynamic system; neglecting unmodeled dynamics may degrade the performance of the proposed controller [19,20]. It is well-known that friction is a phenomenon hard to be modeled correctly because it is dependent on miscellaneous physical factors such as relative velocity, temperature, humidity etc. [21]. There are different proposed models for friction; however, it can be classified as static and dynamic models; for more details on these models please see [21]. Armstrong [22] has proposed a static model which consists of many components: Coulomb friction, viscous friction, Stribeck term, and offset term. Most of these terms are highly nonlinear and could produce miscellaneous instability and divergence problems unless carefully designed controller has been used [23-25]. On the other hand, the adaptive approximation control is a powerful technique for precise tracking of the planned trajectory of robots with uncertain parameters and nonlinear disturbances. The idea behind the adaptive approximation control is to estimate the uncertain term in terms of weighting and basis function matrices, then design a control law based on the update of the weighting matrix. The strength of approximation technique is that it can be applied to different types of robots using the same structure of control law, while this is not the case for a regressor technique in which each robot has its own regressor matrix. In general, there two types of adaptive approximation control: partitioned approximation, and augmented approximation. Partitioned approximation means an estimation of each term on the left-hand side of the equation of motion of the target robot in terms of basis functions and weighting function matrices. Whereas augmented approximation attempts to approximate the whole uncertainty in one term. Lewis et al. [26-28] have applied both the partitioned and augmented approximation control separately on robotic manipulators. Although the authors preferred to use a partitioned approximation, they did not justify the computational complexity inherent in their algorithm when it is used with high DoF robot; the

computational complexity grows exponentially with the DoFs of the robots. Huang and Chien [29] have applied partitioned approximation-based adaptive control for robot manipulators considering different cases such as actuator dynamics, joint flexibility, and impedance control. Al-Shuka et al. [24] have extended the work of [29] to be applied on high DoF robot by using the Virtual Decomposition Control (VDC) [23]; the idea of the VDC is to virtually decompose the whole robotic system into sub-systems and controlling each subsystem separately such that the Lyapunov's stability of the whole system is guaranteed. Liu [30] has applied separately the partitioned and augmented approximation-based adaptive control for nonlinear electro-mechanical systems. Cong et al. [31] have proposed function approximation-based sliding mode adaptive control for DC motor with dead zone uncertainty. The authors have approximated the unmodeled dynamics by using orthogonal Laguerre functions and another sliding mode term. The idea is to estimate the uncertain term by updating the weighting coefficients of Laguerre functions and guaranteeing the output error by using Lyapunov's stability.

This work is concerned with the combination of regressor and approximation techniques for adaptive control of robotic manipulators with constrained motion. The idea behind it is to estimate the known structure of the left-hand side of the equation of motion by using the regressor technique while the uncertain term (unmodeled dynamics such as friction phenomenon) is estimated by using orthogonal Chebyshev functions. Hybrid force/position control based on orthogonalization technique is used as a basis for control of a robot in constrained space [32-36].

### 3.2 Preliminaries

In this section, some details are revisited for dynamics of robots in constrained motion and previous work of regressor and approximation strategies.

#### 3.2.1 Constrained Dynamics of Robotic Manipulators

The 1<sup>st</sup> kind Lagrangian formulation is used for describing constrained dynamics of robotic manipulators. Therefore, the equation of motion can be expressed as [34,35]

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f + \boldsymbol{\tau}_d = \boldsymbol{\tau} + \mathbf{J}^T \boldsymbol{\lambda} \quad (3.1)$$

$$\boldsymbol{\vartheta}(\mathbf{q}) = \mathbf{0} \quad (3.2)$$

where  $\mathbf{D} \in \mathbb{R}^{n_q \times n_q}$  is the inertia matrix with  $n_q$  denoting the number of generalized coordinates,  $\mathbf{q} \in \mathbb{R}^{n_q}$  and their first two derivatives are the state variables of the system,  $\mathbf{H} \in \mathbb{R}^{n_q \times n_q}$  is the Coriolis and centripetal matrix,  $\mathbf{g} \in \mathbb{R}^{n_q}$  is the gravity vector,  $\boldsymbol{\tau}_f \in \mathbb{R}^{n_q}$  represents friction torque vector,  $\boldsymbol{\tau}_d \in \mathbb{R}^{n_q}$  is the disturbance vector,  $\boldsymbol{\tau}$  is the input control vector,  $\mathbf{J} \in \mathbb{R}^{m \times n_q}$  is Jacobian matrix with  $m$  referring to the number of constraints,  $\boldsymbol{\lambda} \in \mathbb{R}^m$  is the Lagrange multipliers associated with the imposed constraint, and  $\boldsymbol{\vartheta}(\mathbf{q})$  denotes the algebraic equation of the imposed constraint.

Some important points should be noted via investigation of Eqs. (3.1) and (3.2):

- Due to the imposed constrained motion, the generalized coordinates ( $n_q$ ) is more than the independent DoFs of the target system ( $n$ ).
- Equations (3.1) and (3.2) represent differential algebraic equations (DAEs) with Lagrange multipliers; the Lagrange multipliers and the constraint algebraic equation of Eq. (3.2) are resulted from the imposed constraint.

In general, there are two approaches for the solution of Eqs. (3.1) and (3.2) which are redundant coordinates approach [37] and minimum coordinates approach [32-36]. Most commercial programs of multibody dynamics (e.g. MSC ADAMS) may depend on redundant coordinates approach in dealing with constrained motion; however, the minimum coordinates approach is so suitable for control purposes and real-time applications. The idea of the minimum coordinates approach is to eliminate Lagrange multipliers from Eq. (3.1) producing constraint-free differential equation. This can be accomplished by using the principle of orthogonalization; please see Ref. [33] for more details.

Therefore, this work exploits the coordinate partitioning, an orthogonalization-based technique, for dimension reduction of the constrained motion. This can be implemented by obtaining a mathematical relationship between the redundant generalized coordinates and the independent coordinates (DoFs) as shown below.

Twice differentiation of Eq. (3.2) leads to

$$\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0} \quad (3.3)$$

$$\mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \mathbf{0} \quad (3.4)$$

Due to redundancy, the independent DoFs  $\boldsymbol{\theta}$  can be mathematically expressed in terms of the redundant generalized coordinates  $\mathbf{q}$  as shown in Eq. (3.5).

$$\boldsymbol{\theta} = \boldsymbol{\rho}(\mathbf{q})\mathbf{q} \quad (3.5)$$

Twice differentiation of Eq. (3.5) gives

$$\dot{\boldsymbol{\theta}} = \mathbf{J}_j(\mathbf{q})\dot{\mathbf{q}} \quad (3.6)$$

$$\ddot{\boldsymbol{\theta}} = \mathbf{J}_j(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}_j(\mathbf{q})\dot{\mathbf{q}} \quad (3.7)$$

with  $J_j(\mathbf{q}) \in \mathbb{R}^{n \times n_q} = \frac{\partial \rho(\mathbf{q})}{\partial \mathbf{q}}$

Blocking Eqs. (3.3) and (3.6) in matrix form

$$\begin{bmatrix} J(\mathbf{q}) \\ J_j(\mathbf{q}) \end{bmatrix} \dot{\mathbf{q}} = \begin{bmatrix} \mathbf{0} \\ \dot{\boldsymbol{\theta}} \end{bmatrix} \quad (3.8)$$

Equation (3.8) can be re-written as

$$\dot{\mathbf{q}} = \begin{bmatrix} J(\mathbf{q}) \\ J_j(\mathbf{q}) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \dot{\boldsymbol{\theta}} \end{bmatrix} = [\mathbf{E}_1 \quad \mathbf{E}_2] \begin{bmatrix} \mathbf{0} \\ \dot{\boldsymbol{\theta}} \end{bmatrix} = \mathbf{E}_2(\mathbf{q})\dot{\boldsymbol{\theta}} \quad (3.9)$$

The matrix  $\mathbf{E}_2 \in \mathbb{R}^{n_q \times n}$  plays important role in eliminating  $\boldsymbol{\lambda}$  by satisfying the following condition

$$J(\mathbf{q})\mathbf{E}_2(\mathbf{q}) = \mathbf{0} \quad (3.10)$$

Differentiating Eq. (3.9) to obtain

$$\ddot{\mathbf{q}} = \mathbf{E}_2(\mathbf{q})\ddot{\boldsymbol{\theta}} + \dot{\mathbf{E}}_2(\mathbf{q}, \dot{\mathbf{q}})\dot{\boldsymbol{\theta}} \quad (3.11)$$

Substituting Eqs. (3.9) and (3.11) into Eq.(3.1) to get

$$\mathbf{D}(\mathbf{q})(\mathbf{E}_2(\mathbf{q})\ddot{\boldsymbol{\theta}} + \dot{\mathbf{E}}_2(\mathbf{q}, \dot{\mathbf{q}})\dot{\boldsymbol{\theta}}) + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{E}_2(\mathbf{q})\dot{\boldsymbol{\theta}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f + \boldsymbol{\tau}_d = \boldsymbol{\tau} + J(\mathbf{q})^T \boldsymbol{\lambda} \quad (3.12)$$

Alternatively, the last equation can be re-written as

$$\bar{\mathbf{D}}(\mathbf{q})\ddot{\boldsymbol{\theta}} + \bar{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\boldsymbol{\theta}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f + \boldsymbol{\tau}_d = \boldsymbol{\tau} + J(\mathbf{q})^T \boldsymbol{\lambda} \quad (3.13)$$

where

$$\bar{\mathbf{D}}(\mathbf{q}) = \mathbf{D}(\mathbf{q})\mathbf{E}_2(\mathbf{q}), \bar{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{D}(\mathbf{q})\dot{\mathbf{E}}_2(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{E}_2(\mathbf{q}) \quad (3.14)$$

Using Eqs. (3.9) and (3.13) yields

$$\boldsymbol{\lambda} = \mathbf{E}_1(\mathbf{q})^T (\bar{\mathbf{D}}(\mathbf{q})\ddot{\boldsymbol{\theta}} + \bar{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\boldsymbol{\theta}} + \mathbf{g}(\mathbf{q}) + \boldsymbol{\tau}_f + \boldsymbol{\tau}_d - \boldsymbol{\tau}) \quad (3.15)$$

Exploiting Eq. (3.10) and pre-multiplying Eq. (3.13) by  $\mathbf{E}_2(\mathbf{q})^T$  to obtain the following constraint-free dynamics

$$\mathbf{E}_2(\mathbf{q})^T \bar{\mathbf{D}}(\mathbf{q})\ddot{\boldsymbol{\theta}} + \mathbf{E}_2(\mathbf{q})^T \bar{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\boldsymbol{\theta}} + \mathbf{E}_2(\mathbf{q})^T \mathbf{g}(\mathbf{q}) + \mathbf{E}_2(\mathbf{q})^T (\boldsymbol{\tau}_f + \boldsymbol{\tau}_d) = \mathbf{E}_2(\mathbf{q})^T \boldsymbol{\tau} \quad (3.16)$$

For more details on properties of Eq. (3.13), e.g. passivity and linear parameterization properties, please see [32,34-36].

**Remark 3.1.** The dynamic coefficient matrices and Jacobian matrices are still in terms of the redundant generalized coordinates  $\mathbf{q}$ ; therefore, they are written in terms of  $\mathbf{q}$ .

### 3.2.2 Regressor-Based Adaptive Control

This section describes briefly adaptive control based on regressor technique. The objective of the controller is to track the desired references of position and contact forces for the investigated robot while estimating the uncertain parameters associated with the robot dynamics. The intuitive control law can be designed as [32, 36]

$$\boldsymbol{\tau} = \hat{\bar{\mathbf{D}}}(\mathbf{q})\dot{\mathbf{r}} + \hat{\bar{\mathbf{H}}}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{r} + \hat{\mathbf{g}}(\mathbf{q}) + \hat{\boldsymbol{\tau}}_f - \mathbf{K}\mathbf{E}_2\mathbf{s} - \underbrace{J(\mathbf{q})^T(\boldsymbol{\lambda}_d - \mathbf{K}_I \int_0^t \mathbf{e}_\lambda dt)}_{\text{Contact force}} \quad (3.17)$$

where the symbol ( $\hat{\cdot}$ ) refers to the estimated values, and

$$\mathbf{r} = \dot{\boldsymbol{\theta}}_d - \boldsymbol{\Lambda}_p \mathbf{e}_p \quad (3.18a)$$

$$\mathbf{s} = \dot{\boldsymbol{\theta}} - \mathbf{r} = \dot{\boldsymbol{\theta}} + \boldsymbol{\Lambda}_p \mathbf{e}_p \quad (3.18b)$$

$$\mathbf{e}_p = \boldsymbol{\theta} - \boldsymbol{\theta}_d \quad (3.18c)$$

$$\mathbf{e}_\lambda = \boldsymbol{\lambda} - \boldsymbol{\lambda}_d \quad (3.18d)$$

where  $\mathbf{K} \in \mathbb{R}^{n_q \times n_q}$  denotes positive definite feedback gain matrix,  $\mathbf{K}_I \in \mathbb{R}^{m \times m}$  is a feedback gain corresponding to integral term for force error ( $\mathbf{e}_\lambda$ ),  $\boldsymbol{\Lambda}_p \in \mathbb{R}^{n \times n}$  refers to feedback gain matrix for position error ( $\mathbf{e}_p$ ), and the subscript ( $d$ ) refers to the desired references. The first four terms of Eq. (3.17) could be linearly parameterized (assuming known structure for friction phenomenon) in terms of the regressor matrix  $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}})$  and the unknown parameter vector  $\hat{\mathbf{p}} \in \mathbb{R}^l$  ( $l$  refers to the number of uncertain parameters) as shown below.

$$\boldsymbol{\tau} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}})\hat{\mathbf{p}} - \mathbf{K}\mathbf{E}_2\mathbf{s} - J(\mathbf{q})^T(\boldsymbol{\lambda}_d - \mathbf{K}_I \int_0^t \mathbf{e}_\lambda dt) \quad (3.19)$$

with the following updating adaptive law

$$\dot{\hat{\mathbf{p}}} = -\boldsymbol{\Gamma}^{-1}\mathbf{Y}^T(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}})\mathbf{E}_2\mathbf{s} \quad (3.20)$$

where  $\boldsymbol{\Gamma} \in \mathbb{R}^{l \times l}$  denotes the adaptation gain. It should be noted that the regressor matrix is a function of the state variables of the target robot. Although it is physics-based technique, it has some inherent limitations such as computational complexity with high DoF robots, the difficulty of dealing with unmodeled dynamics, etc. Therefore, some authors attempted to avoid it by using

different solutions. One of these solutions is introduced in the next section.

**Remark 3.2** The disturbance vector  $\tau_d$  is neglected in Eq. (3.17) because this form of adaptive regressor control cannot deal with disturbance compensation and further work is required.

### 3.2.3 Approximation-Based Adaptive Control

In our previous work [34], we extended the work of Su et al. [32] such that an adaptive approximation control is suggested instead of regressor technique. Here, the controller objective is tracking the desired references of position and contact forces while estimation the weighting coefficient matrices. The control law could be expressed as

$$\tau = \widehat{W}_D^T \Phi_D \dot{r} + \widehat{W}_H^T \Phi_H r + \widehat{W}_g^T \phi_g + \widehat{W}_\tau^T \phi_\tau - K E_2 s - \underbrace{J(q)^T (\lambda_d - K_I \int_0^t e_\lambda dt)}_{\text{Contact force}} \quad (3.21)$$

with the following updating adaptive laws

$$\begin{aligned} \dot{\widehat{W}}_D &= Q_D^{-1} \Phi_D \dot{r} s^T E_2^T \\ \dot{\widehat{W}}_H &= Q_H^{-1} \Phi_H r s^T E_2^T \\ \dot{\widehat{W}}_g &= Q_g^{-1} \phi_g s^T E_2^T \\ \dot{\widehat{W}}_\tau &= Q_\tau^{-1} \phi_\tau s^T E_2^T \end{aligned} \quad (3.22)$$

where  $W_D \in \mathbb{R}^{n\beta \times nq}$ ,  $W_H \in \mathbb{R}^{n\beta \times nq}$ ,  $W_g \in \mathbb{R}^{n\beta \times nq}$ , and  $W_\tau \in \mathbb{R}^{n\beta \times nq}$  are the weighting matrices of the inertia matrix, Coriolis and centripetal matrix, the gravity vector, and disturbance vector respectively with  $\beta$  referring to the number of approximation terms, while  $\Phi_D \in \mathbb{R}^{n\beta \times n}$ ,  $\Phi_H \in \mathbb{R}^{n\beta \times n}$ ,  $\phi_g \in \mathbb{R}^{n\beta \times 1}$ , and  $\phi_\tau \in \mathbb{R}^{n\beta \times 1}$  represent the corresponding basis function matrices and vectors respectively.  $Q_D$ ,  $Q_H$ ,  $Q_g$  and  $Q_\tau \in \mathbb{R}^{n\beta \times n\beta}$  are the corresponding adaptation gains.

**Remark 3.3.** The dimension of the weighting matrices  $W(\cdot)$  is expressed according to the proposed representation of our work [38]; this representation reduces the computational complexity associated with linear parameterization of a matrix.

**Remark 3.4.** The modeling error and the parameter drift problems are not consider in Eqs. (3.21)-(3.22).

### 3.3 Methodology

To attenuate the issues inherent with regressor and approximation techniques, the author proposes a new technique combining the approximation and regressor approaches in one control law for constrained robots. Therefore, the inertia matrix, Coriolis and centripetal matrix, and gravity vector could be estimated based on regressor technique while the friction term and the unmodeled disturbance is estimated by using approximation technique; the details are as follows.

The left-hand side of the reduced order-based equation of motion for robots in constrained motion, Eq. (3.13), can be expressed based on the linear parameterization property as

$$\bar{D}(q)\ddot{\theta} + \bar{H}(q, \dot{q})\dot{\theta} + g(q) + \tau_f + \tau_d = Y(q, \dot{q}, \dot{\theta}, \ddot{\theta})p + W^T \phi + \epsilon \quad (3.23)$$

where

$$\begin{aligned} \bar{D}(q)\ddot{\theta} + \bar{H}(q, \dot{q})\dot{\theta} + g(q) &= Y(q, \dot{q}, \dot{\theta}, \ddot{\theta})p \\ \tau_f + \tau_d &= W^T \phi + \epsilon \end{aligned}$$

with  $\epsilon$  denoting the modeling error. As we see, the known structure of the robot is modeled using the regressor technique, while the approximation technique is used for estimation of the disturbance terms (e.g., friction, etc.).

#### 3.3.1 The Control Law

In light of the above discussions, the modified control law of Eq. (3.17) can be expressed as

$$\tau = \underbrace{Y(q, \dot{q}, r, \dot{r})\hat{p}}_{\text{regressor term}} + \underbrace{\widehat{W}^T \phi}_{\text{approximation term}} - K E_2 s - \tau_{robust} - J(q)^T (\lambda_d - K_I \int_0^t e_\lambda dt) \quad (3.24)$$

where  $\tau_{robust} = k[\text{sgn}(E_1 s_1) \quad \dots \quad \text{sgn}(E_n s_n)]$ , with  $k$  representing positive number that should be greater than  $\|\epsilon\|$ ; this will be proved later, and

$$\begin{aligned} Y(q, \dot{q}, r, \dot{r})\hat{p} &= \widehat{D}(q)\dot{r} + \widehat{H}(q, \dot{q})r + \widehat{g}(q) \\ \widehat{W}^T \phi &= \widehat{\tau}_f + \widehat{\tau}_d \end{aligned} \quad (3.25)$$

Substituting Eq. (13) into above equation to get the following closed loop control system

$$\bar{D}(q)\ddot{\theta} + \bar{H}(q, \dot{q})\dot{\theta} + g(q) + \tau_f + \tau_d - J(q)^T \lambda = Y(q, \dot{q}, r, \dot{r})\hat{p} + \widehat{W}^T \phi - K E_2 s - \tau_{robust} - J(q)^T (\lambda_d - K_I \int_0^t e_\lambda dt) \quad (3.26)$$

The above equation can be further simplified by using Eq. (18)

$$\bar{D}(q)\dot{s} + \bar{H}(q, \dot{q})s + K E_2 s + \tau_{robust} = -(Y(q, \dot{q}, r, \dot{r})\hat{p} + \widehat{W}^T \phi) + \epsilon + J(q)^T (e_\lambda + K_I \int_0^t e_\lambda dt) \quad (3.27)$$

The closed loop control system for the target robot (Eq. (27)) is stable if  $\lim_{t \rightarrow \infty} \hat{\mathbf{p}} \rightarrow \mathbf{p}$ ,  $\lim_{t \rightarrow \infty} \widehat{\mathbf{W}} \rightarrow \mathbf{W}$ ,  $\lim_{t \rightarrow \infty} \mathbf{e}_\lambda \rightarrow \mathbf{0}$ , and the lumped error  $\boldsymbol{\epsilon}$  is compensated by using  $\boldsymbol{\tau}_{robust}$  term. In addition, the precise tracking of the desired contact forces depends on convergence of the estimation process.

**Remark 3.5.** Although the persistent excitation is a necessary condition for estimation convergence, the integral term of force feedback can help to some extent to track the desired forces. However, one of the inherent issue with force feedback is an algebraic loop which is difficult to be treated. There are two possible solutions for dealing with this dilemma: (1) making force observer to estimate the interaction force, and (2) by applying the following equation:

$$\lambda = k_e(x - x_e) \quad (3.28)$$

where  $k_e$  refers to the environment stiffness,  $x$  is the end-effector position, and  $x_e$  denotes to the environment position. Thus with Eq. (28), the measurement of the state variables of the robot is needed rather than force sensors.

### 3.3.2 The Updating Adaptive Law

According to above discussions, to get a stable closed loop system it is necessary to select a suitable updating adaptive law that can estimate both the uncertain parameters vector and the weighting vector. Let us select the following update adaptive laws

$$\dot{\widehat{\mathbf{W}}} = -\mathbf{Q}^{-1} \boldsymbol{\phi} \mathbf{s}^T \mathbf{E}_2^T \quad (3.29a)$$

$$\dot{\hat{\mathbf{p}}} = -\Gamma^{-1} \mathbf{Y}^T(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}}) \mathbf{E}_2 \mathbf{s} \quad (3.29b)$$

To prove the validity of the controller structure (Eqs. (3.24) and (3.29)), let us define the Lyapunov-like function as

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{D} \mathbf{s} + \frac{1}{2} \text{Tr}(\widehat{\mathbf{W}}^T \mathbf{Q} \widehat{\mathbf{W}}) + \frac{1}{2} \tilde{\mathbf{p}}^T \Gamma \tilde{\mathbf{p}} \quad (3.30)$$

where  $\mathbf{D} = \mathbf{E}_2^T \bar{\mathbf{D}}$ .

By differentiating the last equation, we can get

$$\dot{V} = \mathbf{s}^T \mathbf{D} \dot{\mathbf{s}} + \frac{1}{2} \mathbf{s}^T \dot{\mathbf{D}} \mathbf{s} - \text{Tr}(\widehat{\mathbf{W}}^T \mathbf{Q} \dot{\widehat{\mathbf{W}}}) - \tilde{\mathbf{p}}^T \Gamma \dot{\tilde{\mathbf{p}}} \quad (3.31)$$

By pre-multiplying both sides of Eq. (3.27) by  $\mathbf{E}_2^T$  and exploiting the property of Eq. (3.10), we can obtain

$$\mathbf{D} \dot{\mathbf{s}} + \mathbf{E}_2^T \bar{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}}) \mathbf{s} + \mathbf{E}_2^T \mathbf{K} \mathbf{E}_2 \mathbf{s} + \mathbf{E}_2^T \boldsymbol{\tau}_{robust} = -\mathbf{E}_2^T (\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{r}, \dot{\mathbf{r}}) \tilde{\mathbf{p}} + \widehat{\mathbf{W}}^T \boldsymbol{\phi}) + \mathbf{E}_2^T \boldsymbol{\epsilon} \quad (3.31)$$

Substituting the above equation into Eq. (3.31) and using the passivity property of the motion equation (see [32, 34-36]), we can obtain

$$\dot{V} = -\mathbf{s}^T \mathbf{E}_2^T \mathbf{K} \mathbf{E}_2 \mathbf{s} - \mathbf{s}^T \mathbf{E}_2^T (\boldsymbol{\tau}_{robust} - \boldsymbol{\epsilon}) - \text{Tr}(\widehat{\mathbf{W}}^T (\boldsymbol{\phi} \mathbf{s}^T \mathbf{E}_2^T + \mathbf{Q} \dot{\widehat{\mathbf{W}}})) - \tilde{\mathbf{p}}^T (\Gamma \dot{\tilde{\mathbf{p}}} + \mathbf{Y}^T \mathbf{E}_2 \mathbf{s}) \quad (3.32)$$

Inserting the update adaptive laws of Eq. (3.29) into above, we can get

$$\dot{V} = -\mathbf{s}^T \mathbf{E}_2^T \mathbf{K} \mathbf{E}_2 \mathbf{s} - \mathbf{s}^T \mathbf{E}_2^T (\boldsymbol{\tau}_{robust} - \boldsymbol{\epsilon}) \quad (3.33)$$

It should be noted that the robust term which is equal to  $\boldsymbol{\tau}_{robust} = k[\text{sgn}(E_1 s_1) \dots \text{sgn}(E_n s_n)]$  should be larger than the lumped error in order to achieve stability. This can be achieved by selecting  $k \geq \|\boldsymbol{\epsilon}\|$ , so the last equation can be simplified to

$$\begin{aligned} \dot{V} &\leq -\mathbf{s}^T \mathbf{E}_2^T \mathbf{K} \mathbf{E}_2 \mathbf{s} + \|\boldsymbol{\epsilon}\| \|\mathbf{E}_2 \mathbf{s}\| - \mathbf{s}^T \mathbf{E}_2^T \boldsymbol{\tau}_{robust} \\ &\leq -\mathbf{s}^T \mathbf{E}_2^T \mathbf{K} \mathbf{E}_2 \mathbf{s} + k \|\mathbf{E}_2 \mathbf{s}\| - \mathbf{s}^T \mathbf{E}_2^T \boldsymbol{\tau}_{robust} \leq -\mathbf{s}^T \mathbf{E}_2^T \mathbf{K} \mathbf{E}_2 \mathbf{s} \end{aligned} \quad (3.34)$$

Because  $V$  is a positive definite function and  $\dot{V} \leq 0$ , we have  $V \in \mathcal{L}_\infty$  which implies that  $\mathbf{s}$ ,  $\tilde{\mathbf{p}}$ ,  $\widehat{\mathbf{W}} \in \mathcal{L}_\infty$  and  $\mathbf{s} \in \mathcal{L}_2$ . Accordingly, all signals of the investigated system are bounded. So, the latter equation is stable in the sense of Lyapunov's theory.

**Remark 3.6.** In effect, the presence of switching function,  $\text{sgn}(\cdot)$  in the control law could excite the resonance of the system due to the potential chattering. Accordingly, some alternative functions have been used such as saturation function and tangent hyperbolic function; please see [21] for more details.

**Remark 3.7.** In practice, approximation error could be produced due to an insufficient number of approximation terms  $\beta$ . This error could result in parameter drift phenomenon (drift of the predicted parameters to infinity causing instability problems). As a result, modification of adaptive law (Eq. 3.15b) is necessary to compensate for this error. The solution for this problem is performed by using one of robust adaptation law which are [1]: 1. projection modification, 2.  $\sigma$ -modification, 3.  $\epsilon$ -modification, and 4. dead-zone modification. Let us consider the  $\sigma$ -modification for the updating adaptive laws of Eq. (3.29) as follows.

$$\begin{aligned} \dot{\hat{\mathbf{p}}} &= -\Gamma^{-1} \mathbf{Y}^T \mathbf{s} - \Gamma^{-1} \sigma \hat{\mathbf{p}} \\ \dot{\widehat{\mathbf{W}}} &= -\mathbf{Q}^{-1} \boldsymbol{\phi} \mathbf{s}^T - \mathbf{Q}^{-1} \sigma \widehat{\mathbf{W}} \end{aligned} \quad (3.35)$$

where  $\sigma$  refers to a small positive constant. From Eq. (3.16) it is noted that if the modeling error is large, the estimated weighting matrix  $\widehat{W}$  and the uncertain parameters  $\widehat{p}$  drift to infinity. So, the second terms  $(-Q^{-1}\sigma\widehat{W})$  and  $(-\Gamma^{-1}\sigma\widehat{p})$  will be negative large value decreasing the estimated value for the weighting matrix and the uncertain vector respectively. Therefore, the idea of the second term is to prevent the estimation parameters from going to infinity, please see [19] for more details. The designer should keep in mind that these robust adaptive laws do not guarantee that the estimation error converges to zero. In other words, the parameter estimation will be bounded. The reader can follow the work of [29] to prove the definiteness (bounds) of Eq. (3.35).

### 3.4 Simulation Results and Discussion

#### 3.4.1 Two-Link Manipulator in Constrained Circular Arc Space

The two-link manipulator is simulated to verify the effectiveness of the proposed controller, please see Figure 3.1. The target robot is constrained to move along a circular arc. Thus, the DoFs of the target robot are reduced to one generalized coordinate; the robot has one redundant coordinate due to the imposed constraint. Accordingly, the system is controlled based on one generalized coordinate rather than both of the two generalized coordinates. The angular position  $q_1$  is selected as the DoF of the system while  $q_2$  is a redundant coordinate. In the following, some important points are highlighted via simulation to see the validity of the proposed controller.

As stated previously, the task of the controller is to track the desired references of position and contact forces for the investigated robot while estimating the uncertain parameters and the unknown weighting coefficients corresponding to regressor and approximation respectively. It is assumed that there is unmodeled disturbance resulted from viscous friction plus an offset. According to Eqs. (3.24) and (3.29), the regressor technique attempts to estimate the uncertain parameters corresponding to inertia matrix, Coriolis and centripetal matrix and gravity vector. Whereas, the approximation term attempts to compensate for any unmodeled dynamics. One important thing should be noted that the regressor matrix contains terms that are functions of robot state variables and they are highly nonlinear functions. Fortunately, the imposed constraint leads to a very simple  $E_2$  vector which simplifies the equation of motion. For example, if the constraint path represents (vertical or horizontal) straight line, complex regressor may be produced. The strength of the proposed controller lies in that the robot can track the desired references very well although there is an inexact calculation for regressor or other unmodeled dynamics, thanks to the approximation term will make the controller robust for any unconsidered dynamics.

The approximator used in this study is orthogonal Chebyshev polynomials with 11 number of terms ( $\beta$ ) for the approximation of viscous friction and offset term. Increasing the number of terms more than 11 terms may not increase the approximation accuracy. The following feedback and adaptation gains are selected.

$$\Lambda = 100, \mathbf{K} = \text{diag}(25,25), K_I = 100$$

$$\Gamma^{-1} = 30\mathbf{I}_{9 \times 9}, \mathbf{Q}^{-1} = 300\mathbf{I}_{44 \times 44}$$

Figure 3.2 shows the position tracking error for the independent coordinate  $q_1$  (DoF of the robotic system). The position error is relatively small and the robot can track very well the desired angular reference. Figure 3.3 shows the input controls for the target robot. According to Figures 3.4 and 3.5, the estimated values do not converge very well to their actual values; the signals should be persistently excited for good estimation convergence. Despite the problem of convergence of estimation, the control law can work well. On the other hand, the tracking of force references is acceptable, thanks to an integral term which reduces the error as possible.

#### 3.4.2 Significance of the Approximation Term

In effect, the proposed controller is powerful to deal with unmodeled dynamics resulted from the uncertain calculation of regressor or any other disturbances. In light of above, three case studies are compared in terms of position tracking error and input controls as detailed in Table 3.1.

Despite the uncertain calculation of regressor matrix and friction terms, the proposed controller can work well, please see Figures 3.6-8 for position errors and input controls. If the approximation term is neglected from the control law (case study 3), the robot cannot track the desired positions and instability problems may result (see torque response described in Figures 3.7 and 3.9).

### 3.5 Conclusion

This work introduces a robust control strategy that can solve the problem the complexity of regressor. A hybrid regressor and approximation adaptive control is proposed for dealing with uncertainty of robotic systems. The idea behind the proposed controller is that the regressor attempts to estimate the known structure of the investigated system (inertia matrix, Coriolis and centripetal matrix, and gravity vector) while the approximation term of the controller estimates the unmodeled dynamics of the systems resulted from inexact calculation of regressor and other internal/external disturbances.

Extended work is needed to answer the following points:

- i. the best approximator that should be selected to get optimal solution
- ii. compensation technique for the resulted approximation error. Using the continuous hyper tangent function is good selection but with updating gain.
- iii. avoidance of algebraic loop resulted from forces feedback using the contact force estimator.

### Acknowledgements

The author would like to acknowledge Dr. Wen-Hong Zhu at the Canadian Space Agency, for his valuable recommendations and discussions concerning this work.

Table 3.1. The investigated case studies for highlighting significance of the proposed controller

Case study No.	Control Law	Description
1	Eqs. (24) and (29)	In this case study, the regressor is calculated precisely with disturbance resulted from viscous friction and offset term, it is exactly the same case study investigated in previous section.
2	Eqs. (24) and (29)	Here it is assumed that there are some wrong calculations for regressor matrix. Six uncertain parameters are selected arbitrarily with inexact $2 \times 6$ regressor matrix.
3	Eqs. (19) and (20)	In this study, the approximation term is cancelled from the control law, regressor-based control is used for control purposes. The disturbances of the system consist of uncertain calculation of regressor, viscous friction and offset.

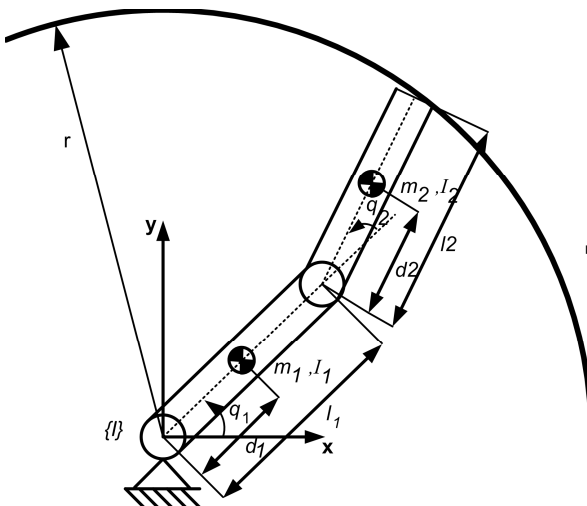


Figure 3.1. Two-link manipulator with the constrained path

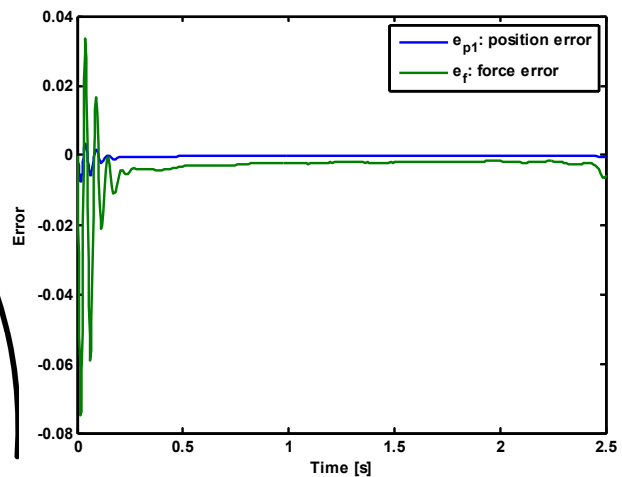


Figure 3.2. Position error for the independent coordinate  $q_1$ , and the contact force error resulted from the imposed constraint

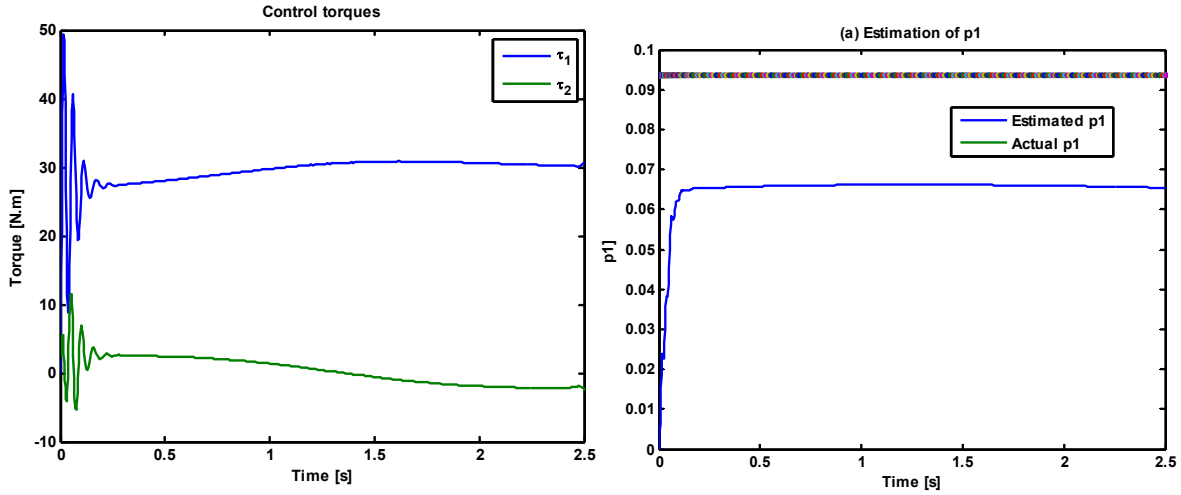
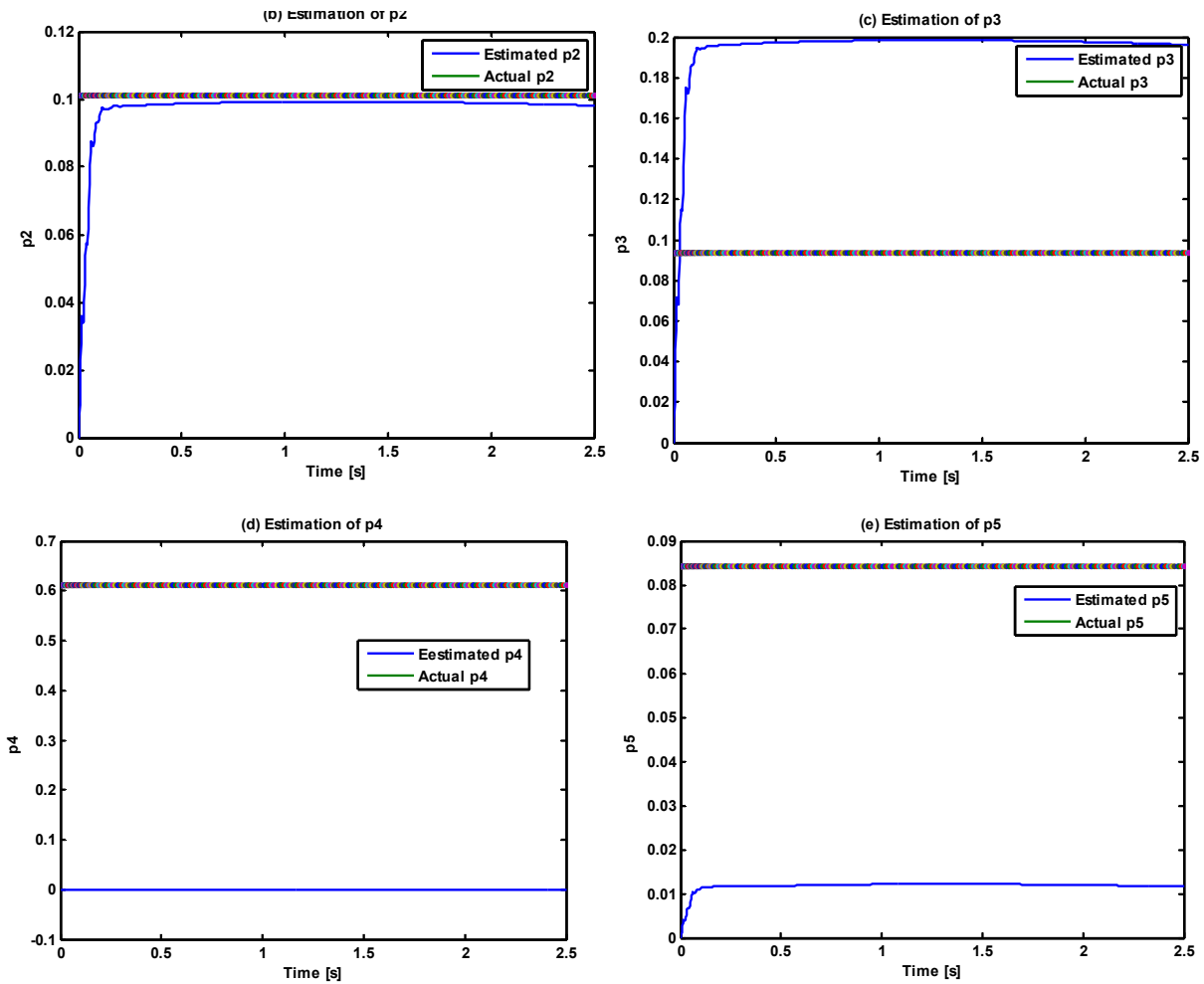


Figure 3.3. Input controls of joint actuators



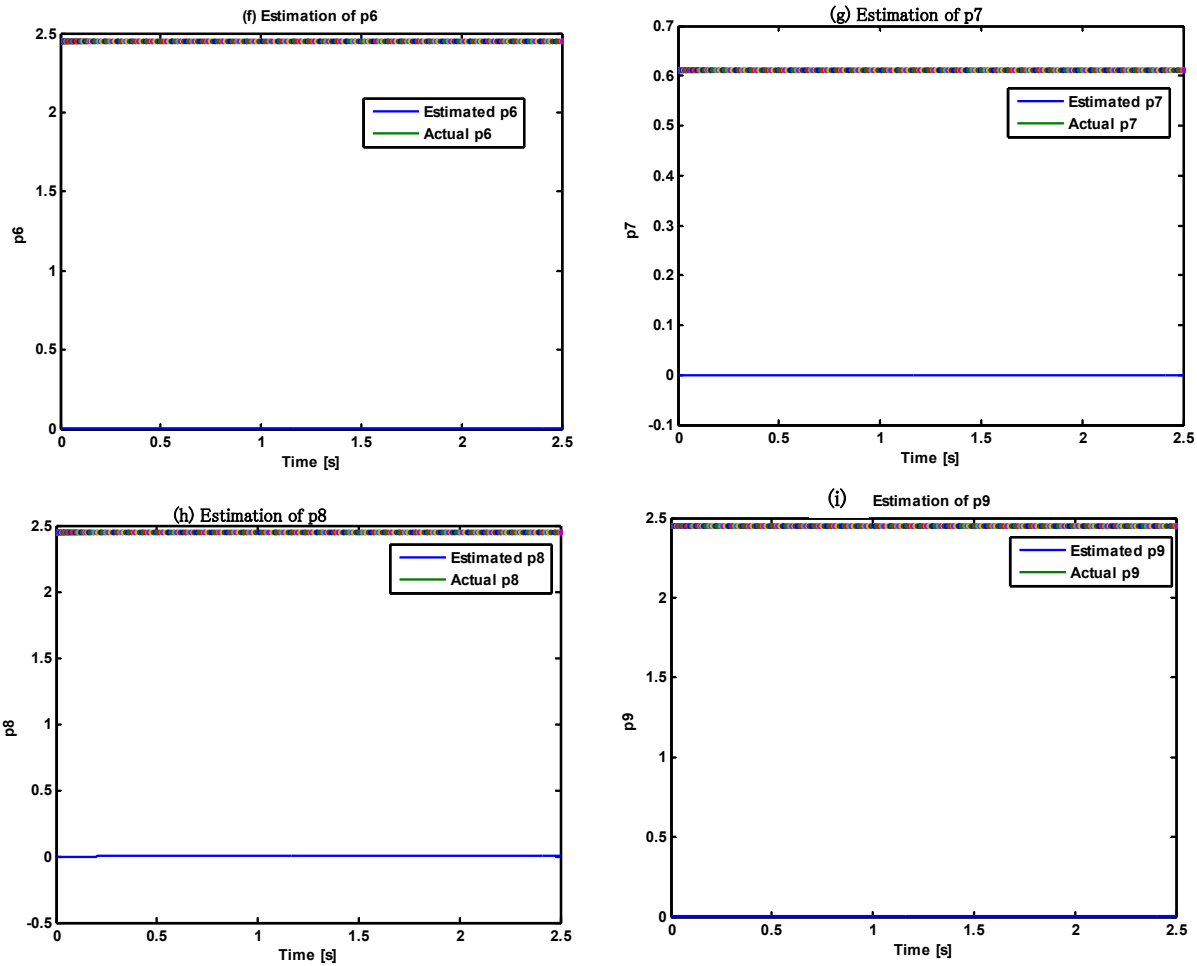


Figure 3.4. Estimation of uncertain parameters vector ( $\mathbf{p}$ ) corresponding to regressor term; they are equal to 9 parameters. Initial values for these parameters are assumed all zero which mean that there is no available information about these parameters.

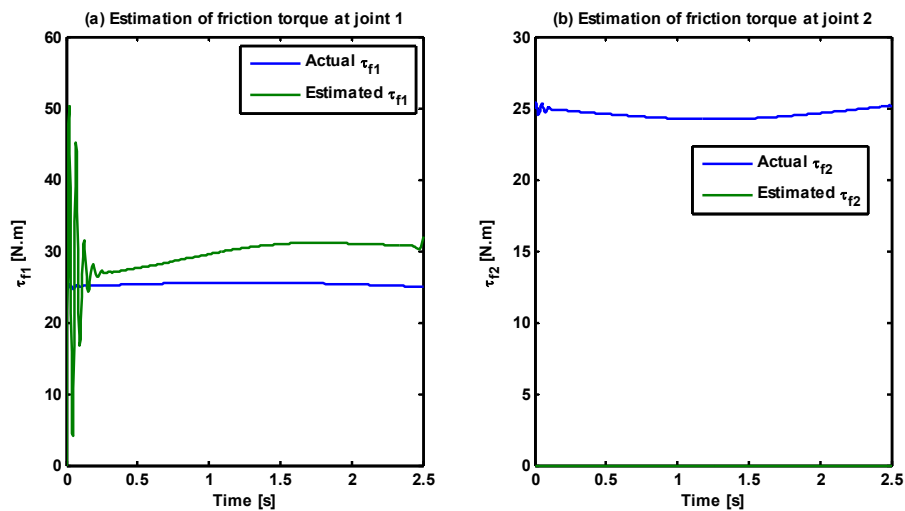


Figure 3.5. The estimation of disturbance resulted form viscous friction and an offset. These estimations are based on approximation technique

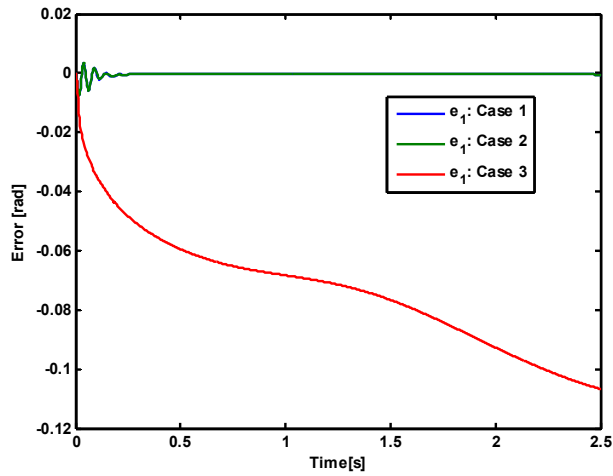


Figure 3.6. Comparison of  $e_1$  for the 3 case studies

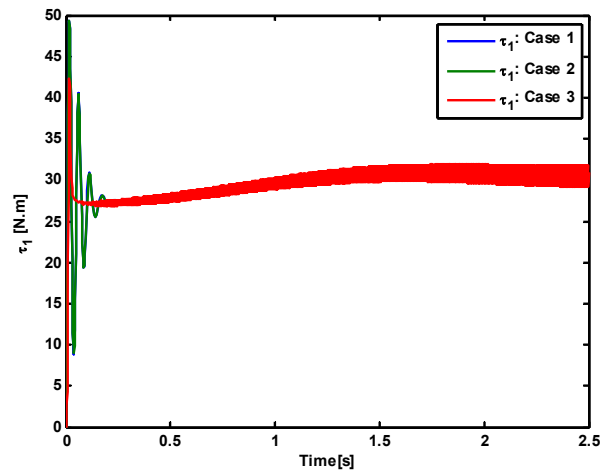
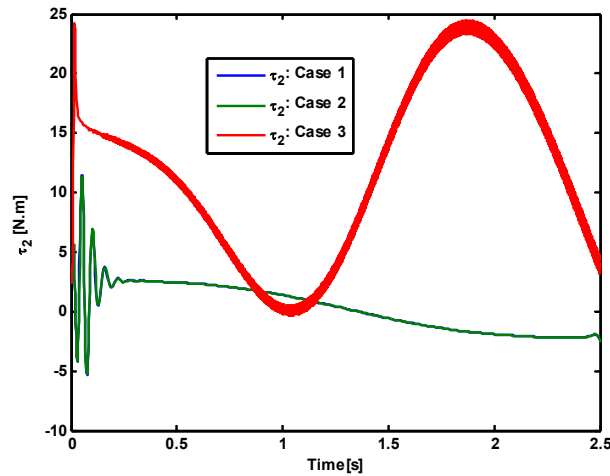


Figure 3.7. Comparison of  $\tau_1$  for the 3 case studies

Figure 3.8. Comparison of  $\tau_2$  for the 3 case studies



## References

- [1] Spong MW, Vidyasagar M. Robot dynamics and control. John Wiley & Sons, USA, 1998.
- [2] Colbaugh R, Seraji H, Glass K. Impedance control for dexterous space manipulators. The 31<sup>st</sup> Conf. on Decision and Control, Tucson, Arizona, pp. 1881–1886; 1992.
- [3] Hogan N. Control of mechanical impedance of prosthetic arms. American Control Conference, San Francisco, USA, pp. 1-5; 1980.
- [4] Whitney DE. Force feedback control of manipulator fine motions. ASME J. of Dync. Sys. Meas. Contr. 99, 91–97; 1977.
- [5] Khatib O. A Unified approach for motion and force control of robot manipulators: The Operational Space Formulation. IEEE J. of Robotics and Automat. 1987; RA-3 (1), 43–53.
- [6] Raibert MH, Craig JJ. Hybrid position / force control of manipulators. ASME J. of Dync. Sys. Meas. Contr. 1981; 102, 126–133.
- [7] Mason MT. Compliance and force control for computer controlled manipulators. IEEE Trans. on Sys. Man. Cyber. 1981; SMC-11, 418–432.
- [8] Anderson R, Spong MW. Hybrid impedance control of robotic manipulators. IEEE J. of Robotics and Automat. 1987; 4, 1073-1080.
- [9] Salisbury JK. Active stiffness control of a manipulator in Cartesian coordinates. The 19<sup>th</sup> IEEE Conf. on Decision and Control, Albuquerque, pp.95 – 100; 1981.
- [10] Salisbury JK, Craig JJ. Articulated hands: Force control and kinematic issues. Int. J. of Robotics Res. 1982; 1, 4–17.
- [11] Whitney DE. Historical perspective and state of the art in robot force control. Int. J. of Robotics Res. 1987; 6, 3 – 14.
- [12] Volpe R., Khosla P. An experimental evaluation and comparison of explicit force control strategies for robotic manipulators. American Control Conference, Chicago IL. pp.758–764; 1992.
- [13] Volpe R, Khosla P. A theoretical and experimental investigation of explicit force control strategies for manipulators. IEEE Trans. on Auto. Contr. 1993; 38 (11), 1634–1650.
- [14] Zeng G, Hemami A. An overview of robot force control. Robotica, 1997; 14, 473-482.
- [15] Arimoto S, Naniwa T, Liu YH. Model-based adaptive hybrid control for manipulators with geometric endpoint constraint. Advanced Robotics, 1995; 9, 67–80.
- [16] Carelli R, Kelly R. An adaptive impedance/force controller for robot manipulators. IEEE Transactions on Automatic Control, 1991; 36, 967–971.
- [17] Parra-Vega V, Arimoto S. A passivity-based adaptive sliding mode position-force control for robot manipulators. International Journal of Adaptive Control and Signal Processing, 1996; 10, 365–377.

- [18] Whitcomb LL, Arimoto S, Naniwa T, Ozaki F. Adaptive model-based hybrid control of geometrically constrained robot arms. *IEEE Transactions on Robotics and Automation*, 1997; 13, 105–116.
- [19] Farrell JA, Polycarpou MM. Adaptive approximation based control: Unifying neural, fuzzy and traditional adaptive approximation approach. John Wiley & Sons, Inc, USA; 2006.
- [20] Ioannou P, Fidan B. Adaptive control tutorial. SIAM; 2006.
- [21] Wang J, Ge SS, Lee TH. Adaptive friction compensation chapter in *Adaptive Control of Nonsmooth Dynamic Systems*, pp. 211-248; 2001.
- [22] Armstrong B. Friction: experimental determination, modeling and compensation. *IEEE International Conference on Robotics and Automation*, 1988; 3, 1422-1427.
- [23] Zhu W-H. Virtual Decomposition control: toward hyper degrees of freedom robots. Springer-Verlag, Berlin, Heidelberg; 2010.
- [24] Al-Shuka HFN, Corves B, Zhu W-H. Function approximation technique-based adaptive virtual decomposition control for a serial-chain manipulator, *Robotica*, 2014; 32 (03), 375-399.
- [25] Olsson H, Aström KJ, Canudas de Wit C, Gäfvert M, Lischinsky P. Friction models and friction compensation. *European Journal of Control*, 1998; 4(3), 176-195.
- [26] Lewis FL, Yesildirek A, Liu K. Multilayer neural net robot controller with guaranteed tracking performance. *IEEE Trans Neural Netw.* 1996; 7, 1–12.
- [27] Lewis FL, Liu K, Yesildirek A. Neural net robot controller with guaranteed tracking performance. *IEEE Trans. Neural Netw.* 1995b ; 6, 703–716.
- [28] Lewis FL, Yesildirek A, Liu K. Neural net robot controller: Structure and stability proofs. *J. Intell Robot Syst.* 1995a; 13, 1–23.
- [29] Huang A-C, Chien M-C. Adaptive control of robot manipulators: A unified regressor-free approach. World Scientific Publishing Co. Pte. Ltd. Singapore; 2010.
- [30] Liu J. Radial basis function (RBF) neural network control for mechanical systems: Design, analysis and Matlab simulation. Tsinghua University Press, Beijing and Springer-Verlag Berlin Heidelberg; 2013.
- [31] Cong S, Liang Y, Shang W. Function approximation-based sliding mode adaptive control for time-varying uncertain nonlinear systems. *Nonlinear Dynamics*, 2008; 54(5), 223-230.
- [32] Su C-Y, Leung TP, Zhou Q-J. Force/motion control of constrained robots using sliding mode. *IEEE Transactions on Automatic Control*, 1992; 37, 668-672.
- [33] Pennestri E, Valentini PP. Coordinate reduction strategies in multibody dynamics: a review, In *Atti Conference on Multibody System Dynamics*; 2007.
- [34] Al-Shuka HFN. Modeling, walking pattern generators, and adaptive control of biped robot. PhD Dissertation, RWTH Aachen University; 2014.
- [35] Al-Shuka HFN, Corves B, Zhu W-H. Dynamic modeling of biped robot using Lagrangian and recursive Newton-Euler formulations. *International Journal of Computer Applications*, 2014; 101, 1-8.
- [36] Lian T, Goldenberg A. Adaptive and sliding control of flexible joint robots in constrained motion. *IEEE International Conference on Systems, Man and Cybernetics*; 1995.
- [37] Al-Shuka HFN, Corves B, Vanderborght B, Zhu W-H. Finite difference-based suboptimal trajectory planning of biped robot with continuous dynamic response. *International Journal of Modeling and Optimization*, 2013; 3, 337-3.
- [38] Al-Shuka HFN. On local approximation-based adaptive control with applications to robotic manipulators and biped robots. *International Journal of Dynamics and Control*. pp. 1-15; 2017

## Chapter 4: Decentralized adaptive partitioned approximation control of robots considering three motor control modes

Partitioned approximation control is avoided in most decentralized control algorithms; however, it is essential to design a feedforward control term for improving the tracking accuracy of the desired references. In addition, consideration of actuator dynamics is important for a robot with high-velocity movement and highly varying loads. As a result, this work is focused on decentralized adaptive partitioned approximation control for complex robotic systems using the orthogonal basis functions as strong approximators. In essence, the partitioned approximation technique is intrinsically decentralized with some modifications. Three actuator control modes are considered in this study: (i) a torque control mode in which the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known, (ii) a current control mode in which the torque/current constant is unknown, and (iii) a voltage control mode with no current servo control being available. The proposed decentralized control law consists of three terms: the partitioned approximation-based feedforward term that is necessary for precise tracking, the high gain-based feedback term, and the adaptive sliding gain-based term for compensation of modeling error. The passivity property is essential to prove the stability of local stability of the individual subsystem with guaranteed global stability. Two case studies are used to prove the validity of the proposed controller: a two-link manipulator and a six-link biped robot.

### 4.1 Introduction

For complex robotic systems, such as humanoid robots or any robot having a number of degrees of freedom (DOFs) larger than 6-DOFs, difficulties are encountered in the implementation of the control algorithms. Therefore, over 30 years, the robotics researchers have focused on the problem of computational efficiency. Many efficient  $O(n)$  algorithms have been developed for inverse [1-6] and forward dynamics [7-10] of robotic systems. For more details on the efficient dynamic algorithms, we refer to [11,12].

However, the adaptive control algorithm that deals with controlling the robotic systems despite their uncertain parameters may decrease the computational efficiency of the dynamics  $O(n)$  algorithms. K. S. Fu et al. [13] have shown that the combined identification and control algorithms can be computed in  $O(n^3)$  time despite using recursive Newton-Euler (NE) formulation. Therefore, most researchers have used a local controller (decentralized controller) such as PID family for controlling complex dynamic systems [14-19]. However, Liu [20] proved that decentralized PD control for robotic manipulator cannot ensure global stability. Therefore, the author proposed decentralized PD + nonlinear cubic term for ensuring global stability. A modified version of Liu's work has been proposed by Hsu and Fu. Yang et al. [21] have used a disturbance observer and an adaptive sliding mode control strategy for compensation of low-pass and high-pass coupled uncertainties respectively. In effect, most available control strategies are based on designing PD control plus a robust and/or adaptive term(s) for compensation of modeling errors and the interconnections. Jasim and Plapper [22] used the sliding common Lyapunov function and sliding mode-based decentralized control for robotic manipulators in free and constrained spaces.

Apart from conventional robust control, the adaptive approximation control is a powerful tool to control robotic manipulator with unknown dynamics. There are two possible categories of adaptive approximation techniques: lumped approximation-based adaptive control [23-26,30,61,62] and partitioned approximation-based adaptive control [27-29,60]. The former deals with collecting the uncertainty in one term and using the linear-in-the-parameters property to represent the uncertainty in terms of weighting and basis-function matrices. Then designing control law based on updating the weighting-coefficient matrix using Lyapunov stability. This strategy requires nominal (ideal) estimation for the unknown parameters. On the other hand, partitioned approximation-based adaptive control approximates each dynamic matrix of the equation of motion of the target robot separately using the linear-in-the-parameters property. The control law is then designed based on updating the weighting-coefficient matrices of the corresponding basis-function matrices using Lyapunov stability. To our knowledge, the partitioned approximation does not require nominal estimation for the dynamic coefficients that eases the control task. However, most decentralized approximation control [20-22,31-37] have used lumped approximation term combined with either PD term or nominal values. Banagi and Polycarpou [31] used the decentralized neural network (NN) control for adaptive control of the large-scale system. The unknown dynamics and the unknown interconnections are approximated by using linear parameterized NN assuming that the interconnection term is bounded by summation of analytic functions in terms of error. The dynamics are formulated in state space model. Tan et al. [32] used decentralized NN adaptive control for robotic manipulators. It is a model-free control strategy that is based on NN control, but the authors did not consider the actuator dynamics in their proposed strategy.

Consideration of actuator dynamics is of importance for a robot with high-velocity movement and highly varying loads [27]. Fateh and Fateh [33] used decentralized fuzzy voltage control for robotic manipulators. The lumped uncertainty of interconnections at voltage level is approximated using fuzzy compensator. The authors considered the actuator dynamics using the voltage control mode. Therefore, Huang and Chen [27] investigated the effect of actuator dynamics on the performance of the actuator dynamics assuming that the motor torque/current constant is known; however, their control strategy was fully centralized. Zhu [42] proposed three motor control modes using the distributed virtual decomposition control. According to his strategy, an electric motor can be in the motor torque control mode when the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known. Otherwise, an electric motor should be in the motor current control

mode when only the armature current is well controlled but the torque/current constant is unknown. Finally, an electric motor must be in the motor voltage control mode when no current servo control is available. For more details on actuators dynamics, the reader is referred to [41]. In addition, for more work on decentralized approximation control techniques, see [34-38].

In view of above, the partitioned approximation control is avoided in most decentralized control algorithms. However, it can be a strong solution to control high DOF robots since it is decentralized in nature but with some modifications. In this study, two important issues are considered: i) design of decentralized control law exploiting the feature of partitioned approximation technique, and ii) consideration of three actuator control modes: motor torque mode, motor current mode, and motor voltage mode. The proposed control law consists of three terms: partitioned approximation-based feedforward term, high-gain-based feedback term, and adaptive sliding mode term for compensating for modeling errors. The second issue coincides with virtual decomposition control proposed by Zhu [42]. These control modes are applied in sequence for adaptive control purposes without considering prior information of robot dynamics parameters, friction, actuator dynamics parameters, etc. In effect, this work is an extended version of the conference paper [60] in which the actuator dynamics are not considered and the focus was on torque control mode only.

The remainder of the chapter is organized as follows. Section 4.2 introduces preliminaries and motivation for the problem. The methodology is presented in Section 2.3. Section 2.4 described simulation results and discussions. Section 2.5 concludes.

## 4.2 Preliminaries and Motivation

### 4.2.1 Preliminaries

As stated previously, the function approximation technique (FAT) is an essential tool for approximating the unknown parameters of a dynamic system for adaptive control purposes. In general, there are two essential adaptive approximation techniques: lumped approximation technique (global approximation) [23-26,30,61,62], and partitioned approximation (local approximation) techniques [27-29,60]. The former approximation technique collects all uncertain parameters in one lumped vector term. The lumped vector can then be represented as linear combinations of basis functions using miscellaneous approximators (splines, orthogonal functions, intelligent techniques etc.). Its main drawback is the need for nominal (simplified) model. On the other hand, the partitioned approximation technique attempts to approximate (linearly parameterize) each dynamic matrix and vector (mass and Coriolis matrices, gravity vector, friction vector etc.) separately. In general, the procedure employed in constructing the FAT-based adaptive control includes a selection of a suitable approximator for the uncertain parameters, choosing an appropriate adaptation law for the weighting coefficient and designing the controller structure [29,30]. There are different classes of approximators such as polynomials, splines, radial basis functions, Cerebellar Model Articulation Controller (CMAC), multilayer perceptron, fuzzy approximation and wavelets. In this chapter, we will focus on orthogonal functions due to their simplicity and capability of achieving minimum approximation errors [27-30].

**Theorem 1 (Weierstrass Approximation Theorem).** [31] Let  $f(x)$  be a continuous function on  $[a, b]$ , then for all  $\epsilon > 0$ , there exists a sequence of polynomials  $p_\beta(x)$  of degree  $< \beta$  that converges uniformly to  $f(x)$  on  $[a, b]$ , i.e.  $\|f(x) - p_\beta(x)\| < \epsilon$  where

$$p_\beta(x) = \sum_{j=1}^{\beta} w_j \phi_j(x) \quad (4.1)$$

where  $w_j$  and  $\phi_j$  are the weighting coefficients and basis functions respectively.

This theorem indicates that each continuous function on a compact interval can be approximated by polynomials with any degree. The scope of this work focuses on orthogonal basis functions.

**Definition 1.** [27] For any set of orthonormal functions  $\{\phi_j(x)\}$ , ( $\forall j = 1, 2, \dots, \beta$ ) on  $[a, b]$ , an arbitrary function  $f(x)$  can be approximated using Eq. (1) under conditions of Theorem 1 with the following weighting coefficients being calculated as

$$w_j = \frac{\int_a^b f(x) \phi_j(x) dx}{\int_a^b \phi_j^2(x)} \quad (4.2)$$

The following important points should be noted [27, 39]:

- Equation (4.2) is computed using the orthogonal property  $\int_a^b f(x) \phi_i \phi_j dx \begin{cases} = 0 & i \neq j \\ \neq 0 & i = j \end{cases}$
- To guarantee convergence of the approximating series, the orthogonal set should be complete.
- There are many orthogonal basis functions: Chebyshev polynomials, Legendre polynomials, Laguerre polynomials, etc.
- The Chebyshev approximation formula is very close to the minimax polynomial since the approximation error is spread smoothly over the period  $[-1, 1]$ . Accordingly, this orthogonal approximation polynomials will be used as strong approximators in the current study.

### 4.2.2 Motivation

To motivate the core of this work, let us consider the following equation of motion of  $n$ -joint robotic manipulators in free space

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_l \quad (4.3)$$

where  $\mathbf{M} \in \mathbb{R}^{n \times n}$  is the inertia matrix of the links,  $\mathbf{C} \in \mathbb{R}^{n \times n}$  is the Coriolis and centripetal matrix of the links,  $\mathbf{g} \in \mathbb{R}^{n \times 1}$  is the gravity vector,  $\mathbf{q} \in \mathbb{R}^{n \times 1}$  represents the displacement output joint variables after gear transmission, and  $\boldsymbol{\tau}_l \in \mathbb{R}^{n \times 1}$  denotes to the output joint torques.

However, the detailed analysis of robotic manipulators dynamics considering actuator dynamics will be introduced in the next section.

The following properties assumptions are necessary for adaptive approximation control (centralized and decentralized version) [28,40].

**Property 4.1.** The inertia matrix, Coriolis and centrifugal matrix, and the gravity vectors are uniformly bounded.

The last property can be extended to include that the dynamic coefficients of the elements of each matrix/vector are uniformly bounded and satisfying theorem 4.1.

**Property 4.2 (Passivity property).** The matrix  $\mathbf{N} = \dot{\mathbf{M}} - 2\mathbf{C}$  is a skew-symmetric matrix, i.e.,  $\mathbf{s}^T \mathbf{N} \mathbf{s} = 0$ , if  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is defined using the Christoffel symbols.

According to the property 4.2, each diagonal element of  $\mathbf{N}$ , with  $k, j \in n$ , is equal to zero. This property is essential to prove the stability of the proposed decentralized control law.

**Assumption 4.1.** The state variables of the target robots are measurable and bounded.

**Assumption 4.2.** Each entry of dynamic matrices of  $\mathbf{M}(\mathbf{q})$ ,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ , and  $\mathbf{g}(\mathbf{q})$  satisfies conditions of Theorem 1, i.e., they can be represented as linear combinations of orthogonal basis functions.

The last assumption includes that each entry of the dynamic matrices can be bounded by analytic and continuous functions. Please for more details on properties of the dynamic matrices, the gravity vector and the residual dynamics, the reader is referred to [28,40,41].

In view of the partitioned approximation technique introduced in [27-29], Eq. (4.3) can be reformulated as

$$\mathbf{W}_M^T \Phi_M \ddot{\mathbf{q}} + \mathbf{W}_C^T \Phi_C \dot{\mathbf{q}} + \mathbf{W}_g^T \Phi_g + \boldsymbol{\epsilon} = \boldsymbol{\tau}_l \quad (4.4)$$

where  $\mathbf{W}_M \in \mathbb{R}^{n\beta \times n}$ ,  $\mathbf{W}_C \in \mathbb{R}^{n\beta \times n}$  and  $\mathbf{W}_g \in \mathbb{R}^{n\beta \times n}$  are the weighting matrices, while  $\Phi_M \in \mathbb{R}^{n\beta \times n}$ ,  $\Phi_C \in \mathbb{R}^{n\beta \times n}$ , and  $\Phi_g \in \mathbb{R}^{n\beta}$  are the basis-function matrices.  $\boldsymbol{\epsilon} \in \mathbb{R}^n$  is the accumulated modeling error vector.

For a 2-DOF robot, Eq. (4.4) can be written as

$$\begin{bmatrix} \mathbf{w}_{m11}^T \Phi_{m11} & \mathbf{w}_{m12}^T \Phi_{m12} \\ \mathbf{w}_{m21}^T \Phi_{m21} & \mathbf{w}_{m22}^T \Phi_{m22} \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{c11}^T \Phi_{c11} & \mathbf{w}_{c12}^T \Phi_{c12} \\ \mathbf{w}_{c21}^T \Phi_{c21} & \mathbf{w}_{c22}^T \Phi_{c22} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{g1}^T \Phi_{g1} \\ \mathbf{w}_{g2}^T \Phi_{g2} \end{bmatrix} = \boldsymbol{\tau}_l \quad (4.5)$$

According to Eq. (4.5), the followings are noted:

- The decoupled dynamics of each DOF can be represented as

$$\begin{aligned} \mathbf{w}_{m11}^T \Phi_{m11} \ddot{q}_1 + \mathbf{w}_{c11}^T \Phi_{c11} \dot{q}_1 + \underbrace{\mathbf{w}_{m12}^T \Phi_{m12} \ddot{q}_2 + \mathbf{w}_{c12}^T \Phi_{c12} \dot{q}_2 + \mathbf{w}_{g1}^T \Phi_{g1}}_{\text{coupling torque}=\tau_{c1}} &= \tau_{l1} \\ \mathbf{w}_{m22}^T \Phi_{m22} \ddot{q}_2 + \mathbf{w}_{c22}^T \Phi_{c22} \dot{q}_2 + \underbrace{\mathbf{w}_{m21}^T \Phi_{m21} \ddot{q}_1 + \mathbf{w}_{c21}^T \Phi_{c21} \dot{q}_1 + \mathbf{w}_{g2}^T \Phi_{g2}}_{\text{coupling torque}=\tau_{c2}} &= \tau_{l2} \end{aligned} \quad (4.6)$$

- The whole dynamics of the target robot is intrinsically decoupled by using the partitioned approximation technique, i.e.

$$\begin{aligned} \mathbf{w}_{m12}^T \Phi_{m12} \ddot{q}_2 + \mathbf{w}_{c12}^T \Phi_{c12} \dot{q}_2 + \mathbf{w}_{g1}^T \Phi_{g1} &= \tau_{c1} \\ \mathbf{w}_{m11}^T \Phi_{m11} \ddot{q}_1 + \mathbf{w}_{c11}^T \Phi_{c11} \dot{q}_1 + \tau_{c1} &= \tau_{l1} \\ \mathbf{w}_{m21}^T \Phi_{m21} \ddot{q}_1 + \mathbf{w}_{c21}^T \Phi_{c21} \dot{q}_1 + \mathbf{w}_{g2}^T \Phi_{g2} &= \tau_{c2} \\ \mathbf{w}_{m22}^T \Phi_{m22} \ddot{q}_2 + \mathbf{w}_{c22}^T \Phi_{c22} \dot{q}_2 + \tau_{c2} &= \tau_{l2} \end{aligned} \quad (4.7)$$

These equations can recursively be solved; however, this technique results in a number of equations that are equal to  $n^2$  which can be computationally tedious.

- The partitioned approximation technique has flexibility in decentralization, e.g., a 6-DOF robot can be decoupled into two 3-DOF subsystems and so on.

### 4.3 Methodology

#### 4.3.1 Dynamics of contact-free motion robots

The dynamics of electrically driven  $n$ -joint robot consist of three subsystems: the link dynamics, the joint dynamics, and armature controlled DC motor dynamics (Fig. 4.1). The classical Euler-Lagrangian formulation of the whole system dynamics can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{g} = \boldsymbol{\tau}_l \quad (4.8a)$$

$$\mathbf{I}_m \ddot{\mathbf{q}}_m + \boldsymbol{\tau}'_f = \boldsymbol{\tau}'_m - \mathbf{G} \boldsymbol{\tau}_l \quad (4.8b)$$

$$\mathbf{L} \dot{\mathbf{i}} + \mathbf{R} \mathbf{i} + \mathbf{B}' \dot{\mathbf{q}}_m = \mathbf{u} \quad (4.8c)$$

where  $\mathbf{q}_m \in \mathbb{R}^n$  is the angular motor displacement before the gear transmission and equal to  $\mathbf{G}^{-1} \mathbf{q}$  with  $\mathbf{G} \in \mathbb{R}^{n \times n}$  referring to the diagonal gear-ratio matrix,  $\mathbf{I}_m \in \mathbb{R}^{n \times n}$  is a diagonal effective inertia matrix of the actuator,  $\boldsymbol{\tau}'_f$  is the joint friction torques, and  $\boldsymbol{\tau}'_m$  is the motor torque vector which is equal to  $\mathbf{K}_\tau \mathbf{i}$ , with  $\mathbf{K}_\tau \in \mathbb{R}^{n \times n}$  denoting to the torque constant, and  $\mathbf{i} \in \mathbb{R}^n$  referring to the armature current,  $\mathbf{L} \in \mathbb{R}^{n \times n}$  is a diagonal inductance matrix,  $\mathbf{R} \in \mathbb{R}^{n \times n}$  is a diagonal resistance matrix,  $\mathbf{B}' \in \mathbb{R}^{n \times n}$  is a diagonal matrix representing the EMF constant, and  $\mathbf{u} \in \mathbb{R}^n$  is the input voltage control. Substituting Eqs. (8b) into Eq. (8a) leads to the simplified full dynamics of robotic manipulators

$$\mathbf{D} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{g} + \boldsymbol{\tau}_f = \boldsymbol{\tau}_m \quad (4.9a)$$

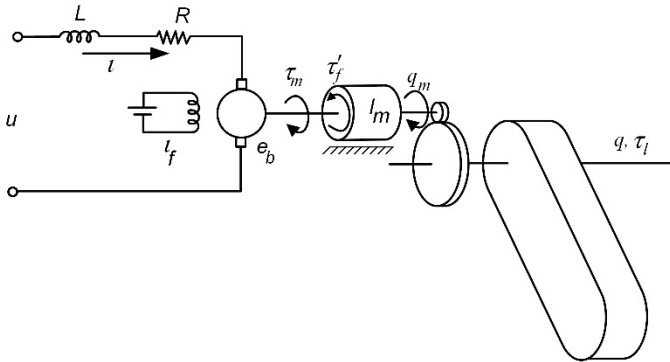
$$\boldsymbol{\tau}_m = \mathbf{H} \mathbf{i} \quad (4.9b)$$

$$\mathbf{L} \dot{\mathbf{i}} + \mathbf{R} \mathbf{i} + \mathbf{B} \dot{\mathbf{q}} = \mathbf{u} \quad (4.9c)$$

with  $\mathbf{D} = \mathbf{M} + \mathbf{G}^{-2} \mathbf{I}_m$ ,  $\boldsymbol{\tau}_f = \mathbf{G}^{-1} \boldsymbol{\tau}'_f$ ,  $\mathbf{H} = \mathbf{G}^{-1} \mathbf{K}_\tau$ ,  $\mathbf{B} = \mathbf{B}' \mathbf{G}^{-1}$

**Remark 4.1.** Equation (4.9) neglects the coupling effects between rotor and link motion [59]. This assumption has been made for electrically driven robots provided with high gear ratio. Due to the high gear ration, the motor rotor moves faster along its axis than other directions and hence its motion is a pure rotation with respect to an inertial frame [42].

**Remark 4.2.** It is well-known that the electric time constant is smaller than the mechanical time constant leading to neglecting the inductance matrix. This assumption leads to simplified robot dynamics with reduced actuator dynamics. In other words, the full dynamics of robots can be simplified to one equation, please see chapter 7 of [15] for more details.



**Fig. 4.1.** Schematic diagram showing the dynamics of the  $k$ -th subsystem consisting of a link, joint, and armature controlled DC motor with  $I_f$  being denoted to the constant field current.

#### 4.3.2 Decomposition approach

Based on Eq. (4.9), the  $k$ -th subsystem of the target robot, in which  $k = 1, 2, \dots, n$ , can be expressed as

$$\sum_{j=1}^n d_{kj}(q) \ddot{q}_j + \sum_{j=1}^n c_{kj}(q, \dot{q}) \dot{q}_j + g_k(q) + \tau_{fk}(q, \dot{q}) = \tau_{mk} \quad (4.10a)$$

$$\tau_{mk} = h_{kk} \iota_k \quad (4.10b)$$

$$L_{kk} \dot{\iota}_k + R_{kk} \iota_k + B_{kk} \dot{q}_k = u_k \quad (4.10c)$$

where  $d_{(\cdot)}$ ,  $c_{(\cdot)}$ ,  $g_k$ ,  $\tau_{fk}(q, \dot{q})$ ,  $\tau_{mk}$ ,  $h_{kk}$ ,  $\iota_k$ ,  $L_{kk}$ ,  $R_{kk}$ ,  $B_{kk}$ ,  $\dot{q}_k$ , and  $u_k$  are the corresponding elements of the target matrix/vector.

Eq. (4.10a) is a highly coupled nonlinear system, whereas Eq. (10b,c) is fully decoupled. Accordingly, Eq. (4.10a) should be modified to meet the conditions of decomposition. As aforementioned, the objective of decoupled control is to control every DOF individually; therefore, Eq. (10a) can be further re-written as

$$d_{kk}(q) \ddot{q}_k + c_{kk}(q, \dot{q}) \dot{q}_j + \sum_{j=1, j \neq k}^n d_{kj}(q) \ddot{q}_j + \sum_{j=1, j \neq k}^n c_{kj}(q, \dot{q}) \dot{q}_j + g_k(q) + \tau_{fk}(q, \dot{q}) = h_{kk} \iota_k \quad (4.11)$$

or alternatively,

$$d_{kk}(q)\ddot{q}_k + c_{kk}(q, \dot{q})\dot{q}_k + \Delta_k(q, \dot{q}) = \tau_{mk} \quad (4.12a)$$

with

$$\Delta_k = \sum_{\substack{j=1 \\ j \neq k}}^n d_{kj}(q)\ddot{q}_j + \sum_{\substack{j=1 \\ j \neq k}}^n c_{kj}(q, \dot{q})\dot{q}_j + g_k(q) + \tau_{fk}(q, \dot{q})$$

$$\tau_{mk} = h_{kk}t_k \quad (4.12b)$$

$$L_{kk}\dot{i}_k + R_{kk}i_k + B_{kk}\dot{q}_k = u_k \quad (4.12c)$$

Equation (4.12) represents the standard dynamic representation that will be adopted to solve the problem of decoupled control.

**Assumption 4.3.** The interconnection/disturbance term  $\Delta_k(q, \dot{q})$  is bounded with analytic and continuous functions satisfying theorem 1.

**Remark 4.3.** The interconnection/disturbance term is assumed bounded with smooth analytic functions, e.g., see [31,32]. Accordingly, this assumption is enough to decompose this term as a linear combination of basis functions without using the inverse analysis proposed in most work.

Let us recall the partitioned approximation technique described in details in [29]. Accordingly, the approximation-based dynamics can be expressed as

$$\begin{bmatrix} \mathbf{w}_{D11}^T \boldsymbol{\varphi}_{D11} & \cdots & \mathbf{w}_{D1n}^T \boldsymbol{\varphi}_{D1n} \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{Dn1}^T \boldsymbol{\varphi}_{Dn1} & \cdots & \mathbf{w}_{Dnn}^T \boldsymbol{\varphi}_{Dnn} \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{C11}^T \boldsymbol{\varphi}_{C11} & \cdots & \mathbf{w}_{C1n}^T \boldsymbol{\varphi}_{C1n} \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{Cn1}^T \boldsymbol{\varphi}_{Cn1} & \cdots & \mathbf{w}_{Cnn}^T \boldsymbol{\varphi}_{Cnn} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{w}_{\Delta 1}^T \boldsymbol{\varphi}_{\Delta 1} \\ \vdots \\ \mathbf{w}_{\Delta n}^T \boldsymbol{\varphi}_{\Delta n} \end{bmatrix} + \boldsymbol{\epsilon} = \boldsymbol{\tau} \quad (4.13)$$

where  $\mathbf{w}_{(\cdot)} \in \mathbb{R}^\beta$ ,  $\boldsymbol{\varphi}_{(\cdot)} \in \mathbb{R}^\beta$  represent the weighting-coefficient and basis-function vectors respectively, and  $\boldsymbol{\epsilon} \in \mathbb{R}^n$  denotes to the accumulated modeling error vector.

According to Eq. (4.13), the partitioned approximation technique has inherently decentralization features that can be exploited for decentralized control. In view of properties 1,2, assumptions 1,2, and Eq. (4.13), Eq. (4.12a) can be reformulated as

$$\mathbf{w}_{Dk}^T \boldsymbol{\varphi}_{Dk} \ddot{q}_k + \mathbf{w}_{Ck}^T \boldsymbol{\varphi}_{Ck} \dot{q}_k + \mathbf{w}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k} + \epsilon_k = \tau_k \quad (4.14)$$

where

$$d_{kk}(q) = \mathbf{w}_{Dk}^T \boldsymbol{\varphi}_{Dk} + \epsilon_{Dk}$$

$$c_{kk}(q) = \mathbf{w}_{Ck}^T \boldsymbol{\varphi}_{Ck} + \epsilon_{Ck}$$

$$\Delta_k(q, \dot{q}) = \mathbf{w}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k} + \epsilon_{\Delta k}$$

with  $\epsilon_{(\cdot)}$  being denoted to the corresponding modeling error. Accordingly, the dynamics of the coupled robotic system is transformed to decoupled subsystems based on linear combinations of basis function with constant weighting coefficients that should be updated using the Lyapunov's theory; the details are next.

### 4.3.3 Controller design

The objective of the proposed controller is to control the  $k$ -th subsystem individually considering the possibly accompanied uncertainties. Accordingly, let us consider the case that  $d_{kk}, c_{kk}, \Delta_k, h_{kk}, L_{kk}, R_{kk}, B_{kk}$  are unknowns. In addition, consideration of actuator dynamics is important for a robot with high-velocity movement and highly varying loads. Three possible actuator control modes are possible for modeling and control of the motor [42]: (i) the torque control mode in which the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known, (ii) current control mode in which only the armature current is well controlled but the torque/current constant is unknown, and (iii) voltage control mode in which no current servo control is available. The solution steps can be summarized in Fig. 4.2.

**Assumption 4.2.** For controller design, it is assumed that there is no servo current amplifier and the current-to-torque constant is unknown. Accordingly, the mentioned three control modes will be introduced in sequence.

#### 4.3.3.1 Motor torque control mode

The aim of this mode is to generate the desired output torque with guaranteed precise tracking for the desired angular joint

displacements. The control law for this mode (see Eq. (4.12a)) can be designed as

$$\tau_{dk} = \hat{d}_{kk}(q)\dot{v}_k + \hat{c}_{kk}(q)v_k + \hat{\Delta}_k - K_k s_k - \hat{\Gamma}_k(t) \text{sgn}(s_k) \quad (4.15)$$

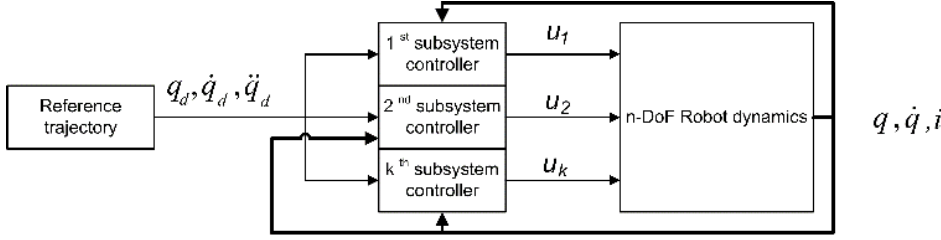
with

$$\hat{d}_{kk} = \hat{\mathbf{w}}_{Dk}^T \boldsymbol{\varphi}_{Dk}$$

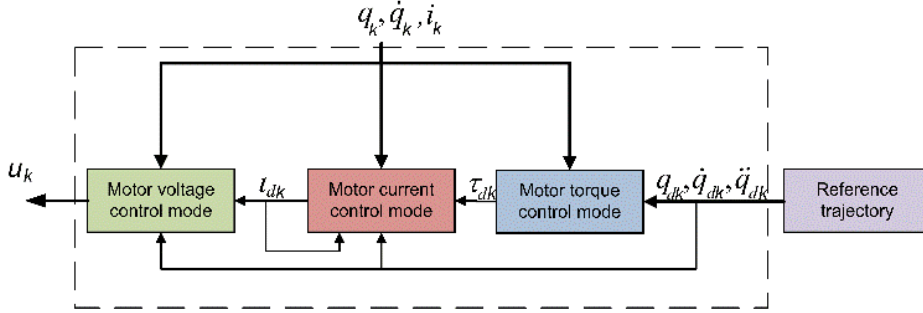
$$\hat{c}_{kk} = \hat{\mathbf{w}}_{Ck}^T \boldsymbol{\varphi}_{Ck}$$

$$\hat{\Delta}_k = \hat{\mathbf{w}}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k}$$

$$v_k = \dot{q}_{dk} - \Lambda_k e_k, \quad s_k = \dot{q}_k - v_k = \dot{e}_k + \Lambda_k e_k, \quad e_k = q_k - q_{dk}$$



(a) The control architecture of the whole system



(b) The k-th subsystem controller

**Fig. 4.2.** (a) The control architecture of the whole system, (b) the k-th subsystem controller. There is an algebraic loop at the motor current control mode due to the dependence of the input signal on the output signal.

where  $K_k$  and  $\Lambda_k$  are positive feedback gains,  $\hat{\Gamma}_k(t)$  denotes to the adaptive sliding gain that should be updated in order to avoid the demand of modeling error bounds. Please for more details on adaptive sliding mode control see [43], and

$$\text{sgn}(s) = \frac{|s|}{s} \quad (4.16)$$

The mathematical relationship of Eq. (4.16) is very useful in the stability proof discussed later. Substituting Eq. (4.15) into Eq. (4.14) leads to the output position closed loop dynamics

$$d_{kk}\dot{s} + c_{kk}s + K_k s_k + \hat{\Gamma}_k(t) \text{sgn}(s_k) = -(\tilde{\mathbf{w}}_{Dk}^T \boldsymbol{\varphi}_{Dk} \dot{v}_k + \tilde{\mathbf{w}}_{Ck}^T \boldsymbol{\varphi}_{Ck} v_k + \tilde{\mathbf{w}}_{\Delta k}^T \boldsymbol{\varphi}_{\Delta k}) - (\tau_{kd} - \tau_k) + \epsilon_k \quad (4.17)$$

From Eq. (17), the closed-loop control system is stable if  $\tilde{\mathbf{w}}_{(\cdot)} \rightarrow 0$ ,  $\tau_{kd} - \tau_k \rightarrow 0$ ,  $\epsilon_k \rightarrow 0$ , and  $\hat{\Gamma}_k(t) \rightarrow \delta_k \geq |\epsilon_k|$ . This requires finding suitable update laws for the weighting coefficients, and the adaptive sliding gain. Let us select the following updating adaptive laws for the weighting vectors and adaptive sliding gain.

$$\dot{\hat{\mathbf{w}}}_{Dk} = -\mathbf{Q}_{Dk} \boldsymbol{\varphi}_{Dk} \dot{v}_k s_k$$

$$\dot{\hat{\mathbf{w}}}_{Ck} = -\mathbf{Q}_{Ck} \boldsymbol{\varphi}_{Ck} v_k s_k$$

$$\dot{\hat{\mathbf{w}}}_{\Delta k} = -\mathbf{Q}_{\Delta k} \boldsymbol{\varphi}_{\Delta k} s_k$$

$$\dot{\hat{\Gamma}}_k(t) = \frac{1}{\rho_k} |s_k| \quad (4.18)$$

where  $\mathbf{Q}_{(\cdot)} \in \mathbb{R}^{\beta \times \beta}$  is a positive-definite adaptation matrix, and  $\rho_k$  is a positive adaptation gain.

**Theorem 4.2.** The  $k$ -th subsystem of the robotic manipulator ( $k = 1, 2, \dots, n$ ) described by Eq. (4.12a) combined with the desired applied torque control law of Eq. (4.15), and with the adaptation laws of the weighting vectors, and the adaptive sliding gain of Eq. (4.18), subject to

$$\tau_{dk} = \tau_k \quad (4.19)$$

is stable in the sense of Lyapunov stability [15,43,44].

**Proof.**

Consider the following Lyapunov-like function candidate ( $V_{\tau}$ ) along the trajectory of Eq. (4.17)

$$V_{\tau k} = \frac{1}{2} d_{kk} s_k^2 + \frac{1}{2} \tilde{\mathbf{w}}_{Dk}^T \mathbf{Q}_{Dk}^{-1} \tilde{\mathbf{w}}_{Dk} + \frac{1}{2} \tilde{\mathbf{w}}_{Ck}^T \mathbf{Q}_{Ck}^{-1} \tilde{\mathbf{w}}_{Ck} + \frac{1}{2} \tilde{\mathbf{w}}_{\Delta k}^T \mathbf{Q}_{\Delta k}^{-1} \tilde{\mathbf{w}}_{\Delta k} + \frac{1}{2} \rho_k \tilde{\Gamma}_k^2 \quad (4.20)$$

Taking the derivative of the last equation leads to

$$\dot{V}_{\tau k} = d_{kk} s_k \dot{s}_k + \frac{1}{2} \dot{d}_{kk} s_k^2 - \tilde{\mathbf{w}}_{Dk}^T \mathbf{Q}_{Dk}^{-1} \dot{\tilde{\mathbf{w}}}_{Dk} - \tilde{\mathbf{w}}_{Ck}^T \mathbf{Q}_{Ck}^{-1} \dot{\tilde{\mathbf{w}}}_{Ck} - \tilde{\mathbf{w}}_{\Delta k}^T \mathbf{Q}_{\Delta k}^{-1} \dot{\tilde{\mathbf{w}}}_{\Delta k} - \rho_k \dot{\tilde{\Gamma}}_k(t) \tilde{\Gamma}_k(t) \quad (4.21)$$

By substituting Eq. (4.17) into Eq. (4.21), using the adaptation laws of Eq. (4.18), and applying the passivity property 4.2 lead to

$$\dot{V}_{\tau k} = -K_k s_k^2 + s_k \epsilon_k - s_k \hat{\Gamma}_k(t) \text{sgn}(s_k) - (\delta_k - \hat{\Gamma}_k(t)) |s_k| - s_k (\tau_{dk} - \tau_k) \quad (4.22)$$

Substituting Eq. (4.19) into Eq. (4.22) and using Eq. (4.16) yield

$$\dot{V}_{\tau k} = -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| < 0 \quad (4.23)$$

Both  $s_k$  and  $\tilde{\Gamma}_k(t)$  reach zero in finite time resulting in position error approaching to zero provided that  $\delta_k \geq |\epsilon_k|$ .

#### 4.3.3.2 Motor current control mode

As aforementioned, the motor current control mode is necessary if the motor current can be controlled directly, and the current-to-torque constant ( $h_{kk}$ ) is not available. Accordingly, the desired torque associated with the unknown  $h_{kk}$  can be expressed as

$$\tau_{dk} = \hat{h}_{kk} l_{dk} \quad (4.24)$$

The updating adaptive law for unknown  $\hat{h}_{kk}$  can be designed as

$$\dot{\hat{h}}_{kk} = Q_{hk} l_{dk} s_k \quad (4.25)$$

**Theorem 4.3.** The  $k$ -th subsystem of the robotic manipulator ( $k = 1, 2, \dots, n$ ) described by Eqs. (4.12a) and (4.12b) combined with the desired applied torque control law of Eq. (4.15), the adaptation laws of the weighting vectors, the adaptive sliding gain of Eq. (4.18), and with updating law of current-to-torque constant of Eq. (4.25), subject to

$$l_{dk} = l_k \quad (4.26)$$

is stable in the sense of Lyapunov stability [15,43,44].

**Proof.**

The Lyapunov-like function candidate along the trajectory of Eq. (4.17) can be expressed as

$$V_{ck} = V_{\tau k} + \frac{1}{2} Q_{hk}^{-1} \tilde{h}_{kk}^2 \quad (4.27)$$

Taking the time derivative of Eq. (4.27) and considering Eq. (4.22) lead to

$$\dot{V}_{ck} = -K_k s_k^2 + s_k \epsilon_k - s_k \hat{\Gamma}_k(t) \text{sgn}(s_k) - (\delta_k - \hat{\Gamma}_k(t)) |s_k| + s_k (\tau_{dk} - \tau_k) - Q_{hk}^{-1} \tilde{h}_{kk} \dot{\tilde{h}}_{kk} \quad (4.28)$$

However, the fifth term can be manipulated as

$$\tau_{dk} - \tau_k = h_{kk}(\iota_{dk} - \iota_k) - \tilde{h}_{kk}\iota_{dk} \quad (4.29)$$

Substituting Eq. (4.29) into Eq. (4.28) and considering the simplification of the third and the fourth terms as made in Eq. (4.23) yield

$$\begin{aligned} \dot{V}_{ck} &= -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - s_k h_{kk}(\iota_{dk} - \iota_k) + s_k \tilde{h}_{kk} \iota_{dk} - Q_{\tilde{h}k}^{-1} \tilde{h}_{kk} \dot{\hat{h}}_{kk} \\ \dot{V}_{ck} &= -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - s_k h_{kk}(\iota_{dk} - \iota_k) + \tilde{h}_{kk}(s_k \iota_{dk} - Q_{\tilde{h}k}^{-1} \dot{\hat{h}}_{kk}) \end{aligned} \quad (4.30)$$

Substituting Eq. (4.25) and Eq. (4.26) into Eq. (4.30) results in the same stability results of Eq. (4.23).

#### 4.3.3.3 Motor voltage control mode

The motor voltage control mode is a complementary control mode necessary if a servo current amplifier is unavailable. The intuitive control law for the armature current dynamics described in Eq. (12c) can be selected as

$$u_{dk} = \hat{\mathbf{w}}_{uk}^T \boldsymbol{\varphi}_{uk} - \hat{h}_{kk} s_k - K_{vk}(\iota_k - \iota_{dk}) \quad (4.31)$$

where

$$L_{kk} \dot{i}_{dk} + R_{kk} \iota_k + B_{kk} \dot{q}_k = \mathbf{w}_{uk}^T \boldsymbol{\varphi}_{uk} + \epsilon_{uk}$$

,  $\hat{h}_{kk}$  can be calculated from Eq. (4.25), and

$$\dot{\hat{\mathbf{w}}}_{uk} = -\mathbf{Q}_{uk} \boldsymbol{\varphi}_{uk}(\iota_{dk} - \iota_k) \quad (4.32)$$

Substituting Eq. (4.31) into Eq. (4.12c) results in the following closed-loop dynamics (neglecting the approximation error associated with (4.31))

$$\tilde{\mathbf{w}}_{uk}^T \boldsymbol{\varphi}_{uk} + K_{vk}(\iota_k - \iota_{dk}) + h_{kk} s_k = L_k(i_{dk} - i_k) + (u_k - u_{dk}) \quad (4.33)$$

**Theorem 4.4.** The  $k$ -th subsystem of the robotic manipulator ( $k = 1, 2, \dots, n$ ) described by Eqs. (4.12a)-(4.12c) combined with the desired motor torque control law of Eq. (4.7), the adaptation laws of the weighting vectors, the adaptive sliding gain of Eq. (4.10), updating law of current-to-torque constant of Eq. (4.25), The desired voltage control input of Eq. (4.31), updating adaptive law of weighting vector of Eq. (4.32), and subject to

$$u_{dk} = u_k \quad (4.34)$$

is stable in the sense of Lyapunov stability.

#### Proof.

To prove the validity of controller law of Eq. (4.31) associated with adaptation law of Eq. (4.32), consider the following Lyapunov-like function candidate

$$V_{uk} = V_{ck} + \frac{1}{2} L_k (\iota_{dk} - \iota_k)^2 + \frac{1}{2} \tilde{\mathbf{w}}_{uk}^T \mathbf{Q}_{uk}^{-1} \tilde{\mathbf{w}}_{uk} \quad (4.35)$$

By taking the time derivative of Eq. (4.35) and considering Eq. (4.30) with adaptation law of Eq. (4.25), we can get

$$\dot{V}_{uk} = -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - s_k h_{kk}(\iota_{dk} - \iota_k) + L_k(\iota_{dk} - \iota_k)(i_{dk} - i_k) - \tilde{\mathbf{w}}_{uk}^T \mathbf{Q}_{uk}^{-1} \dot{\tilde{\mathbf{w}}}_{uk} \quad (4.36)$$

Substituting Eqs. (4.32) and (4.33) into Eq. (4.36) results in

$$\dot{V}_{uk} = -K_k s_k^2 + s_k \epsilon_k - \delta_k |s_k| - K_{vk}(\iota_k - \iota_{dk})^2 - (\iota_{dk} - \iota_k)(u_k - u_{dk}) \quad (4.37)$$

In view of Theorem 4.4, Eq. (4.37) can be expressed as

$$\dot{V}_{uk} = -K_k s_k^2 - K_{vk}(\iota_k - \iota_{dk})^2 + s_k \epsilon_k - \delta_k |s_k| \quad (4.38)$$

The variables  $s_k$ ,  $i_k$  and  $\tilde{\Gamma}_k(t)$  reach zero in finite time resulting in position error approaching zero provided that  $\delta_k \geq |\epsilon_k|$ .

**Remark 4.4.** The stability of the individual subsystems can guarantee the stability of the whole system.

**Remark 4.5.** The function  $sgn(\cdot)$  is a hard nonlinear function that can be approximated by a smoother function such as  $sat(\cdot)$  [27]. However, this can result in a modeling error due to this approximation. Therefore, in order to avoid the possible parameter drift, the robust adaptive laws can be used alternatively such as project modification,  $\sigma$ -modification,  $\varepsilon$ -modification, etc. [45].

**Remark 4.6.** If it is supposed that a sufficient number of basis functions ( $\beta$ ) are used, then the modeling error can ideally be neglected, and hence no adaptive sliding mode term could be required [27].

#### 4.4 Simulation Results and Discussions

Two case studies have been simulated using MATLAB/SIMULINK to prove the validity of the proposed controller: a 2-link manipulator and a 6-link biped robot; the details are as follows.

##### 4.4.1 Two-link manipulator

A fully actuated 2-link manipulator was simulated using MATLAB/SIMULINK to investigate the performance of the proposed controller (Figs. 4.3-5). The parameters of the simulated manipulators are borrowed from [27]. The target manipulator moves freely (without constrained motion) with the following desired trajectories [46]:  $q_{d1} = 30^\circ(1 - \cos(2\pi t))$  and  $q_{d2} = 45^\circ(1 - \cos(2\pi t))$ . However, there are some assumptions made for simulation implementation:

- The current-to-torque constant  $h_{kk}$  is known,
- The joint drive system is directly constructed with gear ratio equal to one. As a result, there are two-stage control modes for the actuators: the motor torque control mode (Eqs. (4.15) and (4.18)) and the motor voltage control mode (Eqs. (31) and (32)).
- It is supposed that a sufficient number of orthogonal Chebyshev basis functions ( $\beta = 15$ ) are used for approximation purposes and hence the approximation error is neglected (it is a typical case used for simplicity in some previous work, see e.g. [27, 31]).
- The friction consists of two terms: Coulomb friction,  $B_c \text{sign}(\dot{\theta})$ , and viscous friction,  $B_v \dot{\theta}$ ; with  $B_c = B_v = 0.5$ . See [49] and the references therein for other possible friction models.
- The saturated input control is not considered and it is assumed that the motors are strong to generate the required voltages.

The feedback and the adaptation gains used are:

$$\Lambda_k = 75, \mathbf{Q}_{Dk} = 10 \mathbf{I}_{15}, \mathbf{Q}_{Ck} = 10 \mathbf{I}_{15}, \mathbf{Q}_{\Delta k} = 25 \mathbf{I}_{15}, K_k = 100, K_{vk} = 50, \mathbf{Q}_{uk} = 50 \mathbf{I}_{15},$$

Figure 4.6 show the position error and the input control voltage for the target robot. The robot tracks well the desired trajectory. One important point should be mentioned is that although the proposed controller does not track precisely the dynamic coefficients of the equation of motion for each subsystem, the controller tracks well the desired references. The objective of adaptive control is to follow the desired trajectory rather than convergence of unknown parameters of the system to their actual values. The system signals should be persistently excited in order to ensure good estimation for unknown parameters, see [44] for more details.

A special case has been applied to the 2-link manipulator in order to test the superiority of the proposed controller comparing with the decoupled PD control. The PD control law can be described as:  $\tau_k = -K_k s_k = -K_k(\dot{e}_k + e_k)$ , with  $K_k = 100$ . Figures 4.7 and 4.8 show the position errors and the input control torques respectively considering both the decentralized adaptive partitioned approximation control (APAC) and PD control. The feedback and adaptive gain matrices are tuned by gradually increasing their values from zero to a limit at which the instability occurs, then the matrices should be scaled down [42]. Both controllers work well with small position error; however, the APAC shows more accuracy than the PD considering the same high gain.

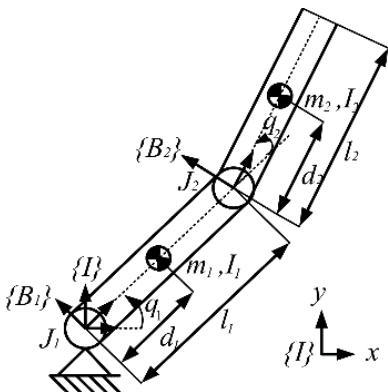


Fig. 4.3. The 2-link manipulator

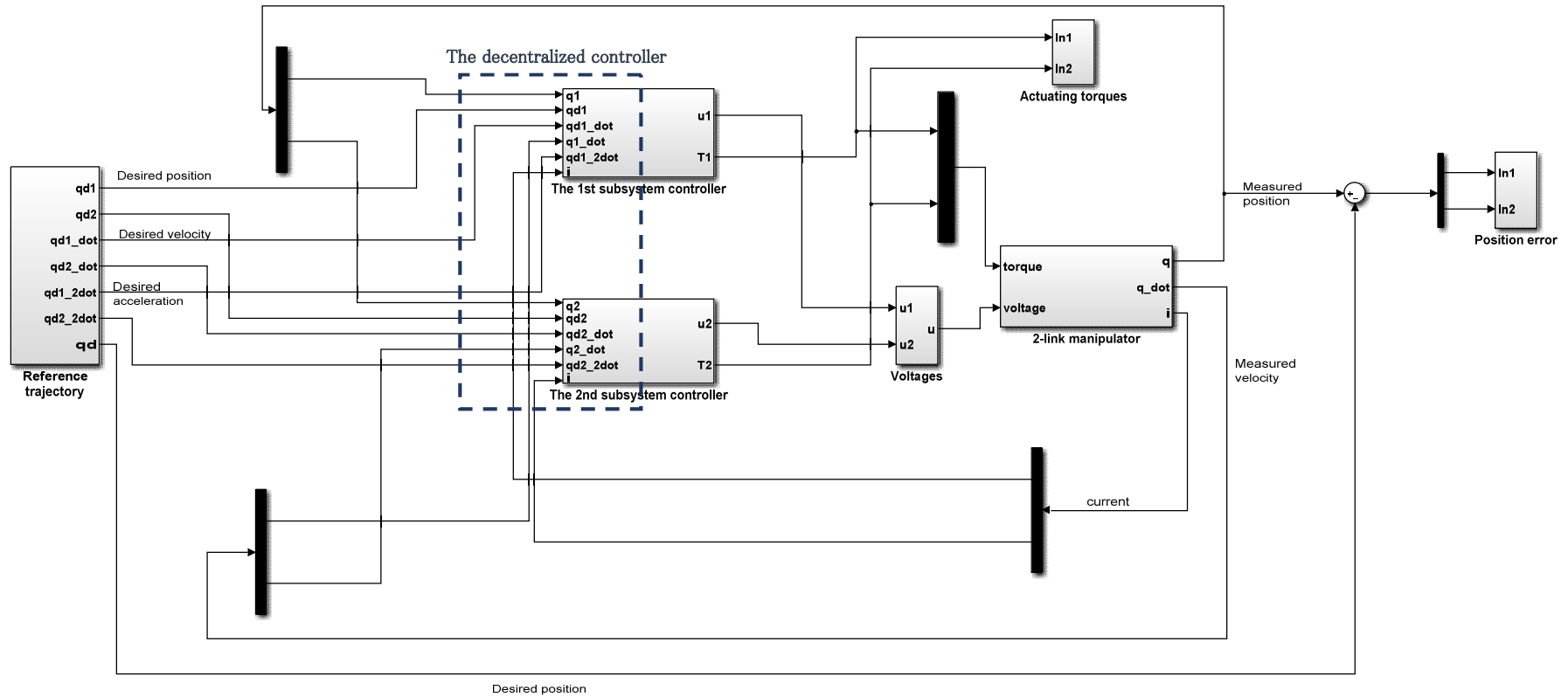


Fig. 4.4 A SIMULINK model describing the proposed control architecture for a two-link manipulator. As noted the control system is decomposed into two subsystems representing the degrees of freedom of the system. See Fig. 5 for more details on the block diagrams used for each subsystem controller.

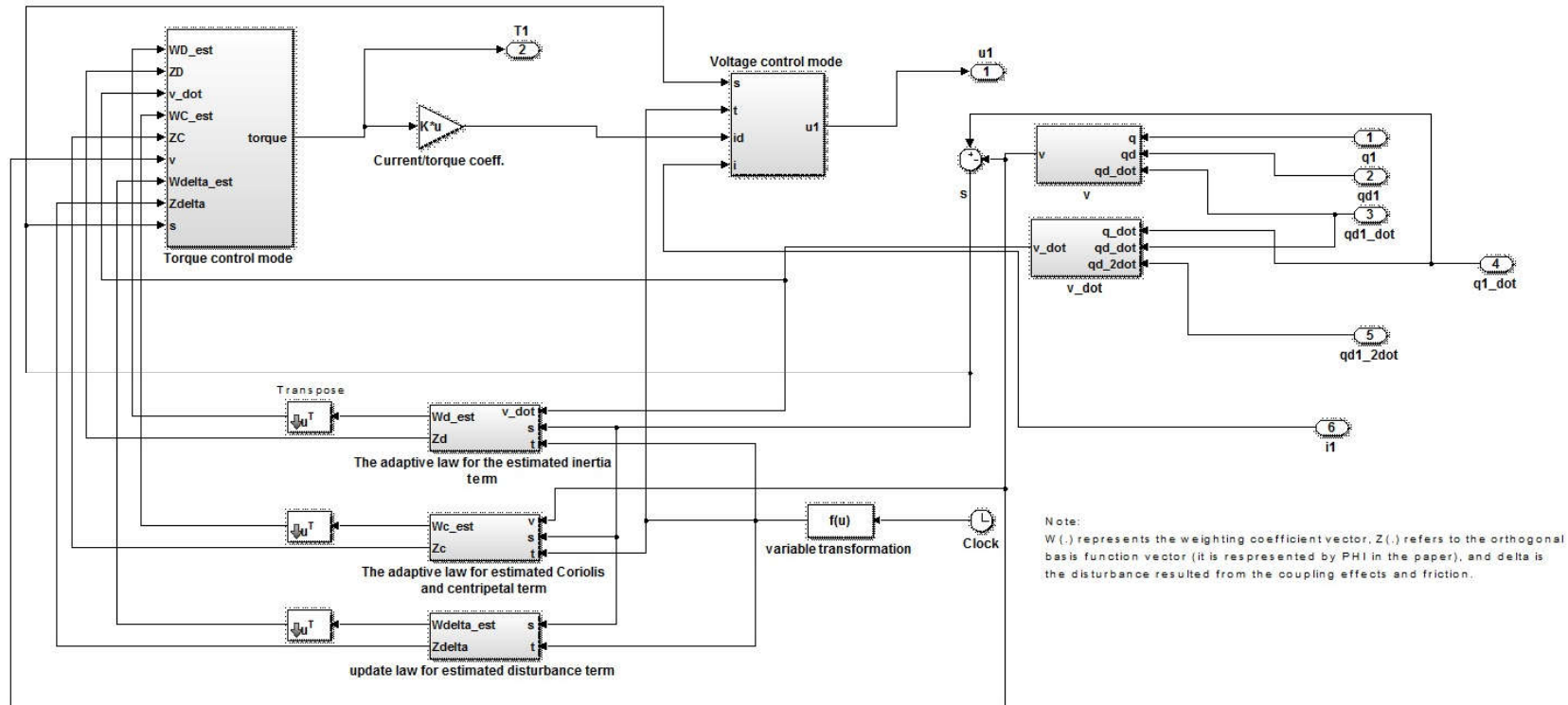


Fig. 4.5. A SIMULINK model describing the 1<sup>st</sup> subsystem decentralized adaptive approximation controller for joint 1. A similar control structure is used for the second subsystem.

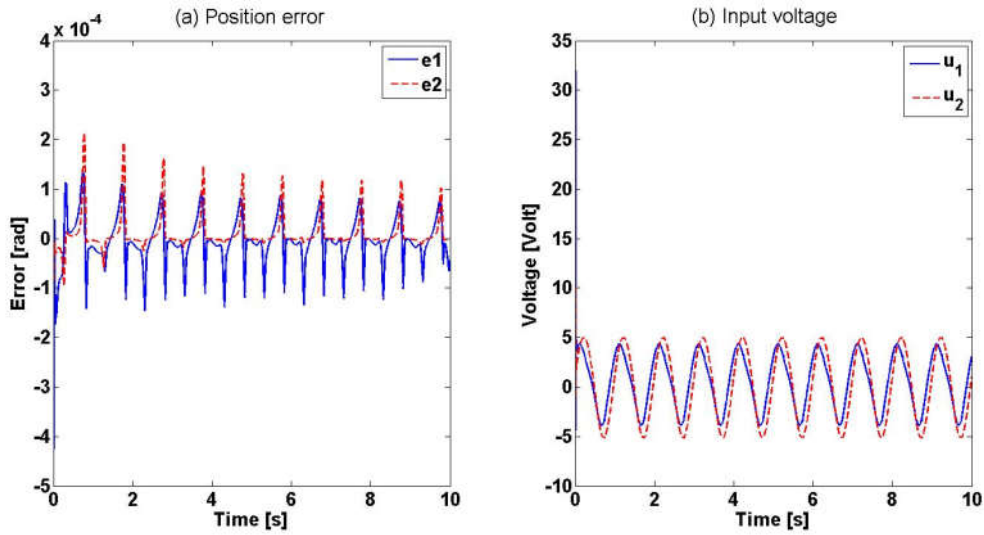


Fig. 4.6. Simulation results for 2-R robot

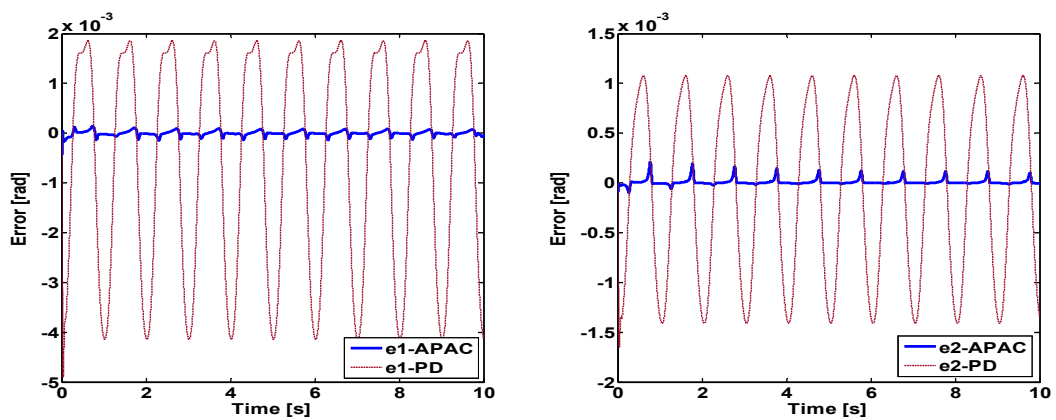


Fig. 4.7. The position errors for the 2-R robot using the PD and APAC controllers

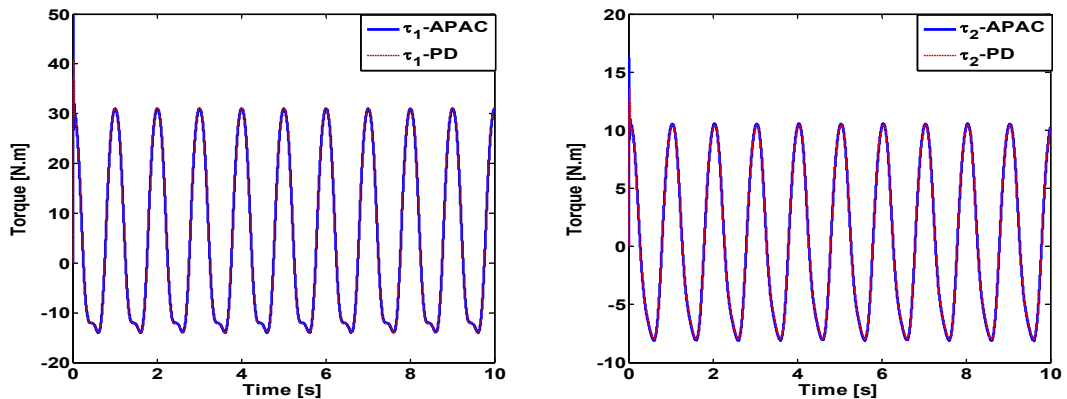


Fig. 4.8. The input control torques for the 2-R robot using the PD and APAC controllers

## 4.2 Six-link biped robot

In this subsection, a fully actuated 6-DOF biped robot is controlled by using the decentralized adaptive approximation control. There are different walking phases for biped locomotion; however, this section concentrates on the single support phase (SSP). The biped is considered an open chain during this walking phase in which the supporting foot is in full contact with the plane ground. This means that the dynamics of the support foot are neglected. This assumption coincides with the concept of zero-moment point (ZMP)-based balance criterion for biped locomotion. If the dynamics of the stance foot are considered

and it is allowed to rotate, the system will be underactuated and the stability of the biped mechanism cannot be governed by the ZMP and another balance criterion, e.g., Poincaré map, is used alternatively. See [48,49] for more details.

Figure 4.9 shows the configuration of the biped robot during the SSP; for more details on modeling and control of biped robots see [47-57]. The physical parameters are borrowed from [58]. It should be noted that this short subsection is focused on low-level control of the biped robot. Control of the biped mechanism may require multi-level control architecture: the walking pattern generator control level, the stabilization controller for compensation of the deviation of the ZMP (as an example of balance criterion), and the low-level control strategy for precise tracking of the desired references (position, torque, etc.). One of the possible strategies for modeling the biped mechanism is to consider the support foot as fixed base and hence the biped mechanism can be dealt as fixed base robots. Therefore, the dynamics and control of robotic manipulators can be applied efficiently. This assumption coincides with ZMP-based walking.

In similar to the previous simulation study, a Matlab/Simulink model used for implementing the simulation experiments. A similar control architecture described in Figs. 4.4,5 and the same assumptions are used for simulation implementation. Therefore, two-stage control modes are used for motion control of the target biped: the motor torque control mode and the motor voltage control mode. The decentralized adaptive approximation control is applied with the following feedback and adaptation gains:

$$\Lambda_k = 50 \mathbf{Q}_{Dk} = 5 \mathbf{I}_{15} \mathbf{Q}_{Ck} = 5 \mathbf{I}_{15} \mathbf{Q}_{\Delta k} = 15 \mathbf{I}_{15}, K_k = 100, K_{vk} = 50, \mathbf{Q}_{uk} = 50 \mathbf{I}_{15},$$

Figures 4.10 and 4.11 show the position error and the voltage input control for the simulated biped robot. The biped can track the desired references very well despite the presence of the unknown parameters of the system and the high degrees of freedom.

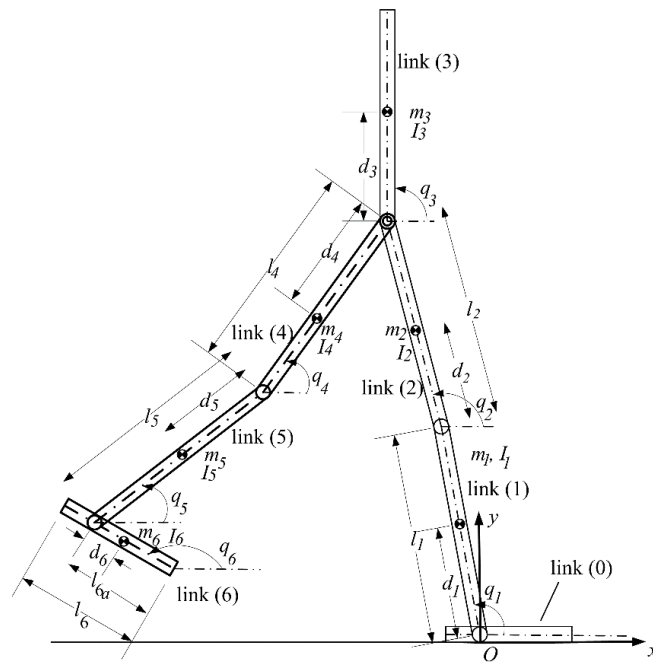


Fig. 4.9. The configuration of the biped robot during the single support phase

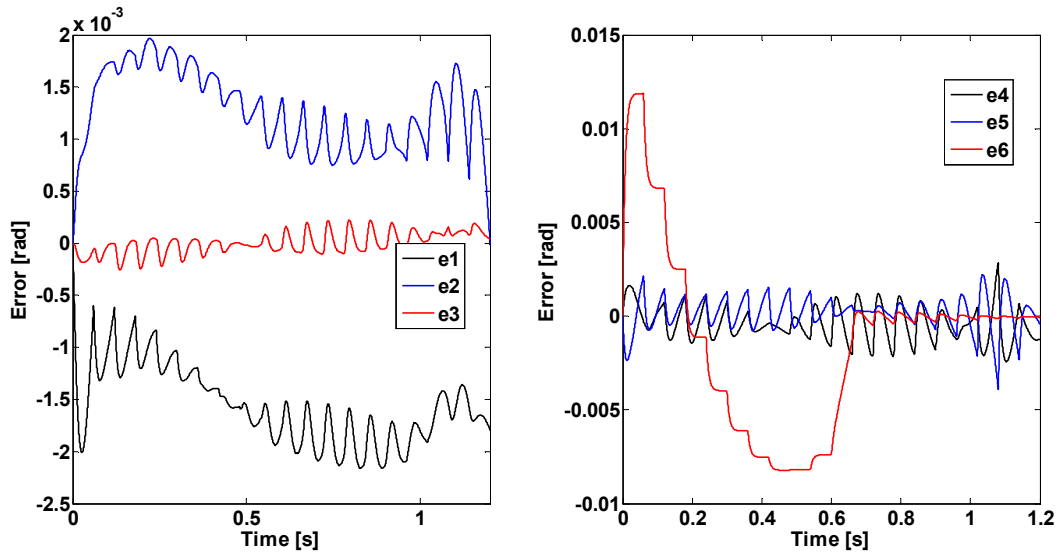


Fig. 4.10. The angular position errors associated with the angular joints of the biped robot depicted in Fig. 4.7.

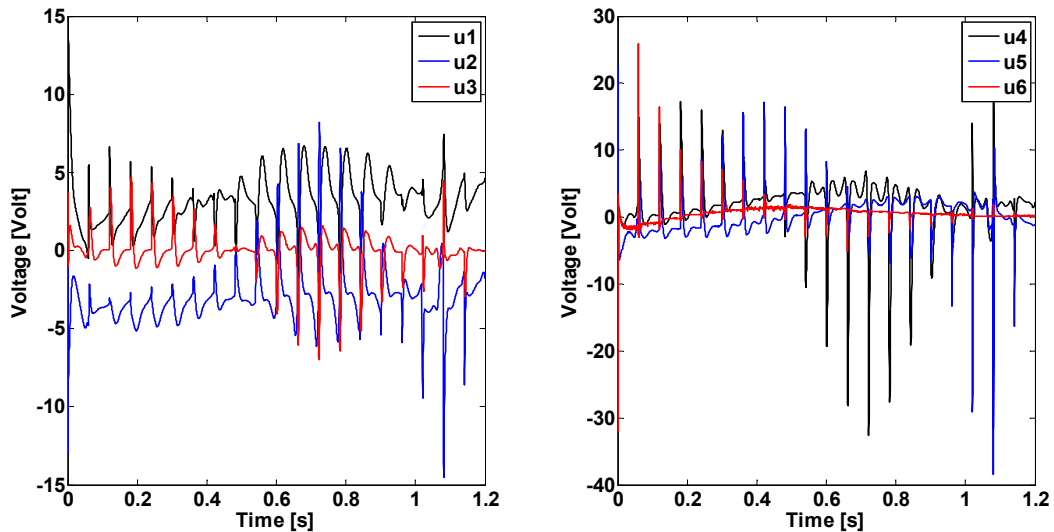


Fig. 4.11. The input voltage control for the biped robot during the SSP

#### 4.5 Conclusion

This chapter proposes a decentralized adaptive partitioned approximation control strategy for robotic manipulators considering actuator dynamics. Three actuator control modes are considered: torque control mode, current control mode, and voltage control mode. In the torque control mode, the armature current is well controlled by a current servo amplifier and the motor torque/current constant is known. Whereas in the current control mode only the armature current is well controlled but the torque/current constant is unknown. On the other hand, the voltage control mode is necessary when no current servo control is available. In general, the proposed control law consists of three terms: a feed-forward control term, a high gain feedback control term, and a robust sliding term. The proposed control architecture has been applied to fully actuated and fixed base robotic systems. Further work is required to consider underactuated and overactuated robotic systems and even the floating base robots. In addition, the design of a decentralized control strategy for a robot moving in constrained space considering hybrid motion/force control or impedance control is necessary for different applications where the computational complexity inherently exists such as cooperating manipulators, robotic arm/hand systems and so on.

Despite most decentralized control strategies are designed based on the Euler-Lagrange (E-L) formulation, exploiting the current strategy with the integration of the Newton-Euler (N-E) formulation is a powerful technique for dealing with

modeling and control of complex robotic systems. Despite the work of [42] has been focused on this point for modeling and decoupled control of N-E based robotic systems, the proposed controller for the whole system is distributed (not fully decentralized). Future work may focus on this point using a fully decentralized control approach considering free and constrained motion with different applications.

## References

1. Luh JYS, Walker MW, Paul RPC (1980) On-line computational scheme for mechanical manipulator. *Trans. ASME, J. Dynamic Systems, Measurements and Control* 102: 69-76
2. Hollerbach JM (1980) A recursive Lagrangian formulation of manipulator dynamics and comparative study of dynamics formulation complexity. *IEEE. Trans. On Systems, Man, and cybernetics SMC-10* (11):730-736
3. Walker M, Orin DE (1982) Efficient dynamic computer simulation of robotic mechanisms. *Trans. ASME, J. Dynamic Systems, Measurements and Control* 104: 205-211.
4. Balafoutis CA, Patel RV (1989) Efficient computation of manipulator, inertia matrices and the direct dynamics problem. *IEEE Transactions on Systems, Man and Cybernetics* 19:1313-1321
5. Angeles J., Ma O, Rojas A (1998) An algorithm for the inverse dynamics of n-axis general manipulator using Kane's formulation of dynamical equations. *Computers and Mathematics with Applications* 17(12):1545-1561
6. Saha SK (1999a) Dynamics of serial multibody systems using the decoupled natural orthogonal complement matrices. *ASME Journal of Applied Mechanics* 66:986-996
7. Featherstone R (1983) The calculation of robot dynamics using articulated-body inertias. *Int. J. Robotic Research* 2 (1):13-30
8. Brandle H, Johann R, Otter M (1986) A very efficient algorithm for the simulation of robots and similar multibody systems without inversion of the mass matrix. *Proc. of IFAC/IFIP/IMACS International Symposium on Theory of Robots, Vienna*, pp.95-100
9. Mohan A, Saha SK (2007) A recursive numerically stable and efficient simulation algorithm for serial robots. *Int. J. Multibody System Dynamics*, 17(4):291-319
10. Lee K, Chirikjian GS (2005) A new perspective on O (n) mass-matrix inversion for serial revolute manipulators" *Proceeding of the IEEE International Conference on Robotics and Automation, Barcelona, Spain*, pp. 4722-4726
11. Featherstone R, Orin D (2000) Robot dynamics: Equations and algorithms. *IEEE International Conference on Robotics and Automation, ICRA* 1:826-834
12. Saha SK (2007) Recursive dynamics algorithms for serial, parallel and closed-chain multibody systems. *Indo-US Workshop on Protein Kinematics and Protein Conformations, IISC, Bangalore*
13. Fu KS, Gonzalez RC, Lee CSG (1987) *Robotics: control, sensing, vision, and intelligence*. McGraw-Hill Book Company, USA
14. Sandell NR, Varaiya P, Athans M, Safonov MG (1978) Survey of decentralized control methods for large scale systems. *IEEE Transactions on Automatic Control*, AC-23(2):108-128
15. Spong MW, Vidyasagar M (1989) *Robot dynamics and control*. John Wiley & Sons, Inc., USA
16. Boha J, Belda K, Valasue M (2002) Decentralized control of redundant parallel robot construction. *Proceeding of the 10<sup>th</sup> Mediterranean Conference on Control and Automation, Lisbon, Portugal*, pp. 9-1
17. Ohri J, Dewan L, Soni MK (2007) Tracking control of robots using decentralized robust PID control for friction and uncertainty compensation. *Proceedings of The World Congress on Engineering and Computer Science, San Francisco, USA*.
18. Xu K (2010) Integrating centralized and decentralized approaches for multi-robot coordination. *Dissertation, Mechanical and Aerospace Engineering, New Brunswick, New Jersey, USA*
19. Leena G, Ray G (2012) A set of decentralized PID controller for an n-link robot manipulator. *Indian Academy of Science* 37:405-423
20. Liu M (1999) Decentralized control of robot manipulators: nonlinear and adaptive approaches. *IEEE Transactions on Automatic Control* 44(2): 357-363
21. Yang Z-J, Fukushima Y, Qin P (2012) Decentralized adaptive robust control of robot manipulators using disturbance observers. *IEEE Transactions on Control Systems Technology* 20(5):1357-1365

22. Jasim IF, Plapper PW (2014) Enhanced decentralized robust adaptive control of robots with arbitrarily-switched unknown constraints, IEEE International Conference on Systems, Man, and Cybernetics October 5-8, San Diego, CA, USA, pp. 4027-4032
23. Lewis FL, Yesildirek A, Liu K (1995) Neural net robot controller: structure and stability proofs. *J. Intell Robot Syst* 13:1–23
24. Lewis FL, Liu K, Yesildirek A (1995) Neural net robot controller with guaranteed tracking performance. *IEEE Trans Neural Netw* 6(3):703–716
25. Lewis FL, Yesildirek A, Liu K (1996) Multilayer neural net robot controller with guaranteed tracking performance. *IEEE Trans Neural Netw* 7(2):1–12
26. Liu J (2013) Radial basis function (RBF) neural network control for mechanical systems: design, analysis and Matlab simulation. Tsinghua University Press, Beijing and Springer-Verlag Berlin Heidelberg
27. Huang A-C, Chien M-C (2010) Adaptive control of robot manipulators: a unified regressor-free approach. World Scientific Publishing Co. Pte. Ltd. Singapore
28. Ge SS, Lee TH, Harris CJ (1998.) Adaptive neural network control of robotic manipulators, World Scientific Publishing Co., Inc. River Edge, NJ, USA
29. Al-Shuka HFN (2018) On local approximation-based adaptive control with applications to robotic manipulators and biped robots. *International Journal of Dynamics and Control*, Springer 6 (1):339-353
30. Al-Shuka HFN, Song R (2017) Adaptive hybrid regressor and approximation control of robotic manipulators in constrained space. *International Journal of Mechanical & Mechatronics Engineering IJMME-IJENS* 17 (3):11-21
31. Panagi P, Polycarpou MM (2008) Decentralized adaptive approximation based control of a class of large-scale systems. American Control Conference Westin Seattle Hotel, Seattle, Washington, USA, pp. 4191–4196
32. Tan KK, Huang S, Lee TH (2009) Decentralized adaptive controller design of large-scale uncertain robotic systems. *Automatica* 45(1): 161-166
33. Fateh MM, Fateh S (2012) Decentralized direct adaptive fuzzy control of robots using voltage control strategy. *Nonlinear Dyn* 70:1919–1930
34. Li T, Li R, Li J (2011) Decentralized adaptive neural control of nonlinear interconnected large-scale systems with unknown time delays and input saturation, *Neurocomputing* 74:2277–2283
35. KIM YT (2010) Decentralized adaptive fuzzy backstepping control of rigid-link electrically driven robots. *Intelligent Automation and Soft Computing*, 16(2):135-149
36. Zhu L, Li Y (2010) Decentralized adaptive neural network control for reconfigurable manipulators, Chinese Control and Decision Conference, pp.1760–1765
37. Zhou Q, Li H, Shi P (2015) Decentralized adaptive fuzzy tracking control for robot finger dynamics, *IEEE Transactions on Fuzzy Systems*, 23(3):501–510
38. Vázquez LA, Jurado F, Alanís AY (2015) Decentralized identification and control in real-time of a robot manipulator via recurrent wavelet first-order neural network. *Mathematical Problems in Engineering*, vol. 2015, Article ID 451049, 12 pages, doi:10.1155/2015/451049
39. Burden RL, Faires J D (1989) Numerical analysis. 4<sup>th</sup> ed, PWS Publishing Co. Boston, MA, USA
40. Mulero-Martínez JI (2007) Uniform bounds of the Coriolis/centripetal matrix of serial robot manipulators. *IEEE Transactions on Robotics* 23(5):1083-1089
41. Kelly R, Santibáñez V, Loria (2005) Control of robot manipulators in joint space. Springer-Verlag London
42. Zhu W-H (2010) Virtual decomposition control, toward hyper degrees of freedom robots. Springer-Verlag Berlin Heidelberg
43. Utkin VI, Poznyak AS (2013) Adaptive sliding mode control. In: Bandyopadhyay B, Janardhanan S, Spurgeon S (eds) *Advances in Sliding Mode Control*. Lecture Notes in Control and Information Sciences, vol 440. Springer, Berlin, Heidelberg
44. Ioannou P, Fidan B (2006) Adaptive control tutorial. SIAM, USA.
45. Farrell JA, Polycarpou MM (2006) Adaptive approximation based control: unifying neural, fuzzy and traditional adaptive approximation approaches. John Wiley & Sons, Inc. USA
46. Slotine J-J, Li W (1991) Applied nonlinear control, Pearson, USA
47. Al-Shuka HFN (2017) An overview on balancing and stabilization control of biped robots. Munich, GRIN Verlag, <http://www.grin.com/en/e-book/375226/an-overview-on-balancing-and-stabilization-control-of-biped-robots>.
48. Al-Shuka HFN, Corves BJ, Zhu W-H, and Vanderborght B (2016) Multi-level control of zero moment point-based biped humanoid robots: a review. *Robotica*, Cambridge Press, 34(11):2440-2466
49. Al-Shuka HFN (2014) Modeling, walking pattern generators and adaptive control of biped robot. PhD Dissertation, RWTH Aachen University, Department of Mechanical Engineering, IGM, Germany

50. Al-Shuka HFN, Corves BJ, Vanderborght B, Zhu W-H (2015) Zero-moment point-based biped robot with different walking patterns. *International Journal of Intelligent Systems and Applications* 7(1):31-41
51. Al-Shuka HFN, Corves BJ, Zhu W-H, Vanderborght B (2014) A Simple algorithm for generating stable biped walking patterns. *International Journal of Computer Applications* 101(4): 29-33
52. Al-Shuka HFN, Corves BJ, Zhu W-H (2014) Dynamic modeling of biped robot using Lagrangian and recursive Newton-Euler formulations. *International Journal of Computer Applications* 101(3):pp. 1-8
53. Al-Shuka HFN, Allmendinger F, Corves BJ, Zhu W-H (2014) Modeling, stability and walking pattern generators of biped robots: a review. *Robotica*, Cambridge Press 32(6)907-934
54. Al-Shuka HFN, Corves BJ, Zhu W-H (2014) Function approximation technique-based adaptive virtual decomposition control for a serial-chain manipulator. *Robotica*, Cambridge Press, 32(3):375-399
55. Al-Shuka HFN, Corves BJ (March 2013) On the walking pattern generators of biped robot. *Journal of Automation and Control*, 1(2):149-156.
56. Al-Shuka HFN, Corves BJ, Zhu W-H (2013) On the dynamic optimization of biped robot. *Lecture Notes on Software Engineering* 1(3)237-243
57. Al-Shuka HFN, Corves BJ, Vanderborght B, Zhu W-H (2013) Finite difference-based suboptimal trajectory planning of biped robot with continuous dynamic response. *International Journal of Modeling and Optimization* 3(4):337-343
58. Vanderborght B (2010) *Dynamic Stabilisation of the biped Lucy powered by actuators with controllable stiffness*. Springer-Verlag Berlin Heidelberg
59. Sciavicco L, Siciliano B, Villani L (1996) Lagrange and Newton-Euler dynamic modeling of a gear-driven robot manipulator with inclusion of motor inertia effects. *Advanced Robotics* 10(3)317-334
60. Al-Shuka HFN, Song R. (2019) Decentralized adaptive partitioned approximation control of robotic manipulators. In: Arakelian V., Wenger P. (eds) *ROMANSY 22 – Robot Design, Dynamics and Control*. CISM International Centre for Mechanical Sciences (Courses and Lectures), vol 584. Springer, Cham
61. Al-Shuka HFN, Song R (2018) Hybrid regressor and approximation-based adaptive control of robotic manipulators with contact-free motion. *The 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC 2018)*, China, pp. 325-329
62. Al-Shuka HFN, Song R (2018) Hybrid regressor and approximation-based adaptive control of piezoelectric flexible beams. *The 2nd IEEE Advanced Information Management, Communicates, Electronic and Automation Control*.