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# Argumentation Frameworks with Higher-Order Attacks: Semantics and Complexity

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## Introduction

**Argumentation frameworks** (AF) are formalisms to express argumentation problems. In Dung's one, they are expressed as directed graph in which nodes represent argument and arrow, attack relations between arguments. Higher-order frameworks, unlike Dung's one, allow to have attacks over attacks. **RAF** are such a framework (see Figures 1 and 2). Arguments are here represented by circles and attack relations by squares.

For future algorithm investigations, we adapted the notion of Dung's AF labellings for RAF. We showed the relation between structures (counterpart of extensions for RAF) and different types of structure labellings.

We studied the complexities of RAF decisions problems and shown that despite the higher expressiveness offered by them, the decision classes stay the same as Dung's AF.

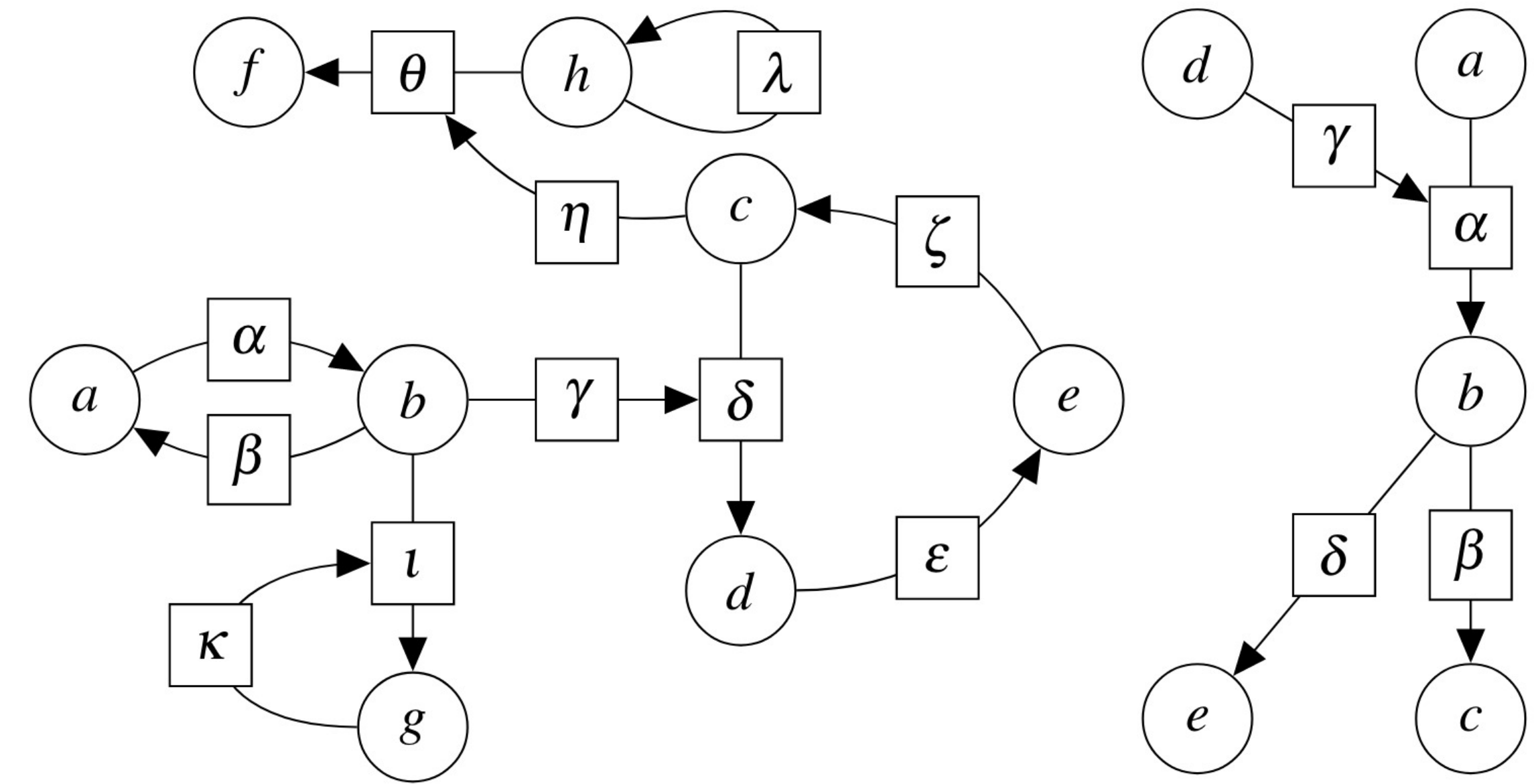


Figure 1

Examples of RAF

Figure 2

## Labellings for RAF

Instead of extensions (set of arguments), RAF solutions are expressed as **structure**: a couple of sets, one of arguments and one of attacks. As for Dung's AF, we introduced **structure labellings** for RAF, a couple of labellings, one for the arguments and the other one for the attacks. They are three value-based: *in* (accepted), *out* (rejected), *und* (undecidable). **Reinstatement RAF labellings** are particular labellings that coincide under some constraints to different RAF semantics (see Tables 1 and 2).

**Definition 14** (Reinstatement RAF labelling). Let  $\Gamma = \langle A, K, s, t \rangle$  be a recursive argumentation framework and  $\mathcal{L} = \langle \ell_A, \ell_K \rangle$  be a RAF labelling.  $\mathcal{L}$  is a reinstatement RAF labelling iff it satisfies the following conditions:  $\forall x \in (A \cup K)$ ,

- $(\mathcal{L}(x) = out) \iff (\exists \alpha \in K \text{ s.t. } t(\alpha) = x, \ell_K(\alpha) = in \text{ and } \ell_A(s(\alpha)) = in)$
- $(\mathcal{L}(x) = in) \iff (\forall \alpha \in K \text{ s.t. } t(\alpha) = x, \ell_K(\alpha) = out \text{ or } \ell_A(s(\alpha)) = out)$

Restriction on Reinstatement RAF labelling	Structure semantics
no restriction	RAF-complete
maximal <i>und</i>	RAF-grounded
minimal <i>in</i>	RAF-grounded
minimal <i>out</i>	RAF-grounded
maximal <i>in</i>	RAF-preferred
maximal <i>out</i>	RAF-preferred
empty <i>und</i>	RAF-stable
minimal <i>und</i>	RAF-semi-stable

Table 2: Reinstatement RAF labellings and structures semantics

		RAF labellings			
		$\mathcal{L}_1$	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_4$
Arguments or attacks	a	und	in	out	out
	b	und	out	in	in
	c	und	und	in	in
	d	und	und	in	in
	e	und	und	out	out
	f	und	und	in	in
	g	und	in	in	out
	h	und	und	und	und
	$\alpha$	in	in	in	in
	$\beta$	in	in	in	in
	$\gamma$	in	in	in	in
	$\delta$	und	in	out	out
	$\epsilon$	in	in	in	in
	$\zeta$	in	in	in	in
	$\eta$	in	in	in	in
	$\theta$	und	und	out	out
$\iota$	und	out	out	in	
$\kappa$	in	in	in	in	
$\lambda$	in	in	in	in	
Semantics with RAF labellings	RAF-complete	✓	✓	✓	✓
	RAF-grounded	✓			
	RAF-preferred		✓	✓	✓
	RAF-arg-preferred		✓	✓	
	RAF-stable				
	RAF-semi-stable			✓	✓

$i \checkmark j$ :  $j$  is a RAF labelling corresponding to semantics  $i$ .

Table 1: RAF labellings for Figure 1

## Complexities of RAF

We introduced a new **flattening** of RAF to Dung's AF (procedure called Raf2Af) in order to prove that it is also the case for RAF complexities.

Table 3 summarises the complexities of the credulous and skeptical acceptance problems, the verification, the existence, the non-empty existence and the uniqueness problems.

Figure 3 shows an example of flattening. For each attack two arguments are created: one, named as the attack, representing the validity of the attack, the other one the validity of both the attack and its source. For each argument, another one is created representing the invalidity of the argument.

Same complexities as Dung's AF

$\sigma$	$Cred_\sigma$	$Skep_\sigma$	$Ver_\sigma$	$Exists_\sigma$	$Exists_\sigma^\emptyset$	$Unique_\sigma$
Co	NP-c	P-c	in L	trivial	NP-c	coNP-c
Gr	P-c	P-c	P-c	trivial	in L	trivial
Pr	NP-c	$\Pi_2^c$ -c	coNP-c	trivial	NP-c	coNP-c
St	NP-c	coNP-c	in L	NP-c	NP-c	DP-c
Ss	$\Sigma_2^c$ -c	$\Pi_2^c$ -c	coNP-c	trivial	NP-c	in $\Theta_2^c$

Table 3: Complexities for RAF decision problems

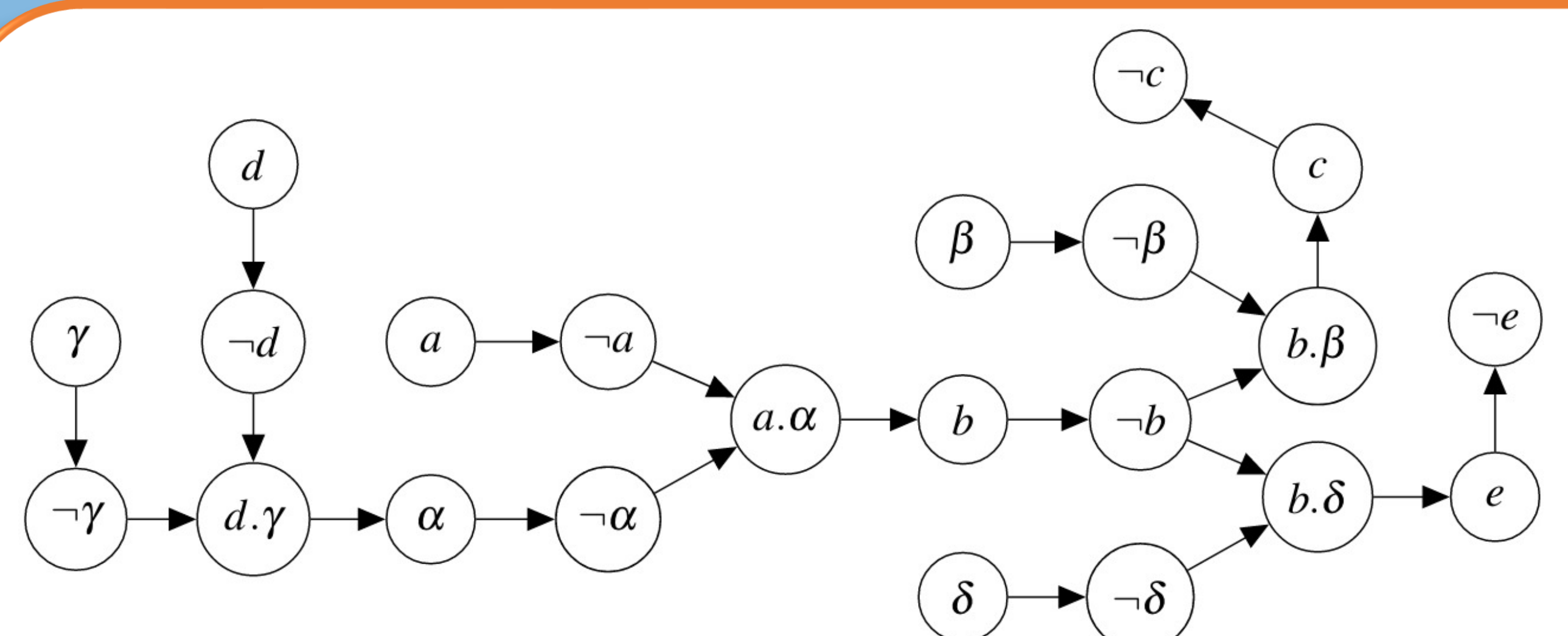


Figure 3: Raf2Af( $\Gamma$ ) of Figure 2 RAF

## Perspectives

- Algorithms for RAF argumentation problems
- Complexities of function problems
- Higher-Order bipolar argumentation framework