GENERATION OF THE RELATIVIST ACCELERATION OF GRAVITY FROM QUANTIFIED DEFORMATIONS OF A QUADRIDIMENSIONAL PLATONIAN SPACE IMMERSED IN A FIVE-DIMENSIONAL SPACE

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Abstract

The articles hal-01970150 and hal-02444592 propose a modeling of various relativistic phenomena from quantified variations of the Platonic absolute time in the presence of a gravitational field.

This article aims to re-establish the uniformity of the flow of this time by attributing these apparent variations to deformations of the Platonic space in a fifth spatial dimension.

As we will explain, these deformations will thus lead, in a purely geometric way, to a quantified modeling of the relativistic acceleration of gravity.

GÉNÉRATION DE L'ACCÉLÉRATION RELATIVISTE DE LA PESANTEUR
À PARTIR DE DÉFORMATIONS QUANTIFIÉES D'UN ESPACE QUADRIDIMENSIONNEL
PLATONICIEN IMMERGÉ DANS UN ESPACE À CINQ DIMENSIONS

Résumé

Les articles hal-01949616 et hal-02397067 proposent une modélisation de divers phénomènes relativistes à partir de variations quantifiées du temps absolu platonicien en présence d'un champ gravitationnel.

Le présent article vise à rétablir l'uniformité de l'écoulement de ce temps en attribuant ces variations apparentes à des déformations de l'espace platonicien dans une cinquième dimension spatiale.

Comme nous allons l'expliquer, ces déformations vont ainsi conduire, de façon purement géométrique, à une modélisation quantifiée de l'accélération relativiste de la pesanteur.

Introductory

Articles hal-01970150 and hal-02444592 (cf. the articles cited in reference to the following paragraph) propose, by means of quantified variations of "absolute time" (cf. article hal-01378215) a modeling of several relativistic phenomena , namely the variations in the measurement of time in a satellite orbiting the Earth, the gravitational redshift (Pound-Rebka experiment), the advance of the perihelion of the planets and the deflection of light in a gravitational field.

However, these quantified variations of absolute time go against one of the fundamental principles of the Platonic model (cf. article hal-01165196) whose framework is a four-dimensional Euclidean space, in which objects ***move uniformly*** (that is to say that between two observations, whatever their trajectory, ***they all travel the same distance*** (*)), but where the events are seen in projection according to a privileged direction.

To restore this principle, we will introduce here a ***fifth spatial dimension*** in which Platonic space will ***deform***, in a quantified way, under the effect of the mass waves generated by the massive bodies in interaction.

We will thus see that the apparent variations of "absolute time" are generated geometrically by these deformations which impose a complementary path to the objects which are subjected to them.

In application, we will show in this article that these deformations shed new light on a modeling of the relativistic acceleration of gravity.

(*) We recall here the ***concept of absolute time*** introduced in the article hal-01340134 :

given the regular motion of the objects observed in the Platonic universe
*(« between two events, the **distances** traveled by all the observed objects are **equal**, regardless of their trajectory »), and with reference to the Newtonian concepts*
*("Absolute, true and mathematical time, of itself, and from its own nature flows equably without regard to anything external"), we can introduce a notion of **absolute time** (measured in meters) in the following terms :*

*« Absolute time ΔT between two events is the **distance** traveled by (all) the mobiles between these two events .»*

1. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

« UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).

« A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pre-publication hal-01165196, version 1).

In the previous articles, the four-dimensional Euclidean space was referred to an orthonormal coordinate system $\left(O,\vec{i},\vec{j},\vec{k},\vec{h}\right)$ whose axes were denoted (OX), (OY), (OZ), (OW); the direction of the projection was that of the vector $\vec{h}$.

The introduction of a fifth dimension will lead us to review these notations.
Thus, the four-dimensional Platonic space considered will be related to an orthonormal coordinate system $\left(O,\vec{e_1},\vec{e_2},\vec{e_3},\vec{e_4}\right)$ whose axes will be denoted (Ox_1), (Ox_2), (Ox_3) and (Ox_4); the direction of the projection becomes that of the vector $\vec{e_4}$; this space is immersed in a five-dimensional space related to an orthonormal frame $\left(O,\vec{e_1},\vec{e_2},\vec{e_3},\vec{e_4},\vec{e_5}\right)$ whose axes will be denoted (Ox_1), (Ox_2), (Ox_3), (Ox_4) and (Ox_5).

Following the hal-01207447 and hal-01213447 articles, the notion of relativistic mass of a particle is described as a result of its interaction with a stratification of the quadridimensional Platonic space by a sequence of hyperplanes $H^{(n)}$ which are orthogonal to the direction of the projection $\vec{e_4}$, regularly spaced by a distance $\Delta w_0 > 0$.

This distance Δw_0 is equal to the ***Compton wavelength*** of the particle in question (for example, for an electron, $\Delta w_e = \dfrac{h}{m_e c} \approx 2,426.10^{-12}\ m$, where m_e denotes the rest mass of the electron).

These concepts are detailed in the Hal articles below:

hal-01165196, v1 : A Platonic (Euclidean-projective) model for the special theory of relativity.

hal-01207447, v1 : Towards a modeling of De Broglie waves in a Platonic quadridimensional space.

hal-01213447, v1 : An idea of the mass of a particle in a Platonic quadridimensional space.

hal-01247385, v1 : A modeling of Michelson-Morley interferometer in a Platonic quadri-dimensional space.

hal-01340134, v1 : One-dimensional elastic collisions in a Platonic quadridimensional space.

hal-01378215, v1 : About time measurement in a Platonic quadridimensional space.

hal-01584918, v1 : A quantified approach to the laws of gravitation in a Platonic quadri-dimensional space.

hal-01756323, v1 : De Broglie waves and quantification of Keplerian orbits in a Platonic quadridimensional space.

hal-01970150, v1 : An introduction to quantified variations of absolute time in the presence of a gravitational field in a Platonic quadridimensional space.

hal-02444592, v1 : Gravitational deflection of light in a Platonic quadridimensional space.

2. Measurement of the deformation of the Platonic space

Taking into account the principles exposed in the introduction, we will evaluate the measure of the deformation of the Platonic space generated in the fifth dimension by an occurrence of the mass wave.

In order to maintain overall consistency with the previous articles, the starting point adopted to determine this measurement will be the first postulate of quantified declination proposed in article hal-01584918; that is to say :

*« the **angular variation** $\Delta\beta_{i,j}$ corresponding to the modification of trajectory undergone by the body C_j in the Platonic space during the **perception of an occurrence of the mass wave** emitted by the body C_i is **independent of the reference frame** R_α and has for value:* $\Delta\beta_{i,j} = \dfrac{Gh}{c^3\left(d_{i,j}\right)^2}\sin\beta_j\cos\psi$ *.»*

It emerges (*) from this starting point that the displacement Δx_5 in the fifth dimension undergone by a body C_j during the perception of an occurrence of the mass wave emitted by the body C_i is given by:

$$\Delta x_5\left(C_j\right) = \frac{Gh}{c^3 d_{i,j}}\sin\beta_i \ .$$

((*) We will see in the ***following paragraph*** how the choice of this measure makes it possible to find the first postulate of quantified declination).

- **Notes**

① By denoting $\vec{v}$ the speed of C_j observed in the reference frame $R_{\beta i}$ linked to the body C_i, we set $\psi = \left(\overrightarrow{C_iC_j'}, \vec{v}\right)$; and the distance $d_{i,j}$ considered in this article (and in those cited in the references) corresponds to the distance measured in the reference frame R_{β_i} between the two interacting bodies. It is defined by the following :

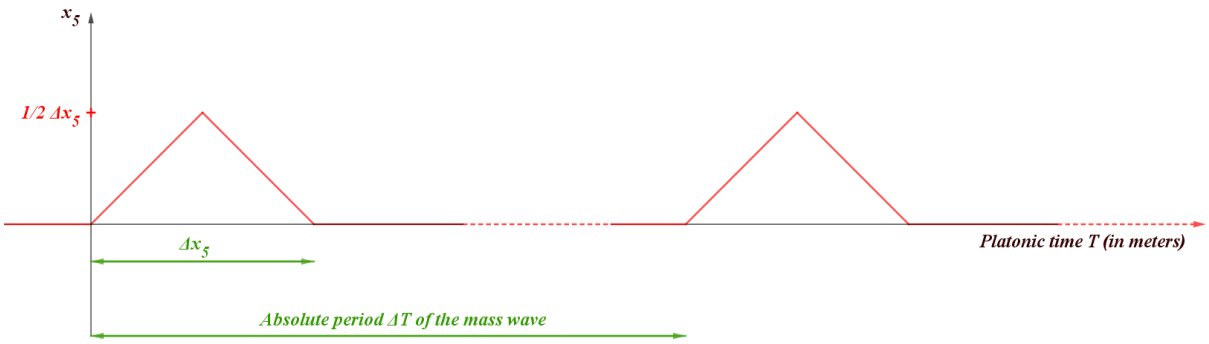
denote by $\vec{v}_i$ the velocity of the body C_i in the Platonic space (see appendix 2 page 15 for a detailed reminder of the concept of Platonic velocity);

by H_i the hyperplane linked to the body C_i, that is, the hyperplane orthogonal to $\vec{v}_i$ and passing through C_i;

by C_j' the projection (according to the direction of the vector $\vec{e_4}$) of the body C_j on H_i.

We define $d_{i,j} = C_iC_j'$.

② In order for the displacement Δx_5 undergone by the body C_j to have the value $\dfrac{Gh}{c^3 d_{i,j}}\sin\beta_i$, the ***deformation*** of the Platonic space in the fifth dimension could be equal to ***half of this value*** and be perceived cyclically as follows:



③ To give an order of the magnitude of this deformation, an electron located at a distance $d_{i,j} = 1\ m$ leads, for $\beta_i = \frac{\pi}{2}$, to:

$$\Delta T = \Delta w_{0i} = \frac{h}{m_e c} \approx 2,428 \times 10^{-12}\ m \quad \text{and} \quad \Delta x_5 \approx 1,64 \times 10^{-69}\ m\,.$$

④ With $d_{i,j} = n_{i,j} \times l_P$, where $n_{i,j} \in \mathbb{N}^*$ and $l_P = \sqrt{\frac{Gh}{2\pi c^3}} \approx 1,62 \times 10^{-35}\ m$ denotes the Planck length, we have: $\frac{1}{2}\Delta x_5 = \frac{1}{2}\frac{Gh}{c^3 d_{i,j}} \sin\beta_i = \frac{\pi l_P}{n_{i,j}} \sin\beta_i$; and consequently this quantity is ***less than the Planck length*** as soon as $n_{i,j} > 3$.

⑤ In appendix 3 on page 18, the mass wave corresponding to the ***apparent stopping*** of the body C_j in the Platonic space is calculated as a special case the absolute period ΔT_0.

Solving the equation $\Delta x_5 = \Delta T_0$ thus leads to:

$$\frac{Gh}{c^3 d_{i,j}} \sin\beta_i = \Delta w_{0i} \sin\beta_i \qquad \text{i.e.} \qquad d_{i,j} = \frac{Gh}{c^3 \Delta w_{0i}}\,.$$

Moreover, in the reference frame R_{β_i}, the mass at rest of the body C_i is $m_{0i} = \frac{h}{c\Delta w_{0i}}$.

So we have $d_{i,j} = \frac{G}{c^2} m_{0i}$, i.e. $d_{i,j} = \frac{1}{2} R_s$, where R_s represents the ***Schwarzschild radius*** of the body C_i.

*In other words, the **apparent stopping** of the body C_j in the Platonic space for $d_{i,j} = \frac{1}{2} R_s$ can be interpreted as a **cyclic displacement** of this body in the **fifth dimension**: after each displacement equal to Δx_5, the body C_j perceives a new occurrence of the mass wave generated by C_i (since $\Delta T_0 = \Delta x_5$) and is forced to move again a distance Δx_5 in the fifth dimension, etc.*

3. Link with the first postulate of quantified declination

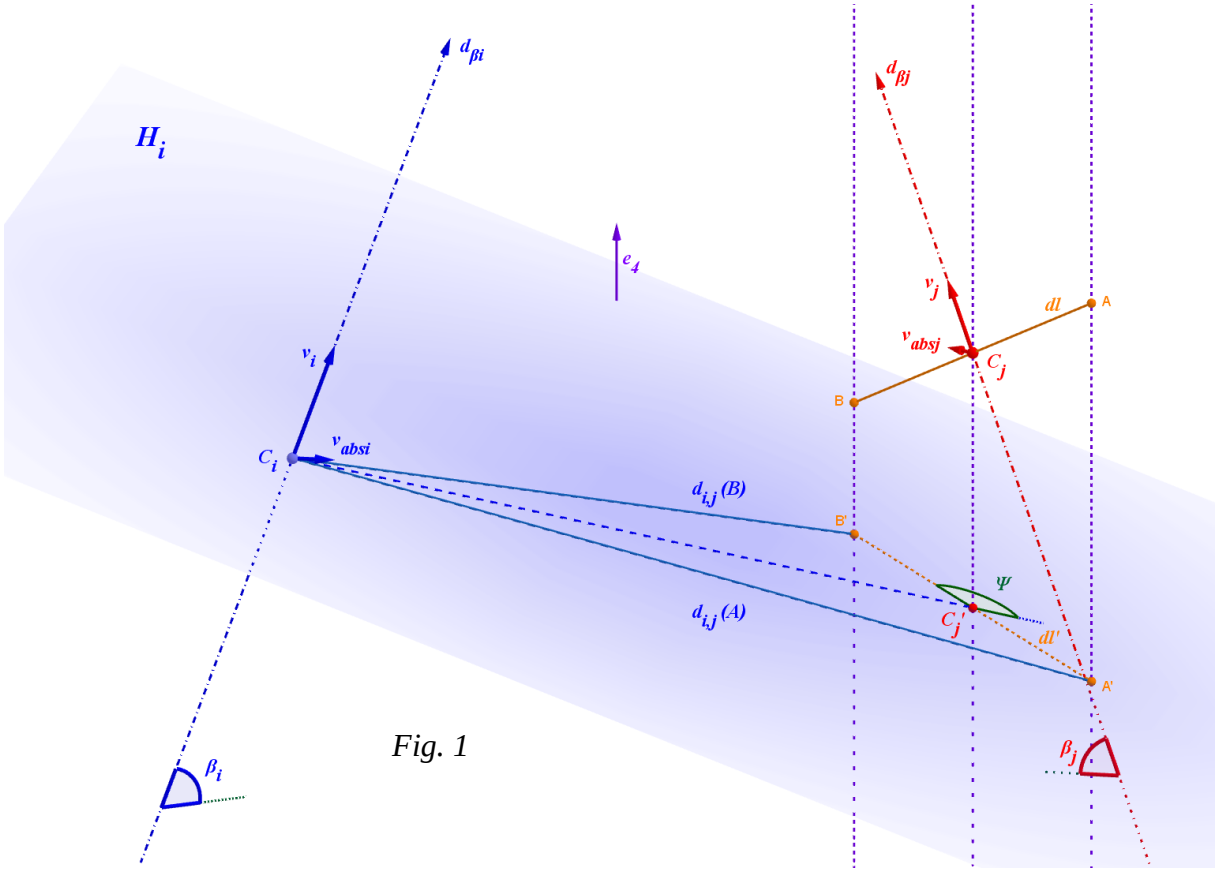


Fig. 1

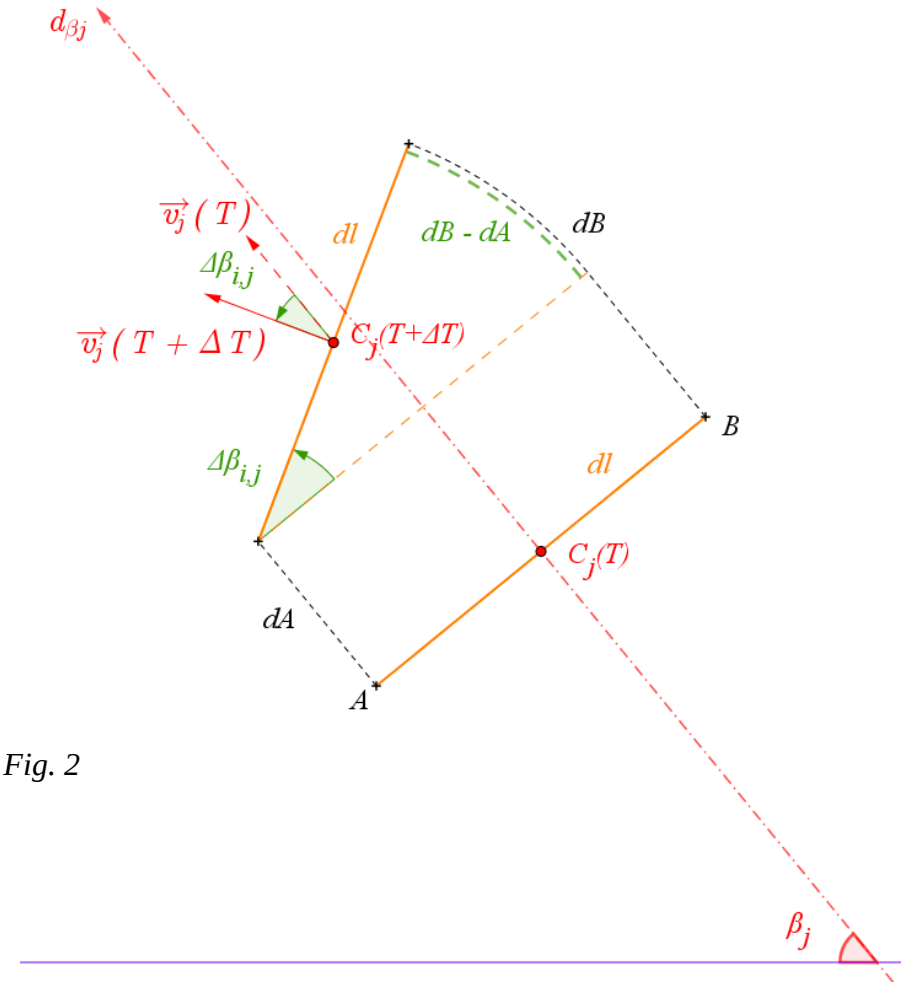


Fig. 2

① Figure 1 shows the bodies C_i and C_j, as well as their directions of displacement in the Platonic space.

The body C_j is assumed to be a ***non-point*** object: the segment $[AB]$, whose length is dl, represents a diameter of C_j in the plane directed by the vectors $\overrightarrow{v_j}$ and $\overrightarrow{e_4}$.

C_j', A' and B' represent the projections of these points in the hyperplane H_i linked to the body C_i and we denote $A'B' = dl'$.

We thus obtain (the details of the calculations are proposed in Appendix 1 on page 12):

❶ $dl' = dl \times \dfrac{\sin\beta_j}{\sin\beta_i} \times \sqrt{\sin^2\beta_i + \cos^2\beta_i \cos^2\sigma}$, where σ designates the angle $\left(\overrightarrow{v_{absi}}, \overrightarrow{v_{absj}}\right)$ formed by the absolute speeds of the bodies C_i and C_j (see appendix 1).

❷ Then, with $dl \approx 0$ and $\overrightarrow{C_iB'} = \overrightarrow{C_iA'} + \overrightarrow{A'B'}$, we obtain :

$$d_{i,j}(B) = d_{i,j}(A) + dl'\cos\psi \text{ , where } \psi = \left(\overrightarrow{C_iA'}, \overrightarrow{A'B'}\right) = \left(\overrightarrow{C_iC_j'}, \vec{v}\right)$$

(by denoting $\vec{v}$ the speed of C_j observed in the reference frame $R_{\beta i}$) ;

$$\text{hence : } \quad d_{i,j}(B) = d_{i,j}(A) + dl \times \frac{\sin\beta_j}{\sin\beta_i} \times \sqrt{\sin^2\beta_i + \cos^2\beta_i \cos^2\sigma} \times \cos\psi \,.$$

② Figure 2 represents the change in direction $\Delta\beta_{i,j}$ of the body C_j caused in the Platonic space by the perception of an occurrence of the mass wave generated by the body C_i.

According to paragraph 2 on page 5, the displacements in the fifth dimension of the ends *A* and *B* are:

- for A : $\Delta x_5(A) = \dfrac{Gh}{c^3 d_{i,j}(A)} \sin\beta_i$;
- for B : $\Delta x_5(B) = \dfrac{Gh}{c^3 d_{i,j}(B)} \sin\beta_i$.

During a period ΔT, it appears that the distances traveled by *A* and *B* in the Platonic space are:

- for A : $dA = \Delta T - \Delta x_5(A)$;
- for B : $dB = \Delta T - \Delta x_5(B)$; hence : $dB - dA = \Delta x_5(A) - \Delta x_5(B)$,

i.e. $$dB - dA = \frac{Gh}{c^3}\sin\beta_i \left(\frac{1}{d_{i,j}(B)} - \frac{1}{d_{i,j}(A)} \right) = \frac{Gh}{c^3}\sin\beta_i \times \frac{dl'\cos\psi}{d_{i,j}(A)^2 + d_{i,j}(A)\,dl'\cos\psi} \text{ ;}$$

then, with $\Delta\beta_{i,j} = \dfrac{dB - dA}{dl}$, we obtain :

$$\Delta\beta_{i,j} = \frac{Gh}{c^3}\sin\beta_i \times \frac{\dfrac{\sin\beta_j}{\sin\beta_i} \times \sqrt{\sin^2\beta_i + \cos^2\beta_i \cos^2\sigma} \times \cos\psi}{d_{i,j}(A)^2 + d_{i,j}(A)\,dl'\cos\psi} \; ;$$

and finally, with $dl' \approx 0$ and noting $d_{i,j}(A) = d_{i,j}$, we have :

$$\Delta\beta_{i,j} = \frac{Gh}{c^3\left(d_{i,j}\right)^2} \times \sin\beta_j \times \sqrt{\sin^2\beta_i + \cos^2\beta_i \cos^2\sigma} \times \cos\psi \; . \qquad [1]$$

This relation corresponds, in a generalized way, to the first postulate of quantified declination proposed in article hal-01584918 where the study carried out pre-assumed $\sigma = \pi$ and, in this particular case, we find:

$$\Delta\beta_{i,j} = \frac{Gh}{c^3\left(d_{i,j}\right)^2} \times \sin\beta_j \times \cos\psi \; .$$

NB. : in article hal-01584918, we notice that $\psi = \left(\overrightarrow{C_i'C_j'}, \overrightarrow{v_{absj}}\right)$ because $\beta_i = \dfrac{\pi}{2}$ and, in this case, the speeds $\overrightarrow{v_{absj}}$ and $\vec{v}$ are collinear.

4. A modeling of the relativistic acceleration of gravity

Keeping the notations of the previous paragraphs, the usual notion of speed (measured in m.s^{-1}) in the reference frame R_{β_i} leads, for the body C_j, to:

$$\frac{v}{c}=\frac{\sqrt{\left(1-\cos\beta_i\cos\beta_j\cos\sigma\right)^2-\sin^2\beta_i\sin^2\beta_j}}{1-\cos\beta_i\cos\beta_j\cos\sigma}$$

(see calculation details in appendix 2 page 15).

To simplify the calculations, we will limit this paragraph to the particular case where the movement of the body C_j takes place in the plane passing through C_i and having as direction vectors $\vec{v_i}$ and $\vec{e_4}$, in other words for $\sigma=0\,[\pi]$, that is to say $\cos\sigma=\pm1$. We then obtain:

$$\frac{v}{c}=\frac{\sqrt{\left(1\pm\cos\beta_i\cos\beta_j\right)^2-\sin^2\beta_i\sin^2\beta_j}}{1\pm\cos\beta_i\cos\beta_j}=\frac{\left|\cos\beta_i\pm\cos\beta_j\right|}{1\pm\cos\beta_i\cos\beta_j}.$$

Furthermore, we will assume that the mass of the body C_i linked to the observation reference frame R_{β_i} is very large compared to that of C_j and that, consequently, the angle β_i remains constant as a function of absolute time *T*.

With the relation $\frac{dT}{cdt}=\frac{\sin\beta_i}{1-\cos\beta_i\cos\beta_j\cos\sigma}$ (see appendix 2), where t measures the usual time (expressed in seconds) into the reference frame R_{β_i}, we thus obtain, for the usual acceleration *a* (measured in m.s^{-2}) of the body C_j :

$$a=\frac{dv}{dt}\;\Rightarrow\;\frac{a}{c^2}=\frac{d\left(\frac{v}{c}\right)}{cdt}=\frac{d\left(\frac{v}{c}\right)}{dT}\times\frac{dT}{cdt}$$

i.e. (with $\cos\sigma=\pm1$) : $$\frac{a}{c^2}=\frac{-d\beta_j}{dT}\times\frac{\sin^2\beta_i\sin\beta_j}{\left(1\pm\cos\beta_i\cos\beta_j\right)^2}\times\frac{\sin\beta_i}{1\pm\cos\beta_i\cos\beta_j}$$

that is to say : $$\frac{a}{c^2}=\frac{-d\beta_j}{dT}\times\frac{\sin^3\beta_i\sin\beta_j}{\left(1\pm\cos\beta_i\cos\beta_j\right)^3} \qquad [2]$$

The factor $\frac{d\beta_j}{dT}$ is obtained from the relation $\frac{d\beta_j}{dT}=f_{i,j}\times\Delta\beta_{i,j}$, where $f_{i,j}$ represents the absolute frequency of the mass wave perceived by the body C_j.

According to Annex 3 page 18, we have : $f_{i,j} = \dfrac{1-\cos\beta_i \cos\beta_j \cos\sigma}{\Delta w_{0i} \sin\beta_i}$. [3]

Which leads to, with [1], [3] and $\cos\sigma = \pm 1$:

$$\frac{d\beta_j}{dT} = \frac{1 \pm \cos\beta_i \cos\beta_j}{\Delta w_{0i} \sin\beta_i} \times \frac{Gh}{c^3 \left(d_{i,j}\right)^2} \times \sin\beta_j \times \cos\psi .$$

The relation [2] thus becomes :

$$\frac{a}{c^2} = -\frac{1 \pm \cos\beta_i \cos\beta_j}{\Delta w_{0i} \sin\beta_i} \times \frac{Gh}{c^3 \left(d_{i,j}\right)^2} \times \sin\beta_j \times \frac{\sin^3\beta_i \sin\beta_j}{\left(1 \pm \cos\beta_i \cos\beta_j\right)^3} \times \cos\psi$$

i.e. $$a = -\frac{Gh}{c\Delta w_{0i} \left(d_{i,j}\right)^2} \times \frac{\sin^2\beta_i \sin^2\beta_j}{\left(1 \pm \cos\beta_i \cos\beta_j\right)^2} \times \cos\psi .$$

(Note that in the particular case studied in this paragraph, we have A = B $\cos\psi = 1$ if $\psi = \left(\overrightarrow{C_i C_j}', \vec{v}\right) = 0$ or $\cos\psi = -1$ if $\psi = \left(\overrightarrow{C_i C_j}', \vec{v}\right) = \pi$).

Finally, we have $m_{0i} = \dfrac{h}{c\Delta w_{0i}}$ (mass at rest of body C_i in the reference frame R_{β_i} - see article hal-01213447 -) and, according to the remark made in appendix 2 page 15:

$$\frac{\sin^2\beta_i \sin^2\beta_j}{\left(1 \pm \cos\beta_i \cos\beta_j\right)^2} = 1 - \frac{v^2}{c^2} .$$

We thus find the relativistic formula: $$|a| = \frac{Gm_{0i}}{\left(d_{i,j}\right)^2}\left(1 - \frac{v^2}{c^2}\right) .$$

5. Appendix 1 : calculation of the ratio *dl'/dl*

- Let us first briefly recall the concept of speed of a mobile M in the Platonic space (see in particular articles hal-01584918 and hal-01756323):

  given the definition of the absolute time T (in m), the ***norm of the speed vector*** *of all the mobiles* ***is equal to 1*** *and the velocity vector* $\vec{v} = \frac{\overrightarrow{dM}}{dT}$ *of any mobile* M *is given by:*

$$\vec{v} = \begin{pmatrix} \cos\beta\cos\gamma\cos\varphi \\ \cos\beta\cos\gamma\sin\varphi \\ \cos\beta\sin\gamma \\ \sin\beta \end{pmatrix},$$

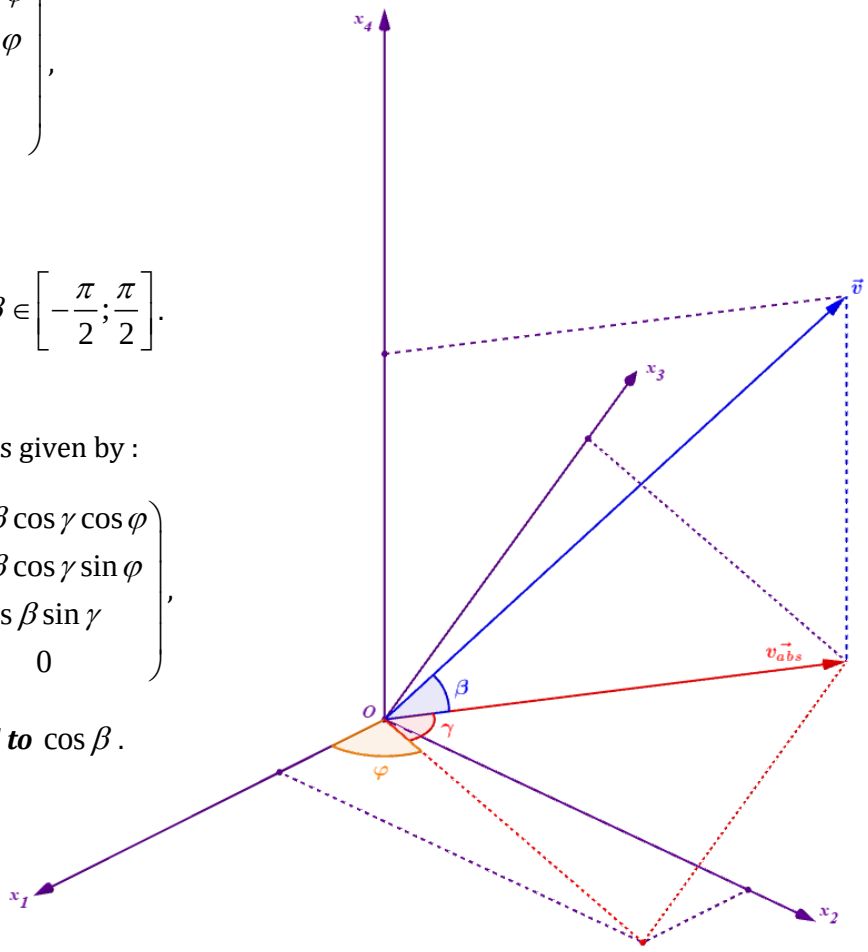
  with any φ,

$$\gamma \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right] \text{ and } \beta \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right].$$

  Its ***absolute speed*** is given by :

$$\overrightarrow{v_{abs}} = \begin{pmatrix} \cos\beta\cos\gamma\cos\varphi \\ \cos\beta\cos\gamma\sin\varphi \\ \cos\beta\sin\gamma \\ 0 \end{pmatrix},$$

  whose ***norm is equal to*** $\cos\beta$.



- To simplify the calculation lines which follow, we will note $\vec{v_i} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$ the velocity vector of the body C_i and $\vec{v_j} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ the velocity vector of the body C_j.

  We will designate by H_i the hyperplane linked to C_i, that is to say the hyperplane orthogonal to $\vec{v_i}$ passing through C_i and by H_j the hyperplane linked to C_j, i.e. the hyperplane orthogonal to $\vec{v_j}$ passing through C_j.

① So we have the following equivalences:

a point $M\left(x_1, x_2, x_3, x_4\right) \in H_i$ if and only if :

$$a_1\left(x_1 - x_1\left(C_i\right)\right) + a_2\left(x_2 - x_2\left(C_i\right)\right) + a_3\left(x_3 - x_3\left(C_i\right)\right) + a_4\left(x_4 - x_4\left(C_i\right)\right) = 0 \qquad \text{[R1]}$$

and $M\left(x_1, x_2, x_3, x_4\right) \in H_j$ if and only if :

$$b_1\left(x_1 - x_1\left(C_j\right)\right) + b_2\left(x_2 - x_2\left(C_j\right)\right) + b_3\left(x_3 - x_3\left(C_j\right)\right) + b_4\left(x_4 - x_4\left(C_j\right)\right) = 0 .$$

② The plane $\left(P_j\right)$ passing through C_j and directed by $\overrightarrow{v_j}$ and $\overrightarrow{e_4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ is parame-

trically represented by : $\begin{cases} x_1 = x_1\left(C_j\right) + \lambda b_1 \\ x_2 = x_2\left(C_j\right) + \lambda b_2 \\ x_3 = x_3\left(C_j\right) + \lambda b_3 \\ x_4 = x_4\left(C_j\right) + \lambda b_4 + \mu \end{cases}$, with $\left(\lambda, \mu\right) \in \mathbb{R}^2$.

Hence $M\left(x_1, x_2, x_3, x_4\right) \in H_j \cap P_j$ if and only if $\lambda {b_1}^2 + \lambda {b_2}^2 + \lambda {b_3}^2 + \lambda {b_4}^2 + \mu b_4 = 0$.

With ${b_1}^2 + {b_2}^2 + {b_3}^2 + {b_4}^2 = \overrightarrow{v_j}^2 = 1$, we obtain $\mu = -\dfrac{\lambda}{b_4}$ et $M\left(x_1, x_2, x_3, x_4\right) \in H_j \cap P_j$ if

and only if : $\begin{cases} x_1 = x_1\left(C_j\right) + \lambda b_1 \\ x_2 = x_2\left(C_j\right) + \lambda b_2 \\ x_3 = x_3\left(C_j\right) + \lambda b_3 \\ x_4 = x_4\left(C_j\right) + \lambda\left(b_4 - \dfrac{1}{b_4}\right) \end{cases}$, with $\lambda \in \mathbb{R}$.

③ The ratio dl'/dl doesn't depend on the length AB, so we can choose for these points two arbitrary values of the parameter λ, for example $\lambda = 0$ for A and $\lambda = 1$ for B ; which leads to $A = \left(x_1\left(C_j\right), x_2\left(C_j\right), x_3\left(C_j\right), x_4\left(C_j\right)\right)$

and $B = \left(x_1\left(C_j\right) + b_1, x_2\left(C_j\right) + b_2, x_3\left(C_j\right) + b_3, x_4\left(C_j\right) + b_4 - \dfrac{1}{b_4}\right)$.

④ By denoting A' the projection of A in the hyperplane H_i, we have :

$A' = \left(x_1\left(C_j\right), x_2\left(C_j\right), x_3\left(C_j\right), x_4\left(A'\right)\right)$, whose coordinates satisfy [R1], which leads to :

$$x_4(A') = \frac{1}{a_4}\left[a_1\left(x_1(C_i) - x_1(C_j)\right) + a_2\left(x_2(C_i) - x_2(C_j)\right) + a_3\left(x_3(C_i) - x_3(C_j)\right) + a_4 x_4(C_i)\right]$$

Likewise, denoting by B' the projection of B in the hyperplane H_i, we have :

$B' = \left(x_1(C_j) + b_1, x_2(C_j) + b_2, x_3(C_j) + b_3, x_4(B')\right)$, whose coordinates satisfy [R1], which leads to :

$$x_4(B') = \frac{1}{a_4}\left[a_1\left(x_1(C_i) - x_1(C_j)\right) + a_2\left(x_2(C_i) - x_2(C_j)\right) + a_3\left(x_3(C_i) - x_3(C_j)\right) + a_4 x_4(C_i) - a_1 b_1 - a_2 b_2 - a_3 b_3\right].$$

⑤ We thus obtain, with $b_1^2 + b_2^2 + b_3^2 + b_4^2 = 1$:

$$dl^2 = AB^2 = b_1^2 + b_2^2 + b_3^2 + \left(b_4 - \frac{1}{b_4}\right)^2 = \frac{1 - b_4^2}{b_4^2}$$

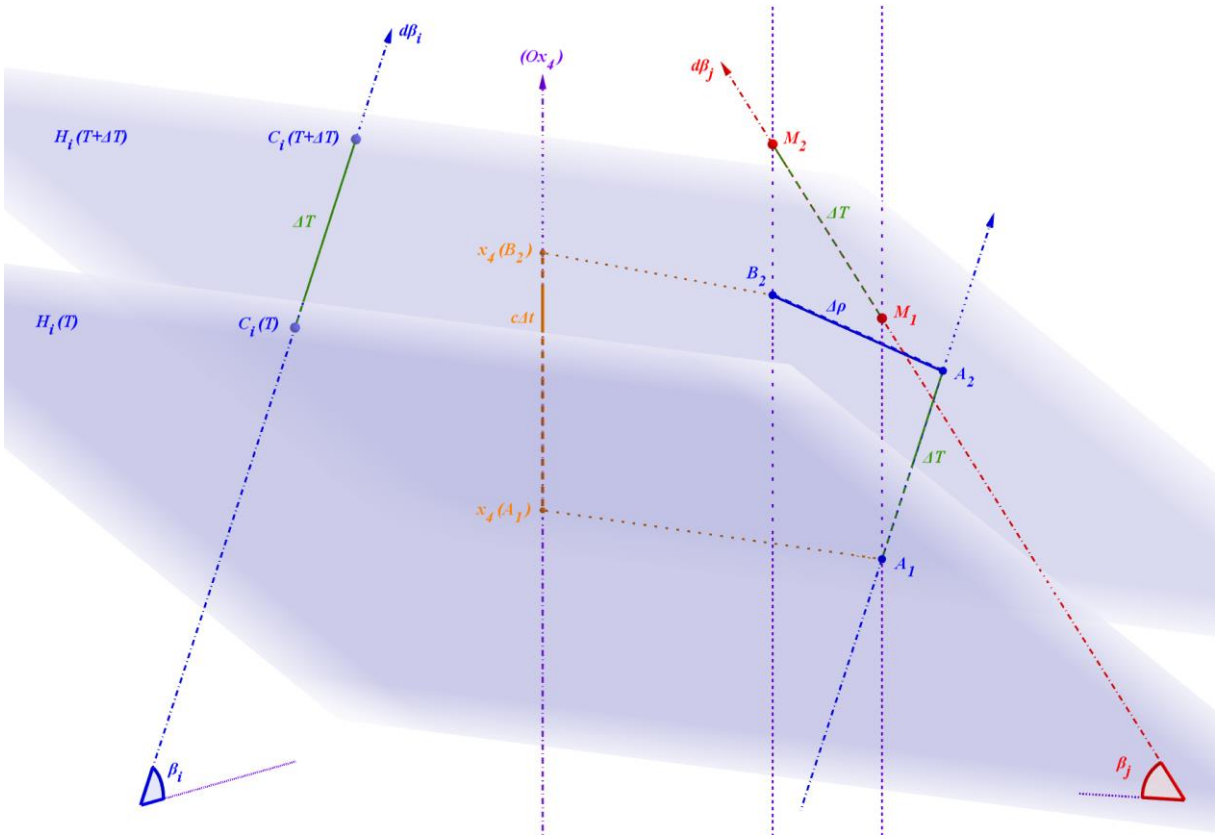
and $dl'^2 = A'B'^2 = b_1^2 + b_2^2 + b_3^2 + \left(\frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{a_4}\right)^2 = 1 - b_4^2 + \frac{\left(a_1 b_1 + a_2 b_2 + a_3 b_3\right)^2}{a_4^2}$.

Hence: $\frac{dl'^2}{dl^2} = \left[1 - b_4^2 + \frac{\left(a_1 b_1 + a_2 b_2 + a_3 b_3\right)^2}{a_4^2}\right] \times \frac{b_4^2}{1 - b_4^2} = \frac{b_4^2}{a_4^2} \times \left[a_4^2 + \frac{\left(a_1 b_1 + a_2 b_2 + a_3 b_3\right)^2}{1 - b_4^2}\right]$.

Finally, we have $a_1 b_1 + a_2 b_2 + a_3 b_3 = \overrightarrow{v_{absi}}.\overrightarrow{v_{absj}} = \cos\beta_i \cos\beta_j \cos\sigma$, where σ denotes the angle $\left(\overrightarrow{v_{absi}}, \overrightarrow{v_{absj}}\right)$; $a_4 = \sin\beta_i$ and $b_4 = \sin\beta_j$, hence the result :

$$\frac{dl'}{dl} = \frac{\sin\beta_j}{\sin\beta_i} \times \sqrt{\sin^2\beta_i + \cos^2\beta_i \cos^2\sigma}\ .$$

6. Appendix 2 : calculation of the speed *v* (in m.s^{-1}) of a mobile M in the observation frame of reference $R_{\beta i}$



In this paragraph, we will determine the speed (in m.s^{-1}, in the classical sense of the term) of a mobile M observed in the reference frame R_{β_i} linked to the body C_i.

In the Platonic space, C_i moves in the direction $d\beta_i$ and M in the direction $d\beta_j$.

As in Appendix 1, to simplify the calculation lines which follow, we will denote $\vec{v_i} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$ the speed vector of the body C_i and $\vec{v_j} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$ the speed vector of the body M.

We will designate by H_i the hyperplane linked to C_i, that is to say the hyperplane orthogonal to $\vec{v_i}$ passing through C_i and, as before, a point $M(x_1, x_2, x_3, x_4) \in H_i$ if and only if :

$$a_1\left(x_1 - x_1(C_i)\right) + a_2\left(x_2 - x_2(C_i)\right) + a_3\left(x_3 - x_3(C_i)\right) + a_4\left(x_4 - x_4(C_i)\right) = 0 \qquad [R1]$$

① At an (absolute) time T, the mobile M is at $M_1 = \left(x_1\left(M_1\right), x_2\left(M_1\right), x_3\left(M_1\right), x_4\left(M_1\right)\right)$ and is perceived in R_{β_i} by the observer $A_1 = \left(x_1\left(M_1\right), x_2\left(M_1\right), x_3\left(M_1\right), x_4\left(A_1\right)\right)$.

As $A_1 \in H_i$, we have, with [R1] :

$$x_4\left(A_1\right) = \frac{1}{a_4}\left[a_4 x_4\left(C_i\right) - a_1\left(x_1\left(M_1\right) - x_1\left(C_i\right)\right) - \ldots - a_3\left(x_3\left(M_1\right) - x_3\left(C_i\right)\right)\right].$$

② At the (absolute) time $T + \Delta T$, the mobile M is at M_2, with :

$$M_2 = \left(x_1\left(M_1\right) + b_1\Delta T, x_2\left(M_1\right) + b_2\Delta T, x_3\left(M_1\right) + b_3\Delta T, x_4\left(M_1\right) + b_4\Delta T\right)$$

and is perceived in R_{β_i} by the observer $B_2 = \left(x_1\left(M_2\right), x_2\left(M_2\right), x_3\left(M_2\right), x_4\left(B_2\right)\right)$.

The observer A is then at $A_2 = \left(x_1\left(M_1\right) + a_1\Delta T, \ldots, x_3\left(M_1\right) + a_3\Delta T, x_4\left(A_1\right) + a_4\Delta T\right)$; and the Cartesian equation of the hyperplane $H_i\left(T + \Delta T\right)$ linked to C_i is :

$M\left(x_1, x_2, x_3, x_4\right) \in H_i\left(T + \Delta T\right)$ if and only if :

$$a_1\left(x_1 - x_1\left(C_i\right) - a_1\Delta T\right) + \ldots + a_4\left(x_4 - x_4\left(C_i\right) - a_4\Delta T\right) = 0. \qquad \text{[R2]}$$

As $B_2 \in H_i\left(T + \Delta T\right)$, we have, with [R2] :

$$x_4\left(B_2\right) = \frac{1}{a_4}\left[a_4 x_4\left(C_i\right) + {a_4}^2\Delta T - a_1\left(x_1\left(M_1\right) + b_1\Delta T - x_1\left(C_i\right) - a_1\Delta T\right) - \ldots - a_3\left(x_3\left(M_1\right) + b_3\Delta T - x_3\left(C_i\right) - a_3\Delta T\right)\right]$$

③ According to the Platonic model (cf. articles hal-01165196 and hal-01378215), the duration Δt measured (in seconds) in the reference frame R_{β_i} between these two events is given by : $c\Delta t = x_4\left(B_2\right) - x_4\left(A_1\right)$. So we have :

$$c\Delta t = \frac{1}{a_4}\left[{a_4}^2\Delta T - a_1 b_1\Delta T + {a_1}^2\Delta T - a_2 b_2\Delta T + {a_2}^2\Delta T - a_3 b_3\Delta T + {a_3}^2\Delta T\right].$$

Furthermore $\sum_{i=1}^{4} {a_i}^2 = 1$, that leads to : $c\Delta t = \frac{1}{a_4}\left[\Delta T - a_1 b_1\Delta T - a_2 b_2\Delta T - a_3 b_3\Delta T\right]$;

and finally, we have $a_1 b_1 + a_2 b_2 + a_3 b_3 = \overrightarrow{v_{absi}}.\overrightarrow{v_{absj}} = \cos\beta_i \cos\beta_j \cos\sigma$, where σ denotes the angle $\left(\overrightarrow{v_{absi}}, \overrightarrow{v_{absj}}\right)$ and $a_4 = \sin\beta_i$, hence the result :

$$c\Delta t = \frac{\Delta T}{\sin\beta_i}\left[1 - \cos\beta_i \cos\beta_j \cos\sigma\right].$$

④ The distance $\Delta\rho$ measured (in meters) in the reference frame R_{β_i} between these two events is given by : $\Delta\rho = A_2 B_2$ (cf. article hal-01165196).

So we have $\Delta\rho = \sqrt{\sum_{i=1}^{4}\left(x_i(B_2) - x_i(A_2)\right)^2}$, which leads, with the previous results, to:

$$\Delta\rho = \frac{\Delta T}{\sin\beta_i}\sqrt{\left(1-\cos\beta_i\cos\beta_j\cos\sigma\right)^2 - \sin^2\beta_i\sin^2\beta_j}\ .$$

⑤ To conclude, we thus obtain the modulus (in the classical sense of the term, expressed in m.s^{-1}) of the speed of the mobile M observed in the reference frame R_{β_i} linked to the body C_i :

$$\frac{v}{c} = \frac{\Delta\rho}{c\Delta t} = \frac{\sqrt{\left(1-\cos\beta_i\cos\beta_j\cos\sigma\right)^2 - \sin^2\beta_i\sin^2\beta_j}}{1-\cos\beta_i\cos\beta_j\cos\sigma}\ . \qquad \text{[R3]}$$

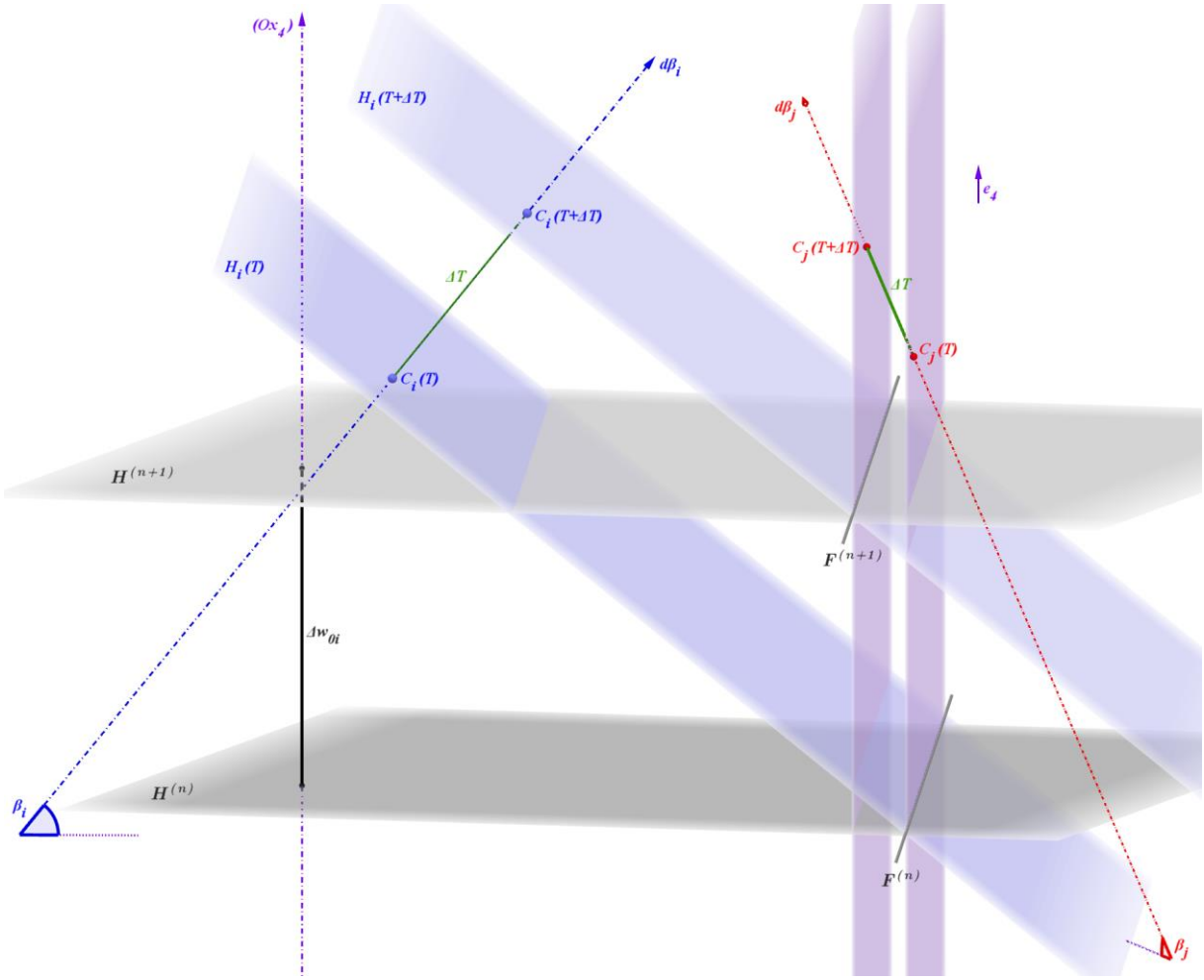
Remarks :

❶ two special cases are to be noted:

- if $\sigma = 0$, we thus obtain : $\frac{v}{c} = \frac{\left|\cos\beta_i - \cos\beta_j\right|}{1-\cos\beta_i\cos\beta_j}$;
- and if $\sigma = \pi$, we have : $\frac{v}{c} = \frac{\cos\beta_i + \cos\beta_j}{1-\cos\beta_i\cos\beta_j}$.

❷ The relation [R3] leads to : $1 - \frac{v^2}{c^2} = \frac{\sin^2\beta_i\sin^2\beta_j}{\left(1-\cos\beta_i\cos\beta_j\cos\sigma\right)^2}$.

7. Appendix 3 : calculation of the absolute frequency of the mass wave generated by the body C_i and perceived by the body C_j



We take up and generalize here the concepts exposed in article hal-01207447: "Towards a modeling of De Broglie waves in a four-dimensional Platonic space".

Following this article and article hal-01213447, the relativistic notion of the mass of a particle is described as a consequence of its interaction with a stratification of the Platonic four-dimensional space by a series of hyperplanes $H^{(n)}$ orthogonal to the direction of projection $\overrightarrow{e_4}$, regularly spaced by a distance $\Delta w_{0i} > 0$.

This distance Δw_{0i} is equal to the ***Compton wavelength*** of the particle considered.

① At an (absolute) time T, the body C_j perceives the front $F^{(n)}$ of the mass wave generated by the massive body C_i ; then, after an absolute period ΔT (measured in meters), the body C_j perceives the occurrence $F^{(n+1)}$.

$F^{(n)}$ corresponds to the intersection of hyperplanes $H_i(T)$ and $H^{(n)}$, of which a Cartesian equation is: $x_4 = x_4\left(H^{(n)}\right)$; and $F^{(n+1)}$ corresponds to the intersection of hyperplanes $H_i(T+\Delta T)$ and $H^{(n+1)}$, of which a Cartesian equation is: $x_4 = x_4\left(H^{(n)}\right) + \Delta w_{0i}$.

② By taking again the concepts and the abbreviated notations used in the two preceding appendices, this leads to the following relations:

- for the Platonic time T :

$$a_1\left(x_1\left(C_j\right)-x_1\left(C_i\right)\right)+\ldots+a_3\left(x_3\left(C_j\right)-x_3\left(C_i\right)\right)+a_4\left(x_4\left(H^{(n)}\right)-x_4\left(C_i\right)\right)=0\ ;$$

- and for the time $T+\Delta T$:

$$a_1\left(x_1\left(C_j\right)+b_1\Delta T-x_1\left(C_i\right)-a_1\Delta T\right)+\ldots+a_3\left(x_3\left(C_j\right)+b_3\Delta T-x_3\left(C_i\right)-a_3\Delta T\right)$$
$$+a_4\left(x_4\left(H^{(n)}\right)+\Delta w_{0i}-x_4\left(C_i\right)-a_4\Delta T\right)=0\,.$$

This system results in: $\Delta T=\dfrac{a_4\Delta w_{0i}}{1-a_1b_1-a_2b_2-a_3b_3}$;

and finally, with $a_1b_1+a_2b_2+a_3b_3=\overrightarrow{v_{absi}}.\overrightarrow{v_{absj}}=\cos\beta_i\cos\beta_j\cos\sigma$, where σ denotes the angle $\left(\overrightarrow{v_{absi}},\overrightarrow{v_{absj}}\right)$ and $a_4=\sin\beta_i$, we obtain :

$$\Delta T=\frac{\Delta w_{0i}\sin\beta_i}{1-\cos\beta_i\cos\beta_j\cos\sigma}\,.$$

③ The absolute frequency $f_{i,j}$ (measured in m^{-1}) of the mass wave generated by the body C_i and perceived by the body C_j is therefore given by:

$$f_{i,j}=\frac{1}{\Delta T}=\frac{1-\cos\beta_i\cos\beta_j\cos\sigma}{\Delta w_{0i}\sin\beta_i}\,.$$

Note

The movements of the body C_j in the fifth dimension generate an apparent decrease of its Platonic velocity v_j .

In the particular case where its apparent speed v_j becomes zero, we then have $\cos\beta_j=0$ (because $v_j=0\Rightarrow b_1=b_2=b_3=b_4=0$) and the absolute period of the mass wave generated by the body C_i and perceived by C_j becomes: $\Delta T_0=\Delta w_{0i}\sin\beta_i$.

The absolute frequency $f_{i,j}$ of the mass wave generated by the body C_i and perceived by the body C_j is then given by: $f_{i,j}=\dfrac{1}{\Delta T_0}=\dfrac{1}{\Delta w_{0i}\sin\beta_i}$.

These results are used on page 6, note ⑤.

8. Conclusion

Since the beginning of the twentieth century, scientific research has often called on additional physical dimensions (Kaluza-Klein's works, String theory, Calabi-Yau spaces, etc.).

The results obtained in the articles cited with reference to page 3 lead here to the introduction of a fifth dimension in which the Platonic four-dimensional space would be immersed and would undergo cyclic deformations, in connection with the De Broglie mass waves generated by the bodies in gravitational interaction.

The Euclidean-projective Platonic model, thus completed, while rediscovering the uniformity of the flow of "absolute time" which is one of its founding principles, offers here from these very small variations a geometric modeling of the relativistic acceleration of gravity.

And this modeling applies equally well to several other relativistic phenomena: variations in the measurement of time in a satellite in orbit around the Earth, the gravitational redshift, the advance of the perihelion of the planets and the deflection of light in a gravitational field (cf. articles hal-01970150 and hal-02444592).

It should also be noted that these very small deformations are less than the Planck length (cf. note ④ on page 6).

This result, in fact, makes this modelization compatible with the theories which introduce additional dimensions whose size is equal to this length.

Given the fruitful results obtained thanks to these multidimensional models, this convergence thus seems to open up promising research in a new geometric framework.

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