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► **To cite this version:**

Sergio Ciliberto. An autonomous out of equilibrium Maxwell's demon for controlling the energy fluxes produced by thermal fluctuations. 2020. hal-02937440

HAL Id: hal-02937440

<https://hal.science/hal-02937440>

Preprint submitted on 14 Sep 2020

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An autonomous out of equilibrium Maxwell's demon for controlling the energy fluxes produced by thermal fluctuations

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(Dated: September 13, 2020)

An autonomous out of equilibrium Maxwell's demon is used to reverse the natural direction of the heat flux between two electric circuits kept at different temperatures and coupled by the electric thermal noise. The demon does not process any information, but it achieves its goal by using a frequency dependent coupling with the two reservoirs of the system. There is no energy flux between the demon and the system, but the total entropy production (system+demon) is positive. The demon can be power supplied by thermocouples. The system and the demon are ruled by equations similar to those of two coupled Brownian particles and of the Brownian gyrator. Thus our results pave the way to the application of autonomous out equilibrium Maxwell demons to coupled nanosystems at different temperatures.

Nowadays the notion of Maxwell's demon (MD) is generically used to indicate mechanisms that allow a system to execute tasks in apparent violation of the second law of thermodynamics, such as for example to produce work from a single heat bath and to transfer heat from cold to hot sources. To obtain this result the demon does not exchange energy with the system but it has a positive entropy production rate, which compensates the negative entropy production of the system. In general the increase in entropy is induced by the fact that the demon needs to analyze the information that it gathers on the system status [1, 2]. In experiments this apparent violation of the second law is obtained by feedback mechanisms which often require the use of external devices such as A/D converters, computers etc. [3–6]. Several smart experiments [7–9] have implemented these feedback locally constructing in this way autonomous Maxwell demons, which do not need the use of external devices as the measure and the feedback are performed in the same place. Several autonomous Maxwell demons have been theoretically developed [10–14], but they can be of difficult practical implementation in several devices such as colloidal particles and mesoscopic electric circuits at room temperature. However it has been recently introduced in ref. [15] a new paradigm of MD based on an out of equilibrium device, which does not elaborate any information about the system status. It has been shown that the parameters of this device can be suitably tuned in such a way that it does not exchange energy (heat or work) with the system but it has a positive entropy production rate. Thus it has the two main requirements of an autonomous MD and it can be more easily experimentally realized because, in contrast to the commonly used definition of MD, it works without acquiring and analyzing any information about the system status.

We discuss here how to implement an out of equilibrium MD (OEMD) [15] in electric circuits, which are

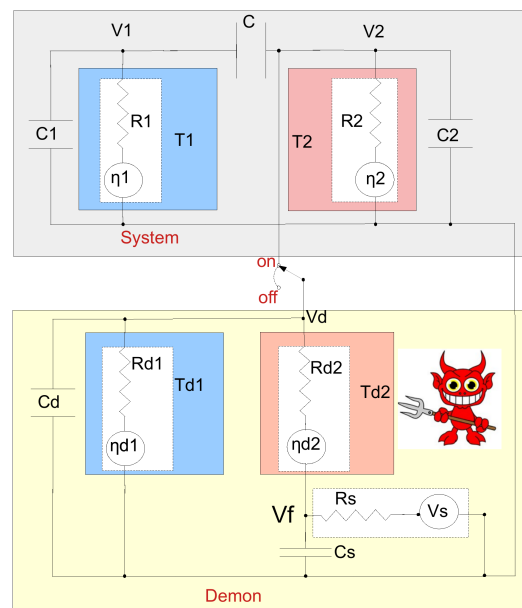


FIG. 1: Diagram of the system (grey box) and of the demon (yellow box). The system is constituted by the two resistances R_1 and R_2 kept respectively at temperature T_1 and T_2 , with $T_2 \geq T_1$. They are coupled via the capacitor C . The capacitors C_1 and C_2 schematize the capacitances of the cables and of the amplifier inputs. The demon (yellow box), is composed by two resistances (R_{d1} and R_{d2}) kept at two different temperatures T_{d1} and T_{d2} . Furthermore the resistance R_{d2} is driven by a voltage generator V_s whose out-put is filtered by the low pas filter composed by the resistance R_s and the capacitance C_s . The four voltage generators η_k ($k=1,2,d1,d2$) represent the Nyquist noise voltages of the resistances at the temperatures of the heat baths.

versatile dynamical systems ruled by coupled Langevin equations [16, 17]. Thus our study is quite general because it opens the way to the application of OEMDs to coupled nanosystems modeled by Langevin equations. As an example we will show in this article how an OEMD can be used to reverse the natural direction of the heat flux

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between two electric circuits kept at different temperatures and coupled by the electric thermal noise [18, 19]. In fig.1 we sketch the system (gray box) and the demon (yellow box). We chose for the system this specific circuit because the statistical properties of the heat flux have been characterized both theoretically and experimentally [18, 19]. Furthermore it is ruled by the same equations of the Brownian gyrator [20, 21] and of two Brownian particles coupled by a harmonic potential and kept at different temperatures[18], making the result rather general

The system (grey box in fig.1) is constituted by two resistances R_1 and R_2 , which are kept at two different temperatures T_1 and $T_2 \geq T_1$. In the figure, the two resistances have been drawn with their associated thermal noise generators η_1 and η_2 , whose power spectral densities are given by the Nyquist formula $|\tilde{\eta}_m|^2 = 4k_B R_m T_m$, with $m = 1, 2$. The coupling capacitance C controls the electrical power exchanged between the resistances and as a consequence the energy exchanged between the two baths. No other coupling exists between the two resistances.. The two capacitors C_1 and C_2 represents the sum of the circuit and cable capacitances. All the relevant energy exchanges in the system can be derived by the simultaneous measurements of the voltage V_m ($m = 1, 2$) across the resistance R_m and the currents i_m flowing through them. When $T_1 = T_2$ the system is in equilibrium and exhibits no net energy flux between the two reservoirs. The circuit equations can be written in terms of charges q_m flowed through the resistances R_m , so the measured instantaneous currents are $i_m = \dot{q}_m$. We make the choice of working with charges because the analogy with a Brownian particle is straightforward as q_m is equivalent to the displacement of the particle m [17–19]. A circuit analysis shows that the equations for the charges are:

$$R_1 \dot{q}_1 = V_1 - \eta_1, \quad \text{and} \quad R_2 \dot{q}_2 = \eta_2 - V_2, \quad (1)$$

with

$$V_1 = \frac{-q_1(C + C_2) + q_2 C}{X} \quad (2)$$

$$V_2 = \frac{-q_1 C + q_2 (C + C_1)}{X}. \quad (3)$$

where $X = C_2 C_1 + C (C_1 + C_2)$ and η_m is the Nyquist white noise: $\langle \eta_i(t) \eta_j(t') \rangle = 2\delta_{ij} k_B T_i R_j \delta(t - t')$. In ref.[19] we have shown that eqs.1 fully characterize all the thermodynamics properties of the system.

In this system the work and the heat are defined as

$$\dot{W}_m = \frac{C}{X} q_{m'} \dot{q}_m \quad (4)$$

$$\dot{Q}_m = V_m i_m = V_m \frac{V_m - \eta_m}{R_m}. \quad (5)$$

The quantities \dot{W}_m is identified as the thermodynamic work performed by the circuit m' on $m \neq m'$ and \dot{Q}_m the heat dissipated by the resistance m [16–19, 22]. As all the variables are fluctuating, the derived quantities

\dot{Q}_m and \dot{W}_m fluctuate too. In ref.[18] we computed and measured the mean heat flux between the two heat baths, which is given by :

$$\langle \dot{Q}_1 \rangle = -\langle \dot{Q}_2 \rangle = \frac{C^2 k_B (T_2 - T_1)}{XY}. \quad (6)$$

where $\langle \cdot \rangle$ stands for mean value and we have introduced the quantity $Y = [(C_1 + C)R_1 + (C_2 + C)R_2]$. We use the convention that the heat extracted from a system reservoir is negative and the heat dissipated is positive.

The out of equilibrium demon is sketched in the yellow box of fig.1 and it is composed by two resistances (R_{d1} and R_{d2}) kept at two different temperatures T_{d1} and T_{d2} (see Appendix A). The two voltage generators η_{d1} and η_{d2} represent the Nyquist noise voltages associated to the two resistances at the heat bath temperatures. Furthermore the resistance R_{d2} is driven by a voltage generator V_s whose out-put is filtered by the low pass filter composed by the resistance R_s and the capacitance C_s (see Appendix A 1). We notice that demon scheme is similar to that of the system, with a coupling capacitance $C \rightarrow \infty$, on which the driving V_s has been added. To design it, we followed the main prescriptions of ref.[15]: 1) It is out of equilibrium; 2) Either T_{d1} or T_{d2} has to be smaller than T_1 ; 3) it produces colored noise, obtained in our case by the source V_s filtered by R_s and C_s ; 4) It is coupled with the two parts of the system on different frequency ranges, specifically high frequencies with subsystem 1 and DC coupled with subsystem 2.

The choice of V_s is very important in order to simplify the experimental configuration. Indeed V_s can be either the thermal fluctuations of R_s with a suitable cut-off imposed by the $R_s C_s$ or an external driving. Many choices are possible and the simplest one is to use $V_s = V_f = \text{constant}$ and $R_s = 0$. In such a way V_f is coupled with R_2 only and the thermal noises η_{d1} and η_{d2} are directly coupled with R_2 and high pass filtered for R_1 (see Appendix A 1) The demon is always out of equilibrium, because, when it is disconnected from the system, the power supplied by V_f is entirely dissipated in the demon resistances producing a mean heat flux towards the demon heat baths, even in the case $T_{d1} = T_{d2}$. This is a simplified version of the original OEMD of ref.[15] because it requires the use of only one cold source at T_d and a DC signal that can be easily generated by thermocouples making the demon fully autonomous. We will demonstrate that this demon can reverse the heat flux of the system in a wide range of parameters with a zero energy flux (heat and work) with the system.

The connection of the demon to the system changes the current distributions and the energy exchanges. The circuit analysis shows (see Appendix B), that the currents \dot{q}_k ($k = 1, 2, d1, d2$) flowing in the resistances R_k are now

ruled by the following equations :

$$R_1 \dot{q}_1 = V_1 - \eta_1 \quad (7)$$

$$R_2 \dot{q}_2 = \eta_2 - V_2 \quad (8)$$

$$R_{d1} \dot{q}_{d1} = \eta_{d1} - V_2 \quad (9)$$

$$R_{d2} \dot{q}_{d2} = \eta_{d2} + V_f - V_2 \quad (10)$$

$$V_1 = \frac{-(C_t + C) q_1 + C q_t}{X_t} \quad (11)$$

$$V_2 = \frac{(C_1 + C) q_t - C q_1}{X_t} \quad (12)$$

where $q_t = (q_2 + q_{d2} + q_{d1})$, $C_t = C_2 + C_d$. and $X_t = C_1 C_t + C(C_1 + C_t)$.

In order to reduce the number of parameters we consider the case $T_d = T_{d1} = T_{d2}$ and $R_{d1} = R_{d2}$. The heat fluxes in the four reservoirs can be computed using $\dot{Q}_k = v_k \dot{q}_k$, where v_k is the potential difference on the resistance R_k (see Appendix C). Introducing the following parameters $R_d = R_{d1} R_{d2} / (R_{d1} + R_{d2})$, $R_t = R_d R_2 / (R_d + R_2)$, $Y_t = R_1 (C + C_1) + R_t (C + C_t)$, $A = C^2 k_B / (X_t Y_t)$, $\langle V_2 \rangle = V_t = V_f R_t / R_{d2}$, $B = A R_t (X_t R_t + R_1 (C_1 + C)^2) / (R_d R_2 C^2)$, we obtain:

$$\langle \dot{Q}_1 \rangle = A \left(\frac{R_t}{R_2} (T_2 - T_1) + \frac{R_t}{R_d} (T_d - T_1) \right) \quad (13)$$

$$\langle \dot{Q}_2 \rangle = -A \frac{R_t}{R_2} (T_2 - T_1) - B (T_2 - T_d) + \frac{V_t^2}{R_2} \quad (14)$$

$$\begin{aligned} \langle \dot{Q}_d \rangle &= -A \frac{R_t}{R_d} (T_d - T_1) - B (T_d - T_2) + \\ &+ \frac{V_f^2 R_t}{R_{d2}} \left(\frac{1}{R_{d1}} + \frac{1}{R_2} \right) - \frac{V_t^2}{R_2} \end{aligned} \quad (15)$$

$$\langle \dot{W}_f \rangle = \frac{V_f^2 R_t}{R_{d2}} \left(\frac{1}{R_{d1}} + \frac{1}{R_2} \right) \quad (16)$$

where $\langle \dot{Q}_d \rangle = \langle \dot{Q}_{d1} \rangle + \langle \dot{Q}_{d2} \rangle$ is the total heat flux in the demon reservoirs and $\langle \dot{W}_f \rangle$ is the total power supplied by the external generator V_f . The total energy balance demon+system is :

$$\langle \dot{Q}_d \rangle - \langle \dot{W}_f \rangle + \langle \dot{Q}_1 \rangle + \langle \dot{Q}_2 \rangle = 0 \quad (17)$$

These equations allow us to define the conditions for which the demon can reverse the flow without any energy exchange with the system.

In absence of the demon the heat flux is given by eqs.6, i.e. $\langle \dot{Q}_2 \rangle = -\langle \dot{Q}_1 \rangle < 0$. Using the demon we want to reverse this flow making $\langle \dot{Q}_2 \rangle > 0$ but keeping $\langle \dot{Q}_1 \rangle = -\langle \dot{Q}_2 \rangle$ because an observer, who measures the heat-flux of the system, has to establish that heat flows from the cold to the hot reservoir. The condition $\langle \dot{Q}_1 \rangle = -\langle \dot{Q}_2 \rangle$

has two important consequences. Firstly it reduces eq.17 to

$$\langle \dot{Q}_d \rangle - \langle \dot{W}_f \rangle = 0, \quad (18)$$

which indicates that all the power supplied by V_f is dissipated in the demon reservoirs and not in the system reservoirs. Secondly applying it to eqs.13,14 we find that:

$$\frac{V_t^2}{R_2} = A \frac{R_t}{R_d} (T_1 - T_d) + B (T_2 - T_d) \quad (19)$$

Finally using eq.19 and the condition $\langle \dot{Q}_2 \rangle > 0$ in eq.14, we compute the range of T_d , where the the spontaneous process is reversed, finding:

$$T_d \leq T_1 - \frac{R_d}{R_2} (T_2 - T_1) \quad (20)$$

The eqs.19 and 20 fix the conditions that allows the demon to reverse the system heat flux without heat exchange (eq.18) between the demon and the system. Eq.19 indicates that the fraction of the power injected by the demon and dissipated in R_2 (V_t^2/R_2 in eq.14) is compensated by the heat extracted from the system baths. We can also prove that thanks to eq.19 the demon does not perform any work on the system. Indeed the total work performed by the demon on the system is : $\langle \dot{W}_{d,s} \rangle = \langle \dot{W}_{d,1} \rangle + \langle \dot{W}_{d,2} \rangle$ where $\langle \dot{W}_{d,1} \rangle$ and $\langle \dot{W}_{d,2} \rangle$ are the works performed on subsystems 1 and 2 respectively. These can be computed using equations eq.7 and eq.8 in which we see that a "force" proportional to $q_{d1} + q_{d2}$ is applied on the two subsystems. Thus the work per unit time of these forces are [19].

$$\langle \dot{W}_{d,1} \rangle = \frac{C}{X_t} \langle \dot{q}_1 (q_{d1} + q_{d2}) \rangle \quad (21)$$

$$\langle \dot{W}_{d,2} \rangle = -\frac{C_1 + C}{X_t} \langle \dot{q}_2 (q_{d1} + q_{d2}) \rangle \quad (22)$$

From these two works (computed in Appendix D) we obtain for the total work :

$$\langle \dot{W}_{d,s} \rangle = -A \frac{R_t}{R_d} (T_1 - T_d) - B (T_2 - T_d) + \frac{V_t^2}{R_2} \quad (23)$$

We clearly see that if the condition on V_t (eq.19) is verified then $\langle \dot{W}_{d,s} \rangle = 0$, i.e. no work is done by the demon on the system. Thus eq.19 and eq.18 insure that total energy flux from the demon to the system is zero.

However the demon produces entropy and the total entropy production rate $\langle \dot{S} \rangle$ is positive in spite of the fact that the system entropy production rate $\langle \dot{S}_s \rangle = \langle \dot{Q}_2 \rangle (1/T_2 - 1/T_1)$ is negative, because $\langle \dot{Q}_2 \rangle > 0$ and $T_2 > T_1$ when the demon is "on". The total entropy production rate is

$$\langle \dot{S} \rangle = \frac{\langle \dot{Q}_d \rangle}{T_d} + \langle \dot{Q}_2 \rangle \left(\frac{1}{T_2} - \frac{1}{T_1} \right) \quad (24)$$

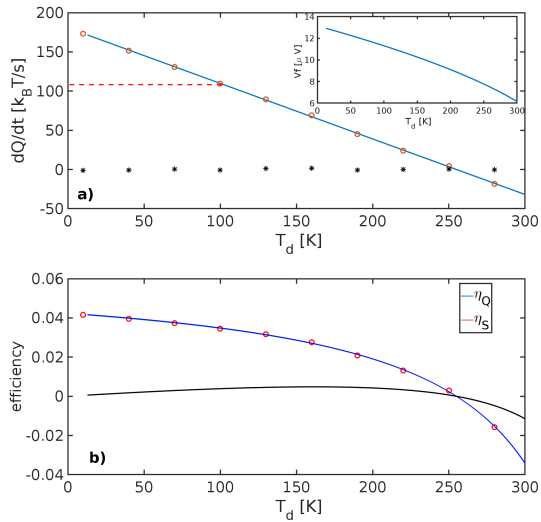


FIG. 2: a) Heat fluxes as a function of the demon temperature T_d : $\langle \dot{Q}_2 \rangle$ (blue line) computed when the demon is on, using eqs.13,...,16 and the condition for V_t^2 eq.19; $\langle \dot{Q}_1 \rangle$ (horizontal red dashed line) computed (eq.6) when the demon is "off"; $\langle \dot{Q}_2 \rangle$ (red circles) and $\langle \dot{Q}_2 \rangle + \langle \dot{Q}_1 \rangle$ (black stars) obtained from the direct numerical simulation of eqs.7,...,10. b) Demon efficiencies as a function of T_d : η_s (black line) and η_Q (blue line, red circles) computed from eqs.14,16,25 (continuous lines) and obtained from the direct numerical simulations of eqs.7,...,10 (red circles). The parameters used to compute the curves in a) and b) are: $T_1 = 300\text{K}$, $T_2 = 450\text{K}$, $C = 1\text{nF}$, $C_1 = C_2 = 100\text{pF}$, $R_1 = R_2 = 10\text{M}\Omega$; $R_d = 3\text{M}\Omega$; $Cd = 50\text{pF}$.

To show that $\langle \dot{S} \rangle > 0$, we start by taking into account that $\langle \dot{Q}_d \rangle = \langle \dot{W}_f \rangle$ (see eq.18) and that $\langle \dot{W}_f \rangle > \frac{V_t^2}{R_2}$ because as we said $\frac{V_t^2}{R_2}$ is a fraction of the total power injected into the system by the demon source (computed Appendix E). Furthermore as we want $\langle \dot{Q}_2 \rangle > 0$ then from eq.14 we have that $V_t^2/R_2 > \langle \dot{Q}_2 \rangle$ as the other terms are negative because $T_2 > T_1 > T_d$. As a consequence $\langle \dot{Q}_d \rangle / T_d > \langle \dot{Q}_d \rangle (1/T_1 - 1/T_2) > \langle \dot{Q}_2 \rangle (1/T_1 - 1/T_2)$ and we find $\langle \dot{S} \rangle > 0$.

These results on the effect of the demon on the system can be checked by comparing the heat fluxes computed from eqs.13,14 with those obtained by the direct numerical integration of eqs.7-10, where the four $\langle \dot{Q}_k \rangle = \langle v_k \dot{q}_k \rangle$ are directly computed using Stratonovich integrals. This comparison is done using for the system components (i.e. R_1, R_2, C_1, C_2, C) the values of the experiment of ref.[18, 19]. For the demon, we chose for C_d a typical wiring value and we fixed $R_d < R_2$ for having a reasonable range of T_d (see eq.20). All the components and temperatures values are indicated in the cap-

tion of fig.2. In fig.2a) the horizontal red dashed line indicates $|\langle \dot{Q}_2 \rangle|$ at $T_2 - T_1 = 150\text{K}$ computed from eq.6 when the demon is "off". When the demon is "on" the value of \dot{Q}_2 computed from eqs.14 and that obtained from direct numerical simulation agree. Most importantly for $T_d < 250\text{K}$ the heat flows from the cold to the hot thermal bath. The values of V_t necessary for implementing the demon conditions (eq.19) are plotted in the inset indeed for these values of V_t we see that $\langle \dot{Q}_2 \rangle + \langle \dot{Q}_1 \rangle = 0$ in the numerical simulation. It is important to notice that the necessary V_t is of the order of a few microvolts meaning that it can be easily obtained by two thermocouples coupled with a cold and an hot bath for example T_1 and T_2 .

The Demon efficiency can be defined in two ways. As the demon does not exchange any work and heat with the system then the efficiency can be defined in terms of entropy production rates, which has been used in other contexts [23–25]. Another way to define efficiency is in terms of the energy fluxes. Specifically these efficiencies are :

$$\eta_s = -\frac{\dot{S}_s}{\dot{S}_d} \quad \text{and} \quad \eta_Q = \frac{\langle \dot{Q}_2 \rangle}{\langle \dot{W}_f \rangle}. \quad (25)$$

We see that η_s is the ratio between the entropy of the non spontaneous process divided by the entropy of the spontaneous process whereas η_Q is the ratio between the reversed heat flux in the system divided by the work performed by the demon to achieve the goal. These two quantities are plotted in fig.2b) as function of T_d , and we observe that $\eta_s < 1\%$ and $\eta_Q < 4\%$, i.e. in order to achieve its goal the demon has to do a lot of work with a very large entropy production rate.

To conclude, we have simplified the original idea of autonomous OEMD because we use a demon with a single bath and a DC forcing (powered by thermocouples) instead of two baths with colored noise as in ref.[15]. We have demonstrated that this autonomous OEMD can be applied to electric circuits in order to reverse the spontaneous heat processes with no energy exchange between the system and demon. The latter has a small efficiency and a very large positive entropy production rate that largely compensates the negative entropy production rate of the system. Our results are very general because they are based on four coupled Langevin equations, which model not only electric circuits, but a lot of micro and nano-systems. Thus this article paves the way to the general applications of OEMDs to the control of these mesoscopic systems. We have chosen this configuration as a proof of principle but other complex circuits can of course be implemented.

Acknowledgments

We acknowledge useful discussion with R.S.Whitney. This work has been supported by the FQXi foundation

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- [1] E. Lutz and S. Ciliberto, *Physics Today* **68**, 30 (2015).
- [2] J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, *Nature Phys.* **11**, 131 (2015).
- [3] S. Toyabe, T. Sagawa, M. Ueda, M. Muneyuki, and M. Sano, *Nature Phys.* **6**, 988 (2010).
- [4] C. Elouard, D. Herrera-Martí, B. Huard, and A. Auffèves, *Phys. Rev. Lett.* **118**, 260603 (2017).
- [5] T. Admon, S. Rahav, and Y. Roichman, *Phys. Rev. Lett.* **121**, 180601 (2018).
- [6] Y. Masuyama, K. Funo, Y. Murashita, A. Noguchi, S. Kono, Y. Tabuchi, R. Yamazaki, M. Ueda, and Y. Nakamura, *Nature Comm.* **9** (2018).
- [7] G. N. Price, S. T. Bannerman, K. Viering, E. Narevicius, and M. G. Raizen, *Phys. Rev. Lett.* **100**, 093004 (2008).
- [8] J. V. Koski, V. F. Maisi, J. P. Pekola, and D. V. Averin, *Proceedings of the National Academy of Sciences* **111**, 13786 (2014).
- [9] J. V. Koski, A. Kutvonen, I. M. Khaymovich, T. Ala-Nissila, and J. P. Pekola, *Phys. Rev. Lett.* **115**, 260602 (2015).
- [10] A. C. Barato and U. Seifert, *EPL*, 60001 (2013).
- [11] N. Shiraishi, S. Ito, K. Kawaguchi, and T. Sagawa, *New Journal of Physics* **17**, 045012 (2015).
- [12] A. B. Boyd, D. Mandal, and J. P. Crutchfield, *New Journal of Physics* **18**, 023049 (2016).
- [13] M. L. Rosinberg and J. M. Horowitz, *EPL* **116**, 10007 (2016).
- [14] Z. Lu and C. A. Jarzynski, *Entropy*, 65 (2019).
- [15] R. Sanchez, J. Splettstoesser, and R. S. Whitney, *Phys. Rev. Lett.* **123**, 216801 (2019).
- [16] N. Freitas, J.-C. Delvenne, and M. Esposito, *Phys. Rev. X* **10**, 031005 (2020).
- [17] R. van Zon, S. Ciliberto, and E. G. D. Cohen, *Phys. Rev. Lett.* **92**, 130601 (2004).
- [18] S. Ciliberto, A. Imparato, A. Naert, and M. Tanase, *Phys. Rev. Lett.* **110**, 180601 (2013).
- [19] S. Ciliberto, A. Imparato, A. Naert, and M. Tanase, *J. Stat. Mech.* **2013**, P12014 (2013).
- [20] R. Filliger and P. Reimann, *Phys. Rev. Lett.* **99**, 230602 (2007).
- [21] S. Cerasoli, V. Dotsenko, G. Oshanin, and L. Rondoni, *Phys. Rev. E* **98**, 042149 (2018).
- [22] N. Garnier and S. Ciliberto, *Phys. Rev. E* **71**, 060101 (2005).
- [23] G. Verley, T. Willaert, C. Van den Broeck, and M. Esposito, *Phys. Rev. E* **90**, 052145 (2014).
- [24] M. Polettini, G. Verley, and M. Esposito, *Phys. Rev. Lett.* **114**, 050601 (2015).
- [25] J.-F. Derivaux and Y. De Decker, *J. Stat. Mech.*, 034002 (2019).
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Appendix A: Several details about the demon

In this section we describe the out of equilibrium demon, composed by two resistances (Rd_1 and Rd_2) kept at two different temperatures Td_1 and Td_2 . The two voltage generators ηd_1 and ηd_2 corresponding to the Nyquist noise of the two resistances at the temperatures of the heat baths. Furthermore the resistance Rd_2 is driven by a voltage generator V_s whose out-put is filtered by the the low pas filter composed by the resistance R_s and the capacitance C_s .

1. The demon as a colored noise generator

The dynamics of the demon can be obtained by writing the Kirchhoff laws for the points V_f and V_d :

$$\frac{V_s - V_f}{R_s} - \dot{V}_f C_f - \frac{V_f + \eta d_2 - V_d}{R_{d2}} = 0 \quad (\text{A1})$$

$$\frac{V_f + \eta d_2 - V_d}{R_{d2}} - \dot{V}_d C_d - \frac{V_d - \eta d_1}{R_{d1}} - i_d = 0 \quad (\text{A2})$$

where i_d is the current flowing between the system and the demon when the latter is "on". From these equations we get:

$$\tau_f \dot{V}_f = -V_f + \frac{R_f}{R_s} V_s + \frac{R_f}{R_{d2}} (V_d - \eta d_2) \quad (\text{A3})$$

$$\tau_d \dot{V}_d = -V_d + \left(\frac{R_d}{R_{d2}} V_f - i_d R_d \right) + \xi_d \quad (\text{A4})$$

$$(\text{A5})$$

where we use :

$$R_d = \frac{R_{d1} R_{d2}}{R_{d1} + R_{d2}}, \quad \tau_d = R_d C_d, \quad R_f = \frac{R_s R_{d2}}{R_{d2} + R_s}, \quad \tau_f = R_f C_s \quad \text{and} \quad \xi_d = \eta_{d1} \frac{R_d}{R_{d1}} + \eta_{d2} \frac{R_d}{R_{d2}}. \quad (\text{A6})$$

As V_s is not necessarily the thermal noise of R_s , it can be a very large external driving that allows the demon to work, we can simplify this circuit if we assume that $R_{d2} \gg R_s$

$$\tau_f \dot{V}_f = -V_f + V_s \quad (\text{A7})$$

$$\tau_d \dot{V}_d = -V_d - \left(\frac{R_d}{R_{d2}} V_f - i_d R_d \right) + \xi_d \quad (\text{A8})$$

These equations show that the demon produces colored noise for the system as it is needed to control the system heat flux.

2. The heat flux in the demon switched off

When $V_s = 0$ and $i_d = 0$ the heat transfers between the two reservoirs of the demon can be obtained from eqs.6 for $C \rightarrow \infty$. Thus the heat transfers in the demon are:

$$\langle \dot{Q}_{d1} \rangle = \frac{k_B(T_{d2} - T_{d1})}{(R_{d1} + R_{d2})C_d}. \quad (\text{A9})$$

$$\langle \dot{Q}_{d2} \rangle = \frac{k_B(T_{d1} - T_{d2})}{(R_{d1} + R_{d2})C_d}. \quad (\text{A10})$$

When there is an external driving one has also to consider the work performed by the external source V_f still in the case $i_d = 0$, i.e. with the demon is not connected to the system. We can use for V_s either thermal fluctuations of R_s with a suitable cut-off imposed by the $R_s C_s$ or an external driving. In the simplest version we make the choice to use a $V_f = V_s$ constant and $R_s = 0$. In such a case the heat dissipated by the two resistances is:

$$\langle \dot{Q}_{d1} \rangle = \frac{k_B(T_{d2} - T_{d1})}{(R_{d1} + R_{d2})C_d} + \frac{V_f^2 R_{d1}}{(R_{d1} + R_{d2})^2} \quad (\text{A11})$$

$$\langle \dot{Q}_{d2} \rangle = \frac{k_B(T_{d1} - T_{d2})}{(R_{d1} + R_{d2})C_d} + \frac{V_f^2 R_{d2}}{(R_{d1} + R_{d2})^2} \quad (\text{A12})$$

and doing the energy balance we have

$$\langle \dot{Q}_{d1} \rangle + \langle \dot{Q}_{d2} \rangle = \frac{V^2}{(R_{d1} + R_{d2})} = \langle \dot{W}_f \rangle \quad (\text{A13})$$

where $\langle \dot{W}_f \rangle$ is the power injected into the demon by the voltage generator. where we use the convention that the heat dissipated in the bath and the work performed on the system are positive. Thus the demon is always out of equilibrium even at $T_{d1} = T_{d2}$

Appendix B: The currents in the system+demon circuit

We now connect the demon to the system and this connection changes the current distributions. We write the equations of the circuits in terms of charges. The relationships between currents and charges are

$$\begin{aligned} 0 &= -\dot{q}_1 - \dot{q}_{c1} + \dot{q}_c, & \dot{q}_2 - \dot{q}_{ct} - \dot{q}_c + \dot{q}_{d2} + \dot{q}_{d1} &= 0, \\ \dot{q}_c &= (V_2 - V_1)C, & V_1 C_1 &= q_{c1} \quad \text{and} \quad V_2 C_t = q_{ct} \end{aligned} \quad (\text{B1})$$

where \dot{q}_k ($k = 1, 2, d1, d2$) are the currents flowing in the resistances R_k . Futhermore \dot{q}_c, \dot{q}_{c1} and \dot{q}_{ct} are the currents flowing respectively in the capacitors C, C_1 and $C_t = C_2 + C_d$. Solving the system for q_{c1} and q_{ct} we find :

$$V_1 = \frac{q_{c1}}{C_1} = \frac{-(C_t + C) q_1 + C q_t}{X_t} \quad (\text{B2})$$

$$V_2 = \frac{q_{ct}}{C_t} = \frac{(C_1 + C) q_t - C q_1}{X_t} \quad (\text{B3})$$

where $q_t = (q_2 + q_{d2} + q_{d1})$ and $X_t = C_1 C_t + C(C_1 + C_t)$. we can now solve for the four currents flowing in the 2 system resistances and the two demon resistances specifically

$$R_1 \dot{q}_1 = V_1 - \eta_1 = \frac{-(C_t + C) q_1 + C (q_2 + q_{d2} + q_{d1})}{X_t} - \eta_1 \quad (\text{B4})$$

$$R_2 \dot{q}_2 = \eta_2 - V_2 = \frac{-(C_1 + C) (q_2 + q_{d2} + q_{d1}) + C q_1}{X_t} + \eta_2 \quad (\text{B5})$$

$$R_{d1} \dot{q}_{d1} = \eta_{d1} - V_2 = \frac{-(C_1 + C) (q_2 + q_{d2} + q_{d1}) + C q_1}{X_t} + \eta_{d1} \quad (\text{B6})$$

$$R_{d2} \dot{q}_{d2} = \eta_{d2} + V_f - V_2 = \frac{-(C_1 + C) (q_2 + q_{d2} + q_{d1}) + C q_1}{X_t} + \eta_{d2} + V_f \quad (\text{B7})$$

We write the equations for the charges because the connection with Brownian particles is straightforward (q correspond to the displacement). Furthermore it is more clear to understand the amount of work performed by the demon on the system and the amount of dissipated heats in the various reservoirs. As the resistances of the demon are just in parallel to R_2 the equations B4.,B7 can be reduced to:

$$R_1 \dot{q}_1 = -\frac{(C_t + C)}{X_t} q_1 + \frac{C}{X_t} q_t - \eta_1 \quad (\text{B8})$$

$$R_t \dot{q}_t = -\frac{(C_1 + C)}{X_t} q_t + \frac{C}{X_t} q_1 + \eta_t + V_t \quad (\text{B9})$$

where we define

$$q_t = (q_2 + q_{d2} + q_{d1}), \quad R_t = (1/R_2 + 1/R_{d2} + 1/R_{d1})^{-1}, \quad \text{and}, \quad (\text{B10})$$

$$V_t = V_f R_t/R_{d2}, \quad \eta_t = (\eta_2/R_2 + \eta_{d1}/R_{d1} + \eta_{d2}/R_{d2})R_t.$$

Using eqs.4,5 we can compute the heat and the work of the different parts of the circuit. We can also use eqs. 6 to compute the total heat exchanged between the reservoir at T_1 with the reservoirs of resistance R_2, R_{d1}, R_{d2} . Defining $T_t = R_t(T_2/R_2 + T_{d1}/R_{d1} + T_{d2}/R_{d2})$ and $Y_t = R_1 (C + C_1) + R_t (C + C_t)$, we get :

$$\langle \dot{Q}_1 \rangle = \frac{C^2 k_B (T_t - T_1)}{X_t Y_t}. \quad (\text{B11})$$

$$\langle \dot{Q}_t \rangle = \frac{C^2 k_B (T_1 - T_t)}{X_t Y_t} \quad (\text{B12})$$

where the contribution of the work performed by V_t cancels out in the stationary regime (see appendix E). Eq. B11 can be decomposed in the various contributions to the heat fluxes from the three reservoir at $(T_2, T_{d1}$ and $T_{d2})$:

$$\langle \dot{Q}_1 \rangle = \frac{C^2 k_B}{X_t Y_t} \left(\frac{R_t}{R_2} (T_2 - T_1) + \frac{R_t}{R_{d1}} (T_{d1} - T_1) + \frac{R_t}{R_{d2}} (T_{d2} - T_1) \right) \quad (\text{B13})$$

Appendix C: Calculation of the heat fluxes

By the definition eq.5 the heat fluxes in the resistances $k = 1, 2, d1, d2$ are given by:

$$\langle \dot{Q}_k \rangle = \left\langle v_k \frac{(v_k - \eta_k)}{R_k} \right\rangle. \quad (\text{C1})$$

where $v_k = V_k - V_f \delta_{k,d2}$ (with $V_{d1} = V_{d2} = V_2$) is the potential difference on the resistance R_k . The mean values can be evaluated by using the Fourier transforms \tilde{v}_k of v_k , which can be written as :

$$\langle \dot{Q}_k \rangle = \int_0^\infty \Re \left\{ \frac{\tilde{v}_k^*}{R_m} (\tilde{v}_k - \tilde{\eta}_k) \right\} \frac{d\omega}{2\pi}. \quad (\text{C2})$$

To evaluate this integral we take into account that the spectral density of $\tilde{\eta}_k$ is $|\tilde{\eta}_k|^2 = 4k_B T_k R_k$ and that $\Re \{ \eta_k \eta_{k'}^* \} = 0$ because different noise sources are uncorrelated We have already computed $\langle \dot{Q}_1 \rangle$ in eq.B11 and eq.B13 using the

eqs.6. We need to estimate all the heat fluxes in the demon reservoirs and in the reservoir 2 by computing V_1 and V_2 given by eqs.B2 and B3. Their values depend on q_1, q_t which can be computed solving the system of eqs.B8 and B9 in Fourier space.

$$Z_1 \tilde{q}_1 - a \tilde{q}_t = -\tilde{\eta}_1 \quad (\text{C3})$$

$$-a \tilde{q}_1 + Z_t \tilde{q}_t = \tilde{\eta}_t + V_t \sqrt{2\pi} \delta(\omega) \quad (\text{C4})$$

where $Z_1 = (C_t + C)/X_t + i\omega R_1$, $Z_t = (C_1 + C)/X_t + i\omega R_t$ and $a = C/X_t$. We find

$$\tilde{q}_1 = \frac{-\tilde{\eta}_1 Z_t + \eta_t a}{\det} \quad (\text{C5})$$

$$\tilde{q}_t = \frac{\tilde{\eta}_t Z_1 - \eta_1 a}{\det} \quad (\text{C6})$$

$$\det = Z_1 Z_t - a^2 = \frac{(1 - \omega^2 R_1 R_t X_t) + i\omega Y_t}{X_t} \quad (\text{C7})$$

Inserting these Fourier transform in eqs.B2 and B3, we get:

$$\tilde{V}_1 = \frac{\tilde{\eta}_1 [1 + i\omega R_t (C_t + C)] + \tilde{\eta}_t i\omega R_1 C}{X_t \det} \quad (\text{C8})$$

$$\tilde{V}_2 = \frac{\tilde{\eta}_t [1 + i\omega R_1 (C_1 + C)] + \tilde{\eta}_1 i\omega R_t C}{X_t \det} + V_t \sqrt{2\pi} \delta(\omega) \quad (\text{C9})$$

We see that

$$|\tilde{V}_1|^2 = \frac{|\tilde{\eta}_1|^2 [1 + (\omega R_t (C_t + C))^2] + |\tilde{\eta}_t|^2 (\omega R_1 C)^2}{(1 - \omega^2 R_1 R_t X_t)^2 + (\omega Y_t)^2} \quad (\text{C10})$$

$$|\tilde{V}_2|^2 = \frac{|\tilde{\eta}_t|^2 [1 + (\omega R_1 (C_1 + C))^2] + |\tilde{\eta}_1|^2 (\omega R_t C)^2}{(1 - \omega^2 R_1 R_t X_t)^2 + (\omega Y_t)^2} + V_t^2 2\pi \delta(\omega) \quad (\text{C11})$$

$$\langle V_1^2 \rangle = \int_0^\infty |\tilde{V}_1|^2 \frac{d\omega}{2\pi} = \frac{|\tilde{\eta}_1|^2 (X_t R_1 R_t + R_t^2 (C_t + C)^2) + |\tilde{\eta}_t|^2 R_1^2 C^2}{4 Y_t X_t R_1 R_t} \quad (\text{C12})$$

$$\langle V_2^2 \rangle = \int_0^\infty |\tilde{V}_2|^2 \frac{d\omega}{2\pi} = \frac{|\tilde{\eta}_t|^2 (X_t R_1 R_t + R_1^2 (C_1 + C)^2) + |\tilde{\eta}_1|^2 R_t^2 C^2}{4 Y_t X_t R_1 R_t} + V_t^2 \quad (\text{C13})$$

where we used the integrals:

$$\int_0^\infty \frac{Y_t}{(1 - \omega^2 R_1 R_t X_t)^2 + (\omega Y_t)^2} \frac{d\omega}{2\pi} = \frac{1}{4} \quad \text{and} \quad \int_0^\infty \frac{Y_t \omega^2}{(1 - \omega^2 R_1 R_t X_t)^2 + (\omega Y_t)^2} \frac{d\omega}{2\pi} = \frac{1}{4 R_1 R_t X_t} \quad (\text{C14})$$

We can now compute the other heat fluxes :

$$\begin{aligned} \tilde{Q}_2 &= \Re \left\{ \tilde{V}_2^* \frac{(\tilde{V}_2 - \tilde{\eta}_2)}{R_2} \right\} = \frac{|\tilde{V}_2|^2}{R_2} - \Re \left\{ \frac{|\tilde{\eta}_2|^2 R_t [1 - i\omega R_1 (C_1 + C)] \det}{R_2^2 X_t |\det|^2} \right\} = \\ &= \frac{|\tilde{V}_2|^2}{R_2} - \frac{|\tilde{\eta}_2|^2 R_t [(1 - \omega^2 R_1 R_t X_t) + \omega^2 Y_t R_1 (C_1 + C)]}{R_2^2 [(1 - \omega^2 R_1 R_t X_t)^2 + (\omega Y_t)^2]} \end{aligned} \quad (\text{C15})$$

$$\begin{aligned} \langle \dot{Q}_2 \rangle &= \int_0^\infty \tilde{Q}_2 \frac{d\omega}{2\pi} = \\ &= \frac{|\tilde{\eta}_t|^2 R_2 (X_t R_1 R_t + R_1^2 (C_1 + C)^2) + R_2 |\tilde{\eta}_1|^2 R_t^2 C^2 - |\tilde{\eta}_2|^2 R_t Y_t R_1 (C_1 + C)}{4 R_2^2 Y_t X_t R_1 R_t} = \\ &= \frac{(|\tilde{\eta}_t|^2 R_2 - |\tilde{\eta}_2|^2 R_t) R_1 (X_t R_t + R_1 (C_1 + C)^2) + R_t^2 C^2 (|\tilde{\eta}_1|^2 R_2 - |\tilde{\eta}_2|^2 R_1)}{4 R_2^2 Y_t X_t R_1 R_t} + \frac{V_t^2}{R_2} \\ &= -\frac{k_B C^2}{X_t Y_t} (T_2 - T_1) \frac{R_t}{R_2} - k_B \frac{X_t R_t + R_1 (C_1 + C)^2}{X_t Y_t R_2} \left(\frac{R_t}{R_{d1}} (T_2 - T_{d1}) + \frac{R_t}{R_{d2}} (T_2 - T_{d2}) \right) + \frac{V_t^2}{R_2} \end{aligned} \quad (\text{C16})$$

$$\begin{aligned}
\tilde{Q}_{d2} &= \Re \left\{ (\tilde{V}_2^* - V_f \sqrt{2\pi} \delta(\omega)) \frac{(\tilde{V}_2 - \tilde{\eta}_{d2} - V_f \sqrt{2\pi} \delta(\omega))}{R_{d2}} \right\} \\
&= \frac{|\tilde{V}_2|^2}{R_{d2}} - \Re \left\{ \frac{|\tilde{\eta}_{d2}|^2 R_t [1 - i\omega R_1 (C_1 + C)] \det}{R_{d2}^2 X_t |\det|^2} \right\} + \frac{V_t (V_t - V_f)}{R_{d2}} 2\pi \delta(\omega) = \\
&= \frac{|\tilde{V}_2|^2}{R_{d2}} - \frac{|\tilde{\eta}_{d2}|^2 R_t [(1 - \omega^2 R_1 R_t X_t) + \omega^2 Y_t R_1 (C_1 + C)]}{R_{d2}^2 [(1 - \omega^2 R_1 R_t X_t)^2 + (\omega Y_t)^2]} + \frac{(V_t - V_f)^2}{R_{d2}} 2\pi \delta(\omega)
\end{aligned} \tag{C17}$$

$$\begin{aligned}
\langle \dot{Q}_{d2} \rangle &= \int_0^\infty \tilde{Q}_{d2} \frac{d\omega}{2\pi} = \\
&= \frac{|\tilde{\eta}_t|^2 R_{d2} (X_t R_1 R_t + R_1^2 (C_1 + C)^2) + R_{d2} |\tilde{\eta}_1|^2 R_t^2 C^2 - |\tilde{\eta}_{d2}|^2 R_t Y_t R_1 (C_1 + C)}{4 R_{d2}^2 Y_t X_t R_1 R_t} + \frac{(V_t - V_f)^2}{R_{d2}} = \\
&= \frac{(|\tilde{\eta}_t|^2 R_{d2} - |\tilde{\eta}_{d2}|^2 R_t) R_1 (X_t R_t + R_1 (C_1 + C)^2) + R_t^2 C^2 (|\tilde{\eta}_1|^2 R_2 - |\tilde{\eta}_{d2}|^2 R_1)}{4 R_{d2}^2 Y_t X_t R_1 R_t} + \frac{(V_t - V_f)^2}{R_{d2}} = \\
&= -A (T_{d2} - T_1) \frac{R_t}{R_{d2}} - B \frac{R_d}{R_{d2}} \left(\frac{R_2}{R_{d1}} (T_{d2} - T_{d1}) + (T_{d2} - T_2) \right) + \frac{(V_t - V_f)^2}{R_{d2}}
\end{aligned} \tag{C18}$$

where

$$A = \frac{k_B C^2}{X_t Y_t} \quad \text{and} \quad B = k_B \frac{R_t (X_t R_t + R_1 (C_1 + C)^2)}{X_t Y_t R_2 R_d} \tag{C19}$$

Finally using the same method we get:

$$\langle \dot{Q}_{d1} \rangle = -A (T_{d1} - T_1) \frac{R_t}{R_{d1}} - B \frac{R_d}{R_{d1}} \left(\frac{R_2}{R_{d2}} (T_{d1} - T_{d2}) + (T_{d1} - T_2) \right) + \frac{V_t^2}{R_{d1}} \tag{C20}$$

Appendix D: The work performed by the demon on the system

We follow refs. [18, 19] to compute the power injected by the demon on R_1 and R_2 with $R_{d1} = R_{d2}$ and $T_{d1} = T_{d2}$

$$\langle \dot{W}_{d,1} \rangle = \frac{C}{X_t} \langle \dot{q}_1 (q_{d1} + q_{d2}) \rangle = \int_0^\infty \Re \left\{ \frac{\tilde{V}_1^* - \tilde{\eta}_1^*}{R_1} \frac{\tilde{\eta}_d - \tilde{V}_2}{R_d i\omega} \right\} \frac{d\omega}{2\pi} = \tag{D1}$$

$$= -\frac{C}{X_t} \int_0^\infty \left[\frac{|\tilde{\eta}_1|^2 R_t C - |\tilde{\eta}_t|^2 R_1 C + (1 - \omega^2 R_1 R_t X_t) R_t C (-|\tilde{\eta}_1|^2 + |\tilde{\eta}_d|^2 R_1/R_d)}{R_1 R_d |\det|^2 X_t^2} \right] \frac{d\omega}{2\pi} = \tag{D2}$$

$$= -k_B \frac{C^2 R_t}{X_t Y_t R_d} (T_1 - T_t) = \frac{A R_t^2}{R_d R_2} (T_2 - T_1) - \frac{A R_t^2}{R_d^2} (T_1 - T_d) \tag{D3}$$

where we used eqs C8, C9, $T_t = R_t (T_2/R_2 + T_d/R_d)$ and eqs C19.

$$\langle \dot{W}_{d,2} \rangle = -\frac{(C_1 + C)}{X_t} \langle \dot{q}_2 (q_{d1} + q_{d2}) \rangle = -\frac{(C_1 + C)}{X_t} \int_0^\infty \Re \left\{ \frac{\tilde{\eta}_2^* - \tilde{V}_2^*}{R_2} \frac{\tilde{\eta}_d - \tilde{V}_2}{R_d i\omega} \right\} \frac{d\omega}{2\pi} + \frac{V_t^2}{R_2} = \tag{D4}$$

$$= -\frac{(C_1 + C)}{X_t R_2 R_d} \int_0^\infty \frac{(|\tilde{\eta}_2|^2 R_t/R_2 - |\tilde{\eta}_d|^2 R_t/R_d) [Y_t - R_1 (C_1 + C) (1 - \omega^2 R_1 R_t X_t)]}{X_t^2 |\det|^2} \frac{d\omega}{2\pi} + \frac{V_t^2}{R_2} \tag{D5}$$

$$= -k_B \frac{(C_1 + C) R_t}{X_t R_2 R_d} (T_2 - T_d) + \frac{V_t^2}{R_2}. \tag{D6}$$

Adding a subtracting $A/R_t^2 T_1/(R_2 R_d)$ from the last expression, taking into account the definition of A and B (eq. C19), that $(C_1 + C)(C_t + C) = X_t + C^2$ and that $k_B (C_1 + C) R_t Y_t / (X_t R_2 R_d Y_t) = B + A/R_t^2 / (R_2 R_d)$ we get

$$\langle \dot{W}_{d,2} \rangle = -B (T_2 - T_d) - \frac{A R_t^2}{R_2 R_d} (T_2 - T_1) - \frac{A R_t^2}{R_2 R_d} (T_1 - T_d) + \frac{V_t^2}{R_2} \tag{D7}$$

The total work performed by the demon on the system is

$$\langle \dot{W}_{d,s} \rangle = \langle \dot{W}_{d,1} \rangle + \langle \dot{W}_{d,2} \rangle = \quad (\text{D8})$$

$$= -A \frac{R_t}{R_d} (T_1 - T_d) - B (T_2 - T_d) + \frac{V_t^2}{R_2} \quad (\text{D9})$$

If the condition on the demon potential and temperature eqs.20 and 19 are satisfied then the last equation imposes that $\langle \dot{W}_{d,s} \rangle = 0$, i.e. the demon does not perform any work on the system.

Appendix E: The power dissipated by the DC currents only

The power dissipated in the system and demon resistances by the DC external generator V_f are:

$$P_1 = 0, \quad P_2 = V_t^2/R_2, \quad P_{d1} = V_t^2/R_{d1}, \quad P_{d2} = (V_f - V_t)^2/R_{d2} \quad (\text{E1})$$

with $V_t = \langle V_2 \rangle$. Taking into account that $V_t = V_f R_t/R_{d2}$ we get the total power supplied by the generator V_f

$$\langle \dot{W}_f \rangle = P_2 + P_{d1} + P_{d2} = \frac{V_f(V_f - V_2)}{R_{d2}} \quad (\text{E2})$$

The dissipation of the demon is $P_d = P_{d1} + P_{d2}$ thus $P_2 = \dot{W}_d - P_d$. This has been used in the text for computing the total entropy

Concerning the work of V_t in eq.B9 we clearly see that for the node 2, $\langle \dot{q}_t \rangle = 0$ if we consider only the DC current imposed by V_f . This means that $V_t \langle \dot{q}_t \rangle = 0$ and there is no contribution to the heat flux \dot{Q}_t which is the heat exchanged by the equivalent R_t circuit with the subsystem 1.
