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# Adaptive Nonlinear Excitation Control of Synchronous Generators with Unknown Mechanical Power

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**Abstract.** A nonlinear adaptive excitation control is designed for a synchronous generator modeled by a standard third order model on the basis of the physically available measurements of relative angular speed, active electric power and terminal voltage. The power angle, which is a crucial variable for the excitation control, is not assumed to be available for feedback. The feedback control is supposed to achieve transient stabilization and voltage regulation when faults occur to the turbines so that the mechanical power may permanently take any (unknown) value within its physical bounds. Transient stabilization and voltage regulation are achieved by a nonlinear adaptive controller, which generates both converging estimates of the mechanical power and the new equilibrium point compatible with the required terminal voltage.

## 1 Introduction

The analysis of transient stability of a synchronous generator connected to an infinite bus when large and sudden faults occur is a classical power system problem, which has been addressed via Lyapunov techniques (see for instance [12], [10], [6] ) in which models of increasing complexity are used) in order to determine the critical clearing time, that is the time before which the fault has to be cleared so that the faulted trajectory still belongs to the stability region of the stable operating condition.

Feedback linearization techniques were proposed in [5], [3], [14] to design stabilizing controls with the purpose of enlarging the stability region of the operating condition. Nonlinear adaptive controls are proposed in [1] which keep the machine in synchronism when short circuits occur in the transmission lines. Nonlinear adaptive controls are also proposed in [15] to improve damping without requiring the knowledge of the operating point. The nonlinear feedback control algorithms so far proposed in the literature make use of power angle measurements which are physically not available and have the

difficulty of determining the faulted equilibrium value which is compatible with the required terminal voltage once the fault (mechanical or electrical failure) has occurred.

In this paper we make use of the standard third order model used in [15] (see [2] , [13] ) to show that the terminal voltage, the relative angular speed and the active electric power (which are actually measurable and available for feedback) are state variables in the physical region of the state space. Since the purpose of the excitation control is to regulate the terminal voltage without loosing the synchronism, we compute the zero dynamics of the system with respect to the terminal voltage and we obtain a highly nonlinear second order dynamics. We then design following [8] a nonlinear adaptive feedback control on the basis of physically available measurements (relative angular speed, active electric power and terminal voltage) which is adaptive with respect to the unknown mechanical power generated by the turbines and achieves transient stability for all physical faults affecting mechanic power generation. The mechanical power estimation quickly recovers the faulted value so that the faulted equilibrium point corresponding to the desired terminal voltage level can be determined and the synchronous generator can be smoothly transferred to the faulted equilibrium point with no loss of synchronism.

## 2 Dynamical model

Consider the simplified mechanical model expressed in per unit as

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}(P_m - P_e) \end{aligned} \quad (1)$$

where:  $\delta(\text{rad})$  is the power angle of the generator relative to the angle of the infinite bus rotating at synchronous speed  $\omega_s$ ;  $\omega(\text{rad/s})$  is the angular speed of the generator relative to the synchronous speed  $\omega_s$  i.e.  $\omega = \omega_g - \omega_s$  with  $\omega_g$  being the generator angular speed;  $H(s)$  is the per unit inertia constant;  $D(p.u.)$  is the per unit damping constant;  $P_m(p.u.)$  is the per unit mechanical input power;  $P_e(p.u.)$  is the per unit active electric power delivered by the generator to the infinite bus. Note that the expression  $\omega_s^2/\omega_g$  is simplified as  $\omega_s^2/\omega_g \simeq \omega_s$  in the right-hand side of (1). The active and reactive powers are given by

$$P_e = \frac{V_s E_q}{X_{ds}} \sin(\delta) \quad (2)$$

$$Q = \frac{V_s}{X_{ds}} E_q \cos(\delta) - \frac{V_s^2}{X_{ds}} \quad (3)$$

where:  $E_q(p.u.)$  is the quadrature's EMF;  $V_s(p.u.)$  is the voltage at the infinite bus;  $X_{ds} = X_T + \frac{1}{2}X_L + X_d(p.u.)$  is the total reactance which takes into account  $X_d(p.u.)$ , the generator direct axis reactance,  $X_L(p.u.)$ , the transmission line reactance, and  $X_T(p.u.)$ , the reactance of the transformer. The quadrature EMF,  $E_q$ , and the transient quadrature EMF,  $E'_q$ , are related by

$$E_q = \frac{X_{ds}}{X'_d} E'_q - \frac{X_d - X'_d}{X'_d} V_s \cos(\delta) \quad (4)$$

while the dynamics of  $E'_q$  are given by

$$\frac{dE'_q}{dt} = \frac{1}{T_{d0}} (K_c u_f - E_q) \quad (5)$$

in which:  $X'_d = X_T + \frac{1}{2}X_L + X'_d(p.u.)$  with  $X'_d$  denoting the generator direct axis transient reactance;  $u_f(p.u.)$  is the input to the (SCR) amplifier of the generator;  $K_c$  is the gain of the excitation amplifier;  $T_{d0}(s)$  is the direct axis short circuit time constant. Substituting (2) into (1) and (4) into (5), we obtain the state space model

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_s}{H} \left( P_m - \frac{V_s}{X'_d} E'_q \sin(\delta) + \frac{X_d - X'_d}{X_{ds} X'_d} V_s^2 \sin(\delta) \cos(\delta) \right) \\ \dot{E}'_q &= \frac{1}{T_{d0}} \left( K_c u_f - \frac{X_{ds}}{X'_d} E'_q + \frac{X_d - X'_d}{X'_d} V_s \cos(\delta) \right) \end{aligned} \quad (6)$$

in which  $(\delta, \omega, E'_q)$  is the state and  $u_f$  is the control input. Since  $P_e$  is measurable while  $E'_q$  is not, it is convenient to express the state space model using  $(\delta, \omega, P_e)$  as states which are equivalent states as long as the power angle  $\delta$  remains in the open set  $0 < \delta < \pi$ . Taking account of the notation

$$T'_{d0} = \frac{X'_d}{X_{ds}} T_{d0}$$

where  $T'_{d0}$  is the direct axis transient short circuit time constant, differentiating (2) with respect to time, and using (1)-(5), we obtain

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega - \frac{\omega_s}{H} (P_e - P_m) \\ \dot{P}_e &= -\frac{1}{T'_{d0}} P_e + \frac{1}{T'_{d0}} \left\{ \frac{V_s}{X_{ds}} \sin(\delta) [K_c u_f + T'_{d0} (X_d - X'_d) \frac{V_s}{X'_d} \omega \sin(\delta)] \right. \\ &\quad \left. + T'_{d0} P_e \omega \cot(\delta) \right\} \end{aligned} \quad (7)$$

which is valid provided that  $0 < \delta < \pi$ . Note that when  $\delta$  is near 0 or near  $\pi$  the effect of the input  $u_f$  on the overall dynamics is greatly reduced. The generator terminal voltage is given by

$$V_t e^{j\varphi} = \frac{jX_s E_q e^{j(\frac{\pi}{2} + \delta)} + jX_d V_s e^{j\frac{\pi}{2}}}{jX_{ds}}$$

where

$$\begin{aligned} X_s &= X_T + \frac{X_L}{2} \\ X_{ds} &= X_d + X_s \end{aligned}$$

so that its modulus is

$$V_t = \frac{1}{X_{ds}} (X_s^2 E_q^2 + V_s^2 X_d^2 + 2X_s X_d E_q V_s \cos(\delta))^{\frac{1}{2}}$$

or in the new state variables

$$V_t = \left( \frac{X_s^2 P_e^2}{V_s^2 \sin^2(\delta)} + \frac{X_d^2 V_s^2}{X_{ds}^2} + \frac{2X_s X_d}{X_{ds}} P_e \cot(\delta) \right)^{\frac{1}{2}} \quad (8)$$

which is the output of the system to be regulated to its reference value  $V_{tr} = 1(p.u.)$

If this is done, the zero dynamics will be

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_s}{H} \left( P_m + \frac{X_d}{X_s X_{ds}} V_s^2 \sin(\delta) \cos(\delta) \right. \\ &\quad \left. - \frac{V_s}{X_s} \sin(\delta) \sqrt{V_{tr}^2 - \frac{X_d^2}{X_{ds}^2} V_s^2 \sin^2(\delta)} \right) \end{aligned}$$

which are very complex, and for some initial conditions or parameters values may become unstable as we may see in Fig. (1) where  $\delta$  goes out of the region  $0 < \delta < 180$  and  $\omega$  grows out of physical boundaries.

The operating conditions  $(\delta_0, \omega_0, P_{e0})$  of the synchronous generator model (7) are given by

$$\begin{aligned} \omega_0 &= 0 \\ P_{e0} &= P_m \\ -P_m + \frac{V_s}{X_{ds}} K_c u_f \sin(\delta) &= 0 \end{aligned}$$

**Fig.1.** Zero Dynamics

Note that while  $\omega_0 = 0, P_{e0} = P_m$  are not affected by  $u_f$ , from the third equation above we see that there are two operating conditions  $\delta_s, \delta_u$ ,  $0 < \delta_s < \frac{\pi}{2}$ ,  $\frac{\pi}{2} < \delta_u < \pi$  for constant inputs  $u_f > (P_m X_{ds}) / (K_c V_s)$ ;  $(\delta_s, 0, P_m)$  is an asymptotically stable equilibrium point while  $(\delta_u, 0, P_m)$  is an unstable equilibrium point. The stable operating condition  $(\delta_s, 0, P_m)$  and the corresponding excitation constant input

$$K_c u_{f0} = \frac{P_m X_{ds}}{V_s \sin(\delta_s)}$$

are chosen so that the modulus of the generator terminal voltage

$$V_t = \frac{1}{X_{ds}} (X_s^2 K_c^2 u_{f0}^2 + V_s^2 X_d^2 + 2X_s X_d K_c u_{f0} V_s \cos(\delta_s))^{\frac{1}{2}}$$

is equal to the prescribed value.

The power angle is not measurable and is also not a physical variable to be regulated; the only physical variable to be regulated is the output  $V_t$ , while  $(V_t, \omega, P_e)$  are measured and are available for feedback action.

As a matter of fact  $(V_t, \omega, P_e)$  is an equivalent state for the models (6) and (7) since (see appendix for the derivation of this formula)

$$\delta = \operatorname{arccotg} \left( \frac{V_s}{X_s P_e} \left( -\frac{X_d V_s}{X_{ds}} + \sqrt{V_t^2 - \frac{X_s^2}{V_s^2} P_e^2} \right) \right) \quad (9)$$

If the parameters  $(V_s, X_s, X_d, X_{ds})$  are known, state measurements are available. From (9) it follows that in order to regulate the terminal voltage  $V_t$  to its reference value ( $V_{tr} = 1(p.u.)$ )  $\delta$  should be regulated to

$$\delta_s = \operatorname{arccotg} \left( \frac{V_s}{X_s P_m} \left( -\frac{X_d V_s}{X_{ds}} + \sqrt{V_{tr}^2 - \frac{X_s^2}{V_s^2} P_m^2} \right) \right) \quad (10)$$

From a physical viewpoint the natural choice of state variables is  $(V_t, \omega, P_e)$  which are measurable. The state feedback control task is to make the stability

region of the stable equilibrium point  $(V_{tr}, 0, P_m)$  as large as possible. In fact the parameter  $P_m$  may abruptly change to an unknown faulted value  $P_{mf}$  due to turbine failures so that  $(V_{tr}, 0, P_m)$  may not belong to the region of attraction of the faulted equilibrium point  $(V_{tr}, 0, P_{mf})$ . The state feedback control should be design so that typical turbine failures do not cause instabilities and consequently loss of synchronism and inability to achieve voltage regulation.

A reduction from  $P_m$  to  $(P_m)_f$  of the mechanical power generated by the turbine, changes the operating condition: the new operating condition  $(\delta_s)_f$  is the solution of

$$-\frac{(P_m)_f}{P_m} + \frac{\sin(\delta)_f}{\sin(\delta_s)} = 0$$

and since  $(P_m)_f$  is typically unknown, the corresponding new stable operating condition  $(\delta_s)_f$  is also unknown.

### 3 Robust and adaptive stabilization

The model (7) is rewritten as

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega - \frac{\omega_s}{H}(P_e - \theta) \\ \dot{P}_e &= -\frac{1}{T'_{d0}}P_e + \frac{V_s}{X_{ds}T'_{d0}}\sin(\delta)K_c u_f + \frac{(X_d - X'_d)V_s^2}{X_{ds}X'_{ds}}\omega \sin^2(\delta) + P_e \omega \cot(\delta) \end{aligned} \quad (11)$$

in which  $\theta(t)$  is a possibly time-varying disturbance: the parameter  $\theta$  is assumed to be unknown and to belong to the known compact set  $[\theta_m, \theta_M]$ : the lower and upper bounds  $\theta_m, \theta_M$  are known.

Let  $\delta_r(t)$  be a smooth reference signal to be tracked. Define  $(\lambda_1 > 0)$

$$\begin{aligned} \tilde{\delta}(t) &= \delta(t) - \delta_r(t) \\ \omega^* &= -\lambda_1 \tilde{\delta} + \dot{\delta}_r \\ \tilde{\omega} &= \omega - \omega^* = \omega + \lambda_1 \tilde{\delta} - \dot{\delta}_r \end{aligned}$$

so that the first two equations in (11) are rewritten as

$$\begin{aligned} \dot{\tilde{\delta}} &= -\lambda_1 \tilde{\delta} + \tilde{\omega} \\ \dot{\tilde{\omega}} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}(\theta(t) - P_e) - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \ddot{\delta}_r \end{aligned}$$

Define  $(\lambda_2 > 0, k > 0)$  the reference signal for  $P_e$  as

$$P_e^* = \frac{H}{\omega_s} \left\{ -\frac{D}{H}\omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \ddot{\delta}_r + \lambda_2 \tilde{\omega} + \tilde{\delta} + \frac{1}{4}k \left( \frac{\omega_s}{H} \right)^2 \tilde{\omega} \right\} + \hat{\theta}$$

while  $\hat{\theta}$  is an estimate of  $\theta = P_m$  and

$$\tilde{P}_e = P_e - P_e^*$$

so that (11) may be rewritten as  $(\tilde{\theta} = \theta - \hat{\theta})$

$$\begin{aligned} \dot{\tilde{\delta}} &= -\lambda_1 \tilde{\delta} + \tilde{\omega} \\ \dot{\tilde{\omega}} &= -\tilde{\delta} - \lambda_2 \tilde{\omega} - \frac{\omega_s}{H} \tilde{P}_e - \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \tilde{\omega} + \frac{\omega_s}{H} \tilde{\theta} \\ \dot{\tilde{P}}_e &= -\frac{1}{T'_{d0}} P_e + \frac{V_s}{X_{ds} T'_{d0}} \sin(\delta) K_c u_f + \frac{(X_d - X'_d) V_s^2}{X_{ds} X'_{ds}} \omega \sin^2(\delta) + P_e \omega \cot(\delta) \\ &\quad - \frac{H}{\omega_s} \left\{ \left( -\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right) (-\lambda_1 \tilde{\delta} + \tilde{\omega}) \right. \\ &\quad \left. + \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right) \left( -\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_s}{H} P_e - \ddot{\delta}_r \right) \right\} \\ &\quad - \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right) \hat{\theta} - \dot{\hat{\theta}} \\ &\quad - \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right) \tilde{\theta} + \frac{D}{\omega_s} \ddot{\delta}_r + \frac{H}{\omega_s} \dot{\delta}_r \end{aligned}$$

Defining  $(\lambda_3 > 0)$

$$\begin{aligned} \tilde{\theta} &= \theta - \hat{\theta} \\ u_f &= \frac{T'_{d0} X_{ds}}{V_s K_c \sin(\delta)} \phi_0 \\ \phi_0 &= \frac{1}{T'_{d0}} P_e - \frac{(X_d - X'_d)}{X_{ds} X'_{ds}} V_s^2 \omega \sin^2(\delta) - P_e \omega \cot(\delta) \\ &\quad + \frac{H}{\omega_s} \left\{ \left( -\lambda_1^2 + 1 + \lambda_1 \frac{D}{H} \right) (-\lambda_1 \tilde{\delta} + \tilde{\omega}) \right. \\ &\quad \left. + \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right) \left( -\frac{D}{H} \omega - \lambda_1^2 \tilde{\delta} + \lambda_1 \tilde{\omega} - \frac{\omega_s}{H} P_e - \ddot{\delta}_r \right) \right\} \\ &\quad + \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right) \hat{\theta} + \dot{\hat{\theta}} \\ &\quad - \frac{k}{4} \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right)^2 \tilde{P}_e - \frac{D}{\omega_s} \ddot{\delta}_r - \frac{H}{\omega_s} \dot{\delta}_r - \lambda_3 \tilde{P}_e + \frac{\omega_s}{H} \tilde{\omega} \end{aligned}$$



the closed loop system becomes

$$\begin{aligned}\dot{\tilde{\delta}} &= -\lambda_1 \tilde{\delta} + \tilde{\omega} \\ \dot{\tilde{\omega}} &= -\tilde{\delta} - \lambda_2 \tilde{\omega} - \frac{\omega_s}{H} \tilde{P}_e - \frac{k}{4} \left(\frac{\omega_s}{H}\right)^2 \tilde{\omega} + \frac{\omega_s}{H} \tilde{\theta} \\ \dot{\tilde{P}}_e &= \frac{\omega_s}{H} \tilde{\omega} - \lambda_3 \tilde{P}_e \\ &\quad - \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left(\frac{\omega_s}{H}\right)^2\right) \tilde{\theta} \\ &\quad - \frac{k}{4} \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left(\frac{\omega_s}{H}\right)^2\right)^2 \tilde{P}_e\end{aligned}$$

The adaptation law is ( $\gamma$  is a positive adaptation gain)

$$\dot{\hat{\theta}} = \gamma Proj \left( \left( \tilde{P}_e \left( \frac{D}{H} - \lambda_1 - \lambda_2 - \frac{k}{4} \left(\frac{\omega_s}{H}\right)^2 \right) + \tilde{\omega} \frac{\omega_s}{H} \right), \hat{\theta} \right)$$

where  $Proj(y, \hat{\theta})$  is the smooth projection algorithm introduced in [11]

$$\begin{aligned}Proj(y, \hat{\theta}) &= y, & \text{if } p(\hat{\theta}) \leq 0 \\ Proj(y, \hat{\theta}) &= y, & \text{if } p(\hat{\theta}) \geq 0 \text{ and } \langle grad p(\hat{\theta}), y \rangle \leq 0 \\ Proj(y, \hat{\theta}) &= [1 - p(\hat{\theta}) | grad p(\hat{\theta}) |], & \text{otherwise}\end{aligned}\tag{12}$$

with

$$p(\theta) = \frac{(\theta - \frac{\theta_M + \theta_m}{2})^2 - (\frac{\theta_M - \theta_m}{2})}{\epsilon^2 + 2\epsilon(\frac{\theta_M - \theta_m}{2})}$$

for  $\epsilon$  an arbitrary positive constant which guarantees in particular that:

- i)  $\theta_m - \epsilon \leq \hat{\theta}(t) \leq \theta_M + \epsilon$
- ii)  $|Proj(y, \hat{\theta})| \leq |y|$
- iii)  $(\theta - \hat{\theta}) Proj(y, \hat{\theta}) \geq (\theta - \hat{\theta}) y$

Consider the function

$$W = \frac{1}{2} (\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P}_e^2)\tag{13}$$

whose time derivative, according to (11), is

$$\begin{aligned} \dot{W} = & -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P}_e^2 + \tilde{\omega} \frac{\omega_s}{H} \tilde{\theta} - \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \tilde{\omega}^2 \\ & - \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right) \tilde{\theta} \tilde{P}_e - \frac{k}{4} \left( -\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4} \left( \frac{\omega_s}{H} \right)^2 \right)^2 \tilde{P}_e^2 \end{aligned}$$

Completing the squares, we obtain the inequality

$$\dot{W} \leq -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P}_e^2 + \frac{2}{k} \tilde{\theta}^2 \quad (14)$$

which guarantees arbitrary  $\mathcal{L}_\infty$  robustness from the parameter error  $\tilde{\theta}$  to the tracking errors  $\tilde{\delta}, \tilde{\omega}, \tilde{P}_e$ .

The projection algorithms (12) guarantee that  $\tilde{\theta}$  is bounded, and, by virtue of (13) and (14), that  $\tilde{\delta}, \tilde{\omega}$  and  $\tilde{P}_e$  are bounded. Therefore,  $\dot{\tilde{\theta}}$  is bounded. Integrating (14), we have for every  $t \geq t_0 \geq 0$

$$-\int_{t_0}^t (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2 + \lambda_3 \tilde{P}_e^2) d\tau + \frac{2}{k} \int_{t_0}^t \tilde{\theta}^2 d\tau \geq W(t) - W(t_0)$$

Since  $W(t) \geq 0$  and, by virtue of the projection algorithm (12),

$$\tilde{\theta}(t) \leq \theta_M - \theta_m + \epsilon$$

it follows that

$$\int_{t_0}^t (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2) d\tau \leq W(t_0) + \frac{2}{k} (\theta_M - \theta_m + \epsilon)^2 (t - t_0)$$

which, if  $W(t_0) = 0$  (i.e.  $t_0$  is a time before the occurrence of the fault), implies arbitrary  $\mathcal{L}_2$  attenuation (by a factor  $k$ ) of the errors  $\tilde{\delta}$  and  $\tilde{\omega}$  caused by the fault. To analyze the asymptotic behavior of the adaptive control, we consider the function

$$V = \frac{1}{2} (\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P}_e^2) + \frac{1}{2} \frac{1}{\gamma} \tilde{\theta}^2$$

The projection estimation algorithm (12) is designed so that the time derivative of  $V$  satisfies

$$\dot{V} \leq -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P}_e^2 \quad (15)$$

Integrating (15), we have

$$\lim_{t \rightarrow \infty} \int_{t_0}^t (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2 + \lambda_3 \tilde{P}_e^2) d\tau \leq V(0) - V(\infty) < \infty$$

From the boundedness of  $\dot{\tilde{\delta}}, \dot{\tilde{\omega}}$  and  $\dot{\tilde{P}}_e$ , and Barbalat's Lemma (see [9], [7]) it follows that

$$\lim_{t \rightarrow \infty} \left\| \begin{bmatrix} \tilde{\delta}(t) \\ \tilde{\omega}(t) \\ \tilde{P}_e(t) \end{bmatrix} \right\| = 0$$

## 4 Simulation results

In this section some simulation results are given with reference to the eight-machine power system network reported in [3] with the following data:

$$\begin{aligned} \omega_s &= 314.159 \text{ rad/s} & D &= 5 \text{ p.u.} & H &= 8s \\ T_{d0} &= 6.9s & K_c &= 1 & X_d &= 1.863 \text{ p.u.} \\ X'_d &= 0.257 \text{ p.u.} & X_T &= 0.127 \text{ p.u.} & X_L &= 0.4853 \text{ p.u.} \end{aligned}$$

The operating point is  $\delta_s = 72^\circ$ ,  $P_m = 0.9$  p.u.,  $\omega_0 = 0$  to which corresponds  $V_t = 1$  p.u., with  $V_s = 1$  p.u..

It was considered a fast reduction of the mechanical input power, and simulated according to the following sequences

1. The system is in pre-faulted state.
2. At  $t = 0.5s$  the mechanical input power begins to decrease.
3. At  $t = 1.5s$  the mechanical input power is 50% of the initial value.
4. At  $t = 2s$  the controller starts to drive the system to the new predicted equilibrium power angle.
5. At  $t = 4s$  the system finishes to drive the system to the new equilibrium.

The simulations were carried out using as control parameters

$$\begin{aligned} \lambda_i &= 20 & 1 \leq i \leq 3 \\ \gamma &= 1 & k = 0.1 \end{aligned}$$

Fig. 2a) shows that the predicted equilibrium power angle  $((\delta_s)_f)$  is available since  $t = 1.5s$  i.e. from the moment when the mechanical power arrives at its final value. One may see that  $\delta$  follows perfectly its reference value  $\delta_r$ .

Actually,  $\delta_r$  may be very fast, but the magnitude of the control signal will grow as well. In these simulations, the control signal was kept lower than twice of the necessary to keep the system at its original equilibrium point. If larger controls are acceptable, the system may be driven faster to its equilibrium.

In Fig. 2b) one may see that the estimation of the mechanical power is very fast and accurate, and that the electrical power is correctly driven to the mechanical one. The electrical power just leaves its equilibrium value during the time that the system is driven for its new equilibrium point. One can see that before being driven to its final equilibrium value,  $\delta$  is kept at its original one ( $0.5 \leq t \leq 2$ ). The same may be observed in Fig. 2c) for the rotor velocity.

Fig. 3a) shows how the output voltage drops during the fault, and goes to its correct value when the system is driven to the predicted equilibrium point. Note that during the time ( $1.5 \leq t \leq 2$ ) the system is stable, but the output voltage is not anymore the correct one.

Finally, one can see in Fig. 3b) that the control signal is very smooth and is kept inside the prescribed bounds.

(a) (b)

**Fig.2.** a1)  $\delta$  (-),  $\delta_r$  (- -),  $(\delta_s)_f$  (-.) a2)  $P_m$  (-),  $\hat{\theta}$  (-.),  $P_e$  (- -) a3)  $\omega$   
 b1)  $V_t$  b2) Control signal

## 5 Conclusions

In this work we have computed the zero dynamics of the system with respect to the terminal voltage having then obtained a highly nonlinear second order dynamics. We show by simulations that this zero dynamics may become unstable by a simple change of the input mechanical power. This is our motivation to control the power angle and the relative angular speed as well as the terminal voltage.

We then show that the terminal voltage, the relative angular speed and the active electric power are an equivalent set of state variables for the system. They have the advantage of being measurable standard outputs from the system.

We then present the relation between the terminal voltage and power angle which allows us to avoid measurement of the power angle as well as to compute the new equilibrium angle that produces the correct terminal voltage.

Finally we present a nonlinear adaptive feedback control that stabilizes the system to the pre-faulted equilibrium point, recovers the correct value for the mechanical power and then, the correct value of the faulted equilibrium power angle, driving the system smoothly to this new equilibrium point.

The system may be kept arbitrarily close to the original equilibrium point and may be driven arbitrarily fast to the new equilibrium point. The only restriction will be the magnitude of the control signal.

As a continuation of this research, the problem of transmission line failure will be addressed. We will as well make a deeper study on the behavior of the zero dynamics of the system with respect to the terminal voltage.

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### Appendix

Substituting (2) into (3) we have that

$$Q = P_e \cot(\delta) - \frac{V_s^2}{X_{ds}} \quad (16)$$

Then one will find that

$$Q^2 = \frac{P_e^2}{\sin^2(\delta)} - P_e^2 - 2P_e \cot(\delta) \frac{V_s^2}{X_{ds}} + \frac{V_s^4}{X_{ds}^2}$$

where we used the relation

$$\cot^2 = \frac{1}{\sin^2} - 1$$

Then, it is easy to find that

$$\begin{aligned} & \frac{X_s^2}{V_s^2} (Q^2 + P_e^2) + \frac{V_s^2}{X_{ds}^2} (X_d^2 - X_s^2) + 2 \frac{(X_s^2 + X_s X_d)}{X_{ds}} P_e \cot(\delta) \\ &= \frac{X_s^2}{V_s^2} \frac{P_e^2}{\sin^2(\delta)} + \frac{V_s^2}{X_{ds}^2} X_d^2 + 2 \frac{X_s X_d}{X_{ds}} P_e \cot(\delta) \\ &= V_t^2 \end{aligned}$$

where the last relation may be verified looking at (8). We then may rewrite

$$V_t^2 = \frac{X_s^2}{V_s^2} (Q^2 + P_e^2) + \frac{V_s^2}{X_{ds}^2} (X_d^2 - X_s^2) + 2X_s P_e \cot(\delta)$$

Then, we have

$$\cot(\delta) = \frac{1}{2X_s P_e} V_t^2 - \frac{1}{2X_s P_e} \frac{X_s^2}{V_s^2} (Q^2 + P_e^2) - \frac{1}{2X_s P_e} \frac{V_s^2}{X_{ds}^2} (X_d^2 - X_s^2) \quad (17)$$

Substituting (17) into (16) one will find that

$$Q^2 + \frac{2V_s^2}{X_s} Q + P_e^2 + \frac{V_s^2}{X_s^2} (V_s^2 - V_t^2) = 0$$

and consequently we may find its roots. By physical bounds on  $Q$  (it must assume positive and negative values while  $V_s$  and  $X_s$  are positive) we find that

$$Q = -\frac{V_s^2}{X_s} + \frac{V_s}{X_s} \sqrt{V_t^2 - \frac{X_s^2}{V_s^2} P_e^2}$$

and then we have that

$$Q^2 = \frac{V_s^2}{X_s^2} \left( V_t^2 - \frac{X_s^2}{V_s^2} P_e^2 \right) - 2 \frac{V_s^2}{X_s} \frac{V_s}{X_s} \sqrt{V_t^2 - \frac{X_s^2}{V_s^2} P_e^2} + \frac{V_s^4}{X_s^2} \quad (18)$$

Substituting (18) into (17) one will find that

$$\cot(\delta) = \left( \frac{V_s}{X_s P_e} \right) \left( -\frac{V_s}{X_{ds}} X_d + \sqrt{V_t^2 - \frac{X_s^2}{V_s^2} P_e^2} \right)$$

and finally we have that

$$\delta = \operatorname{arccot} \left( \left( \frac{V_s}{X_s P_e} \right) \left( -\frac{V_s}{X_{ds}} X_d + \sqrt{V_t^2 - \frac{X_s^2}{V_s^2} P_e^2} \right) \right)$$

For the equilibrium value, we just replace  $P_e$  and  $V_t$  by its reference values  $P_m$  and  $V_{tr}$  and then we find

$$\delta_s = \operatorname{arccot} \left( \left( \frac{V_s}{X_s P_m} \right) \left( -\frac{V_s}{X_{ds}} X_d + \sqrt{V_{tr}^2 - \frac{X_s^2}{V_s^2} P_m^2} \right) \right)$$