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INSTITUT FEMTO-ST

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*On the Heterogeneity Bias of Cost Matrices when
Assessing Scheduling Algorithms*

Version 2

Louis-Claude CANON — Laurent PHILIPPE

Rapport de Recherche n° RR-FEMTO-ST-8663

DÉPARTEMENT DISC – November 7, 2016

On the Heterogeneity Bias of Cost Matrices when Assessing Scheduling Algorithms

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Département DISC

Rapport de Recherche n° RR-FEMTO-ST-8663 November 7, 2016 (53 pages)

Abstract: Assessing the performance of scheduling heuristics through simulation requires one to generate synthetic instances of tasks and machines with well-identified properties. Carefully controlling these properties is mandatory to avoid any bias. We consider the scheduling problem consisting of allocating independent sequential tasks on unrelated machines while minimizing the maximum execution time. In this problem, the instance is a cost matrix that specifies the execution cost of any task on any machine. This report proposes two measures for quantifying the heterogeneity properties of a cost matrix. An analysis of two classical methods used in the literature reveals a bias in previous studies. Two new methods are proposed to generate instances with given heterogeneity properties and it is shown that they have a significant impact on several heuristics.

Key-words: Scheduling; Cost Matrix; Heterogeneity; Bias; Parallelism; Unrelated; Measure.

Sur le biais d'hétérogénéité dans les matrices de coût lors de l'évaluation des algorithmes d'ordonnancement

Version 2

Résumé : Évaluer la performance des heuristiques d'ordonnancement avec des simulations nécessite de générer des instances synthétiques de tâches et de machines avec des propriétés clairement identifiées. Contrôler ces propriétés avec soin est indispensable pour éviter tout biais. Nous considérons le problème d'ordonnancement qui consiste à allouer des tâches séquentielles indépendantes sur des machines indépendantes tout en minimisant le temps d'exécution maximum. Dans ce problème, l'instance est une matrice de coût qui fournit le temps d'exécution de chaque tâche sur chaque machine. Ce rapport propose deux mesures pour quantifier les propriétés d'hétérogénéité d'une matrice de coût. Une analyse de deux méthodes classiques utilisées dans la littérature montre un biais dans les études précédentes. Deux nouvelles méthodes sont proposées pour générer des instances avec des propriétés d'hétérogénéité données et nous montrons qu'elles ont un impact significatif sur plusieurs heuristiques.

Mots-clés : ordonnancement; matrices de coût; hétérogénéité; biais; parallélisme; mesures.

1 Introduction

Leveraging the parallelism of multi-core distributed platforms involves efficiently scheduling applications on several machines [105]. Current studies rely on performance evaluation to determine the best solution for any underlying problem. This process can be divided into several categories: formal analysis, experiments, simulations, etc. In the case of simulations, a scheduling strategy is tested in a virtual environment with a given workload. This paper focuses on the generation of synthetic instances.

Synthetic instances of workload allow a more general evaluation than with specific traces. They are particularly useful for sensitivity analysis [139], which consists in assessing the impact of the instance properties on the algorithms. However, the lack of control on the instance properties makes it difficult to confront the results of independent studies. For instance, although many papers have compared several scheduling heuristics [22, 26, 43, 109], predicting their performance is still an issue. These problems can be tackled by carefully controlling the instance properties.

Specifically, we consider the scheduling problem noted $R||C_{\max}$ in $\alpha|\beta|\gamma$ notation [63]. It consists in scheduling n independent sequential tasks on m unrelated machines. All tasks are available simultaneously and preemption is not possible. The instance is a *cost matrix* where each element $e_{i,j}$ is a positive integer that represents the execution cost of task i on machine j . The objective is to allocate each task to a machine such that the maximum execution time on any machine is minimized. More formally, we want to minimize $\max(\sum \pi(i,j) \times e_{i,j})$ where $\pi(i,j)$ is equal to one if task i is scheduled on machine j and zero otherwise.

For this problem, the range-based and CVB (Coefficient of Variation Based) methods proposed in [17, 18] are currently the standard methods used in the literature to generate instances. However, the properties of the matrices generated with these methods have never been formally analysed and previous studies may thus be exposed to a bias.

This paper provides the following contributions:¹

- a statistical description of the use of the range-based and CVB methods in the literature (Section 3);
- a study of how to quantify the heterogeneity properties of a cost matrix (Section 4);
- a formal analysis of the range-based and CVB methods and the identification of a bias that impacts several studies (Section 4);
- a new method with control over heterogeneity properties (Section 5);
- and, an assessment of the impact of these properties on several heuristics (Section 6).

2 Related Work

The concept of heterogeneity was first introduced in the context of cost matrix by Armstrong [20]. He described the *heterogeneity quadrant* in which cost matrices are divided into four categories depending on their heterogeneity properties regarding tasks and machines: low/low, low/high, high/low, and high/high. For instance low/high refers to low task heterogeneity and high machine heterogeneity. However, no method for generating such matrices was proposed.

The range-based and CVB methods were first proposed to fill this gap in [8] and then in [17, 18]. However, task and machine heterogeneities were not formally defined and analyzed. The methods were assumed to generate matrices with the expected properties and only validated through some examples.

¹The related code, data and analysis are available in [37].

The limits of these methods were later acknowledged in [5], which proposed to consider the average coefficient of variation², skewness and kurtosis of the costs for each task and for each machine. The proposed scheme (based on decision trees) uses these additional information to predict heuristic performance. Despite a wide experimentation plan, the study lacks discussion and interpretation in particular on the relative importance of the considered measures. Additionally, no formal analysis was provided. The exhibited decision trees suggest that the average coefficient of variation plays a significant role, which supports the current work.

The MPH (Machine Performance Homogeneity) is introduced in [3] for capturing the heterogeneity between the machines while its counterpart for the tasks, the TDH (Task Difficulty Homogeneity), appears in [2]. We discuss them more extensively in Section 4. In addition, the TMA (Task-Machine Affinity) is also defined in [3]: it quantifies the specialisation of the system (i.e., whether some machines are particularly efficient for some specific tasks). Although the three measures are applied to a real benchmark, no method is proposed for generating matrices with given MPH, TDH and TMA. It is thus unclear what is the impact of the proposed measures on heuristic performance. Finally, they show that the range-based and CVB methods do not cover the entire range of possible values for the MPH and the TMA, which is consistent with the conclusion of Section 4.

Friese et al [53] present a method for adding tasks in a given cost matrix while preserving some statistical properties on each column (mean, coefficient of variation, skewness and kurtosis). It ignores the properties on each row however.

A method for generating matrices with varying affinities (similar to the TMA) is proposed in [6]. It is similar to the noise-based method described in Section 5, but no formal analysis is provided.

Khemka et al [82] propose a method for changing the TMA of an existing matrix while keeping the same MPH and TDH. TMA is mentioned to be related to the correlation. Investigating the correlation properties is left for future work. There is also another field of studies dedicated to the generation of matrices with given correlation and covariance matrices [59].

3 Matrix Generation Methods

The most used methods for generating cost matrices are the range-based and the CVB (Coefficient of Variation Based) methods [8, 17, 18]. The most frequent notations are summarized in Appendix A.

3.1 Range-Based Method

The range-based method generates n vectors of m values that follow a uniform distribution in the range $[1, R_{mach}]$ (see Algorithm 1). Each row is then multiplied by a random value that follows a uniform distribution in the range $[1, R_{task}]$ (Line 2). The resulting cost matrix is similar to the following (where τ is a vector of n uniform values in $[1, R_{task}]$):

$$\begin{pmatrix} \tau[1]U(1, R_{mach}) & \cdots & \tau[1]U(1, R_{mach}) \\ \vdots & \ddots & \vdots \\ \tau[n]U(1, R_{mach}) & \cdots & \tau[n]U(1, R_{mach}) \end{pmatrix}$$

Proposition 1. *When used with parameters R_{task} and R_{mach} , the range-based method generates costs with expected value $\frac{1}{4}(R_{task} + 1)(R_{mach} + 1)$ and standard deviation $\frac{1}{12}[(R_{task} - 1)^2(R_{mach} - 1)^2 + 3(R_{mach} - 1)^2(R_{task} + 1)^2 + 3(R_{task} - 1)^2(R_{mach} + 1)^2]^{1/2}$.*

²Ratio of the standard deviation to the mean.

Algorithm 1 Range-based cost matrix generation with the uniform distribution

Input: n, m, R_{task}, R_{mach}

Output: a $n \times m$ cost matrix

```

1: for all  $1 \leq i \leq n$  do                                     {Generate each row}
2:    $\tau[i] \leftarrow U(1, R_{task})$ 
3:   for all  $1 \leq j \leq m$  do                                   {Generate each value of the row}
4:      $e_{i,j} \leftarrow \tau[i] \times U(1, R_{mach})$ 
5:   end for
6: end for
7: return  $\{e_{i,j}\}_{1 \leq i \leq n, 1 \leq j \leq m}$ 

```

Property	Value
Expected value	$\frac{1}{4}(R_{task} + 1)(R_{mach} + 1)$
Standard deviation	$\frac{1}{12} \sqrt{(R_{task} - 1)^2(R_{mach} - 1)^2 + 3(R_{mach} - 1)^2(R_{task} + 1)^2 + 3(R_{task} - 1)^2(R_{mach} + 1)^2}$
CV	$\frac{1}{3} \sqrt{\frac{(R_{task}-1)^2(R_{mach}-1)^2}{(R_{task}+1)^2(R_{mach}+1)^2} + 3\frac{(R_{task}-1)^2}{(R_{task}+1)^2} + 3\frac{(R_{mach}-1)^2}{(R_{mach}+1)^2}}$
Distribution	Product of two uniform laws
Asymptotic expected value	$\frac{1}{4}R_{task}R_{mach}$
Asymptotic standard deviation	$\frac{\sqrt{7}}{12}R_{task}R_{mach}$
Asymptotic CV	$\frac{\sqrt{7}}{3} \approx 0.88$

Table 1: Summary of the cost matrix properties with the range-based method. Asymptotic values are when both R_{task} and R_{mach} are large.

Proof. Each cost is the product of $\tau[i]$, which follows a uniform law in the range $[1, R_{task}]$, and a random variable that follows a uniform law in the range $[1, R_{mach}]$. Therefore, the expected value of the costs is the product of the expected values of both distributions, namely $(R_{task} + 1)/2$ and $(R_{mach} + 1)/2$.

The standard deviation of the product of two random variables with means μ_1 and μ_2 , and standard deviations σ_1 and σ_2 is $\sqrt{\sigma_1^2\sigma_2^2 + \mu_1^2\sigma_2^2 + \sigma_1^2\mu_2^2}$. With a similar argument as for the expected value, we can derive the standard deviation of the costs. \square

Table 1 summarizes the properties of this method. Except for low values of R_{task} and R_{mach} , the CV (Coefficient of Variation) remains close to a constant. For instance, when $R_{task} = R_{mach} = 100$, then the CV is around 0.86. As shown in Section 4, this method is not well-suited to control the heterogeneity of the resulting cost matrix. Also, given that this method is asymmetric, it may be expected to handle task heterogeneity differently from machine heterogeneity.

3.2 CVB Method

The CVB method is based on the same principle except it uses parameters that are distinct from the underlying distribution parameters. In particular, it requires two CV (V_{task} for the tasks and V_{mach} for the machines) and one mean (μ_{task} for the tasks). The random values follow a gamma distribution whose parameters are computed such that the provided CV and mean are respected.

Algorithm 2 CVB cost matrix generation with the gamma distribution**Input:** $n, m, V_{task}, V_{mach}, \mu_{task}$ **Output:** a $n \times m$ cost matrix

```

1:  $\alpha_{task} \leftarrow 1/V_{task}^2$ 
2:  $\alpha_{mach} \leftarrow 1/V_{mach}^2$ 
3:  $\beta_{task} \leftarrow \mu_{task}/\alpha_{task}$ 
4: for all  $1 \leq i \leq n$  do
5:    $q[i] \leftarrow G(\alpha_{task}, \beta_{task})$ 
6:    $\beta_{mach}[i] \leftarrow q[i]/\alpha_{mach}$ 
7:   for all  $1 \leq j \leq m$  do
8:      $e_{i,j} \leftarrow G(\alpha_{mach}, \beta_{mach}[i])$ 
9:   end for
10: end for
11: return  $\{e_{i,j}\}_{1 \leq i \leq n, 1 \leq j \leq m}$ 

```

Property	Value
Expected value	μ_{task}
CV	$\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$
Distribution	Product of two gamma laws

Table 2: Summary of the cost matrix properties with the CVB method.

Proposition 2. *When used with parameters V_{task} , V_{mach} and μ_{task} , the CVB method generates costs with expected value μ_{task} and coefficient of variation $\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$.*

Proof. In order to apply the same analysis as in the proof of Proposition 1, we need to prove that any cost is the product of two gamma distributions. More precisely, we need to prove that the random generation on Line 8 is equivalent to multiplying $q[i]$ by a gamma law with mean one and CV V_{mach} .

Each cost $e_{i,j}$ is a random variable that follows a gamma distribution with mean $q[i]$ and CV V_{mach} . The probability that $e_{i,j}$ is no more than x is given by $\frac{1}{\Gamma(\alpha)}\gamma(\alpha, \frac{x}{\beta})$ where $\alpha = 1/V_{mach}^2$, $\beta = q[i]/\alpha$, $\Gamma(\alpha)$ is the gamma function and $\Gamma(\alpha, \frac{x}{\beta})$ is the lower incomplete gamma function.

By contrast, let X be a random variable that follows a gamma distribution with mean one and CV V_{mach} . Then, the probability that $q[i]X$ is no more than x is the probability that X is no more than $x/q[i]$: $\frac{1}{\Gamma(\alpha)}\gamma(\alpha, \frac{x/q[i]}{\beta})$ where $\alpha = 1/V_{mach}^2$ and $\beta = 1/\alpha$. It is thus the same as for $e_{i,j}$.

Thus, Line 8 can be replaced by the product of $q[i]$ by a gamma law with mean one and CV V_{mach} (i.e., $e_{i,j} \leftarrow q[i]G(\alpha_{mach}, 1/\alpha_{mach})$), which is the product of two gamma distributions.

The proof is then analogous to the proof of Proposition 1. \square

Table 2 summarizes the properties of this method, which is more adapted to control the heterogeneity of the resulting cost matrix. However, it is still asymmetric. Note that the CV is the same as with the range-based method when we replace V_{task} by the CV of the first uniform law, $\frac{\sqrt{12}}{6} \frac{R_{task}-1}{R_{task}+1}$, and V_{mach} by the CV of the second uniform law, $\frac{\sqrt{12}}{6} \frac{R_{mach}-1}{R_{mach}+1}$.

3.3 Consistency Extension

Both the previous methods produce cost matrices that may not be representative of realistic settings. For instance, the costs of a given task is not correlated to the costs of another task, which may often be the case in practice. The consistency extension consists in reordering the costs in the generated matrix to have an instance that is closer to the uniform case. Specifically, the rows of a submatrix of an rows and bm columns are sorted. Thus, a machine that is faster for a given task than another machine will likely be also faster for another task. Inconsistent matrices have $a = b = 0$ while consistent matrices have $a = b = 1$ (other matrices are either called semiconsistent or partially consistent).

3.4 Usage in the Literature

We covered the English articles that cite at least one of the references in which the methods were initially presented [8,17,18] and that were freely available. For each reference, we extracted all the distinct sets of parameters. Additionally, we differentiated between example cost matrices that illustrate the generation methods from cost matrices that are used in actual sets of experiments to study scheduling algorithms. However, the size was ignored as we only consider asymptotic properties (Section 4.6 assesses the impact of the size).

Some data were not specifically provided. The parameters that could be directly inferred from the article or from similar works are emphasized: this concerns mostly missing parameters for the consistency extension (the ones from the cited article were taken). Otherwise, they are treated as missing values (denoted by NA). Some articles lack enough information, which prevented any parameter extraction.

On the 160 analysed articles, 78 provide exploitable information on the cost matrix instances. The rest consists of 40 articles with no description, but which refer to instances described in other articles and 42 articles with unclear descriptions or approaches that do not fit the current study. The extracted data are provided in Appendix B and summarized below. While most articles fail to precisely describe the used method, only the range and CV parameters are crucial for reproducing similar instances. In the end, 342 sets of parameters were extracted in 78 articles for a total of 210 unique settings: 37 for the range-based method and 173 for the CVB one.

Figure 1 depicts the values used with both methods. Although there is no clear agreement on which precise parameters are the most relevant, there are some common tendencies. Values for low heterogeneity are usually 10 and 100 for the range-based method and .1, .25 and .3 for the CVB method. Values for high heterogeneity are usually 100, 1e3, 3e3 and 1e5 for the range-based method and .3, .35, .4, .5, .6, .7, .9, 1 and 2 for the CVB method.

4 Heterogeneity Measures

Assessing the impact of heterogeneity on heuristic performance requires a method for quantifying the heterogeneity of the generated cost matrices.

4.1 TDH and MPH

The closest related measures are the TDH (Task Difficulty Homogeneity) and the MPH (Machine Performance Homogeneity) [2,3]. The TDH computation is described in Algorithm 3. The value $TD[i]$ represents the difficulty of task i , namely whether it has small costs. After the ordering, the final sum computes the average ratio between similar tasks in terms of difficulty (which lies in the interval $(0, 1]$). If this average is one, then tasks are all similar. If it is close to zero, then the task heterogeneity is large.

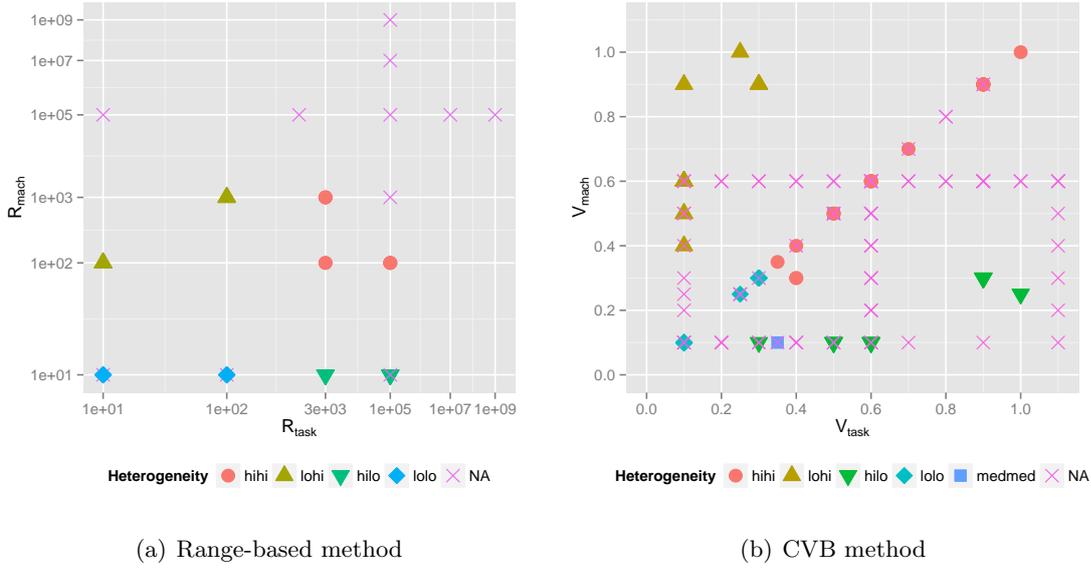


Figure 1: Parameters used in the literature. Three points are not shown for the CVB method: (1.4, 0.4), (1.8, 0.4) and (0.1, 2).

Algorithm 3 TDH computation

Input: a $n \times m$ cost matrix

Output: the TDH of this matrix

- 1: **for all** $1 \leq i \leq n$ **do**
 - 2: $TD[i] \leftarrow \sum_{j=1}^m \frac{1}{e_{i,j}}$
 - 3: **end for**
 - 4: sort TD in ascending order
 - 5: **return** $\frac{1}{n-1} \sum_{i=1}^{n-1} \frac{TD[i]}{TD[i+1]}$
-

The MPH computation is analogous except that the sum on Line 2 is performed on each row instead of each column. This results in a measure of the machine heterogeneity.

These measures have three major shortcomings (as mentioned in Section 2). First, they are not intuitive (they require to invert costs, to order sums and to average ratios). Also, they do not rely on classical statistical measures, which makes deriving formal results more difficult. In particular, the ordering on Line 4 complicates formal analysis. A last notable problem is that the resulting values depend on the size of the matrix. In particular, it is close to one when the matrix is large (even if it is generated with the same parameters and has, intuitively, the same characteristics). For instance, if we consider only one machine, the following matrices (cost vectors in this case) have the same TDH: $[1, 2]$ and $[0.125, 0.25, 0.5, 1, 2, 4]$. The second vector, however, seems more heterogeneous. As another example, let the minimum TD be 1 and the maximum TD be 100. Given Proposition 3, the TDH is always greater than 0.60 when there are 10 tasks and it is always greater than 0.95 when there are 100 tasks. This measure is thus relevant only for comparing small cost matrices with similar sizes.

Proposition 3. *The TDH cannot be lower than $e^{\log\left(\frac{\min(TD)}{\max(TD)}\right)/(n-1)}$.*

Proof. The minimum TDH is achieved when the sum $\sum_{i=1}^{n-1} \frac{a_i}{a_{i+1}}$ where $a_i = TD[i]$ is minimum. Let $f : [a_1, a_n]^{n-2} \rightarrow (0, \infty)$ be the corresponding multivariate function with a_1 and a_n being

constant. Each value a_i for $1 < i < n$ is greater than or equal to a_1 because the a_i are ordered. As a_1 represents an average cost and is thus strictly greater than zero, all nominators and all denominators are strictly greater than zero. Therefore, f is a continuous function from the compact $[a_1, a_n]^{n-2}$. The extreme value theorem states that a continuous function from a non-empty compact space to a subset of the real numbers attains a maximum and a minimum. This proves the existence of a minimum.

We now show by contradiction that this minimum is achieved when the ratios $\frac{a_i}{a_{i+1}}$ are all equal for $1 \leq i < n$. Assume it is not the case and let i be the lowest value for which $\frac{a_i}{a_{i+1}} \neq \frac{a_{i+1}}{a_{i+2}}$, which can be rewritten as $a_{i+1} \neq \sqrt{a_i a_{i+2}}$. A lower value is attained when $a_{i+1} = \sqrt{a_i a_{i+2}}$ because the partial derivate of f with respect to a_{i+1} (i.e., $-\frac{a_{i-1}}{a_i^2} + \frac{1}{a_{i+1}}$) is zero with this value. Therefore, the minimum is achieved when all ratios $\frac{a_i}{a_{i+1}}$ are equal. This is the case when $a_i = e^{\log(TD[1]) + \frac{i-1}{n-1} \log(\frac{a_n}{a_1})}$ for $1 \leq i \leq n$.

When replacing a_i by $TD[i]$, the TDH simplifies as $e^{\log(\frac{TD[1]}{TD[n]})/(n-1)}$ or $e^{\log(\frac{\min(TD)}{\max(TD)})/(n-1)}$ if the vector TD is not sorted. \square

4.2 Intuitive Measures of Heterogeneity

We identify below two intuitive measures of task and machine heterogeneity that rely on classic properties:

- Assuming that the mean of each row represents a task weight, the task heterogeneity may be defined as the CV (Coefficient of Variation) of the means of the rows (noted $V\mu_{task}$). Analogously, the machine heterogeneity may be measured as the CV of the means of the columns (noted $V\mu_{mach}$).

$$CV \left\{ \begin{array}{l} \mu_1 \\ \vdots \\ \mu_n \end{array} \right. \left(\begin{array}{ccc} e_{1,1} & \cdots & e_{1,m} \\ \vdots & \ddots & \vdots \\ e_{n,1} & \cdots & e_{n,m} \end{array} \right)$$

- Alternatively, the CV of one column may represent the task heterogeneity for a given machine. Therefore, the mean of the CV of the columns may measure the task heterogeneity (noted μV_{task}) while the mean of the CV of the rows may measure the machine heterogeneity (noted μV_{mach}).

The first measure of task and machine heterogeneity has been criticized for small instances [2]. It is argued that the MPH is better than the CV as it is less sensitive to outliers. In this case, the CV can be replaced by the quartile coefficient of dispersion, which is a similar standard statistical measure but is more difficult to formally analyze. Finally, the decision trees in [5] suggest that varying this measure has an impact on the heuristic performance and is thus significant.

With both measures, it is possible to use the standard deviation instead of the CV. However, the CV provides a relative measure that is independent from the cost mean. If an absolute measure is deemed more meaningful, the proposed measures can be adapted by using the standard deviation.

4.3 Coherence with the Uniform Model

The previous measures do not only rely on intuition, they are also consistent with the expectation when we consider the uniform model. In this model, the cost of executing a task i on a machine j is given by the product of the task weight, w_i , and the cycle time, b_j . The concept of task

and machine heterogeneity is easy to grasp in the uniform model: it is given by the statistical dispersion of the weights and the speeds, respectively. We assume that the CV of the weights, noted CV_{task} , is a relevant measure of the task heterogeneity. Analogously, the CV of the speeds, noted CV_{mach} , represents the machine heterogeneity.

It is possible to convert an instance of the uniform model to the unrelated model because this last model is more general. The cost matrix is generated by combining both vectors $\{w_i\}_{1 \leq i \leq n}$ and $\{b_j\}_{1 \leq j \leq m}$ such that $e_{i,j} = w_i b_j$. As we know the heterogeneity properties of a uniform instance, we expect our proposed measures for the unrelated model to be consistent when applied on the converted instance.

Proposition 4. *Let $U = (\{w_i\}_{1 \leq i \leq n}, \{b_j\}_{1 \leq j \leq m})$ be a uniform instance and $E = \{e_{i,j}\}_{1 \leq i \leq n, 1 \leq j \leq m}$ be the corresponding unrelated instance such that $e_{i,j} = w_i b_j$. Then, $CV_{task}(U) = V\mu_{task}(E) = \mu V_{task}(E)$ and $CV_{mach}(U) = V\mu_{mach}(E) = \mu V_{mach}(E)$.*

Proof. By definition, $CV_{task}(U) = \frac{\sqrt{\sum_{i=1}^n w_i^2/n - (\sum_{i=1}^n w_i/n)^2}}{\sum_{i=1}^n w_i/n}$ whereas $V\mu_{task}(E)$ is the CV of the means of the rows. The mean of row i is $\sum_{j=1}^m e_{i,j}/m = w_i/m \sum_{j=1}^m b_j$. Then, $V\mu_{task}(E) = \frac{\sqrt{\sum_{i=1}^n (w_i \phi)^2/n - (\sum_{i=1}^n w_i \phi/n)^2}}{\sum_{i=1}^n w_i \phi/n}$ where $\phi = \sum_{j=1}^m b_j/m$ is the mean of the inverse speeds. Therefore, $V\mu_{task}(E) = CV_{task}(U)$.

Remember that $\mu V_{task}(E)$ is the mean of the CV of the columns. The CV of column j is

$$\begin{aligned} \frac{\sqrt{\sum_{i=1}^n e_{i,j}^2/n - (\sum_{i=1}^n e_{i,j}/n)^2}}{\sum_{i=1}^n e_{i,j}/n} &= \frac{\sqrt{\sum_{i=1}^n (w_i b_j)^2/n - (\sum_{i=1}^n (w_i b_j)/n)^2}}{\sum_{i=1}^n (w_i b_j)/n} \\ &= CV_{task}(U) \end{aligned}$$

The mean of these CV is thus also $CV_{task}(U)$.

The demonstration is analogous for the machine heterogeneity measures. \square

Proposition 4 shows that our proposed measures are consistent with the intuition on uniform instances.

4.4 Heterogeneity of the Range-Based and CVB Methods

We analyse the asymptotic heterogeneity properties of the CVB method with the proposed measures depending on the parameters V_{task} and V_{mach} . An estimator T converges to θ when the expected value of T tends to θ as the number of samples (n and m in our case) tends to ∞ .

Proposition 5. *The measure $V\mu_{task}$ of a cost matrix generated using the CVB method with the parameters V_{task} and V_{mach} converges to V_{task} as $n \rightarrow \infty$ and $m \rightarrow \infty$.*

Proof. This proof assumes that the mean of a set of n samples (called the sample mean) of a random variable with mean μ and standard deviation σ is a random variable with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. Moreover, the CV of a set of n samples (called the sample CV) of a random variable with CV V converges to V as $n \rightarrow \infty$.

Let μ_i be the sample mean of the costs on row i . This row is the product of $q[i]$, which is a random variable that follows a distribution with mean μ_{task} and CV V_{task} , and m values that follow a distribution with mean one and CV V_{mach} . μ_i is thus also the product of the first random variable and the sample mean of the other m values, which follows a random variable with mean one and CV $\frac{V_{mach}}{\sqrt{m}}$. Therefore, the mean of μ_i is μ_{task} and its CV is $\sqrt{V_{task}^2 \frac{V_{mach}^2}{m} + \frac{V_{mach}^2}{m} + V_{task}^2}$, which tends to V_{task} as $m \rightarrow \infty$. The consistency properties have no impact on μ_i because only values on the same row are ordered. \square

Proposition 6. *The measure $V\mu_{mach}$ of a cost matrix generated using the CVB method with the parameters V_{task} and V_{mach} converges to $a\sqrt{b}V_{mach}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$.*

Proof. Let μ_j be the sample mean of the costs on column j . The measure $V\mu_{mach}$ is the ratio of the sample standard deviation of all μ_j , noted $\sigma\mu_{mach}$, to the sample mean of all μ_j , noted $\mu\mu_{mach}$.

Let's distinguish the columns where the costs are consistent ($1 \leq j \leq bm$) from the inconsistent columns ($bm < j \leq m$). For the inconsistent columns, μ_j is the sample mean of n values that follow a product between a distribution with mean μ_{task} and CV V_{task} , and a distribution with mean one and CV V_{mach} . Thus, μ_j follows a distribution with mean μ_{task} and CV $\sqrt{\frac{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}{n}}$ for $bm < j \leq m$. Therefore, the sample mean of μ_j converges to μ_{task} and its sample standard deviation converges to zero as $n \rightarrow \infty$ for $bm < j \leq m$.

For the consistent columns, an rows are sorted. Let q_p denotes the p -quantile of a distribution with mean one and CV V_{mach} (it is the value x for which $F(x) = p$ where F is the cumulative distribution function). Note that $e_{i,j} \rightarrow q[i]q_{j/(bm)}$ as $m \rightarrow \infty$ for $1 \leq i \leq an$ and $1 \leq j \leq bm$. Therefore, μ_j can be decomposed as a weighted sum of sample means (one for the sorted rows and another for the last rows): the first sample mean follows a distribution with mean $\mu_{task}q_{j/(bm)}$ and CV $\frac{V_{task}}{\sqrt{an}}$ while the second follows a distribution with mean μ_{task} and CV $\sqrt{\frac{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}{(1-a)n}}$. Therefore, the sample mean of μ_j converges to $a\mu_{task}q_{j/(bm)} + (1-a)\mu_{task}$ and its sample standard deviation converges to zero as $n \rightarrow \infty$ for $1 \leq j \leq bm$.

On one hand, $\mu\mu_{mach} = \frac{1}{m} \sum_{j=1}^m \mu_j = \frac{1}{m} (\sum_{j=1}^{bm} (a\mu_{task}q_{j/(bm)} + (1-a)\mu_{task}) + (1-b)m\mu_{task}) = ab\mu_{task} \frac{1}{bm} \sum_{j=1}^{bm} q_{j/(bm)} + (1-a)b\mu_{task} + (1-b)\mu_{task}$ as $n \rightarrow \infty$. Note that $\frac{1}{bm} \sum_{j=1}^{bm} q_{j/(bm)} = \int_0^1 q_p dp = 1$ as $m \rightarrow \infty$. Thus, $\mu\mu_{mach} = \mu_{task}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$. On the other hand, as $n \rightarrow \infty$ and $m \rightarrow \infty$:

$$\begin{aligned}
 \sigma\mu_{mach} &= \sqrt{\frac{1}{m} \sum_{j=1}^m \mu_j^2 - \left(\frac{1}{m} \sum_{j=1}^m \mu_j \right)^2} \\
 &= \sqrt{\frac{1}{m} \sum_{j=1}^{bm} \mu_j^2 + \frac{1}{m} \sum_{j=bm+1}^m \mu_j^2 - \mu_{task}^2} \\
 &= \sqrt{\frac{1}{m} \sum_{j=1}^{bm} (a\mu_{task}q_{j/(bm)} + (1-a)\mu_{task})^2 + (1-b)\mu_{task}^2 - \mu_{task}^2} \\
 &= \mu_{task} \sqrt{\frac{1}{m} \sum_{j=1}^{bm} (a^2 q_{j/(bm)}^2 + 2aq_{j/(bm)}(1-a) + (1-a)^2) - b} \\
 &= \mu_{task} \sqrt{a^2 b \frac{1}{bm} \sum_{j=1}^{bm} q_{j/(bm)}^2 + 2a(1-a)b \frac{1}{bm} \sum_{j=1}^{bm} q_{j/(bm)} + (1-a)^2 b - b} \\
 &= a\sqrt{b}\mu_{task} \sqrt{\frac{1}{bm} \sum_{j=1}^{bm} q_{j/(bm)}^2 - 1}
 \end{aligned}$$

Note that $\frac{1}{bm} \sum_{j=1}^{bm} q_{j/(bm)}^2 = \int_0^1 q_p^2 dp = \int_{-\infty}^{\infty} x^2 f(x) dx = V_{mach}^2 + 1$ as $m \rightarrow \infty$ with the substitution $p = F(x)$ and $dp = f(x) dx$ where f is the probability density function of a distribution with mean one and CV V_{mach} . This requires the distribution to be continuous,

Measure	Value
$V\mu_{task}$	V_{task}
μV_{task}	$\begin{cases} \Phi = \sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2} & \text{if } a = 0 \\ bV_{task} + (1 - b)\Phi & \text{if } a = 1 \end{cases}$
$V\mu_{mach}$	$a\sqrt{b}V_{mach}$
μV_{mach}	V_{mach}

Table 3: Summary of the heterogeneity properties of the CVB method.

which is the case for the gamma distribution. Therefore, $\sigma\mu_{mach} = a\sqrt{b}\mu_{task}V_{mach}$ and $V\mu_{mach} = a\sqrt{b}V_{mach}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$. \square

Proposition 7. *The measure μV_{task} of a cost matrix generated using the CVB method with the parameters V_{task} and V_{mach} converges to $\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$ as $n \rightarrow \infty$ if the matrix is inconsistent and to $bV_{task} + (1 - b)\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$ if $a = 1$.*

Proof. Let V_j be the sample CV of column j . When $a = 0$, the values on column j follow a distribution that is the product of a distribution with mean μ_{task} and CV V_{task} , and a distribution with mean one and CV V_{mach} . Therefore, V_j converges to $\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$ as $n \rightarrow \infty$. Since this value does not depend on j , μV_{task} (the sample mean of these sample CV) also converges to $\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$ as $n \rightarrow \infty$.

When $a = 1$, V_j still converges to $\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$ as $n \rightarrow \infty$ for $bm < j \leq m$. However, μ_j (the sample mean of column j) converges to $\mu_{task}q_{j/(bm)}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$ while σ_j (the sample standard deviation of column j) converges to $\mu_{task}V_{task}q_{j/(bm)}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$ for $1 \leq j \leq bm$. Thus, V_j converges to V_{task} as $n \rightarrow \infty$ and $m \rightarrow \infty$ for $1 \leq j \leq bm$. Therefore, μV_{task} converges to $bCV_{task} + (1 - b)\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$ as $n \rightarrow \infty$ and $m \rightarrow \infty$. \square

Proposition 8. *The measure μV_{mach} of a cost matrix generated using the CVB method with the parameters V_{task} and V_{mach} converges to V_{mach} as $m \rightarrow \infty$.*

Proof. Let V_i be the sample CV of row i . The values on row i follow a distribution that is the product of $q[i]$ and a distribution with mean one and CV V_{mach} . Therefore, V_i converges to V_{mach} as $m \rightarrow \infty$. Since this value does not depend on i , μV_{mach} (the sample mean of these sample CV) also converges to V_{mach} as $m \rightarrow \infty$. \square

Table 3 synthesises the previous formal results. They can be extended to the range-based method by replacing V_{task} by the CV of the first random variable ($\frac{\sqrt{12} R_{task}-1}{6 R_{task}+1}$) and V_{mach} by the CV of the second one ($\frac{\sqrt{12} R_{mach}-1}{6 R_{mach}+1}$). Indeed, the proofs only use the mean and the CV of the underlying distributions. Moreover, the uniform distribution is also continuous. Although the formal analysis of μV_{task} for arbitrary values of a was unsuccessful, the following formula provides a close estimate: $a^2bV_{task} + (1 - a^2b)\sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$.

In the case of complete consistency (i.e., when $a = b = 1$), $V\mu_{task} = \mu V_{task} = V_{task}$ and $V\mu_{mach} = \mu V_{mach} = V_{mach}$, which supports the proposed heterogeneity measures. This special

case is due to the fact that consistent cost matrices are closer to uniform instances than inconsistent ones and both measures are equivalent for uniform instances.

However, the CVB method has two issues. As a consequence of the asymmetry of the generation method, the task heterogeneity is not symmetric to the machine heterogeneity. For instance, we have $\mu V_{task} = \sqrt{V_{task}^2 V_{mach}^2 + V_{task}^2 + V_{mach}^2}$, whereas $V \mu_{mach} = V_{mach}$ for inconsistent matrices. This makes the generation method less direct as the parameters must be chosen such as to circumvent this asymmetry. In particular, if a high machine heterogeneity is required, then the task heterogeneity will also be high.

The second issue is related to the impact of the consistency parameters on the heterogeneity properties. It biases comparisons of scheduling methods when cost matrices are used with different consistency settings because these matrices will also have different heterogeneity properties. The range-based method presents an even stronger bias as both V_{task} and V_{mach} tend to $\frac{\sqrt{12}}{6}$ as $R_{task} \rightarrow \infty$ and $R_{mach} \rightarrow \infty$ (the heterogeneity properties are thus often similar).

4.5 Task and Machine Heterogeneity in Previous Studies

For each of the instances summarized in Section 3, we computed both heterogeneity measures using the formulas of Table 3 and the input parameters: R_{task} and R_{mach} for the range-based method; V_{task} and V_{mach} for the CVB method; and the consistency parameters, a and b , for both methods. For the case when $0 < a < 1$, μV_{task} was measured on a single 1000×1000 cost matrix that was generated with the range-based or the CVB method. When the consistency values are missing, matrices are assumed to be inconsistent. Finally, the mean is set to 1 when it is not given with the CVB method because it has no impact on any measure.

Figures 2 and 3 depict the values for the measures proposed above. The range-based method has a clear bias because many heterogeneity properties have never been obtained. Also, the consistency parameters invalidate the claimed properties of the cost matrices relatively to the heterogeneity quadrant for both heterogeneity measures: some *hihi* instances have the same machine heterogeneity as *lolo* instances on Figure 2, whereas some *lohi* instances have the same task heterogeneity as *hilo* instances on Figure 3.

This analysis is also consistent with the observation made in [3] about the fact that the range-based and CVB methods do not cover the entire range of possible values for the MPH.

As mentioned in Section 4, both proposed heterogeneity measures are relative. This allows a direct comparison between each heterogeneity value. Using the standard deviation instead would require normalizing them for this analysis.

4.6 Non-asymptotic properties

The previous analysis only considers asymptotic heterogeneity properties (i.e., when $n \rightarrow \infty$ and $m \rightarrow \infty$). We study how far are these properties from the asymptotic ones by generating several small cost matrices for each setting used in the literature and by applying all the measures on them. Figure 4 depicts the heterogeneous properties of cost matrices with different sizes. We see that the column corresponding to 256 tasks with 64 machines is close to the asymptotic properties presented in Figures 2 and 3. Although the properties of smaller cost matrices are more dispersed, their central tendency remain similar to the asymptotic ones and the previous biases also occur with small matrices.

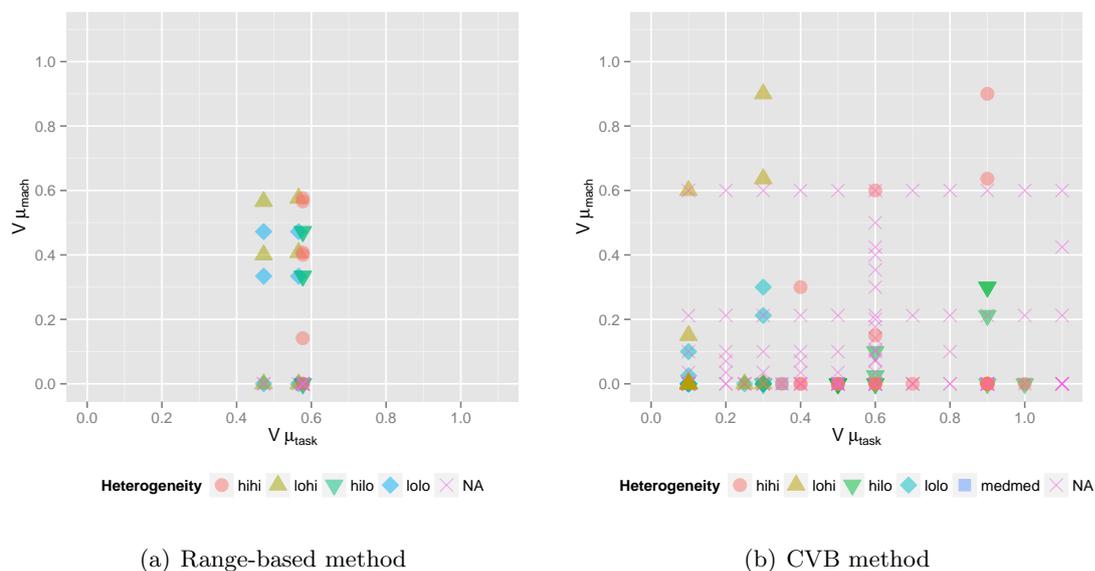


Figure 2: Heterogeneity properties ($V\mu_{task}$ and $V\mu_{mach}$) of cost matrices used in the literature. Two points are not shown for the CVB method: $(1.4, 0)$ and $(1.8, 0)$.

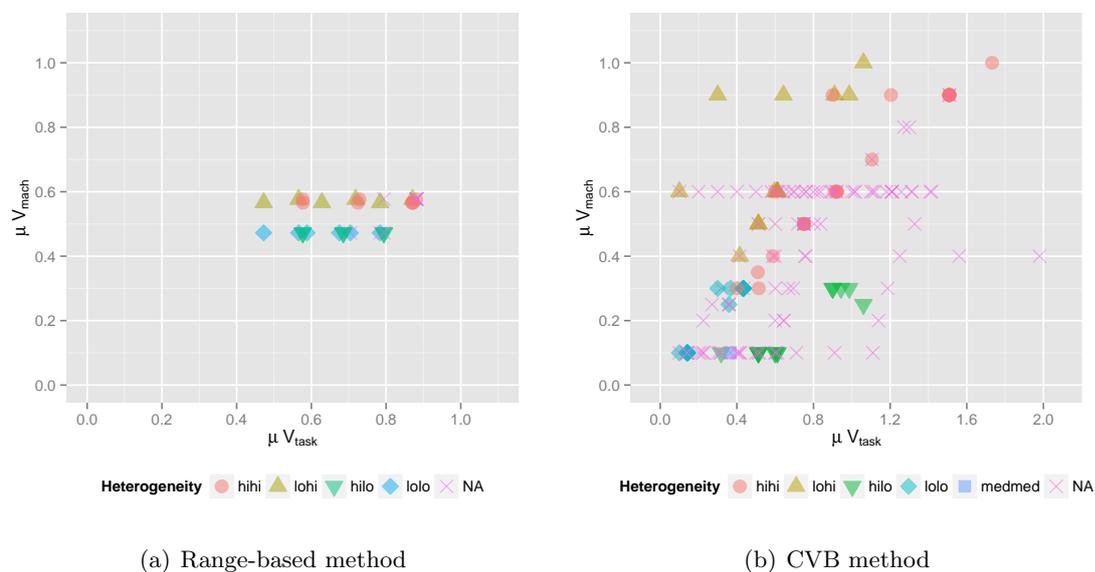


Figure 3: Heterogeneity properties (μV_{task} and μV_{mach}) of cost matrices used in the literature. The x -scale is twice as large as in Figure 2 for the CVB method because large values of V_{mach} tends to increase the measure μV_{task} . One point is not shown for the CVB method: $(2.01, 2)$.

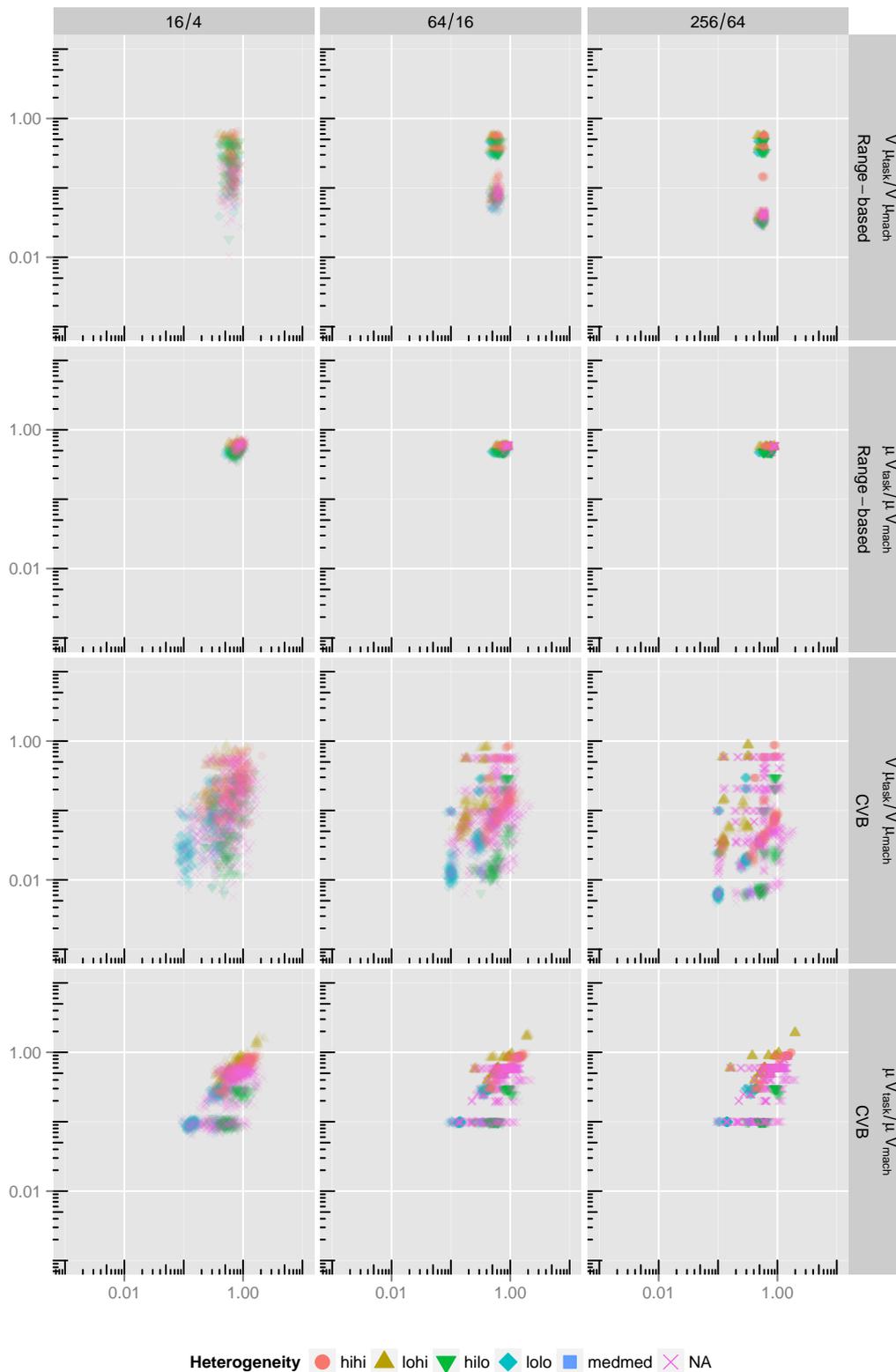


Figure 4: Heterogeneity properties (with respect to all measures) of cost matrices used in the literature. For each setting, 10 cost matrices are generated with different sizes: 16 tasks and 4 machines; 64 tasks and 16 machines; and, 256 tasks and 64 machines. The logarithmic scale is the same that is used in the figures of Section 6.

5 Controlling the Heterogeneity

We are interested in generating cost matrices that have specific heterogeneity properties according to the measures introduced in Section 4. We propose two methods that both alter a cost matrix generated from uniform instances for which we control the task and machine heterogeneities. These cost matrices have specific properties in terms of consistency and correlation between each row and each column, and the proposed methods introduce some randomness in it. They both possess the same time complexity (i.e., $O(nm)$).

5.1 Shuffling Method

The first proposed method shuffles the costs in the matrix that corresponds to a uniform instance (see Algorithm 4). It first generates the task weights on Line 2 and the inverse of the machine speeds on Line 5. The corresponding matrix is computed on Line 9 before starting the shuffling part. For each cost $e_{i,j}$, another cost $e_{i',j'}$ is selected on a different row and column (Lines 14 and 15). The same amount is then removed from these costs and is added to two other costs, $e_{i,j'}$ and $e_{i',j}$ (one that is on the same row as the first cost and on the same column as the second, and another one that is on the same row as the second cost and on the same column as the first). This step (Lines 25 to 28) preserves the mean of each row and the mean of each column. The heterogeneity properties thus remain the same.

The transferred amount is the largest value (in absolute) such that no cost among the four considered costs becomes lower than the minimum one among them (this prevents costs to be arbitrarily low). For instance, if $e_{i,j}$ is the minimum cost (i.e., $e_{i,j} = \min(e_{i,j}, e_{i',j}, e_{i,j'}, e_{i',j'})$), there are two cases: if $e_{i,j'} < e_{i',j}$, then $e_{i,j'}$ becomes the new minimum and the added value to $e_{i,j}$ and to $e_{i',j'}$ is $e_{i,j'} - e_{i,j}$; otherwise, it is $e_{i',j} - e_{i,j}$.

Maintaining both the minimum and the maximum cost is not possible because the cost matrix is generated from a uniform instance. This method focuses only on preventing costs to be arbitrarily low because it is critical to guarantee positive costs.

Proposition 9. *When used with parameters V_{task} and V_{mach} , the shuffling method generates costs with expected value 1.*

Proof. Costs in the matrix corresponding to the uniform matrix follow a distribution that is the product of two distributions with mean one. Therefore, the expected value of the costs in the matrix before the shuffling step is also one. The shuffling step does not change the expected value of the costs because the amount that is taken on any cost is given to another cost. \square

Proposition 10. *The measure $V\mu_{task}$ of a cost matrix generated using the shuffling method with the parameters V_{task} and V_{mach} converges to V_{task} as $n \rightarrow \infty$.*

Proof. Analogously to the proof of Proposition 9, the shuffling step has no impact on the mean of each row and each column. The measure $V\mu_{task}$ is thus the same for the final cost matrix as for the intermediate matrix that corresponds to a uniform instance.

As a corollary of Proposition 4, the sample CV of the sample means of all rows in this intermediate matrix is equal to the sample CV of the vector $\{w_i\}_{1 \leq i \leq n}$. This last sample CV converges to V_{task} as $n \rightarrow \infty$. \square

Proposition 11. *The measure $V\mu_{mach}$ of a cost matrix generated using the shuffling method with the parameters V_{task} and V_{mach} converges to V_{mach} as $m \rightarrow \infty$.*

Proof. Due to the symmetry of the shuffling method, the proof is analogous to the proof of Proposition 10. \square

Table 4 summarizes the formal results related to the shuffling method.

Algorithm 4 Shuffling cost matrix generation with gamma distribution**Input:** n, m, V_{task}, V_{mach} **Output:** a $n \times m$ cost matrix

```

1: for all  $1 \leq i \leq n$  do
2:    $w_i \leftarrow G(1/V_{task}^2, V_{task}^2)$ 
3: end for
4: for all  $1 \leq j \leq m$  do
5:    $b_j \leftarrow G(1/V_{mach}^2, V_{mach}^2)$ 
6: end for
7: for all  $1 \leq i \leq n$  do
8:   for all  $1 \leq j \leq m$  do
9:      $e_{i,j} \leftarrow w_i b_j$ 
10:   end for
11: end for
12: for all  $1 \leq i \leq n$  do
13:   for all  $1 \leq j \leq m$  do
14:      $i' \leftarrow (U(1, n-1) + i - 1 \bmod n) + 1$ 
15:      $j' \leftarrow (U(1, m-1) + j - 1 \bmod m) + 1$ 
16:     if  $e_{i,j} = \min(e_{i,j}, e_{i',j}, e_{i,j'}, e_{i',j'})$  then
17:        $d \leftarrow \min(e_{i',j} - e_{i,j}, e_{i,j'} - e_{i,j})$ 
18:     else if  $e_{i',j} = \min(e_{i',j}, e_{i,j'}, e_{i',j'})$  then
19:        $d \leftarrow -\min(e_{i,j} - e_{i',j}, e_{i,j'} - e_{i',j})$ 
20:     else if  $e_{i,j'} = \min(e_{i,j'}, e_{i',j'})$  then
21:        $d \leftarrow -\min(e_{i,j} - e_{i,j'}, e_{i',j'} - e_{i,j'})$ 
22:     else
23:        $d \leftarrow \min(e_{i',j} - e_{i',j'}, e_{i,j'} - e_{i',j'})$ 
24:     end if
25:      $e_{i,j} \leftarrow e_{i,j} + d$ 
26:      $e_{i',j} \leftarrow e_{i',j} - d$ 
27:      $e_{i,j'} \leftarrow e_{i,j'} - d$ 
28:      $e_{i',j'} \leftarrow e_{i',j'} + d$ 
29:   end for
30: end for
31: return  $\{e_{i,j}\}_{1 \leq i \leq n, 1 \leq j \leq m}$ 

```

Property	Value
Expected value	1
$V\mu_{task}$	V_{task}
$V\mu_{mach}$	V_{mach}

Table 4: Summary of the cost matrix properties with the shuffling method.

5.2 Noise-Based Method

The second method, described in Algorithm 5, relies on a simple idea, which was also used in [6]: each cost of a matrix, which corresponds to a uniform instance, is multiplied by a matrix with a random variable with mean one (Line 9).

Algorithm 5 Noise-based cost matrix generation with gamma distribution**Input:** $n, m, V_{task}, V_{mach}, V_{noise}$ **Output:** a $n \times m$ cost matrix

```

1: for all  $1 \leq i \leq n$  do
2:    $w_i \leftarrow G(1/V_{task}^2, V_{task}^2)$ 
3: end for
4: for all  $1 \leq j \leq m$  do
5:    $b_j \leftarrow G(1/V_{mach}^2, V_{mach}^2)$ 
6: end for
7: for all  $1 \leq i \leq n$  do
8:   for all  $1 \leq j \leq m$  do
9:      $e_{i,j} \leftarrow w_i b_j \times G(1/V_{noise}^2, V_{noise}^2)$ 
10:  end for
11: end for
12: return  $\{e_{i,j}\}_{1 \leq i \leq n, 1 \leq j \leq m}$ 

```

Proposition 12. *When used with parameters V_{task}, V_{mach} and V_{noise} , the noise-based method generates costs with expected value one and CV $\sqrt{V_{task}^2 V_{mach}^2 V_{noise}^2 + V_{task}^2 V_{mach}^2 + V_{task}^2 V_{noise}^2 + V_{mach}^2 V_{noise}^2 + V_{task}^2 + V_{mach}^2 + V_{noise}^2}$.*

Proof. Each cost is the product of three random variables that have all the same mean one. Additionally, their CV (and standard deviations in this case) are V_{task}, V_{mach} and V_{noise} . The global CV can be derived by remarking that the CV of the product of two random variables with CV V_1 and V_2 is $\sqrt{V_1^2 V_2^2 + V_1^2 + V_2^2}$. \square

Proposition 13. *The measure $V\mu_{task}$ of a cost matrix generated using the noise-based method with the parameters V_{task}, V_{mach} and V_{noise} converges to V_{task} as $n \rightarrow \infty$ and $m \rightarrow \infty$.*

Proof. Let μ_i be the sample mean of row i . This row is the product of w_i , which follows a distribution with mean one and CV V_{task} , and m values that are each the product of a random variable with mean one and CV V_{mach} and a random variable with mean one and CV V_{noise} . μ_i is thus also the product of w_i and the sample mean of the other m values, which follows a random variable with mean one and CV $\sqrt{\frac{V_{mach}^2 V_{noise}^2 + V_{mach}^2 + V_{noise}^2}{m}}$. Therefore, the mean of μ_i is one and its CV is $\sqrt{V_{task}^2 \frac{V_{mach}^2 V_{noise}^2 + V_{mach}^2 + V_{noise}^2}{m} + V_{task}^2 + \frac{V_{mach}^2 V_{noise}^2 + V_{mach}^2 + V_{noise}^2}{m}}$, which tends to V_{task} as $m \rightarrow \infty$. Therefore, the sample CV of all μ_i converges to V_{task} as $n \rightarrow \infty$ and $m \rightarrow \infty$. \square

Proposition 14. *The measure $V\mu_{mach}$ of a cost matrix generated using the noise-based method with the parameters V_{task}, V_{mach} and V_{noise} converges to V_{mach} as $n \rightarrow \infty$ and $m \rightarrow \infty$.*

Proof. Due to the symmetry of the noise-based method, the proof is analogous to the proof of Proposition 13. \square

Proposition 15. *The measure μV_{task} of a cost matrix generated using the noise-based method with the parameters V_{task}, V_{mach} and V_{noise} converges to $\sqrt{V_{task}^2 V_{noise}^2 + V_{task}^2 + V_{noise}^2}$ as $n \rightarrow \infty$.*

Proof. Let V_j be the sample CV of column j . Each column is the product of b_j and n values that are each the product of a random variable with mean one and CV V_{task} and a random variable with mean one and CV V_{noise} . Thus, V_j converges to the CV of this product

Property	Value
Expected value	1
CV	$\sqrt{\frac{V_{task}^2 V_{mach}^2 V_{noise}^2 + V_{task}^2 V_{mach}^2 + V_{task}^2 V_{noise}^2 + V_{mach}^2 V_{noise}^2 + V_{task}^2 + V_{mach}^2 + V_{noise}^2}{V_{mach}^2 V_{noise}^2 + V_{task}^2 + V_{mach}^2 + V_{noise}^2}}$
Distribution	Product of three gamma laws
$V\mu_{task}$	V_{task}
μV_{task}	$\sqrt{V_{task}^2 V_{noise}^2 + V_{task}^2 + V_{noise}^2}$
$V\mu_{mach}$	V_{mach}
μV_{mach}	$\sqrt{V_{mach}^2 V_{noise}^2 + V_{mach}^2 + V_{noise}^2}$

Table 5: Summary of the cost matrix properties with the noise-based method.

(i.e., $\sqrt{V_{task}^2 V_{noise}^2 + V_{task}^2 + V_{noise}^2}$) as $n \rightarrow \infty$. Therefore, the measure μV_{task} converges to $\sqrt{V_{task}^2 V_{noise}^2 + V_{task}^2 + V_{noise}^2}$ as $n \rightarrow \infty$. \square

Proposition 16. *The measure μV_{mach} of a cost matrix generated using the noise-based method with the parameters V_{task} , V_{mach} and V_{noise} converges to $\sqrt{V_{mach}^2 V_{noise}^2 + V_{mach}^2 + V_{noise}^2}$ as $m \rightarrow \infty$.*

Proof. Due to the symmetry of the noise-based method, the proof is analogous to the proof of Proposition 15. \square

Table 5 summarizes the formal results related to the noise-based method.

This method requires one additional parameter: V_{noise} . When the objective is to have cost matrices with specific values of $V\mu_{task}$ and $V\mu_{mach}$ (for large n and m), we propose to set V_{noise} to $\min(V_{task}, V_{mach})$. This limits the amount of noise in the costs.

Contrary to the shuffling method, the noise-based method can also generate cost matrices with specific values of μV_{task} and μV_{mach} (asymptotically). The parameters can be fixed as follow: if $\mu V_{task} < \mu V_{mach}$, then $V_{task} = 0$, $V_{noise} = \mu V_{task}$ and $V_{mach} = \sqrt{(\mu V_{mach}^2 - \mu V_{task}^2)/(\mu V_{task}^2 + 1)}$; otherwise, $V_{mach} = 0$, $V_{noise} = \mu V_{mach}$ and $V_{task} = \sqrt{(\mu V_{task}^2 - \mu V_{mach}^2)/(\mu V_{mach}^2 + 1)}$. This setting maximizes the amount of noise.

Even though the shuffling method has less formal results (probably due to its combinatoric operations), the noise-based method has two drawbacks: the additional parameter is not trivial to determine and the method introduces more variation in the costs than the shuffling method. This makes this method more complex to use.

6 Impact on Scheduling Heuristics

This section assesses the impact of the heterogeneity properties defined in Section 4 on the relative performance of some classic heuristics.

6.1 Scheduling Heuristics

Our intention here is not to find the best heuristic but rather to show the impact of the cost matrix generation method on the performance results. We use classical heuristics from the

Name	Ref	Complexity	Remark	Algo
OLB	[26]	nm	Opportunistic Load Balancing	6
MET	[26]	nm	Minimum Execution Time	7
MCT	[26]	nm	Minimum Completion Time	8
Min-min	[26]	n^2m	Earliest finish time of smallest task	9
Max-min	[26]	n^2m	Earliest finish time of largest task	10
Suff	[38]	n^2m	Task that will suffer most first	11
GA	[26]	–	Genetic Algorithm	12
HLPT	[62]	$nm + n \log(n)$	Heterogeneous version of LPT	13
GreedySuff		$nm \log(m)$	Greedy allocation based on sufferage	14
BalSuff	–		Reconsider MET mapping	15
BalEFT	–		Reconsider MET mapping	16

Table 6: Summary of the scheduling heuristics for the $R||C_{\max}$ problem.

literature summarized in Table 6. Most of them (OLB, MET, MCT, Min-min, Max-min, HLPT³, Suff) are list-based algorithms. The Genetic Algorithm (GA) relies on an initial population containing a solution obtained with Min-min. In addition to these classic heuristics, we added two more elaborated methods (the Bal prefixed methods) that try to reconsider an initial mapping obtained from MET (Minimum Execution Time) mapping: any task is moved to the machine that will finish it the earliest if it does not increase the maximum completion time. These heuristics are described in Appendix C.

Getting reference values (lower bounds on the makespan) for our performance measures is not straightforward in practice due to the heterogeneity of the problem. We thus rely on a variation of the genetic algorithm to provide an estimation of these values. The initial population is initialized, in addition to other random individuals, with all the solutions obtained by the other algorithms. The population evolution is based on the algorithm description given in [26]. An elite chromosome is maintained so that the resulting solution cannot be worse than any of the initial solutions and thus the genetic algorithm is no worse than any of the other algorithms.

6.2 Settings

Cost matrices are generated with three different methods: the shuffling method and the noise-based method with two approaches to set the noise (see Section 5.2). In all cases, there are two parameters: $V\mu_{task}$ and $V\mu_{proc}$ for the first two methods and μV_{task} and μV_{proc} for the last one. These two parameters are distributed in the range $[0.001, 10]$ with 30 equidistant values using a probit scale (i.e., 0.001, 0.0014, 0.0019, 0.0026, ..., 5.3, 7.3, 10).

For each pair of parameters, 200 cost matrices are generated with $n = 100$ tasks and $m = 30$ machines. For each scenario, we compute the makespan of each heuristic. We only consider the relative difference from the reference makespan: $|C - C_{\min}|/C_{\min}$ where C is the makespan of a given heuristic and C_{\min} is the best makespan we obtained (the genetic algorithm initialized with all the solutions obtained by the other heuristics). The closer to zero, the better the performance.

6.3 Results

Figures 5 to 7 are heat maps of the relative performance for each algorithm. On each figure, we use a logarithmic scale on both axes: the x -axis gives the heterogeneity value for the tasks

³The variant HLPT (mean) is equivalent to HEFT [154].

($V\mu_{task}$ or μV_{task}) while the y -axis gives the heterogeneity value for the machines ($V\mu_{mach}$ or μV_{mach}). The bottom-left area represents almost homogeneous instances (same cost for each execution) while the top-right area is the most heterogeneous one. The heterogeneity values covered by the range-based and CVB methods in the literature are represented with dark rectangles on each sub-figure.

The scales on each heat map start at 0.001. We assume that an heterogeneity that is below this value may be considered negligible and that a heuristic that is closer to the reference makespan than this value is good enough. For instance, BalSuff may be considered near-optimal when the heterogeneity values are below 1%.

In Figure 7, both V_{task} and V_{mach} are zero on the diagonal, which may cause the irregularity for almost all heuristics. We suspect that using larger matrices would smooth this effect.

Figure 8 plots the best heuristic depending on the heterogeneity properties. Contour lines show the number of heuristics which performance is closer to the best heuristics than 0.001. For instance, there are at least 5 heuristics whose relative performances are almost equivalent when task heterogeneity is high (i.e., if the best heuristic average relative difference from the reference value is 0.004, then at least 5 other heuristics have a relative difference lower than 0.005).

The heuristics are ordered by the number of instances for which no other heuristic produces a better solution. When several heuristics are equivalent for a given tile, the appearing heuristic is the one that is the best the least often. This allows one to see even the settings for which the worst heuristics may be good.

Note that GA is close to Min-min (we observed that it returns the same solution in more than half of the cases) but improves it when the task heterogeneity is high. This proximity is explained by the inclusion of the solution provided by Min-min in the initial population of GA.

6.4 Analysis

The settings cover a large part of the possible instances for the $R||C_{\max}$ problem. Some areas on the figures may be associated to specific scheduling problems: the $Q|p_i = p|C_{\max}$ problem (top-left area), the $P|p_i = p|C_{\max}$ problem (bottom-left area) and the $P||C_{\max}$ problem (bottom-right area). While the first two problems can be solved in polynomial time, the last problem is NP-complete.

The heat maps suggest that the area where the heterogeneity values are between 0.1 and 1 is more challenging for most heuristics (areas in purple on the heat maps are 30% far from the reference). This is confirmed by Figure 8 where there is often a single best heuristic with these settings. Oppositely, many heuristics are close to the best one when the task heterogeneity is low or high, or when the machine heterogeneity is high. On one hand, execution costs are quite similar when the coefficient of variation is below 0.1. A non-optimal allocation will thus have a lower impact than with higher heterogeneity. On the other hand, most execution costs are close to zero when the coefficient of variation is higher than 1 and bad allocations may be easy to avoid because there are few allocations that are extremely critical while most of them are not. It is thus easier to generate a reasonable schedule.

When the machine heterogeneity is low (with medium task heterogeneity), there is often a single best heuristic. This suggests that these settings leads to difficult instances. As mentioned above, this is close to the $P||C_{\max}$ problem. We may conclude that dealing with heterogeneous tasks is more difficult than with heterogeneous machines, which is also supported by the asymmetry of the heat maps.

Finally, Figure 8 shows the best heuristics: BalSuff when both heterogeneity properties are comparable, BalEFT when the machine heterogeneity is higher than the task heterogeneity and HLPT when the task heterogeneity is high.

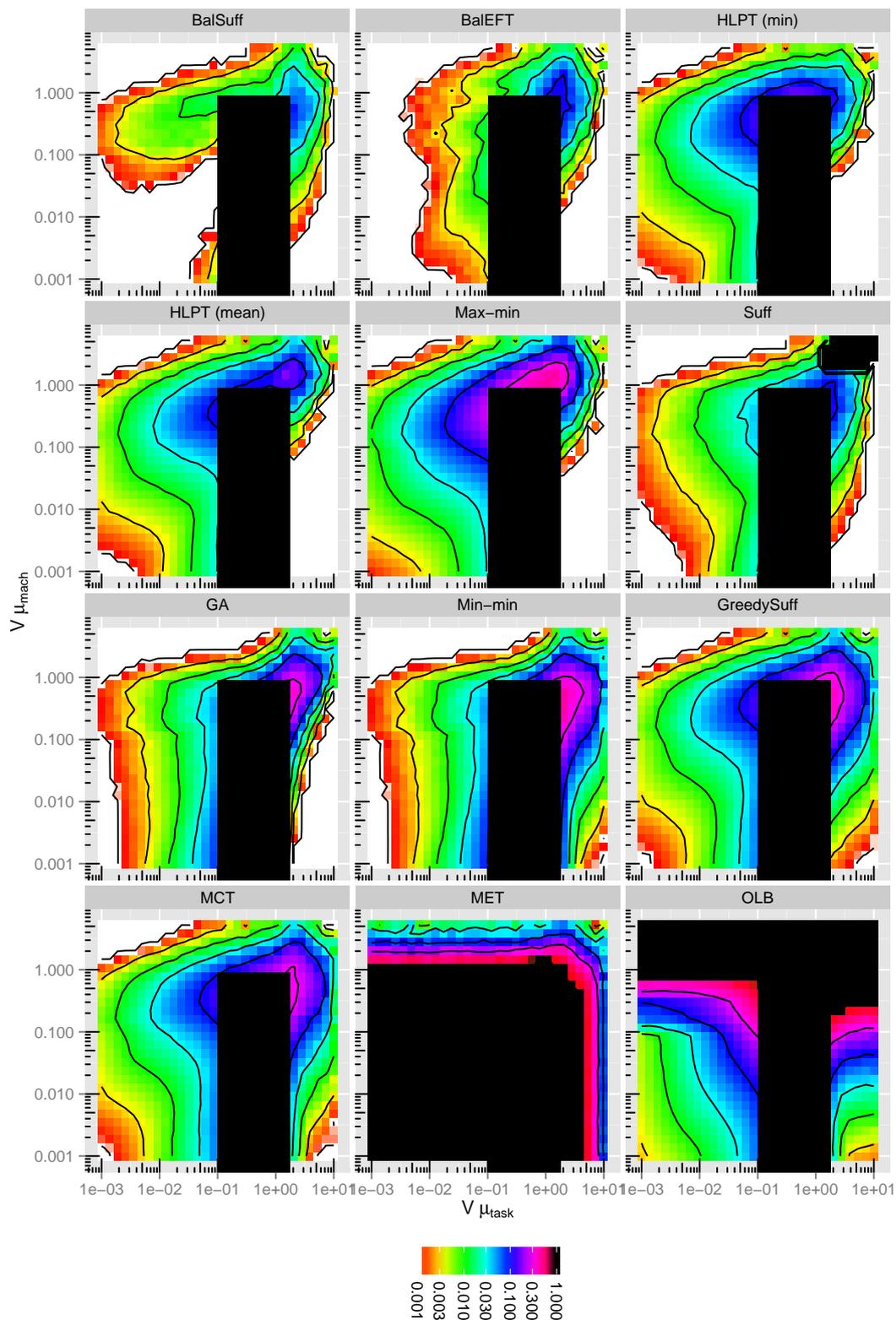


Figure 5: Heuristic performance relative to the best case with the shuffling method. Values below 0.001 are shown in white and values above 1 are shown in black. Contour lines correspond to the levels in the legend (0.001, 0.003, ...). The dark rectangles correspond to the properties covered by the range-based and CVB methods in the literature (see Figure 2).

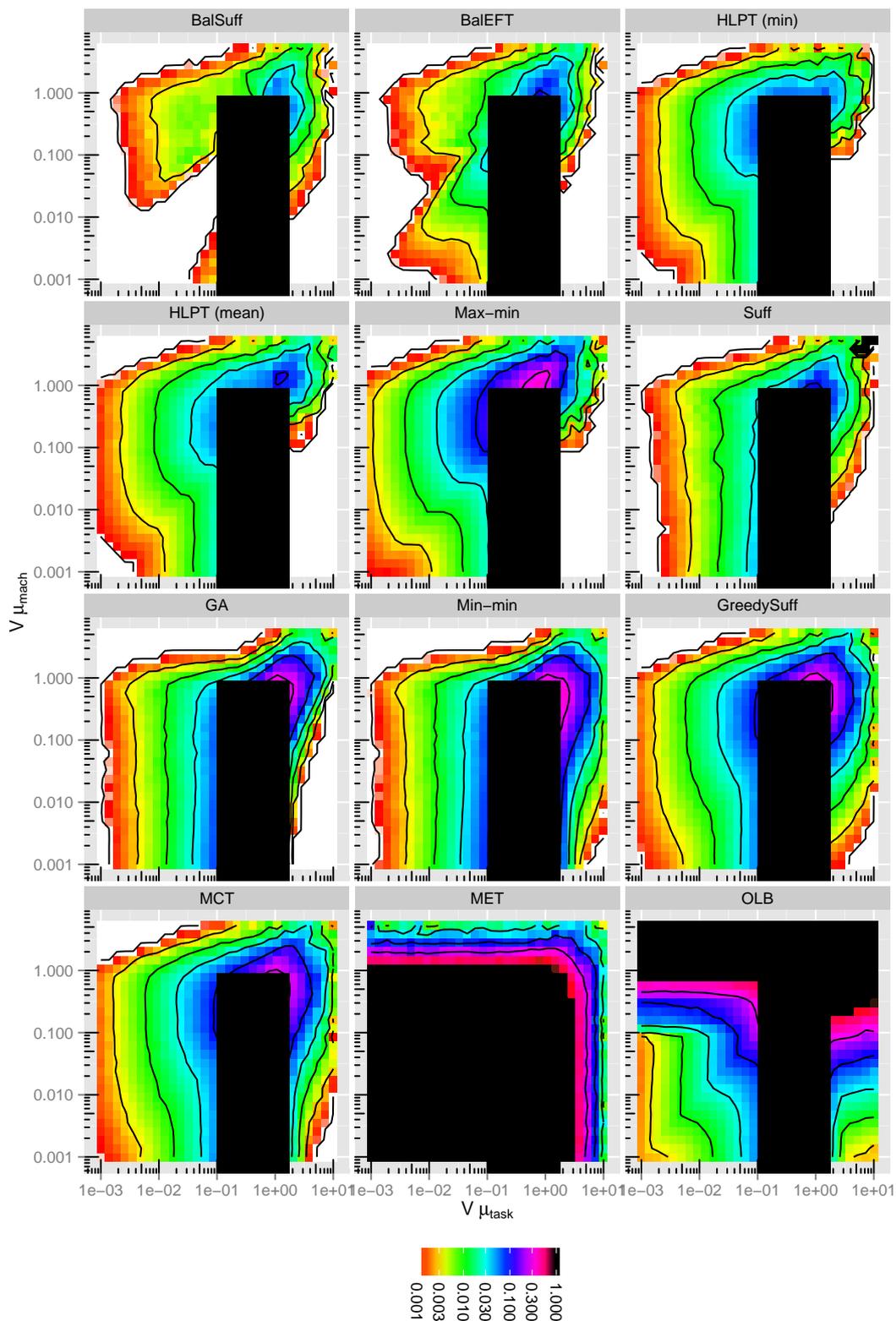


Figure 6: Heuristic performance relatively to the best case with the noise-based method with $V \mu_{task}$ and $V \mu_{mach}$ as parameters. Values below 0.001 are shown in white and values above 1 are shown in black. Contour lines correspond to the levels in the legend (0.001, 0.003, ...). The dark rectangles correspond to the properties covered by the range-based and CVB methods in the literature (see Figure 2).

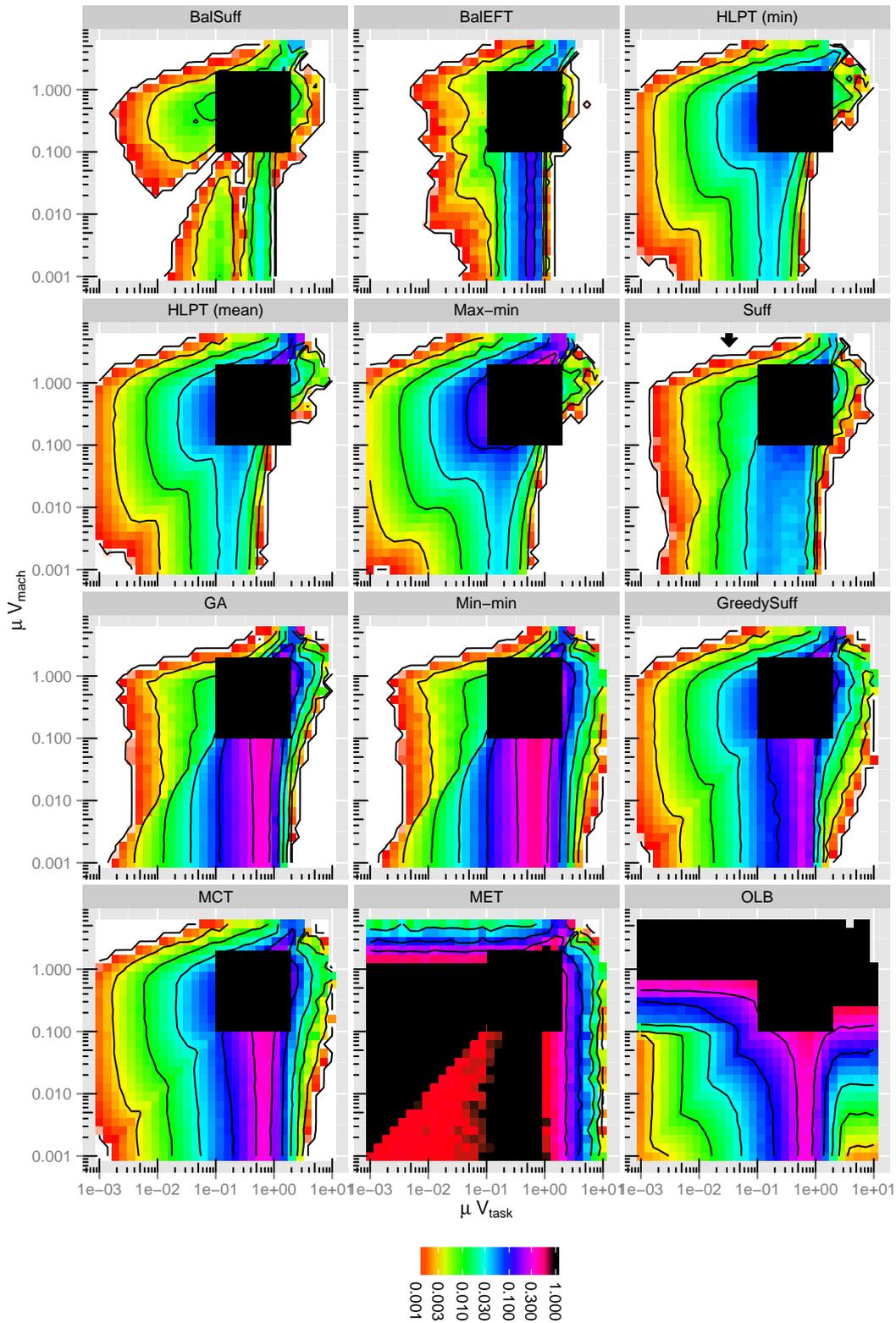


Figure 7: Heuristic performance relative to the best case with the noise-based method with μV_{task} and μV_{mach} as parameters. Values below 0.001 are shown in white and values above 1 are shown in black. Contour lines correspond to the levels in the legend (0.001 , 0.003 , \dots). The dark rectangles correspond to the properties covered by the range-based and CVB methods in the literature (see Figure 3).

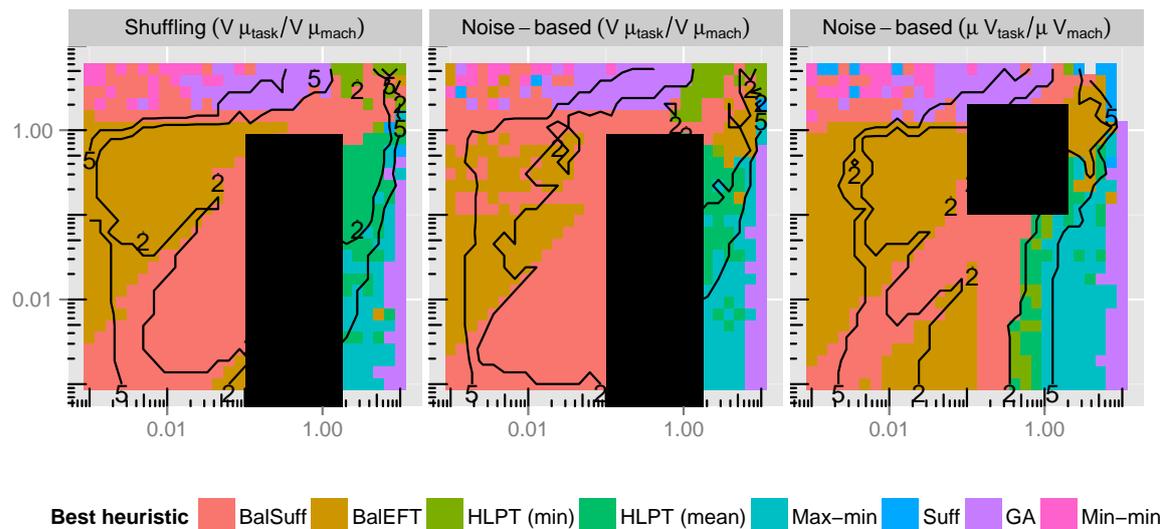


Figure 8: Best heuristic in the average case with the shuffling and the noise-based method with $V\mu_{task}$ and $V\mu_{mach}$ or μV_{task} and μV_{mach} as parameters. Contour lines correspond to the number of heuristics with a performance closer to the best heuristic performance than 0.001. The dark rectangles correspond to the properties covered by the range-based and CVB methods in the literature (see Figure 3).

Overall, we used two generation methods and two heterogeneity measures (one with the shuffling method and two with the noise-based method) and this analysis stands in all cases.

The range-based and CVB generation methods used in the literature could not provide these results due to two factors: the heterogeneity properties of the generated instances have a limited coverage (shown by the dark rectangles) and the erroneous claimed properties of these matrices prevent an unbiased analysis.

6.5 Discussion

This study focuses on the impact of some measures (either $V\mu_{task}$ and $V\mu_{mach}$, or μV_{task} and μV_{mach}) on the performance of several heuristics. However, there are many other properties that could be measured. If we consider the skewness and the kurtosis as in [5], we can think of 4×4 measures for the rows and as many for the columns. The main limitation of this study is to ignore the effect of all these possible measures. In addition, this study cannot be directly extended to assess all the possible interactions between them.

Another limitation is related to the effect of outliers. For large instances, the law of large number applies and the measures proposed in Section 4 correspond to the characteristics of the cost matrices. However, for small instances, we suggest switching to robust measures such as the median, the interquartile range and the quartile coefficient of dispersion instead of the mean, the standard deviation and the CV, respectively.

7 Conclusion

This study shows that the methods used in the literature for generating cost matrices are biased: the claimed heterogeneity properties of these instances are invalidated by the two measures we proposed to quantify them. We also show that the range of instances that has been used are restricted. It is specifically the case for the range-based method that covers only a minor fraction of all the possible settings in terms of heterogeneity. By providing new cost matrix generation methods, we show that heuristics for the $R||C_{\max}$ problem have interesting behavior outside this restriction. For instance, BalEFT is the best heuristic when the task heterogeneity is low and this could not have been shown with the instances used in the literature.

In addition to all the possible measures mentioned in Section 6.5, we plan to analyse other properties, in particular the correlation. It would also be interesting to see if the conclusions hold for some variations of the $R||C_{\max}$ problem such as considering arrival times or online scheduling.

Acknowledgments & Contributions

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Computations have been performed on the supercomputer facilities of the Mésocentre de calcul de Franche-Comté.

Analyzed previous generation methods: LCC. Designed generation methods: LCC. Implemented heuristics: LP. Performed the experiments: LP. Analyzed the data: LCC LP. Wrote the paper: LCC LP.

A Notation

Symbol	Definition
i	index of the tasks
j	index of the machines
n	number of tasks
m	number of machines
$e_{i,j}$	execution cost of task i on machine j
w_i	weight of task i
b_j	inverse speed of machine j
$U(A, B)$	uniform distribution between A and B
$G(\alpha, \beta)$	gamma distribution with shape α and scale β
R_{task}	parameter for the range-based method
R_{mach}	parameter for the range-based method
V_{task}	parameter for the CVB, shuffling and noise-based methods
V_{mach}	parameter for the CVB, shuffling and noise-based methods
V_{noise}	parameter for the noise-based method
a	fraction of the consistent rows
b	fraction of the consistent columns
$V\mu_{task}$	first measure of task heterogeneity
$V\mu_{mach}$	first measure of machine heterogeneity
μV_{task}	second measure of task heterogeneity
μV_{mach}	second measure of machine heterogeneity

Table 7: List of notations

B Usage of the Range-Based and CVB Methods in the Literature

B.1 Range-Based Method

The following table summarizes the studies that used the range-based method for generating cost matrices. Each row correspond to a cost matrix instance generated with the range-based method. I (resp. T) denotes matrices that are used for Illustration (resp. Testing algorithm performance). The range and consistency columns are the input parameters. The last columns represent the expected heterogeneity properties with two possible levels: low and high.

Ref.	Context		Range		Consistency		Heterogeneity	
	Use		Task	Mach.	Task	Mach.	Task	Mach.
[8, 17, 18]	I		[1;10]	[1;10]	0	0	low	low
[170]	T		[1;10]	[1;10]	0	0	low	low
[36, 171, 172]	T		[1;10]	[1;10]	NA	NA	NA	NA
[170]	T		[1;10]	[1;10]	1	.5	low	low
[170]	T		[1;10]	[1;10]	1	1	low	low
[8, 17, 18]	I		[1;10]	[1;100]	0	0	low	high

Table 8 – *Continued from previous page*

Context		Range		Consistency		Heterogeneity	
Ref.	Use	Task	Mach.	Task	Mach.	Task	Mach.
[170]	T	[1;10]	[1;100]	0	0	low	high
[170]	T	[1;10]	[1;100]	1	.5	low	high
[170]	T	[1;10]	[1;100]	1	1	low	high
[156, 157]	T	[1;10]	$[1;1e5]$	NA	NA	NA	NA
[25, 153]	I	[1;100]	[1;10]	0	0	low	low
[25, 26]	T	[1;100]	[1;10]	0	0	low	low
[23]	T	[1;100]	[1;10]	0	0	NA	NA
[25, 26]	T	[1;100]	[1;10]	1	.5	low	low
[25, 26]	T	[1;100]	[1;10]	1	1	low	low
[25]	I	[1;100]	[1;1e3]	0	0	low	high
[25, 26, 153]	T	[1;100]	[1;1e3]	0	0	low	high
[25, 26, 153]	T	[1;100]	[1;1e3]	1	.5	low	high
[25, 26, 153]	T	[1;100]	[1;1e3]	1	1	low	high
[156, 157]	T	[1;1e3]	$[1;1e5]$	NA	NA	NA	NA
[25]	I	[1;3e3]	[1;10]	0	0	high	low
[8, 25, 26, 147]	T	[1;3e3]	[1;10]	0	0	high	low
[25, 26]	T	[1;3e3]	[1;10]	1	.5	high	low
[25, 26]	T	[1;3e3]	[1;10]	1	1	high	low
[8, 110, 111, 147]	T	[1;3e3]	[1;100]	0	0	high	high
[8]	T	[1;3e3]	[1;100]	.5	.25	high	high
[110, 111]	T	[1;3e3]	[1;100]	.5	.25	high	high
[25, 26, 153]	I	[1;3e3]	[1;1e3]	0	0	high	high
[25, 26]	T	[1;3e3]	[1;1e3]	0	0	high	high
[26]	I	[1;3e3]	[1;1e3]	1	.5	high	high
[25, 26]	T	[1;3e3]	[1;1e3]	1	.5	high	high
[26]	I	[1;3e3]	[1;1e3]	1	1	high	high
[25, 26]	T	[1;3e3]	[1;1e3]	1	1	high	high
[8, 17, 18]	I	[1;1e5]	[1;10]	0	0	high	low
[170]	T	[1;1e5]	[1;10]	0	0	high	low
[170]	T	[1;1e5]	[1;10]	1	.5	high	low
[170]	T	[1;1e5]	[1;10]	1	1	high	low
[156, 157]	T	$[1;1e5]$	[1;10]	NA	NA	NA	NA
[8, 17, 18]	I	[1;1e5]	[1;100]	0	0	high	high
[170]	T	[1;1e5]	[1;100]	0	0	high	high
[170]	T	[1;1e5]	[1;100]	1	.5	high	high
[170]	T	[1;1e5]	[1;100]	1	1	high	high
[156, 157]	T	$[1;1e5]$	[1;1e3]	NA	NA	NA	NA
[156, 157]	T	[1;1e5]	[1;1e5]	NA	NA	NA	NA
[156, 157]	T	$[1;1e5]$	[1;1e7]	NA	NA	NA	NA
[156, 157]	T	$[1;1e5]$	[1;1e9]	NA	NA	NA	NA
[156, 157]	T	[1;1e7]	$[1;1e5]$	NA	NA	NA	NA
[156, 157]	T	[1;1e9]	$[1;1e5]$	NA	NA	NA	NA

B.2 CVB Method

This second table is for the CVB method with the gamma distribution. There is one more parameter, the mean, and the expected heterogeneity has a new level: *med*, for medium.

Context		CV		Global	Consistency		Heterogeneity	
Ref.	Use	Task	Mach.	Mean	Task	Mach.	Task	Mach.
[8, 17, 18]	I	.1	.1	1e3	0	0	low	low
[41, 42, 116]	T	.1	.1	100	0	0	low	low
[41, 42]	T	.1	.1	100	.5	.25	low	low
[41, 42]	T	.1	.1	100	1	1	low	low
[107]	T	.1	.1	10	NA	NA	low	low
[58]	T	.1	.1	50	NA	NA	low	low
[68, 164]	T	.1	.1	100	NA	NA	low	low
[57]	T	.1	.1	1e3	NA	NA	low	low
[3]	T	.1	.1	NA	NA	NA	low	low
[109, 115]	T	.1	.1	1e3	0	0	NA	NA
[109]	T	.1	.1	1e3	.5	.5	NA	NA
[109]	T	.1	.1	1e3	1	1	NA	NA
[78, 80]	T	.1	.1	2e12	NA	NA	NA	NA
[115]	T	.1	.2	1e3	0	0	NA	NA
[52]	T	.1	.25	10	NA	NA	NA	NA
[115]	T	.1	.3	1e3	0	0	NA	NA
[31]	T	.1	.4	7.5	NA	NA	low	high
[115]	T	.1	.4	1e3	0	0	NA	NA
[58]	T	.1	.5	30	NA	NA	low	high
[57]	T	.1	.5	1e3	NA	NA	low	high
[115]	T	.1	.5	1e3	0	0	NA	NA
[78, 80]	T	.1	.5	2e12	NA	NA	NA	NA
[41, 42, 116]	T	.1	.6	100	0	0	low	high
[8, 17, 18]	I	.1	.6	1e3	0	0	low	high
[41, 42]	T	.1	.6	100	.5	.25	low	high
[41, 42]	T	.1	.6	100	1	1	low	high
[68, 164]	T	.1	.6	100	NA	NA	low	high
[109, 115]	T	.1	.6	1e3	0	0	NA	NA
[109]	T	.1	.6	1e3	.5	.5	NA	NA
[109]	T	.1	.6	1e3	1	1	NA	NA
[3]	T	.1	.9	NA	NA	NA	low	high
[8, 17, 18]	I	.1	2	1e3	0	0	low	high
[43, 109]	T	.2	.1	1e3	0	0	NA	NA
[109]	T	.2	.1	1e3	.5	.5	NA	NA
[43]	T	.2	.1	1e3	1	.5	NA	NA
[109]	T	.2	.1	1e3	1	1	NA	NA
[109]	T	.2	.6	1e3	0	0	NA	NA
[109]	T	.2	.6	1e3	.5	.5	NA	NA
[109]	T	.2	.6	1e3	1	1	NA	NA
[49, 50]	T	.25	.25	100	NA	NA	low	low
[167]	T	.25	.25	750	0	0	NA	NA
[168]	T	.25	.25	NA	0	0	NA	NA

Table 9 – Continued from previous page

Context		CV		Global	Consistency		Heterogeneity	
Ref.	Use	Task	Mach.	Mean	Task	Mach.	Task	Mach.
[166]	T	.25	.25	750	NA	NA	NA	NA
[49]	T	.25	1	100	NA	NA	low	high
[8, 17, 18]	I	.3	.1	1e3	0	0	high	low
[109, 115]	T	.3	.1	1e3	0	0	NA	NA
[109]	T	.3	.1	1e3	.5	.5	NA	NA
[109]	T	.3	.1	1e3	1	1	NA	NA
[150, 151]	T	.3	.3	30	0	0	low	low
[113, 114]	T	.3	.3	100	0	0	low	low
[112, 114]	T	.3	.3	120	0	0	low	low
[71]	T	.3	.3	1e3	0	0	low	low
[71]	T	.3	.3	1e3	1	.5	low	low
[71]	T	.3	.3	1e3	1	1	low	low
[84, 85]	T	.3	.3	3	NA	NA	low	low
[12]	T	.3	.3	NA	NA	NA	NA	NA
[109, 115]	T	.3	.6	1e3	0	0	NA	NA
[109]	T	.3	.6	1e3	.5	.5	NA	NA
[109]	T	.3	.6	1e3	1	1	NA	NA
[71]	T	.3	.9	1e3	0	0	low	high
[71]	T	.3	.9	1e3	1	.5	low	high
[71]	T	.3	.9	1e3	1	1	low	high
[108]	T	.35	.1	10	NA	NA	med	med
[107]	T	.35	.1	10	NA	NA	NA	NA
[107, 108]	T	.35	.35	10	NA	NA	high	high
[43, 109]	T	.4	.1	1e3	0	0	NA	NA
[109]	T	.4	.1	1e3	.5	.5	NA	NA
[43]	T	.4	.1	1e3	1	.5	NA	NA
[109]	T	.4	.1	1e3	1	1	NA	NA
[32]	T	.4	.3	1	1	1	high	high
[33]	T	.4	.3	1	NA	NA	high	high
[10]	T	.4	.4	12	NA	NA	high	high
[159]	T	.4	.4	20	0	0	NA	NA
[109]	T	.4	.6	1e3	0	0	NA	NA
[109]	T	.4	.6	1e3	.5	.5	NA	NA
[109]	T	.4	.6	1e3	1	1	NA	NA
[29]	I	.5	.1	100	0	0	high	low
[8, 17, 18]	I	.5	.1	1e3	0	0	high	low
[58]	T	.5	.1	50	NA	NA	high	low
[57]	T	.5	.1	1e3	NA	NA	high	low
[109, 115]	T	.5	.1	1e3	0	0	NA	NA
[109]	T	.5	.1	1e3	.5	.5	NA	NA
[109]	T	.5	.1	1e3	1	1	NA	NA
[142, 143]	T	.5	.5	20	NA	NA	med	med
[3]	T	.5	.5	NA	NA	NA	med	med
[77]	T	.5	.5	10	NA	NA	high	high
[58]	T	.5	.5	50	NA	NA	high	high

Table 9 – Continued from previous page

Context		CV		Global	Consistency		Heterogeneity	
Ref.	Use	Task	Mach.	Mean	Task	Mach.	Task	Mach.
[57]	T	.5	.5	1e3	NA	NA	high	high
[35]	T	.5	.5	20	NA	NA	NA	NA
[61, 67]	T	.5	.5	100	NA	NA	NA	NA
[169]	T	.5	.5	200	NA	NA	NA	NA
[109, 115]	T	.5	.6	1e3	0	0	NA	NA
[109]	T	.5	.6	1e3	.5	.5	NA	NA
[109]	T	.5	.6	1e3	1	1	NA	NA
[41, 42, 116]	T	.6	.1	100	0	0	high	low
[41, 42]	T	.6	.1	100	.5	.25	high	low
[41, 42]	T	.6	.1	100	1	1	high	low
[68, 164]	T	.6	.1	100	NA	NA	high	low
[109, 115]	T	.6	.1	1e3	0	0	NA	NA
[109]	T	.6	.1	1e3	.5	.5	NA	NA
[109]	T	.6	.1	1e3	1	1	NA	NA
[109, 115]	T	.6	.2	1e3	0	0	NA	NA
[109]	T	.6	.2	1e3	.5	.5	NA	NA
[109]	T	.6	.2	1e3	1	1	NA	NA
[109, 115]	T	.6	.3	1e3	0	0	NA	NA
[109]	T	.6	.3	1e3	.5	.5	NA	NA
[109]	T	.6	.3	1e3	1	1	NA	NA
[109, 115]	T	.6	.4	1e3	0	0	NA	NA
[109]	T	.6	.4	1e3	.5	.5	NA	NA
[109]	T	.6	.4	1e3	1	1	NA	NA
[43, 109, 115]	T	.6	.5	1e3	0	0	NA	NA
[109]	T	.6	.5	1e3	.5	.5	NA	NA
[43]	T	.6	.5	1e3	1	.5	NA	NA
[109]	T	.6	.5	1e3	1	1	NA	NA
[41, 42, 116]	T	.6	.6	100	0	0	high	high
[8, 17, 18]	I	.6	.6	1e3	0	0	high	high
[41, 42]	T	.6	.6	100	.5	.25	high	high
[41, 42]	T	.6	.6	100	1	1	high	high
[68, 164]	T	.6	.6	100	NA	NA	high	high
[109, 115]	T	.6	.6	1e3	0	0	NA	NA
[28]	T	.6	.6	1e3	.25	.25	NA	NA
[109]	T	.6	.6	1e3	.5	.5	NA	NA
[43]	T	.6	.6	1e3	1	.5	NA	NA
[43, 109]	T	.6	.6	1e3	1	1	NA	NA
[115]	T	.7	.1	1e3	0	0	NA	NA
[109, 115]	T	.7	.6	1e3	0	0	NA	NA
[109]	T	.7	.6	1e3	.5	.5	NA	NA
[109]	T	.7	.6	1e3	1	1	NA	NA
[9]	T	.7	.7	10	NA	NA	high	high
[13, 14, 16, 51]	T	.7	.7	10	NA	NA	NA	NA
[109]	T	.8	.6	1e3	0	0	NA	NA
[109]	T	.8	.6	1e3	.5	.5	NA	NA

Table 9 – *Continued from previous page*

Context		CV		Global	Consistency		Heterogeneity	
Ref.	Use	Task	Mach.	Mean	Task	Mach.	Task	Mach.
[109]	T	.8	.6	1e3	1	1	NA	NA
[125]	T	.8	.8	15	.25	.25	NA	NA
[12]	T	.8	.8	NA	NA	NA	NA	NA
[115]	T	.9	.1	1e3	0	0	NA	NA
[71]	T	.9	.3	1e3	0	0	high	low
[71]	T	.9	.3	1e3	1	.5	high	low
[71]	T	.9	.3	1e3	1	1	high	low
[151, 152]	T	.9	.3	180	1	1	high	low
[43, 109, 115]	T	.9	.6	1e3	0	0	NA	NA
[109]	T	.9	.6	1e3	.5	.5	NA	NA
[109]	T	.9	.6	1e3	1	1	NA	NA
[87]	T	.9	.6	60	NA	NA	NA	NA
[87]	T	.9	.6	600	NA	NA	NA	NA
[86]	T	.9	.6	NA	NA	NA	NA	NA
[150, 151]	T	.9	.9	30	0	0	high	high
[113, 114]	T	.9	.9	100	0	0	high	high
[112, 114]	T	.9	.9	120	0	0	high	high
[71]	T	.9	.9	1e3	0	0	high	high
[71]	T	.9	.9	1e3	1	.5	high	high
[71]	T	.9	.9	1e3	1	1	high	high
[84, 85]	T	.9	.9	3	NA	NA	high	high
[144–146]	T	.9	.9	100	NA	NA	high	high
[3]	T	.9	.9	NA	NA	NA	high	high
[73, 74]	T	.9	.9	20	0	0	NA	NA
[72]	T	.9	.9	30	0	0	NA	NA
[49]	T	1	.25	100	NA	NA	high	low
[109]	T	1	.6	1e3	0	0	NA	NA
[109]	T	1	.6	1e3	.5	.5	NA	NA
[109]	T	1	.6	1e3	1	1	NA	NA
[44, 48–50]	T	1	1	100	NA	NA	high	high
[115]	T	1.1	.1	1e3	0	0	NA	NA
[115]	T	1.1	.2	1e3	0	0	NA	NA
[115]	T	1.1	.3	1e3	0	0	NA	NA
[115]	T	1.1	.4	1e3	0	0	NA	NA
[115]	T	1.1	.5	1e3	0	0	NA	NA
[43, 109, 115]	T	1.1	.6	1e3	0	0	NA	NA
[109]	T	1.1	.6	1e3	.5	.5	NA	NA
[43]	T	1.1	.6	1e3	1	.5	NA	NA
[43, 109]	T	1.1	.6	1e3	1	1	NA	NA
[11]	T	1.4	.4	NA	0	0	NA	NA
[11]	T	1.8	.4	NA	0	0	NA	NA

B.3 Missing Parameters of Named Instances

The following articles mention some specific instances without providing the exact parameters used to generate them (they may thus be the same as the referenced articles):

- missing values, probably the same as [18]: [15, 27, 45–47, 79, 83, 88, 128, 130–132, 140, 141]
- missing values, probably the same as [17]: [34, 66, 69, 70, 76, 158]
- missing values, probably the same as [26]: [1, 22, 24, 65, 69, 70, 97, 101, 103, 117, 121, 127, 129, 133, 160]
- missing values, probably the same as either [18] or [26]: [21]
- missing values, probably the same as either [17] or [26]: [7, 118, 120, 122–124]

B.4 Discarded Articles

The following references were discarded either because they lack information on the instance generation or because they do not fit our study:

- missing values and unclear generation method: [19, 39, 40, 56, 60, 64, 75, 81, 89–96, 98–100, 102, 104, 126, 135–138, 155, 161, 163, 165, 173]
- no value for the shape parameter and range for the scale parameter: [148, 149]
- only one CV is given: [54, 55, 134]
- mismatched citation: [162]
- mismatched value for high machine heterogeneity: [119]
- use bi or tri-modal distributions with infinite mean: [30]
- range for CV: [4, 5]
- ranges for mean and CV: [106]

C Scheduling Heuristics

In this part, we describe the scheduling algorithms used for the experiments. Part of them are classical algorithms as OLB or Min-Min taken for other research works. We also introduce less classical algorithms as HLPT, an adaptation of LPT to the heterogeneous context, algorithms based on sufferage values for which a sufferage matrix is computed and balance algorithms that try to improve an initial solution by moving the tasks that will not lose too much from changing their mapping.

C.1 Related Work

The OLB algorithm (Opportunistic Load Balancing) takes a list of tasks sorted in random order and allocates them to the first ready machine. This algorithm generates mostly random schedules as they depend of the task list order. On the one hand, it tries to keep the machines busy as it allocates a task to a machine as soon as the former is free. On the other hand, it does not take the $e_{i,j}$ value into account. Thus, it may map a task to its worst machine.

Algorithm 6 Opportunistic Load Balancing (OLB)

Input: T a set of n tasks in an arbitrary order, M a set of m machines, E a cost matrix**Output:** an allocation function π

- 1: Init $\forall p \in M, RT[p] \leftarrow 0$ {earliest ready time for each machine}
 - 2: **for all** $t \in T$ **do**
 - 3: $\pi_t \leftarrow p$ s.t. $RT[p] = \min_k(RT[k])$ {first ready machine}
 - 4: $RT[\pi_t] \leftarrow RT[\pi_t] + e_{t,\pi_t}$
 - 5: **end for**
 - 6: **return** π
-

Algorithm 7 Minimum Execution Time (MET)

Input: T a set of n tasks in an arbitrary order, M a set of m machines, E a matrix of the execution costs of the tasks on the machines**Output:** an allocation function π

- 1: **for all** $t \in T$ **do** {earliest ready time for each machine}
 - 2: $\pi_k \leftarrow p$ s.t. $e_{t,p} = \min_k(e_{t,k})$ {best machine for the task}
 - 3: **end for**
 - 4: **return** π
-

The MET algorithm (Minimum Execution Time) maps each task, from a randomly ordered task list, to its best machine, regardless of the machine load. This is a more shorted-viewed algorithm, even more than random, because a single powerful will concentrate all the load.

The MCT algorithm (Minimum Completion Time) maps each task, from a randomly ordered task list, to the machine that will end it the soonest. This is a little smarter than the previous ones as it takes both the load of the machines and the $e_{i,j}$ value into account.

Algorithm 8 Minimum Completion Time (MCT)

Input: T a set of n tasks in an arbitrary order, M a set of m machines, E a matrix of the execution costs of the tasks on the machines**Output:** an allocation function π

- 1: Init $\forall p \in M, RT[p] \leftarrow 0$ {earliest ready time for each machine}
 - 2: **for all** $t \in T$ **do**
 - 3: $\pi_t \leftarrow p$ s.t. $e_{t,p} = \min_k(RT[k] + e_{t,k})$ {min completion time for the task}
 - 4: $RT[\pi_t] \leftarrow RT[\pi_t] + e_{t,\pi_t}$
 - 5: **end for**
 - 6: **return** π
-

The Min-min algorithm corresponds to the classical EFT algorithm (Earliest Finish Time). Actually, the Min-min algorithm presented in [26] first computes the set M of minimum completion times for each unmapped task, then takes the task with the min completion time, hence the Min-Min name. The version presented here does not compute M but rather finds the task to allocate by computing all the possible completion times and taking the earliest one. So the Min-Min algorithm just searches for the earliest finish task and allocates it to the corresponding machine. Intuitively, this algorithm will not generate good schedules when the

task heterogeneity is high as it first schedules the smallest tasks, leaving the largest ones for the end.

Algorithm 9 Earliest Finish Time or Min-min

Input: T a set of n tasks,

M a set of m machines,

E a matrix of the execution costs of the tasks on the machines

Output: an allocation function π

```

1: Init  $\forall p \in M, RT[p] \leftarrow 0$                                 {earliest ready time for each machine}
2: while  $T \neq \emptyset$  do
3:    $eft \leftarrow t_1$                                            {earliest finish task}
4:    $efm \leftarrow 1$                                              {allocation of earliest finish task}
5:   for all  $t \in T$  do                                          {find the task with earliest finish time}
6:     for all  $p \in M$  do
7:       if  $RT[p] + e_{t,p} < RT[efm] + e_{eft,efm}$  then
8:          $eft \leftarrow t$ 
9:          $efm \leftarrow p$ 
10:      end if
11:    end for
12:     $\pi_{eft} \leftarrow efm$ 
13:     $RT[efm] \leftarrow RT[efm] + e_{eft,efm}$ 
14:     $T \leftarrow T - \{eft\}$ 
15:  end for
16: end while
17: return  $\pi$ 

```

The Max-min algorithm is close to the Min-min one except that it chooses the largest task instead of the smallest from the set M . It is thus close to the classical LPT that gives good results in homogeneous platforms.

The Suff algorithm relies on a sufferage value that is computed as the difference between the best and the second best finish time for the task. The algorithm first allocates the task that will suffer the most from not being allocated to its best machine, which is the one that will finish it the earliest.

The genetic algorithm is based on the genetic algorithm described by Siegel in [26]. This is a classical genetic algorithm that represents a schedule as an individual with one chromosome: each gene is a task and its associated value is the machine where the task is mapped. Note that there is no need to define the start time of the tasks as we are only interested by the $C_{\max} = \max_j (\sum_{i=1, \pi(i)=j}^n e_{i,j})$, which only depends on the execution duration thanks to the commutativity of sum. The population is composed of 200 chromosomes. The algorithm iterates a thousand times unless the elite chromosome does not change during 150 iterations or all the chromosomes are the same. Elitism is implemented in the algorithm. The best chromosome of each iteration becomes the elite chromosome if it gets a better fitness than the previous elite one. At each iteration the population is filtered using a wheel selection. In wheel selection the probability that a chromosome is maintained in the population is inversely proportional to its fitness. Then a crossover step, with a probability of 0.6, and mutation step, with a probability of 0.4, are applied to generate new chromosomes. Two configurations of the initial population are used. In the first one the initial population is initialised only with random individuals, the tasks are randomly assigned to machines. In the second configuration one of the chromosomes

Algorithm 10 Max-min**Input:** T a set of n tasks, M a set of m machines, E a matrix of the execution costs of the tasks on the machines**Output:** an allocation function π

```

1: Init  $\forall p \in M, RT[p] \leftarrow 0$  {earliest ready time for each machine}
2: while  $T \neq \emptyset$  do
3:   Init  $\forall t \in T, CT[t] \leftarrow \infty$  {earliest completion time for each task}
4:   for all  $t \in T$  do {find the task with earliest finish time}
5:     for all  $p \in M$  do
6:       if  $RT[p] + e_{t,p} < CT[t]$  then
7:          $CT[t] \leftarrow RT[p] + e_{t,p}$ 
8:          $CP[t] \leftarrow p$  {the completing machine for each task}
9:       end if
10:    end for
11:  end for
12:   $mct \leftarrow t$  s.t.  $CT[t] = \max_k(CT[k])$  {maximum completing task}
13:   $\pi_{mct} \leftarrow CP[mct]$ 
14:   $RT[\pi_{mct}] \leftarrow CT[\pi_{mct}]$ 
15:   $T \leftarrow T - \{mct\}$ 
16: end while
17: return  $\pi$ 

```

Algorithm 11 First allocate the task that would suffer most (Suff)**Input:** T a set of n tasks, M a set of m machines, E a matrix of the execution costs of the tasks on the machines**Output:** an allocation function π

```

1: Init  $\forall p \in M, RT[p] \leftarrow 0$  {earliest ready time for each machine}
2: while  $T \neq \emptyset$  do
3:    $maxSuff \leftarrow 0$ 
4:    $maxSuffT \leftarrow t_1$ 
5:    $maxSuffM \leftarrow 1$ 
6:   for all  $t \in T$  do
7:      $finishTime_t \leftarrow sort_k(RT[k] + e_{t,k})$  {sort in non-decreasing order of the sufferage for  $t$ }
8:     if  $maxSuff < finishTime_t[2] - finishTime_t[1]$  then
9:        $maxSuff \leftarrow finishTime_t[2] - finishTime_t[1]$ 
10:       $maxSuffT \leftarrow t$ 
11:       $maxSuffM \leftarrow p$  s.t.  $RT[p] + e_{t,p} = finishTime_t[1]$ 
12:    end if
13:  end for
14:   $RT[maxSuffM] \leftarrow RT[maxSuffM] + e_{maxSuffT, maxSuffM}$ 
15:   $\pi_{maxSuffT} \leftarrow maxSuffM$ 
16:   $T \leftarrow T - \{maxSuffT\}$ 
17: end while
18: return  $\pi$ 

```

is based on the mapping generated by the Min-Min algorithm to have a good solution. The algorithm is run 4 times for each configurations.

Algorithm 12 GA

Input: T a set of n tasks,
 M a set of m machines,
 E a matrix of the execution costs of the tasks on the machines

Output: an allocation function π

```

1: Init  $nbChrom \leftarrow 200$ 
2:  $pop \leftarrow InitPop(Rand, MinMin)$  {Initialize population}
3: while  $nbIter < 1000 \wedge lastEliteChange < 150 \wedge noChange = TRUE$  do
4:    $sortedChrom \leftarrow sortByFitness(pop)$ 
5:   if  $fitness(sortedChrom[1]) < fitness(elit)$  then
6:      $elit \leftarrow sortedChrom[1]$ 
7:      $lastEliteChange \leftarrow 1$ 
8:   else
9:      $lastEliteChange \leftarrow lastEliteChange + 1$ 
10:  end if
11:   $pop \leftarrow wheelSelection(pop)$ 
12:  for all  $indiv \in pop$  do {Select individuals for cross-over}
13:    if  $rand(0,1) > crossOverProba$  then
14:       $crossOverPop \leftarrow crossOverPop + indiv$ 
15:    end if
16:  end for
17:  for all  $indiv1, indiv2 \in crossOverPop$  do {cross-over}
18:     $crossOverPlace \leftarrow rand(1, n)$ 
19:     $indiv1 \leftarrow crossOver(indiv1, indiv2, crossOverPlace)$ 
20:     $indiv2 \leftarrow crossOver(indiv2, indiv1, crossOverPlace)$ 
21:  end for
22:  for all  $indiv \in pop$  do {Select individuals for mutation}
23:    if  $rand(0,1) > mutationProba$  then
24:       $indiv \leftarrow mutation(indiv)$ 
25:    end if
26:  end for
27:   $nbIter \leftarrow nbIter + 1$  :
28:   $noChange \leftarrow compareChrom(pop)$ 
29: end while
30: return  $\pi$ 

```

C.2 Other Heuristics

In this part, we present other algorithms used in our simulations. They are not referenced even though they may already be defined in the literature. Since there exists so many papers on heterogeneous scheduling, we did not check them all.

The Heterogeneous Largest Processing Time (HLPT) is an adaptation of the well-known Largest Processing Time where the tasks are sorted in non-increasing order depending of their different processing times. HLPT can be used either with *func* set to the min or *mean* function to sort the tasks in non-increasing order. HLPT differs from Max-Min in the way it first sorts the tasks, depending on the chosen function. It then tries to find the best allocation for each

task depending on the machine load, while Max-Min recomputes M set at each iteration and thus sorts the tasks depending on the machine load.

Algorithm 13 Heterogeneous Largest Processing Time (HLPT)

Input: T a set of n tasks,

M a set of m machines,

E a matrix of the execution costs of the tasks on the machines

Output: an allocation function π

```

1: Init  $\forall p \in M, RT[p] \leftarrow 0$  {earliest ready time for each machine}
2:  $T \leftarrow \{t_1, t_2, \dots\}$  such that  $func_t(e) \geq func_{t+1}(e)$  {sort tasks in non-increasing order using a
   function of their processing costs}
3: for all  $t \in T$  do {consider each task in given order}
4:    $efm \leftarrow 1$ 
5:   for all  $p \in M$  do {find the earliest finish time}
6:     if  $RT[p] + e_{t,p} < RT[efm] + e_{t,efm}$  then
7:        $efm \leftarrow p$ 
8:     end if
9:   end for
10:   $\pi_t \leftarrow efm$ 
11:   $RT[efm] \leftarrow RT[efm] + e_{t,efm}$ 
12: end for
13: return  $\pi$ 

```

The GreedySuff algorithm relies on the sufferage matrix. In the sufferage matrix each value is the difference between the processing time on the current machine $e_{i,j}$ and the minimum processing time of the task. The higher the value, the more the task will suffer from being mapped on this machine. A null value indicates that the task is processed by the fastest machine for this task.

The GreedySuff algorithm takes the task in the cost matrix order, so it does not take the tasks in a particular order. For each task, it tries to map it on the machine that generates the lower sufferage and does not increase the C_{\max} . If the algorithm does not find any machine, then it uses the machine with the earliest finish time, hence the one that increases C_{\max} the least.

The BalSuff algorithm, as the GreedySuff, uses the sufferage matrix. It starts from an initial mapping where the tasks are mapped on their best machine (MET algorithm). Then, the algorithm tries to rearrange the tasks in a way such that the makespan is improved and the chosen task is the one that suffers the least from moving. The algorithm stops when there is no more task on the most loaded machine that could benefit from moving. Note that if two tasks have the same sufferage value on the most loaded machine, the tasks are considered in an arbitrary order

The BalEFT algorithm is a variant of the BalSuff algorithm. After the initial mapping based on the shortest processing time (MET algorithm), this algorithm tries to rearrange tasks using an early finish time criterion: it takes all the tasks of the most loaded machine and tries to find a new allocation such that the global makespan is decreased. The algorithm stops when there is no more task than can decrease the C_{\max} by moving.

Although not very costly in practice, we were not able to determine the worst-case time complexity of either BalSuff or BalEFT.

Eventually we also have implemented a second version of the GA algorithm, initialized with a random initial population where we added one chromosome for each mapping generated by the other algorithms. This algorithm gives of course always the best result as the elitism procedure

Algorithm 14 Greedily allocate tasks on suffering (GreedySuff)

Input: T a set of n tasks, M a set of m machines, E a matrix of the execution costs of the tasks on the machines**Output:** an allocation function π

```

1: Init  $\forall p \in M, RT[p] \leftarrow 0$                                 {earliest ready time for each machine}
2: for all  $t \in T, p \in M$  do                                  {generate the sufferage matrix S}
3:    $S_{t,p} \leftarrow e_{t,p} - \min_k(e_{t,k})$ 
4: end for
5:  $C_{\max} \leftarrow 0$ 
6: for all  $t \in T$  do                                         {allocate tasks}
7:    $SortSuff \leftarrow sort(S_t)$                              {sort in non-decreasing order of the sufferage for  $t$ }
8:    $k \leftarrow 1$ 
9:    $found \leftarrow false$ 
10:  repeat
11:     $p \leftarrow SortSuff[k]$ 
12:    if  $RT[p] + e_{t,p} \leq C_{\max}$  then
13:       $\pi_t \leftarrow p$ 
14:       $RT[p] \leftarrow RT[p] + e_{t,p}$ 
15:       $found \leftarrow true$ 
16:    end if
17:     $k \leftarrow k + 1$ 
18:  until  $k \geq Card(M)$  or  $found$ 
19:  if  $\neg found$  then                                         {if no allocation found use EFT}
20:     $\pi_t \leftarrow \arg \min_i (RT[i] + e_{t,i})$ 
21:     $RT[\pi_t] \leftarrow RT[\pi_t] + e_{t,\pi_t}$ 
22:  end if
23:   $C_{\max} \leftarrow \max_i (RT[i])$ 
24: end for
25: return  $\pi$ 

```

Algorithm 15 Sufferage with machine balancing (BalSuff)**Input:** T a set of n tasks, M a set of m machines, E a matrix of the execution costs of the tasks on the machines**Output:** an allocation function π

```

1: for all  $t \in T, p \in M$  do                                     {generate the sufferage matrix S}
2:    $S_{t,p} \leftarrow e_{t,p} - \min_k(e_{t,k})$ 
3: end for
4: Init  $\forall p \in M, RT[p] \leftarrow 0$                                {earliest ready time for each machine}
5: for all  $t \in T$  do                                           {initial allocation based on MET}
6:    $\pi_k \leftarrow p$  s.t.  $e_{t,p} = \min_k(e_{t,k})$ 
7:    $RT[\pi_t] \leftarrow RT[\pi_t] + e_{t,\pi_t}$ 
8: end for
9:  $C_{\max} \leftarrow \max_k(RT[k])$ 
10:  $newC_{\max} \leftarrow \max_k(RT[k])$ 
11:  $M_{\max} \leftarrow p$  s.t.  $RT[p] = C_{\max}$ 
12: repeat                                                         {try to re-arrange task allocations}
13:    $MinSuff \leftarrow \max_{i,j}(S_{i,j}) + 1$ ;  $mst \leftarrow 0$ ;  $msm \leftarrow 0$ 
14:   for all  $t \in T$  such that  $\pi_t = M_{\max}$  do                 {find the task that suffers less from moving}
15:     for all  $p \in M - \{M_{\max}\}$  do
16:       if  $(MinSuff > S_{t,p})$  and  $(RT[p] + e_{t,p} \leq C_{\max})$  then
17:          $MinSuff \leftarrow S_{t,p}$ ;  $mst \leftarrow t$ ;  $msm \leftarrow p$ 
18:       end if
19:     end for
20:   end for
21:   if  $RT[msm] + e_{mst,msm} \leq C_{\max}$  then                       {re-allocation improves  $C_{\max}$ }
22:      $newC_{\max} \leftarrow \max(C_{\max} - e_{mst,msm}, RT[msm] + e_{mst,msm})$ 
23:      $\pi_t \leftarrow msm$                                          {task is moved}
24:      $RT[msm] \leftarrow RT[msm] + e_{mst,msm}$ 
25:      $RT[M_{\max}] \leftarrow RT[M_{\max}] - e_{mst,msm}$ 
26:     if  $newC_{\max} = RT[msm] + e_{mst,msm}$  then                 {find last finishing machine}
27:        $M_{\max} \leftarrow msm$ 
28:     end if
29:   end if
30:    $C_{\max} = newC_{\max}$ 
31: until  $mst \neq 0$ 
32: return  $\pi$ 

```

Algorithm 16 Earliest Finish Time with machine balancing (BalEFT)

Input: T a set of n tasks,
 M a set of m machines,
 E a matrix of the execution costs of the tasks on the machines

Output: an allocation function π

- 1: **for all** $t \in T, p \in M$ **do** {generate the sufferage matrix S }
- 2: $S_{t,p} \leftarrow e_{t,p} - \min_k(e_{t,k})$
- 3: **end for**
- 4: Init $\forall p \in M, RT[p] \leftarrow 0$ {earliest ready time for each machine}
- 5: **for all** $t \in T$ **do** {initial allocation based on MET}
- 6: $\pi_k \leftarrow p$ s.t. $e_{t,p} = \min_k(e_{t,k})$
- 7: $RT[\pi_t] \leftarrow RT[\pi_t] + e_{t,\pi_t}$
- 8: **end for**
- 9: **repeat** {try to re-arrange task allocations}
- 10: $M_{\max} \leftarrow p$ s.t. $p = \max_k(RT[k])$
- 11: $MinRT \leftarrow RT[M_{\max}]$; $eft \leftarrow 0$; $efm \leftarrow 0$
- 12: **for all** $t \in T$ such that $\pi_t = M_{\max}$ **do** {find the task that benefit most from moving}
- 13: **for all** $p \in M - \{M_{\max}\}$ **do**
- 14: $newRT \leftarrow RT[p] + e_{t,p}$
- 15: **if** $newRT < minRT$ **then**
- 16: $minRT \leftarrow newRT$; $eft \leftarrow t$; $efm \leftarrow p$
- 17: **end if**
- 18: **end for**
- 19: **end for**
- 20: **if** $RT[efm] + e_{eft,efm} < C_{\max}$ **then** {re-allocation improves C_{\max} }
- 21: $newC_{\max} \leftarrow \max(C_{\max} - e_{eft,efm}, RT[efm] + e_{eft,efm})$
- 22: $\pi_{eft} \leftarrow efm$ {task is moved}
- 23: $RT[efm] \leftarrow RT[efm] + e_{eft,efm}$
- 24: $RT[M_{\max}] \leftarrow RT[M_{\max}] - e_{eft,efm}$
- 25: **if** $newC_{\max} = RT[efm] + e_{eft,efm}$ **then** {find last finishing machine}
- 26: $M_{\max} \leftarrow efm$
- 27: **end if**
- 28: **end if**
- 29: $C_{\max} = newC_{\max}$
- 30: **until** $eft \neq 0$
- 31: **return** π

guaranties that we keep the best chromosome (at least the best of the results of the other algorithms).

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