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A hybrid genetic algorithm for a multilevel assembly replenishment planning problem with stochastic lead times

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Abstract

This paper on replenishment planning for multi-level assembly systems with several components at each level deals with the problem of calculating planned lead-times when the real lead-times for all components are assumed to be stochastic. This problem is already treated in [Ben-Ammar et al. \(2018\)](#) by using a recursive procedure and a Branch and Bound algorithm. Here, in order to decrease the computation time, a novel generalized probabilistic model based on an iterative approach is developed. The proposed model calculates the expected total cost, which is composed of the inventory holding cost for components and the backlogging and inventory holding costs for the finished product. An iterative approach and a hybrid genetic algorithm are introduced to determine the planned order release dates for components at the last level of the bill of materials that minimizes the expected total cost. Experimental results show that the proposed optimization algorithm efficiently finds good-quality approximate solutions regardless of the type of assembly system, the number of components at the last level and the variability of the finished product–related costs.

Keywords: Assembly systems, Assemble-to-order, Replenishment planning, Stochastic lead-times, Genetic algorithms, Discrete optimization

1. Introduction

In an assemble-to-order (ATO) environment, the manufacturer needs several components to assemble a finished customized product. The demand is not known in advance and there is no stock of finished products to anticipate it. The customer asks for a product composed of a given set of standard or personalized components. The producer uses information on the lead-times of the components (time to order them from suppliers or to make and assemble them) and the assembly process to set the delivery time to the customer ([d'Avino et al., 2013](#)). This delivery time is often used as the due date ([Hammami et al., 2017](#)). The producer's Manufacturing Resource Planning (MRP) system uses the input data (e.g., demand and its due date, assembly times, delivery times) to calculate the assembly order release dates for components. If the actual lead times are stochastic, the calculation of planned lead times, and thus, release dates for the components, is a complex problem.

11 ATO strategies are widely used in industry. However, ATO systems are very sensitive to uncertainty, which can
12 disrupt the ATO process and its MRP system (Milne et al., 2015). The assembly process can be interrupted due to
13 supply variations (Flynn et al., 2016; Simangunsong et al., 2012; Wazed et al., 2009) or disruptions (Snyder et al.,
14 2016; Speier et al., 2011; Kleindorfer & Saad, 2005). In practice, machine breakdowns can interrupt the assembly
15 process. Component customization and replenishment lead-times may also be uncertain and significantly longer or
16 shorter than planned. This leads to stochastic component replenishment lead times. If only one component is delayed,
17 the entire assembly process is stopped. The holding cost for other components, already delivered, increases, and
18 delivery of the finished product may be delayed.

19 MRP logic with deterministic assumptions often proves to be too limited in ATO environments. The majority of
20 the existing academic literature over the past 30 years has argued that MRP logic with deterministic assumptions about
21 lead time is too restrictive. In fact, the random variability of lead-times drastically decreases the system's performance
22 and this leads to a need to estimate planned lead times far more precisely (Bandaly et al., 2016). A possible solution
23 to cope with this difficulty consists of introducing safety lead-times: planned lead-times are assumed to be equal to the
24 contractual plus additional safety lead times (Dolgui & Prodhon, 2007). Nevertheless, in several survey papers (Dolgui
25 et al., 2013; Dolgui & Prodhon, 2007; Damand et al., 2013; Koh et al., 2002) which have investigated how MRP
26 systems address lead-time uncertainties, it was concluded that a vast amount of literature highlighting the use of safety
27 stocks to handle them, and that the safety lead-times have long been neglected (van Kampen et al., 2010). Furthermore,
28 there have been very few studies focusing on how to minimize the additional costs caused by the lead-time uncertainty
29 (Dolgui & Prodhon, 2007) or how to anticipate them (Jansen et al., 2013). For more details on replenishment planning
30 models under uncertainty, interested readers can refer to Aloulou et al. (2014), Díaz-Madroño et al. (2014), Dolgui
31 et al. (2013), Dolgui & Proth (2010), Ko et al. (2010), Peidro et al. (2009), Mula et al. (2006b) and Koh et al. (2002).

32 Despite the importance of this issue, as far as we know, only a small number of researchers have studied multi-level
33 assembly systems in an ATO environment with non-deterministic lead-time behavior (Atan et al., 2017). In this paper,
34 this gap is addressed by investigating an ATO supply network to assemble a given tailored finished product under
35 lead time uncertainty. The customer's request defines this finished product and the set of customizable components
36 needed to customize it. In the considered case, there are no product or component stocks available to anticipate this
37 demand and so it is necessary to set a due date for client delivery. To do this, the planners use information about
38 the tailored product, customizable components, supplier availabilities and the assembly process in order to design the
39 whole supply network, define the list of partners (local assembly units or external suppliers) and estimate the planned
40 lead-times. As highlighted by Golini & Kalchschmidt (2011), Chandra & Grabis (2008), Berlec et al. (2008) and Arda
41 & Hennet (2006), this information is essential at the negotiation step to design the supply network, estimate lead-times
42 and release the orders.

43 There is usually little information available on the capacity and state of resources and production management at
44 supply chain partner enterprises (suppliers). The only information available to help assess the lead-time of each partner
45 is its probability distribution based on past statistics. These include not only processing times but also additional times
46 related to workload, capacity constraints, local planning decisions, variations, etc. Due to the complexity and, often,

47 the impossibility (e.g. confidentiality) of considering all the details on the state of supply chain partners, this approach
48 has escaped criticism and has been highly recommended in the literature on contracting and planning under uncertainty
49 (Chen et al., 2019; Jabbarzadeh et al., 2018; Giri & Sarker, 2017; Carvalho et al., 2012; Berlec et al., 2008; Fiala, 2005;
50 Song et al., 2000).

51 In this paper, a multi-level assembly system with stochastic component lead-times is studied. The aim is to calculate
52 planned lead-times for all components of a given product order at a given due date, in the case of random variability of
53 real component lead-times. The study is composed of the following steps:

- 54 • a new approach and efficient model for performance evaluation (expected cost calculation) is suggested;
- 55 • an overall discrete optimisation model is proposed;
- 56 • an efficient approximate algorithm to solve the proposed model is developed;
- 57 • numerical tests and analysis of model behaviour are carried out;
- 58 • conclusions and managerial insights are given.

59 The remainder of the paper is organized as follows: Section 2 reviews the relevant literature; Section 3 describes
60 the problem; Section 4 presents the generalized probabilistic model; Section 5 describes the optimization approach;
61 Section 6 gives performance tests; and Section 7 rounds off with a conclusion; managerial insights; and perspectives
62 for future research.

63 2. Related publications

64 The literature features various probabilistic approaches for dealing with lead-time uncertainty in the case of an MRP
65 environment (readers can refer to Díaz-Madroñero et al. (2014); Dolgui et al. (2013); Dolgui & Prodhon (2007); Mula
66 et al. (2006b)). Analytical probabilistic approaches often model the simplest structures (e.g. for one-level systems) and
67 give way to simulation methodology for more complex structures (multi-level systems, multi-period planning, etc.).
68 Several studies have modeled one- or two-level assembly systems under a probabilistic distribution of lead-times (see
69 Table 1). The proposed models consider continuous or discrete random lead-times, but remain limited:

- 70 • To one-level in the bill of materials (BOM), with constant demand for all periods;
- 71 • Or to two-levels in the BOM, with mono-period planning (including one customer demand).

72 For the case of multi-level BOMs, Ben-Ammar et al. (2016, 2018) were the first to propose an analytical approach
73 to model a one customer demand. There are also studies based on fuzzy logic. In the area of fuzzy reasoning, statistical
74 data are assumed to be either too unreliable or insufficient to obtain reliable probability distributions. Mula et al.
75 (2006b) presented an excellent review of supply and production planning under uncertainty, in which they noted that
76 most studies have focused on handling the uncertainty of demand (Guillaume et al., 2013; Peidro et al., 2009; Grabot
77 et al., 2005; Mula et al., 2006a). This under-focus on the supply side is explained by the difficulty of modeling lead-
78 time variability. Most studies have considered lead-times as nil or fixed. Without attempting an exhaustive review

79 of the literature, we specifically focus here on probabilistic approaches. [Dolgui et al. \(2013\)](#) presented a literature
80 review classifying the techniques and approaches in the field of supply planning and inventory control systems under
81 stochastic lead-times. As mentioned earlier and described in [Table 1](#), most models have assumed (i) one-level assembly
82 systems and mono- or multi-period planning or (ii) two-level assembly systems and mono-period planning.

83 [Yano \(1987a,b\)](#) were among the first to note the effects of the variability of lead-times. They proposed the case of
84 serial production systems and mono-period planning, which was later generalized by [Elhafsi \(2002\)](#). In a third study
85 [Yano \(1987c\)](#) studied a two-level assembly system and one-period planning. One component is assembled from two
86 components and lead-times follow stochastic Poisson and negative binomial distributions. The objective function is
87 the expected total cost (ETC), which is composed of tardiness and inventory holding costs. An algorithm based on
88 an exploration of the properties of the objective function determines the optimal planned lead-times. [Kumar \(1989\)](#)
89 studied the case of one-period planning and one-level assembly systems. Like [Yano \(1987a\)](#), he minimized the total
90 expected cost and proposed optimal order release dates. An exact analysis was proposed with assumptions of special
91 types of distribution (normal, uniform and exponential). A few years later, [Chu et al. \(1993\)](#) looked at the case of a
92 one-level assembly system: they proved the convexity of the mathematical expectation of total cost, and proposed an
93 iterative algorithm to minimize it. To solve this issue for the case of a two-stage model production model, [Hegedus
94 & Hopp \(2001\)](#) proposed an approach based on a Newsboy-like analytical formulation. The main weakness in these
95 studies is that they are limited to one- or two-level assembly systems.

96 Next, [Axsäter \(2006\)](#) considered a three-level assembly network with independent stochastic operation times. The
97 objective was to choose starting times for different operations in order to minimize the ETC, composed of the inventory
98 holding costs of components and the tardiness cost of end items. An approximate decomposition technique, based on
99 the repeat application of the solution of a single-stage problem, was suggested. In the numerical result for the first
100 example of a two-stage problem, the error was only 1% error, i.e. the relative cost increase due to approximation.
101 However, for the second three-stage example, the error was about 10%, which cannot be disregarded.

102 Earlier, [Dolgui et al. \(1995\)](#) and [Dolgui & Ould-Louly \(2002\)](#) had developed an approach to multi-period planning
103 based on coupling simulation with integer linear programming to model one-level assembly systems. Several types of
104 finished product were produced and assembled from different types of components. An inventory holding cost for each
105 component and a backlogging cost for the finished products were introduced. For each period, the authors optimized
106 the quantity of finished products to be assembled and the quantity of components to be ordered. [Proth et al. \(1997\)](#)
107 studied the same problem and added a heuristic to select which products were to be assembled and which components
108 were to be ordered.

109 For the case of multi-period models and one-level assembly systems, [Ould-Louly & Dolgui \(2002a,b, 2004, 2009,
110 2011, 2013\)](#), [Ould-Louly et al. \(2008b,a\)](#) and [Shojaie et al. \(2015\)](#) presented mathematical models with random lead-
111 times, a known finished product demand (the same for all periods) and unlimited assembly capacity. [Hnaien & Afsar
112 \(2017\)](#) recently considered the min-max robust lot-sizing problem with discrete lead-time scenarios, and provided a
113 complexity analysis proving that robust lot-sizing problems are NP-hard even when there are two scenarios. They
114 showed that several optimality conditions for the deterministic cases provided in [Wagner & Whitin \(1958\)](#), as well

Authors	Type of system			Criteria	Lot sizing rules	Comments
	#Levels	#Periods	#Demand			
Kumar (1989)	1	1	1	Hc, Tc	L4L policy	Generic study of inventor control.
Chu et al. (1993)	1	1	1	Hc, Tc	L4L policy	Iterative algorithm.
Hegedus & Hopp (2001)	1	1	1	Hc, Sl	FPO policy	Combinatorial optimization method.
Hnaien & Afsar (2017)	1	1	1	Sc, Hc, Sl	L4L policy	Mixed integer programming models.
Chauhan et al. (2009)	1	1	1	Hc, Tc	L4L policy	ATO environment, continuous model, simulated annealing.
Atan et al. (2016); Jansen et al. (2019, 2018)	1	1	1	Hc, Tc	L4L policy	CTO environment, assembly process fed by a multi-stage parallel process, EV, recursive equations, iterative heuristic.
Jansen et al. (2019, 2018)	1	1	1	Hc, Tc	L4L policy	CTO environment, EV, Newboy formulae, commitment lead-time decisions, blame policy.
Borodin et al. (2016)	1	1	<i>c</i>	Hc, Tc	L4L policy	Joint chance constrained model, MIP.
Dolgui (2001)	1	<i>p</i>	<i>c</i>	Hc, Tc	Reorder point	ILP models, simulation, heuristics.
Ould-Louly & Dolgui (2002a)	1	<i>p</i>	<i>c</i>	Hc, Tc	L4L policy	EV, Markov, Newsboy formulae, all components share identical properties.
Ould-Louly & Dolgui (2004)	1	<i>p</i>	<i>c</i>	Hc, Tc	POQ policy	EV, Markov, Newsboy formulae, specific assumptions on costs and probability distributions.
Ould-Louly & Dolgui (2009)	1	<i>p</i>	<i>c</i>	Hc, Tc	EOQ policy	EV, B&B.
Ould-Louly et al. (2008a)	1	<i>p</i>	<i>c</i>	Hc, Sl	L4L policy	Generalization of Ould-Louly & Dolgui (2002a) .
Ould-Louly & Dolgui (2013)	1	<i>p</i>	<i>c</i>	Sc, Hc, Sl	POQ policy	EV, Newsboy formulae.
Shojaie et al. (2015)	1	<i>p</i>	<i>c</i>	Hc, Sl	POQ policy	EV, POQ policy, optimization.
Proth et al. (1997)	1	<i>p</i>	<i>d</i>	Hc, Tc	L4L Policy	Disc. event systems, heuristic algorithm, gradient descent.
Yano (1987c)	2	1	1	Hc, Tc	L4L policy	EV, Nonlinear programming.
Tang & Grubbström (2003)	2	1	1	Hc, Tc	L4L policy	Laplace transform procedure.
Hnaien et al. (2009)	2	1	1	Hc, Tc	L4L policy	EV, GA, B&B.
Hnaien et al. (2010)	2	1	1	Hc, Sl	L4L policy	EV, GA, multi-objective, elitist selection.
Fallah-Jamshidi et al. (2011)	2	1	1	Hc, Tc	L4L policy	EV, Hybrid GA.
Sakiani et al. (2012)	2	1	1	Hc, Sl	L4L policy	EV, GA, multi-objective, tournament selection.
Guiras et al. (2019)	2	1	1	Mc, Hc, Tc	L4L policy	EV, GA, maintenance cost.
Ben-Ammar & Dolgui (2018)	2	1	1	Hc, Tc	L4L policy	EV, B&B, GA.
Axsäter (2006)	3	1	1	Hc, Tc	L4L policy	Approximate decomposition technique, continuous distrib.
Ben-Ammar et al. (2018)	<i>m</i>	1	1	Hc, Tc	L4L policy	ATO environment, EV, recursive formula, B&B.
Current paper	<i>m</i>	1	1	Hc, Tc	L4L policy	ATO environment, EV, generic model, iterative process, hybrid GA.

m: Multi-level, *p* Multi-period, *c*: Constant demand, *d*: Dynamic demand, *Tc*: Tardiness cost, *Hc*: Holding cost, *Sl*: Service level, *Sc*: Set-up cost,

Mc: Maintenance cost, L4L: Lot for lot policy, FPO: Fixed Period Ordering policy, POQ: periodic order quantity, EOQ: economic order quantity,

MIP: Mixed integer programming, EV: Expectation value.

Table 1: Assembly systems: summary and classification of publications.

115 as a classic facility location-based model, were no longer valid. To analyze the effect of lead-time uncertainty, they
116 measured different indicators (backlogging costs, inventory-holding costs, service level and set-up cost), modeled
117 some policies (periodic order quantity (POQ), lot for lot (L4L), economic order quantity (EOQ)) and used several
118 optimization approaches to find optimal release dates for components.

119 In the cases of one-period planning and two-level assembly systems, [Tang & Grubbström \(2003\)](#) modeled both
120 process times and lead-times for components. They considered a fixed demand, unlimited capacity, and a known
121 due date. A Laplace procedure was used to calculate the optimal safety lead-times to minimize backlogging and
122 inventory holding costs. Later, [Hnaïen et al. \(2009\)](#) and [Fallah-Jamshidi et al. \(2011\)](#) treated the same problem using
123 genetic algorithms (GAs) to minimize the expected value of the same total cost. [Hnaïen et al. \(2009\)](#) supposed that
124 components at level 1 of the BOM were stored and that the finished product was only assembled after the given due
125 date. [Fallah-Jamshidi et al. \(2011\)](#) explored the same problem but in a multi-objective context, and reinforced the GA
126 by a reliable technique called the electromagnetism-like mechanism.

127 [Hnaïen et al. \(2016\)](#) studied the case of a one-period inventory model for a one-level assembly system under
128 stochastic demand and lead-times. An analytical model and a Branch and Bound (B&B) approach were found to
129 optimize the component quantities and planned lead-times. Although the authors limited their model to one period,
130 it can be extended to multi-period planning to account for possible trade-offs between stocks from different periods.
131 Building on this work, [Borodin et al. \(2016\)](#) proposed a new approach based on chance-constrained programming and
132 a linear model to solve the same problem. However, they limited themselves to one-level assembly systems because
133 of the dependency between levels. This obstacle can easily be overcome, as in this paper, using separate lead-time
134 uncertainty models at each level.

135 [Atan et al. \(2016\)](#) recently considered a final assembly process fed by a multi-stage parallel process. Each stage
136 had a stochastic throughput time and the system was controlled by planned lead-times at each stage. The authors
137 developed an iterative heuristic to optimize the planned lead-times for different stages and to minimize the expected
138 cost of a customer order. The problem studied contained a one-assembly process which, if modeled differently by
139 adding planned assembly dates at intermediate levels, could be considered a special case of our study in this paper.
140 Therefore, it could be modeled in this way and resolved using our optimization approach. More recent papers ([Jansen
141 et al., 2019, 2018](#)) feature an interesting investigation into the combination of planned lead-time and commitment
142 lead-time decisions. In [Ben-Ammar et al. \(2018\)](#), the authors provided a generalized probabilistic model and a B&B
143 algorithm to optimize multi-level assembly systems for the case of one customer demand. The objective function is
144 composed of inventory holding costs for components at all levels, and holding and backlogging costs for the finished
145 product. This total cost is expressed as a recursive procedure taking into account inter-level dependency. However,
146 the limitation of this earlier study was that each lead-time varies between 1 and a known upper limit, whereas we
147 observe that the recursive function requires a recursively enumerable domain and depends on the lead-time distribution
148 functions. The major drawback of this recursive procedure is its influence on computation time.

149 The purposes of this paper, based on the modeling approach presented in [Ben-Ammar et al. \(2018\)](#), are (i) to
150 develop a more general approach than is found in the models introduced by [Yano \(1987a\)](#), [Chu et al. \(1993\)](#), [Tang](#)

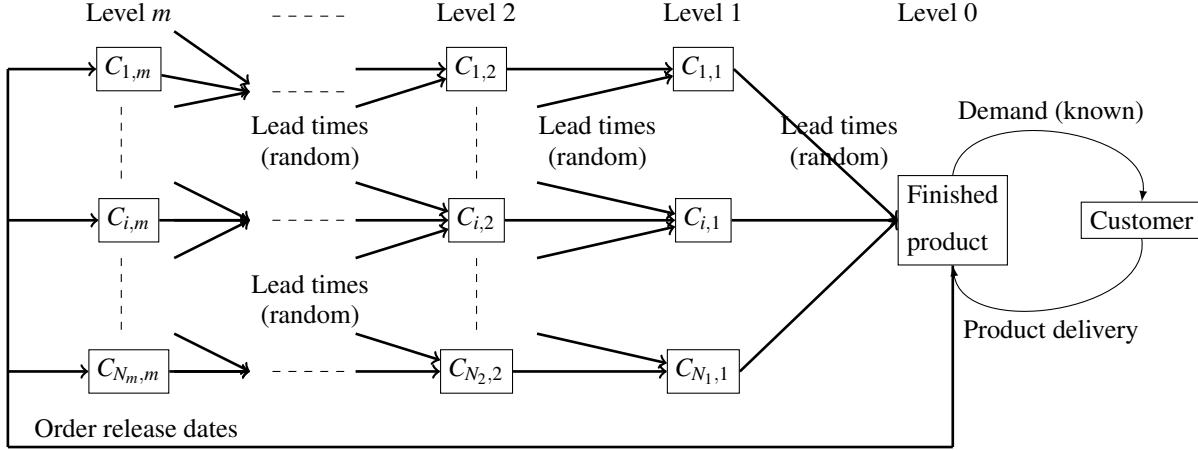


Figure 1: A multi-level assemble-to-order system.

151 & Grubbström (2003), Hnaïen et al. (2009), Fallah-Jamshidi et al. (2011), Ben-Ammar et al. (2018) and Ben-Ammar
 152 & Dolgui (2018), (ii) to propose a more efficient mathematical model, and (iii) to develop efficient algorithms to
 153 resolve large problems. Moreover, this paper extends several works. In Ben-Ammar & Dolgui (2018), the proposed
 154 mathematical model only serves to study two-level assembly systems. In Ben-Ammar et al. (2018), the mathematical
 155 model based on recursive formulae serves to study multi-level assembly systems, but only with lead-times varying
 156 between one and a fixed upper limit. Here, to eliminate this assumption but reduce the initial search space as in Ben-
 157 Ammar et al. (2018), we introduce an original technique based on the Newsboy model (see Section 4). In Ben-Ammar
 158 et al. (2016), the authors indicated that integrating several techniques such as a local search and perturbation is crucial
 159 to converge to solution values quickly, but little attention has been given to the theoretical basis of algorithm parameter
 160 setting nor to the computation time required to solve large problems with more than 6 levels in the BOM. Here, we
 161 address this gap by giving these analyses.

162 3. Problem assumptions

163 To be closer to real-world industrial planning methods, we consider a discrete temporal environment and integer
 164 decision variables. Figure 1 shows that the finished product is produced from components that are themselves obtained
 165 from the following level's components and so on.

166 To satisfy the customer's request for a tailored finished product, the planner configures the whole supply network
 167 according to the desired delivery date and so has to know when the overall processes should be released. Note that
 168 our approach is focused on the case of contracting with our customer in an ATO environment. We suppose that all
 169 supply chain partners are independent enterprises and that the supply chain will be managed in a decentralized manner.
 170 As in Ben-Ammar et al. (2018), we only determine order release dates for components at the last level of the BOM
 171 and we assume that there is no decision variable for internal levels (no possibility for accounting for future local
 172 decisions by supply chain partners, and we only know the lead-time probability distributions of our partners). The
 173 production cycle thus extends from order release dates of components at the last level to finished-product delivery

174 date. The assembly system is composed of m levels. At each level, we consider $\forall l = 1, \dots, m, N_l$ components. In this
 175 way, $\sum_{l=1}^m N_l$ components are needed to assemble the finished product. Let us assume that: (i) only components $c_{i,m}$
 176 ($i = 1, \dots, N_m$) at the last level m are ordered from external suppliers and their order release dates $X_{i,m}$ ($i = 1, \dots, N_m$)
 177 have to be defined; (ii) in local assembly units, components $c_{i,l}$ ($i = 1, \dots, N_l$) at intermediate levels ($l = 1, \dots, m-1$) are
 178 assembled without any decision possible on their start dates; (iii) suppliers and local assembly units are independent.

179 As in [Ben-Ammar et al. \(2018\)](#), without loss of generality, we introduce the following assumptions: the finished
 180 product demand D is known and equal to 1, and to assemble it, only one unit of each component is required. The unit
 181 backlogging cost b and the unit inventory holding cost r for the finished product, and the unit inventory holding cost
 182 $h_{i,l}$ for the component $c_{i,l}$, are known.

183 The following notations are used in this paper:

Parameters

T	Due date for the finished product, $T > 0$
D	Demand (known) for the finished product at the date T , without loss of generality, let $D = 1$
l	Level in a bill of material (BOM), $l = 1, \dots, m$
$c_{i,l}$	Component i of level l of the BOM
N_l	Number of components of level l
$S_{i,l}$	Set of components needed to assemble component $c_{i,l}$
$L_{i,l}$	Random lead-time for component $c_{i,l}$
$t_{i,l}$	Minimum value of $c_{i,l}$
$u_{i,l}$	Maximum value of $L_{i,l}$; each $L_{i,l}$ varies in $[t_{i,l}, u_{i,l}]$
$U_{i,m}$	The longest time between the release date for component $c_{i,m}$ and T . Equal to the maximum value of $\sum_{v=1}^m L_{i_v,v}$; $\forall \theta \in [1, m-1]$ and $\forall c_{i_{\theta+1},l} \in S_{i_{\theta},l-1}$
$T_{i,m}$	The shortest time between the release date for component $c_{i,m}$ and T . Equal to the minimum value of $\sum_{v=1}^m L_{i_v,v}$; $\forall \theta \in [1, m-1]$ and $\forall c_{i_{\theta+1},l} \in S_{i_{\theta},l-1}$
$h_{i,l}$	Unit holding cost for component $c_{i,l}$ per period
b	Unit backlogging cost of the finished product per period
r	Unit inventory holding cost for the finished product per period

Variables

$X_{i,m}$	Decision variable: release date for component $c_{i,m}$ (this type of variable is only defined for components at level m), $X_{i,m} \in [T - U_{i,m}; T - T_{i,m}]$
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Functions

$\mathbb{P}(\cdot)$	Probability value
$\mathbb{E}(\cdot)$	Expected value
$\mathbb{F}(\cdot)$	Cumulative distribution function

Table 2: Model notation and definition

184 We use the following notations to simplify several expressions:

- 185 • Assembly date for $c_{i,m-1}$: $M_{i,m-1} = \max_{c_{k,m} \in S_{i,m-1}} (X_{k,m} + L_{k,m})$
- 186 • Assembly date for $c_{i,l-1}$, $\forall 2 \leq l \leq m-1$, $\forall i \in \llbracket 1, N_l \rrbracket$: $M_{i,l-1} = \max_{c_{k,l} \in S_{i,l-1}} (M_{k,l} + L_{k,l})$
- 187 • Assembly date for the finished product: $M_{FP} = \max_{i \in \llbracket 1, N_1 \rrbracket} (M_{i,1} + L_{i,1})$
- 188 • Maximum between M_{FP} and due date T : $M_{FP}^+ = \max(M_{FP}, T)$
- 189 • Minimum between M_{FP} and due date T : $M_{FP}^- = \min(M_{FP}, T)$
- 190 • $\sum_{i=1}^{N_l} H_i = \sum_{i=1}^{N_l} (h_{i,l} - \sum_{c_{k,l+1} \in S_{i,l}} h_{k,l+1})$
- 191 • $H = \sum_{i=1}^{N_1} h_{i,1} + b$
- 192 • $R = r - \sum_{i=1}^{N_1} h_{i,1}$

193 4. Expected cost calculation

194 The main issue for this problem is how calculate the expected cost for different values of decision variables. The
 195 total cost $C(X, L)$ is the sum of the inventory holding or backlogging cost for the finished product and the inventory
 196 holding cost for components. Figure 2 gives an example of when the finished product is assembled after the due date.
 197 We note that $p_{i,j}$ is the planned lead time for component $c_{i,j}$ and $PM_{i,j}$ is its planned assembly date.

198 Because of the lead-time uncertainties:

- 199 • There is a backlog for the finished product if at least one type of component at level 1 is delivered after the due
 200 date T . Then the corresponding backlogging cost is equal to $b(M_{FP}^+ - T)$.
- 201 • If all components $c_{i,1}, \forall i = 1, \dots, N_1$, at level one are available before T , the finished product may be assembled
 202 and stored. The corresponding inventory holding cost is equal to $r(T - M_{FP}^-)$.
- The components $c_{k,m}$ at the last level m are ordered at date $X_{k,m}$ and delivered at date $L_{k,m} + X_{k,m}$. Assembly of
 component $c_{i,m-1}$ begins when all the necessary components $c_{k,m} \in S_{i,m-1}$ are available, i.e. at the date $M_{i,m-1}$.
 The holding cost of components $c_{k,m}$ at the last level m is equal to:

$$\sum_{i=1}^{N_{m-1}} \sum_{c_{k,m} \in S_{i,m-1}} h_{k,m} (M_{i,m-1} - (X_{k,m} + L_{k,m}))$$

- There are inventories for components $c_{i,l}$ ($l = 2, \dots, m-1$ and $c_{k,l} \in S_{i,l-1}$) during the time period between their
 arrival at $M_{k,l} + L_{k,l}$ and $M_{i,l-1}$, which is the assembly date for component $c_{i,l-1}$. The corresponding holding cost
 is equal to:

$$\sum_{l=2}^{m-1} \sum_{i=1}^{N_{l-1}} \sum_{c_{k,l} \in S_{i,l-1}} h_{k,l} (M_{i,l-1} - (M_{k,l} + L_{k,l}))$$

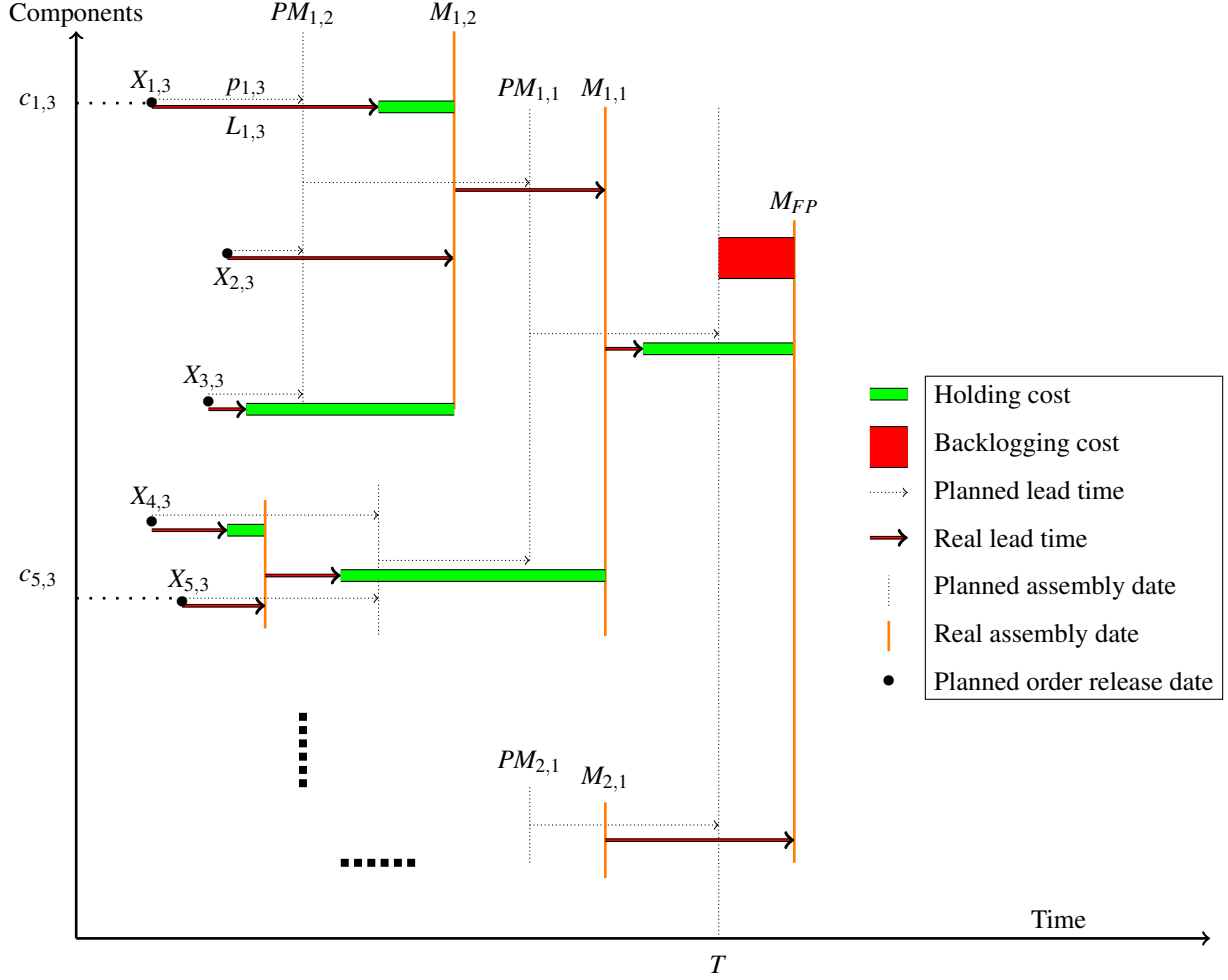


Figure 2: Composition of total cost (in the case of a backlog).

- There are inventories for components $c_{i,1}, \forall i = 1, \dots, N_1$ during the time period between their arrival at $M_{i,1} + L_{i,1}$ and M_{FP} , which is the assembly date for the finished product. The corresponding inventory holding cost is equal to:

$$\sum_{i=1}^{N_1} h_{i,1} (M_{FP} - (M_{i,1} + L_{i,1}))$$

The total cost $C(X, L)$ is a random discrete variable (because the lead-times $L_{i,j}, \forall i = 1, \dots, N_j$ and $\forall j = 1, \dots, m$, and assembly dates $M_{i,j}$ are random variables). Its explicit form is demonstrated in [Ben-Ammar et al. \(2018\)](#) and it reads as follows:

$$C(X, L) = \sum_{i=1}^{N_1} h_{i,1} M_{FP} + \sum_{l=1}^{m-1} \sum_{i=1}^{N_l} H_i M_{i,l} - \sum_{l=1}^m \sum_{i=1}^{N_l} h_{i,l} L_{i,l} - \sum_{i=1}^{N_m} h_{i,m} X_{i,m} + b(M_{FP}^+ - T) - r(T - M_{FP}^-) \quad (1)$$

with $L = (L_{1,1}, \dots, L_{i,1}, \dots, L_{N_1,1}, \dots, L_{1,m}, \dots, L_{i,m}, \dots, L_{N_m,m})$ and $X = (X_{1,m}, \dots, X_{i,m}, \dots, X_{N_m,m})$.

An explicit form of the mathematical expectation of total cost $\mathbb{E}(C(X, L))$ was proposed in [Ben-Ammar et al. \(2018\)](#). Here we develop a new explicit form that avoids using a recursive function and allows us to model actual lead-times $L_{i,l}$ whose variations are between any two limits $t_{i,l}$ and $u_{i,l}$. Note that integrating these two limits, which

are time intervals in which assembly dates $M_{i,l}$ vary, is calculated to substantially reduce computation times. Let Γ be a positive random discrete variable with a finite number of possible values and $\mathbb{F}_\Gamma(\cdot)$ be its cumulative distribution function. Its expected value is equal to:

$$\mathbb{E}(\Gamma) = \sum_{s \geq 0} (1 - \mathbb{P}(\Gamma \geq s)) = \sum_{s \geq 0} (1 - \mathbb{F}_\Gamma(s)) \quad (2)$$

204 In Hnaïen et al. (2009, 2010); Fallah-Jamshidi et al. (2011); Sakiani et al. (2012); Hnaïen et al. (2016); Borodin
 205 et al. (2016); Guiras et al. (2019); Ben-Ammar & Dolgui (2018), this expression (2) is used to calculate $\mathbb{E}(M_{i,l})$,
 206 $\mathbb{E}(M_{FP}^-)$, $\mathbb{E}(M_{FP}^+)$ and $\mathbb{E}(M_{FP}^-)$ for two-level assembly systems. In Ben-Ammar et al. (2018), the authors were the first
 207 to propose a recursive function that expressed the dependence among levels and enabled us to study assembly systems
 208 with more than two levels.

209 **Definition 4.1.** (Ben-Ammar et al., 2018) Let Q^+ be the recursive function that serves to express the dependence
 210 among levels. All assembly dates $M_{i,l}$, M_{FP}^- and M_{FP}^+ are positive random discrete variables with a finite number of
 211 possible values. Their expected values are as follows:

$$\mathbb{E}(M_{i,l}) = \sum_{s \geq 0} (1 - Q^+(L_{i,l}, s, l)) \quad (3)$$

$$\mathbb{E}(M_{FP}^-) = \sum_{0 \leq s \leq T-1} (1 - Q^+(L_{i,1}, s, 1)) \quad (4)$$

$$\mathbb{E}(M_{FP}^+) = T + \sum_{s \geq T} (1 - Q^+(L_{i,1}, s, 1)) \quad (5)$$

212 Nevertheless, as mentioned in the Appendix in Ben-Ammar et al. (2018), calculating the expected value for an
 213 assembly date of a given item (component or finished product) requires using the probability distributions of all the
 214 components of which it is composed. Here we propose an iterative approach to calculate the cumulative distribution
 215 function for each assembly date.

Proposition 4.1.

$$\mathbb{F}_{i,m-j-1}(s) = \begin{cases} \prod_{c_{k,m} \in S_{i,m-1}} \mathbb{F}_{k,m}(-X_{k,m} + s) & \text{if } j = 0 \\ \prod_{c_{k,m-j} \in S_{i,m-j-1}} \sum_{\substack{o_j + w_j = s \\ o_j + w_j \in \mathbb{N}}} \mathbb{P}(L_{k,m-j} = o_j) \mathbb{F}_{k,m-j}(w_j) & \text{if } j \in \llbracket 1, \dots, m-1 \rrbracket \end{cases} \quad (6)$$

216 where $\mathbb{F}_{0,0}(s)$ is the cumulative distribution function of the finished-product assembly date.

217 *Proof.* see Appendix. ■

218 In Hnaïen et al. (2009, 2010); Fallah-Jamshidi et al. (2011); Sakiani et al. (2012); Hnaïen et al. (2016); Borodin
 219 et al. (2016); Guiras et al. (2019); Ben-Ammar & Dolgui (2018), the authors assumed that all lead-times $L_{i,l}, \forall l \in \llbracket 1, 2 \rrbracket$
 220 and $\forall i \in \llbracket 1, N_2 \rrbracket$, varied between 1 and a fixed upper limit. These studies considered assembly dates as random discrete
 221 variables s varying between 0 and $T + U - 2$ with $U = \max(U_{i,2})$ and $U_{i,2} = u_{k,2} + u_{i,1}$ the maximum value of $L_{k,2} + L_{i,1}$
 222 ($c_{k,2} \in S_{i,1}, L_{k,2} + L_{i,1} \in \llbracket 2, U_{k,2} \rrbracket$). In Ben-Ammar et al. (2018), the authors generalized these limits and defined s as a

223 natural number such as $0 \leq s \leq T + m(u - 1)$. Here we suppose that each lead-time $L_{i,l}$, $\forall l \in \llbracket 1, m \rrbracket$ and $\forall i \in \llbracket 1, N_m \rrbracket$
 224 varies between $t_{i,l}$ and $u_{i,l}$. The mathematical expectation of the total cost, introduced in Ben-Ammar et al. (2018),
 225 can be simplified using expression (6).

Proposition 4.2. *The mathematical expectation of the total cost, noted $\mathbb{E}(C(X, L))$, can be simplified and written as follows:*

$$\begin{aligned} \mathbb{E}(C(X, L)) = & R \cdot \left(\sum_{s=\alpha_{0,0}}^{T-1} \mathbb{F}_{0,0}(s) \right) - \sum_{l=1}^{m-1} \sum_{i=1}^{N_l} H_i \cdot \left(\beta_{i,l} - \sum_{s=\alpha_{i,l}}^{\beta_{i,l}-1} \mathbb{F}_{i,l}(s) \right) \\ & + H \cdot \left(\beta_{0,0} - \sum_{s=T}^{\beta_{0,0}-1} \mathbb{F}_{0,0}(s) \right) - b \cdot T - \sum_{l=1}^m \sum_{i=1}^{N_l} h_{i,l} \mathbb{E}(L_{i,l}) - \sum_{i=1}^{N_m} h_{i,m} \mathbb{E}(X_{i,m}) \end{aligned} \quad (7)$$

226 where:

$$227 \alpha_{i,l} = \max_{c_{k,l+1} \in \mathcal{S}_{i,l}} (\alpha_{k,l+1} + t_{k,l+1}) \text{ with } \alpha_{i,m} = X_{i,m} \text{ and } \alpha_{0,0} = \max_{i \in \llbracket 1, N_1 \rrbracket} (\alpha_{i,1} + t_{i,1}).$$

$$228 \beta_{i,l} = \max_{c_{k,l+1} \in \mathcal{S}_{i,l}} (\beta_{k,l+1} + u_{k,l+1}) \text{ with } \beta_{i,m} = X_{i,m}.$$

229 *Proof.* see Appendix. ■

230 The intervals $T - U_{i,m} \leq X_{i,m} \leq T - m$ are the initial search space, which depends on maximum and minimum
 231 lead-times and on the number of levels. To reduce their upper limits, the multi-level assembly system is decomposed
 232 to N_m (the number of components at level m) multi-level linear supply chains. A finished product is delivered by each
 233 linear chain i , $i \in \llbracket 1, N_m \rrbracket$ on a specified delivery date ψ_i . Two costs are taken into account: (i) if a given finished
 234 product is delivered after the due date T , a backlogging cost is considered, and (ii) if it arrives before T , an inventory
 235 holding cost is assumed. The optimal order release date, noted X_i^{**} , for one linear chain is used to reduce the initial
 236 search space for the corresponding component release date at the last level in the BOM.

Definition 4.2. (Ben-Ammar et al., 2018)

$$T - U_{i,m} \leq X_{i,m} \leq X_i^{**} \quad \forall i \in \llbracket 1, N_m \rrbracket$$

where the optimal order release date X_i^{**} satisfies the optimality condition for the discrete Newsboy model:

$$\mathbb{F}(T - X_i^{**} - 1) \leq \frac{b}{b+r} \leq \mathbb{F}(T - X_i^{**}) \quad \forall i \in \llbracket 1, N_m \rrbracket \quad (8)$$

237 and where $\mathbb{F}(\cdot)$ is the cumulative distribution function of the total lead-time L associated with linear chain i .

238 In order to solve this non-linear probabilistic problem, in Ben-Ammar et al. (2018) the authors developed bounds
 239 and proposed a B&B procedure. Its efficiency depends on the number of levels and the ratio between b (the unit
 240 backlogging cost of the finished product per period) and r (the unit inventory holding cost for the finished product per
 241 period). In this work, we observed that optimal solutions are computed exactly for small and medium-sized problems
 242 with at most 40 components and 3 levels in the BOM. Nevertheless, this branch-and-bound-based approach may not
 243 be practical for assembly systems with more than 3 levels in the BOM, as it carries the key limitation that it depends
 244 on both (i) backlogging and inventory holding costs and (ii) the number of levels in the structure.

245 In Ben-Ammar et al. (2013), the authors tried to develop metaheuristics to determine good quality approximate
 246 solutions for larger problems. To achieve a more effective search, a first optimization approach was based on the
 247 integration of the classic genetic algorithm (CGA). However, we observed major drawbacks with this approach: (i)
 248 CPU time depends heavily on the number of levels in the assembly system, and (ii) a CGA is not guaranteed to
 249 converge to a global minimum - even though the stopping criterion is fixed to 1000 iterations and the number of
 250 components at the last level m (size of each individual). In Ben-Ammar et al. (2016), several techniques were proposed
 251 to reinforce the CGA, such as a reduction in the initial research space (RSR), a local search (LS), and an integration
 252 of perturbation (P). These techniques only seem to be efficient enough to optimize two-level assembly systems with
 253 less than 100 components at the last level of the BOM, even though the authors did not determine the set of parameter
 254 values that yields the best performance of the hybrid genetic algorithm (HGA).

255 Even though these works improved the efficiency of the HGA, most of the improvements were scattered across
 256 several studies that are not realistically comparable in terms of methodology and results. Nonetheless, it is possible to
 257 improve the efficiency of the approach further, and group the results. With this goal in mind, the next section describes
 258 the techniques used to reinforce the CGA, and explains the experiments carried out in order to study the behavior of
 259 the HGA.

260 5. Optimization model and approach

The goal is to find the optimal order release dates for the components at level m to minimize the total expected cost $\mathbb{E}(C(X, L))$. The optimisation problem is as follows:

$$\min(\mathbb{E}(C(X, L)))$$

Subject to constraints:

$$\mathbb{E}(M_{FP}) = \beta_{0,0} - \sum_{s=T}^{\beta_{0,0}-1} \mathbb{F}_{0,0}(s) - \sum_{s=\alpha_{0,0}}^{T-1} \mathbb{F}_{0,0}(s) \quad (9)$$

$$\mathbb{E}(M_{FP}^+) = \beta_{0,0} - \sum_{s=T}^{\beta_{0,0}-1} \mathbb{F}_{0,0}(s) \quad (10)$$

$$\mathbb{E}(M_{FP}^-) = T - \sum_{s=\alpha_{0,0}}^{T-1} \mathbb{F}_{0,0}(s) \quad (11)$$

$$\mathbb{E}(M_{i,l}) = \beta_{i,l} - \sum_{s=\alpha_{i,l}}^{\beta_{i,l}-1} \mathbb{F}_{i,l}(s) \quad \forall i \in \llbracket 1, N_l \rrbracket, \forall l \in \llbracket 1, m-1 \rrbracket \quad (12)$$

$$T - U_{i,m} \leq X_{i,m} \leq X_i^{**} \quad \forall i \in \llbracket 1, N_m \rrbracket \quad (13)$$

$$X_{i,m} \in \mathbb{N} \quad \forall i \in \llbracket 1, N_m \rrbracket \quad (14)$$

261 where:

262 • $\alpha_{i,l} = \max_{c_{k,l+1} \in S_{i,l}} (\alpha_{k,l+1} + t_{k,l+1})$ with $\alpha_{i,m} = X_{i,m}$ and $\alpha_{0,0} = \max_{i \in \llbracket 1, N_1 \rrbracket} (\alpha_{i,1} + t_{i,1})$,

263 • $\beta_{i,l} = \max_{c_{k,l+1} \in \mathcal{S}_{i,l}} (\beta_{k,l+1} + u_{k,l+1})$ with $\beta_{i,m} = X_{i,m}$.

264 The objective function which expresses the expected cost is nonlinear with integer variables, see (7). The assem-
 265 bly date for the finished product is equal to $\max_{i \in \llbracket 1, N_1 \rrbracket} (M_{i,1} + L_{i,1})$. Constraint (9) gives its mathematical expectation.
 266 Constraints (10) and (11), respectively, gives the mathematical expectation of the maximum/minimum, between the
 267 assembly date M_{FP} and the due date T . Constraints (12) express the mathematical expectation of assembly dates for
 268 components $c_{i,l}$. The search spaces for order release dates are defined in constraints (13). Finally, constraints (14) are
 269 the integrity constraints.

270 This study made partial use of the optimization approach introduced by Ben-Ammar et al. (2016). In our new
 271 approach, the CGA presented by Hnaïen et al. (2009) is reinforced by several techniques such as a reduced space of
 272 research (RSR), a local search (LS), and the integration of perturbation (P).

273 Let us use the following set of parameters:

- 274 • x_1 Maximum iteration number,
- 275 • x_2 Population size,
- 276 • x_3 Crossing-over probability,
- 277 • x_4 Mutation probability,
- 278 • x_5 Number of individuals from the initial population that can be a local minimum,
- 279 • x_6 Number of iterations that have passed without the best solution being improved (to apply the perturbation
 280 approach),
- 281 • x_7 Number of iterations that have passed without the best solution being improved (to apply the local
 282 search).

283 The various operations of the proposed HGA are described in Algorithm 1. The stop criterion is that the maximum
 284 iteration number, noted x_1 , is reached. The initial population is formed by x_2 individuals. Note that the chromosome
 285 representation, the mutation, the crossover and the neighborhood search are done in the same manner as in Hnaïen
 286 et al. (2009). However, the only differences concerning these operations are that (i) the length of a given chromosome
 287 is N_m , (ii) a given chromosome contains a sequence of order release dates $X_{i,m}$ (integer decision variables), (iii) the
 288 crossover and mutation probabilities, noted x_3 and x_4 , will be recalibrated.

289 As shown in Algorithm 1, the proposed HGA includes several steps. The following subsections discuss some of
 290 these steps in detail.

Algorithm 1: Proposed genetic algorithm

```

1 Function Best_Subset( $A, n$ ):
2   return  $S \subseteq A, S = n$  and  $s \in S, \nexists s' \in A \setminus S, \text{Fitness}(s') < \text{Fitness}(s)$ ;
3 End Function
4  $\text{card}(S_{LO}) \leftarrow x_5$ ;
5  $S_{Pop} \leftarrow \text{Initial\_Population}(x_2, x_5)$ ;
6 foreach  $j \in \llbracket 1, x_1 \rrbracket$  do
7   /* Reproduction Selection */
8    $S_{Parents} \leftarrow \text{Best\_Subset}(S_{Pop}, \frac{x_2}{2})$ ;
9   /* Evolutionary Operations */
10   $S_{Cross} \leftarrow \text{Crossover}(S_{Parents}, x_3)$ ;
11   $S_{Mut} \leftarrow \text{Mutation}(S_{Parents}, x_4)$ ;
12  /* Perturbation */
13  if  $\text{Not\_Convergence}(x_6 \text{ iterations})$  then
14     $S_{Pop} \leftarrow \text{perturbation}(S_{Parents} \cup S_{Cross} \cup S_{Mut} \setminus S_{LO}, \text{duplications})$ ;
15  end if
16  /* Local Search */
17  if  $\text{Not\_Convergence}(x_7 \text{ iterations})$  then
18     $\text{Best\_Solution} \leftarrow \text{Best\_Subset}(S_{Parents} \cup S_{Cross} \cup S_{Mut} \setminus S_{LO}, 1)$ ;
19    if  $\text{Best\_Solution} \notin S_{LO}$  then
20       $LS\_Sol \leftarrow \text{Best\_In\_Neighborhood}(\text{Best\_Solution})$ ;
21    end if
22    if  $LS\_Solution > \text{Best\_Solution}$  and  $\text{Best\_Solution} \notin S_{LO}$  then
23       $S_{LO} \leftarrow S_{LO} \cup \text{Best\_Solution}$ ;
24    else
25       $S_{Parents} \leftarrow S_{Parents} \cup LS\_Sol$ ;
26    end if
27  end if
28  /* Replacement Selection */
29   $S_{Pop} \leftarrow \text{Best\_Subset}(S_{Parents} \cup S_{Cross} \cup S_{Mut} \cup S_{LO}, \frac{N}{2})$ ;
30 end foreach

```

293 *5.1. Generation of initial population and local search*

294 Choosing the right population, of the right size, is crucial because it can affect the performance of the algorithm.
 295 We use the RSR (see Definition 4.2) exploring the property related to the distribution function of lead-times, and the
 296 initial population is generated randomly with uniform distribution as follows:

$$X_{i,m} = Rnd(T - U_{i,m}, X_i^{**}) \quad \forall i \in \llbracket 1, N_m \rrbracket$$

where $Rnd(T - U_{i,m}; X_i^{**})$ is a discrete random value between $T - U_{i,m}$ and X_i^{**} .

In this suggested algorithm, we assume that x_5 individuals can be a local minimum. The procedure, introduced in Hnaïen et al. (2009) and applied one time to obtain the best neighbor solution (*Best_In_Neighborhood*), is considered. Starting out from one individual, all neighbouring solutions are explored and the best one is selected to undergo the same operation again. This procedure is applied several times until a local minimum is met. Note that this procedure (*Best_In_Neighborhood*) is applied to the best solution at a given iteration if, and only if, (i) the best solution is not a local minimum, and (ii) x_7 iterations have passed without the best solution being improved.

5.2. Perturbation approach

The perturbation consists of replacing the solutions which have the same cost by solutions undergoing a special mutation (using block mutation). Each duplication $X = (X_{1,m}, \dots, X_{i,m}, \dots, X_{N_m,m})$ undergoes a modification. To do so, an item (finished product or component) is selected randomly. The order release dates for components $c_{v,m}, \dots, c_{w,m}$ at the last level m needed to assemble this item undergo a block mutation. Note that this approach is applied to duplications at a given iteration if, and only if, x_6 iterations have passed without the best solution being improved.

6. Computational experiments

In this section, we first present the data generation process, then we illustrate the main contribution of this paper and go on to describe the experiments carried out in order to study the effect of multiple factors on HGA performance. We finish by analyzing the performance of the HGA.

6.1. Random test instances

We compared our HGA against a heuristic on 140 randomly generated test instances, defined in the following way. First, we consider 14 different BOM, and randomly generate 10 instances for each BOM. For different BOMs, the total number of levels is equal to 8, and the total number of components ($Comp_{total}$) at all levels is equal to 39. We distinguish two types of BOM: B_m and C_m . In the intermediate levels ($l \in \llbracket 1, m - 1 \rrbracket$) of B_m , the number of components at each level N_l is equal to 2, for C_m , $N_l \geq 2$. Table 3 gives the BOM of each assembly system.

Each component lead-time $L_{i,j}, \forall j \in \llbracket 1, m \rrbracket, \forall i \in \llbracket 1, N_j \rrbracket$ varies between two limits $t_{i,j}$ and $u_{i,j}$ with a discrete uniform probability distribution. These limits are generated randomly between 1 and 10 with $t_{i,j} < u_{i,j}$. The unit inventory holding costs $h_{i,j}, \forall j \in \llbracket 1, m \rrbracket, \forall i \in \llbracket 1, N_j \rrbracket$ for components $c_{i,j}$ and the unit inventory holding and backlogging costs for the finished product are generated as follows: (i) for components at the last level m , $h_{i,m}$ are random between 1 and 40; (ii) for components at level $l \in \llbracket 1, m - 1 \rrbracket$, $h_{i,l}$ is random between $\alpha = \sum_{c_{k,l} \in S_{i,l}} h_{k,l}$ and 1.2α ; (iii) for the finished product, the unit inventory holding cost r is random between $\alpha = \sum_{c_{k,l} \in S_{FP}} h_{k,l}$ and 1.2α ; and (iv) the backlogging cost b is made to vary in $\{0.1r, 0.5r, r, 5r, 10r\}$.

	B_2	C_2	B_3	C_3	B_4	C_4	B_5	C_5	B_6	C_6	B_7	C_7	B_8	C_8
N_1	2	4	2	2	2	4	2	2	2	2	2	2	2	2
N_2	37	35	2	4	2	7	2	4	2	4	2	3	2	3
N_3			35	33	2	9	2	8	2	8	2	4	2	3
N_4					33	19	2	9	2	8	2	5	2	4
N_5							31	16	2	8	2	7	2	5
N_6									29	9	2	8	2	5
N_7											27	10	2	6
N_8													25	11
$Compt_{total}$	39	39	39	39	39	39	39	39	39	39	39	39	39	39

Table 3: The BOM of each assembly system

327 6.2. Comparison between fitness functions

328 Before we go on to explain the theoretical basis of our HGA parameter setting, we first present the main contribution
329 of this paper. In order to evaluate the efficiency of the proposed model, we carried out several experiments based on
330 the test instances described above.

331 In this paper, the fitness function, which is an important part of the GA, is the expected total cost $\mathbb{E}(C(X,L))$. A
332 first explicit form of this expected value, noted $V(\cdot)$, was proposed in Ben-Ammar et al. (2018). Here we propose a
333 new explicit form, noted $V^*(\cdot)$ (see Propositions 4.1 and 4.2), which avoids using a recursive function, and can model
334 real-world lead-times $L_{i,l}$ whose variations are logically between any two limits $t_{i,l}$ and $u_{i,l}$.

335 We design a CGA to compute each instance 5 times using $V(\cdot)$ or $V^*(\cdot)$ as the fitness function. Without preliminary
336 testing or parameter calibration, the following parameters are fixed: maximum iteration number; noted x_1 ; is equal to
337 1000; population size x_2 is equal to 60 chromosomes; crossing-over probability x_3 is fixed to 0.95; and mutation
338 probability x_4 is equal to 0.05. The CGA and the mathematical models, which calculate the fitness functions $V(\cdot)$ and
339 $V^*(\cdot)$, are coded in C++. The experiments were carried out on a computer with a 1.66 GHz Intel Core 2 Duo CPU and
340 4 GB of RAM.

341 Fig. 3 illustrates the average CGA execution time after 1000 iterations. $V(B)$ and $V^*(B)$ use the first type of BOM
342 (B_m) and $V(C)$, and $V^*(C)$ use the second type (C_m). Figure 4 shows that the CGA using our new explicit form of
343 $\mathbb{E}(C(X,L))$ as fitness function is less sensitive to the increase in number of levels m than the CGA which uses the
344 earlier form given in our last work. Thus, $V^*(\cdot)$ will be used as the fitness function in the HGA.

345 6.3. Calibration of parameters and performance measures.

346 In order to find the set of HGA parameters that guarantees optimal performance for our algorithm, we decided
347 to use a ‘design of experiments’ approach based on a standard central composite design (CDD). Based on a reduced
348 number of experiments (see below for details), this kind of 3 level experimental design allows us to fit efficiently a
349 quadratic multivariate regression model whose optimum can be searched for. The set of parameters being investigated

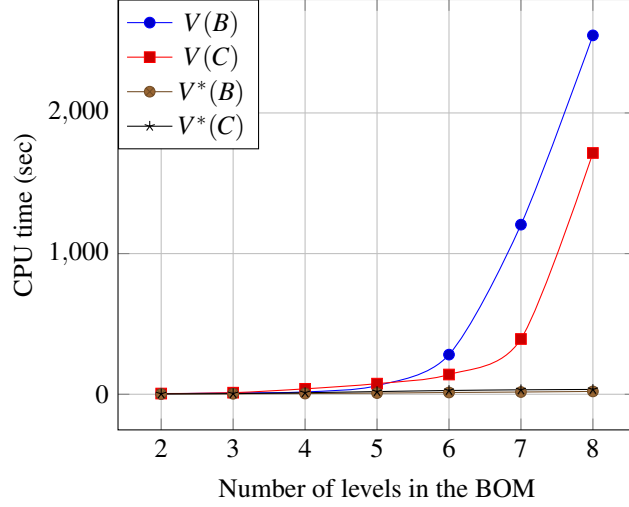


Figure 3: Comparison between fitness functions.

350 is x_1, x_2, \dots, x_7 , which are set out in Section 5. The min, middle and max levels for these studied parameters are given
 351 in Table 4:

Parameters	Levels		
	Min	Middle	Max
x_1	50	100	150
x_2	40	60	80
x_3	0.85	0.90	0.95
x_4	0.05	0.10	0.15
x_5	0	1	2
x_6	0	10	20
x_7	0	10	20

Table 4: Parameters and levels tested.

Note that the backloging cost b is equal to r and a single set of instances was considered. There are 42 instances in each set ($3B_2, \dots, 3B_8, 3C_2, \dots, 3C_8$). Using each combination of parameter values, the algorithm was applied in the following way. The set of instances was executed 5 times, and, for each execution, we determined the same single performance indicator i.e. the average expected total cost $\mathbb{E}(TC)$ of the best solution found. In order to obtain the $p = 7$ optimal parameters $x^* = (x_1^*, \dots, x_7^*)$ of HGA for $\mathbb{E}(TC)$, we used a standard CCD with $n = 79$ experiments and $k = 5$ replicates, i.e. 395 observations. The CCD structure thus generated allows us to estimate the following quadratic model with 36 coefficients (a_0, a_1, \dots, a_{35}):

$$y = a_0 + a_1x_1 + \dots + a_7x_7 + a_8x_1x_2 + a_9x_1x_3 + \dots + a_{28}x_6x_7 + a_{29}x_1^2 + \dots + a_{35}x_7^2 + \varepsilon$$

352 where ε is an error term. Once this model was obtained, we searched for the solution x^* minimizing this quadratic

353 model in the hypercube defined by the *min* and *max* levels listed in Table 4. For $\mathbb{E}(TC)$, Table 5 (column headed
 354 ‘coeff’) gives the coefficients $(a_0, a_1, \dots, a_{35})$.

355 All the rows in Table 5 marked with ‘ $a = 0$ ’ correspond to the regression coefficients (‘coeff’) a_0, a_1, \dots, a_{35} for
 356 which the p-value (p-val) is larger than the standard threshold value $\alpha = 0.05(5\%)$. As these coefficients are considered
 357 statistically equal to 0, their corresponding parameters or parameter combinations should therefore have no influence
 358 on response y . For example, concerning $\mathbb{E}(TC)$, parameters x_3, x_4 and x_6 do not seem to be influential. The estimated
 359 model appears to fit the experiments well, as the coefficient of determination $R^2 = 0.98216$ is very close to 1. Thus, the
 360 vector of optimal parameters is $x^* = (150, 80, 0.9057, 0.141, 1.20, 11.87, 10.93)$. Knowing that x_5, x_6 and x_7 must be
 361 integer values, they are then rounded to the nearest whole the number. Finally, $x^* = (150, 80, 0.9057, 0.141, 1, 12, 11)$.

362 6.4. Performance analysis of HGA

363 For each instance, 100 tests were carried out. The B&B method proposed in Ben-Ammar et al. (2018) requires
 364 more than 10 hours to find exact solutions. The CPU times depend not only on the number of components in the last
 365 level m , the number of levels m and the different costs, but are also significantly dependant on $t_{i,l}$ and $u_{i,l}$, the lower
 366 and upper limits of the actual lead-times $L_{i,l}$.

367 Note that the HGA is reinforced by several techniques, such as a reduced space of research (RSR), a local search
 368 (LS) and the integration of perturbation (P). We analyzed the influence of several parameters: (i) type of BOM; (ii)
 369 number of levels; and (iii) ratio of backlogging-to-inventory costs (b/r) for the finished product. To analyze the
 370 performance of the optimization approach, several notions were introduced:

- 371 • Average number of iterations in which the best solution is found;
- 372 • Average *gap* between the best solution in the initial population and the best solution $best_{sol_{150}}$ found by the
 373 algorithm after 150 iterations: $gap = \frac{best_{sol_0} - best_{sol_{150}}}{best_{sol_{150}}} \cdot 100$;
- 374 • Average gap^* between $best_{sol_{150}}$ and the best-known solution $best_{BKS}$ found among the 100 tests: $gap^* =$
 375 $\frac{best_{sol_{1000}} - best_{BKS}}{best_{BKS}} \cdot 100$;
- 376 • Average CPU time of the HGA when the best solution is found.

377 Table 6 shows that the number of iterations needed to find the best known solution (BKS) is unaffected by the type
 378 of BOM and the number of levels. The efficiency of RSR translates through the fact that a big backlogging-to-inventory
 379 costs ratio (b/r) slightly reduces the average number of iterations needed to find the BKS. Note too that regardless of
 380 the variation of parameters, fewer than 60 iterations are required.

381 Table 7 shows the evolution of the average gap as a function of all parameters. This evolution becomes substantial
 382 in the case of either a small ratio b/r and/or small number of levels in the BOM. This is explained by the fact that the
 383 search space is biggest for a small b/r , and the number of components at the last level is big for the case of a small m .

384 Table 8 shows that, for assembly systems composed of 40 items, the HGA seems to be efficient and is not parameter-
 385 dependent. Nevertheless, its performance needs to be further evaluated for more complex systems involving more than
 386 40 items, and for a small backlogging cost for the finished product.

Parameters (individual or combinations)	coeff	95% confidence interval		p-val
1	$a_0=4928.12$	809.436	9046.81	0.01916
x_1	$a_1=-2.51915$	-3.88487	-1.15343	0.00033
x_2	$a_2=-7.52967$	-11.9796	-3.07973	0.00097
x_3	$a_3=-5967.82$	-15201.8	3266.11	0.20455 = 0
x_4	$a_4=-12721.2$	-26378.4	936.033	0.06781 = 0
x_5	$a_5=-638.405$	-690.446	-586.364	0.00000
x_6	$a_6=-2.00088$	-7.205	3.20325	0.45008 = 0
x_7	$a_7=-7.00097$	-12.2051	-1.79685	0.00851
x_1x_2	$a_8=0.00871653$	0.0062748	0.0111583	0.00000
x_1x_3	$a_9=-0.202188$	-1.17888	0.774504	0.68417 = 0
x_1x_4	$a_{10}=-6.90688$	-16.6738	2.86004	0.16517 = 0
x_1x_5	$a_{11}=0.892039$	0.843205	0.940874	0.00000
x_1x_6	$a_{12}=-0.0058632$	-0.0107466	-0.00097973	0.01875
x_1x_7	$a_{13}=-0.0068445$	-0.0117279	-0.00196098	0.00614
x_2x_3	$a_{14}=0.366719$	-2.07501	2.80845	0.76789 = 0
x_2x_4	$a_{15}=-24.2203$	-48.6376	0.196984	0.05187 = 0
x_2x_5	$a_{16}=2.00968$	1.88759	2.13176	0.00000
x_2x_6	$a_{17}=0.0171098$	0.0049012	0.0293185	0.00615
x_2x_7	$a_{18}=0.00766734$	-0.0045413	0.019876	0.21761 = 0
x_3x_4	$a_{19}=3452.37$	-6314.54	13219.3	0.48741 = 0
x_3x_5	$a_{20}=65.6406$	16.806	114.475	0.00857
x_3x_6	$a_{21}=-2.27581$	-7.15927	2.60765	0.36003 = 0
x_3x_7	$a_{22}=5.11219$	0.228728	9.99565	0.04024
x_4x_5	$a_{23}=2397.03$	1908.69	2885.38	0.00000
x_4x_6	$a_{24}=46.2944$	-2.54022	95.129	0.06310 = 0
x_4x_7	$a_{25}=37.1494$	-11.6852	85.984	0.13553 = 0
x_5x_6	$a_{26}=0.637528$	0.393355	0.881701	0.00000
x_5x_7	$a_{27}=0.225116$	-0.0190573	0.469289	0.07065 = 0
x_6x_7	$a_{28}=0.0270297$	0.00261239	0.051447	0.03013
x_1^2	$a_{29}=0.0030473$	-0.002081	0.0081758	0.24336 = 0
x_2^2	$a_{30}=0.0195109$	-0.012542	0.0515638	0.23206 = 0
x_3^2	$a_{31}=3208.54$	-1919.91	8337	0.21936 = 0
x_4^2	$a_{32}=309094$	-203751	821940	0.23669 = 0
x_5^2	$a_{33}=99.8044$	86.9832	112.626	0.00000
x_6^2	$a_{34}=0.0781836$	-0.0500278	0.206395	0.23123 = 0
x_7^2	$a_{35}=0.0763036$	-0.0519078	0.204515	0.24262 = 0

Table 5: Standard CDD results for $\mathbb{E}(TC)$.

		<i>m</i>						
<i>b/r</i>	2	3	4	5	6	7	8	
B	0.1	37.10	49.09	58.56	45.95	36.88	45.09	17.42
	0.5	50.31	21.24	18.42	31.21	14.97	5.75	7.37
	1	2.22	1.61	10.54	16.43	11.88	16.43	4.27
	5	16.92	4.26	17.14	18.67	28.47	15.76	7.66
	10	9.22	9.48	3.63	1.00	1.00	3.37	14.42
	25	17.23	10.76	16.55	5.07	6.04	1.00	4.11
	50	1.33	6.40	7.66	15.37	33.84	14.33	12.38
	100	1.00	7.23	9.81	2.26	2.81	8.29	5.33
	200	1.00	1.00	1.00	11.72	1.00	1.00	1.10
	500	1.00	1.00	1.93	5.22	12.98	1.00	9.67
C	0.1	24.16	31.45	29.89	3.64	10.40	12.22	7.98
	0.5	42.57	50.21	43.51	25.47	24.93	9.20	9.29
	1	18.65	23.52	18.19	6.63	5.49	6.46	12.58
	5	7.79	1.00	9.09	2.95	1.00	2.23	2.81
	10	6.14	4.62	9.58	8.24	7.93	1.06	3.56
	25	2.87	7.99	6.81	1.32	2.59	5.48	5.73
	50	1.00	3.38	3.30	2.48	1.00	1.76	6.08
	100	1.00	9.13	12.61	5.00	4.39	1.18	1.88
	200	1.00	1.00	3.31	1.00	1.61	2.80	8.69
	500	1.00	1.00	4.88	2.67	1.00	2.04	3.60

Table 6: Average number of iterations in which the best solution is found.

		<i>m</i>						
<i>b/r</i>	2	3	4	5	6	7	8	
B	0.1	5.77	4.96	5.36	3.44	2.96	3.39	0.45
	0.5	12.76	1.50	2.19	1.33	1.97	0.03	0.30
	1	0.00	0.16	0.51	0.09	1.61	0.09	0.08
	5	2.12	1.99	0.21	1.23	0.52	0.92	0.00
	10	1.40	0.70	0.05	0.00	0.00	0.01	0.11
	25	3.52	0.41	0.19	0.00	0.02	0.00	0.00
	50	0.64	0.12	0.43	0.87	0.46	0.16	0.08
	100	0.00	0.17	0.12	0.00	0.33	0.11	0.17
	200	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	500	0.00	0.00	0.19	0.36	0.09	0.00	0.00
C	0.1	3.61	4.10	0.78	0.00	0.14	0.20	0.42
	0.5	9.25	3.81	1.78	2.48	6.83	0.09	0.01
	1	3.15	0.57	0.94	0.16	0.19	0.74	1.16
	5	0.95	0.00	0.17	0.00	0.00	0.00	0.00
	10	2.94	0.86	0.64	0.18	1.54	0.00	0.21
	25	1.04	0.00	0.01	0.00	0.00	0.29	0.37
	50	0.00	0.00	0.02	0.03	0.00	0.02	0.06
	100	0.00	0.25	0.45	0.26	0.37	0.00	0.01
	200	0.00	0.00	0.08	0.00	0.00	0.16	0.41
	500	0.00	0.00	0.02	0.00	0.00	0.17	0.21

Table 7: Average *gap* (%).

		<i>m</i>						
<i>b/r</i>		2	3	4	5	6	7	8
B	1.21	0.00	0.14	1.73	0.86	0.89	0.16	0.45
	0.66	1.26	0.12	0.00	0.33	0.00	0.00	0.30
	0.00	0.16	0.00	0.01	1.61	0.07	0.03	0.08
	0.00	0.00	0.00	0.09	0.45	0.00	0.00	0.00
	0.00	0.05	0.02	0.00	0.00	0.00	0.00	0.11
	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00
	0.00	0.00	0.15	0.00	0.00	0.00	0.00	0.08
	0.00	0.00	0.02	0.00	0.15	0.00	0.06	0.17
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C	3.60	1.26	0.14	0.00	0.02	0.00	0.33	0.42
	2.51	0.00	0.24	0.00	0.00	0.00	0.00	0.01
	0.00	0.00	0.07	0.00	0.00	0.00	0.00	1.16
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.21
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.01
	0.00	0.00	0.03	0.00	0.00	0.00	0.02	0.41
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21

Table 8: Average *gap** (%).

387 Table 9 presents the execution times. Note that the best solutions are always found in less than a second and that the
388 convergence to a good solution does not depend on number of levels, nor the type of the BOM, nor the backlogging-
389 to-inventory holding costs ratio.

390 For this problem, we used a heuristic and compared its performance against the HGA. It is the upper bound in-
391 troduced in Ben-Ammar et al. (2018) and detailed in Algorithm 2. The solution proposed by this heuristic is equal
392 to the minimum between two variables V_1 and V_2 . As mentioned in Definition 4.2, we decompose the multi-level
393 assembly system to N_m (the number of components at level m) multi-level linear supply chains. The different $X_{i_m,m}$
394 are ranked ($\text{Classify}(X_{i_m,m}, \omega_{i_m})$) in descending order according to the costs of the linear chains. So, the first $X_{1_m,m}$
395 has the largest cost $\omega_{i_m} = h_{i_1,1} + h_{i_2,2} + \dots + h_{i_{m-1},m-1} + h_{i_m,m}$. Let us consider two vectors $\Phi = (\phi_1, \phi_2, \dots, \phi_{N_m}) =$
396 $(T - U_{1_m,m}, T - U_{2_m,m}, \dots, T - U_{N_m,m})$ and $\Psi = (\psi_1, \psi_2, \dots, \psi_{N_m}) = (X_1^{**}, X_2^{**}, \dots, X_{N_m}^{**})$. We start by delaying the order
397 release date ϕ_1 (by advancing ψ_1), and the same operation is executed until the $\mathbb{E}(C(\Phi, L))$ no longer decreases further.
398 We then repeat the same operations for order release date ϕ_2 of the next component.

		<i>m</i>						
<i>b/r</i>	2	3	4	5	6	7	8	
B	0.07	0.14	0.29	0.34	0.34	0.66	0.42	0.45
	0.07	0.07	0.12	0.24	0.21	0.17	0.24	0.30
	0.01	0.02	0.08	0.15	0.19	0.28	0.20	0.08
	0.03	0.02	0.10	0.15	0.32	0.26	0.23	0.00
	0.02	0.03	0.04	0.04	0.08	0.11	0.35	0.11
	0.02	0.03	0.09	0.06	0.11	0.08	0.16	0.00
	0.01	0.02	0.05	0.11	0.34	0.22	0.27	0.08
	0.02	0.04	0.06	0.04	0.08	0.15	0.16	0.17
	0.02	0.03	0.02	0.09	0.06	0.07	0.10	0.00
	0.03	0.03	0.02	0.05	0.17	0.06	0.22	0.00
C	0.08	0.13	0.29	0.13	0.44	0.43	0.35	0.42
	0.08	0.19	0.41	0.43	0.66	0.32	0.39	0.01
	0.04	0.10	0.18	0.17	0.24	0.24	0.44	1.16
	0.02	0.02	0.11	0.11	0.09	0.15	0.18	0.00
	0.01	0.03	0.10	0.16	0.25	0.11	0.19	0.21
	0.01	0.03	0.08	0.07	0.11	0.22	0.23	0.37
	0.02	0.02	0.05	0.08	0.08	0.11	0.23	0.06
	0.07	0.04	0.12	0.11	0.13	0.10	0.14	0.01
	0.07	0.01	0.05	0.06	0.08	0.14	0.38	0.41
	0.07	0.04	0.07	0.10	0.08	0.12	0.19	0.21

Table 9: CPU time (s).

Algorithm 2: Proposed heuristic

```

1 Classify( $X_{i_m, m}, \omega_{i_m}$ );
2  $V_1 \leftarrow \emptyset$ ;  $V_2 \leftarrow \emptyset$ ;
3 foreach  $k \in \llbracket 1, N_m \rrbracket$  do
4   while  $\mathbb{E}(C(\Phi, L))$  is decreasing &  $\phi_k \leq X_{k_m}^{**}$  do
5      $V_1 \leftarrow \mathbb{E}(C(\Phi, L))$ ;
6      $\phi_k \leftarrow \phi_k + 1$ ;
7   end while
8   while  $\mathbb{E}(C(\Psi, L))$  is decreasing &  $\psi_k \geq T - U_{k_m}$  do
9      $V_2 \leftarrow \mathbb{E}(C(\Psi, L))$ ;
10     $\psi_k \leftarrow \psi_k - 1$ ;
11  end while
12 end foreach
13  $UB \rightarrow \min(V_1, V_2)$ ;

```

To analyze the performance of the optimization approaches, we introduce the following notions:

- $AGap^*$, average value of all gap^* ;
- $AGapH^*$, average gap between the solution proposed by the heuristic ($best_H$) and the best-known solution $best_{BKS}$ found among the 100 tests: $AGapH^* = \frac{best_H - best_{BKS}}{best_{BKS}} \cdot 100$;
- $ACPU$ times (s), the average execution time of all CPU times (s) with the HGA when the best solution is found;
- $ACPUH$ times (s), the average execution time of the proposed heuristic.

In Table 10, looking at the performances of HGA and the proposed heuristic, the b/r ratio seems to influence the quality of the best solutions and the CPU times. Nevertheless, the solutions proposed by the HGA are the best, they are still less than 1% from the best-known solutions found among the 100 tests, and less than 6% for the heuristic. For execution times, both approaches require less than a second (on average) to propose the best solution.

b/r	0.1	0.5	1	5	10	25	50	100	200	500
$AGap^*$ (%)	0.74	0.37	0.14	0.04	0.01	0.00	0.01	0.03	0.00	0.00
$AGapH^*$ (%)	5.19	3.73	2.95	2.87	2.27	1.45	1.17	0.83	0.84	0.64
$ACPU$ times (s)	0.29	0.26	0.17	0.13	0.11	0.09	0.12	0.09	0.08	0.09
$ACPUH$ times (s)	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02

Table 10: Performances of HGA and the proposed heuristic.

411 7. Conclusion and future research

412 In this work, a generalized probabilistic model, and algorithms to optimize the planned lead-times for multi-level
413 ATO systems working in an MRP environment under uncertainty of actual lead-times are proposed. Planned lead-times
414 were determined for the case of multi-level assembly systems with a one-period approach, i.e. for a given customer
415 demand. An infinite assembly capacity at all levels was assumed, a known and fixed demand for the finished product,
416 and independent and identically distributed discrete component procurement times were considered. Therefore, the
417 proposed analytical model calculates the mathematical expectation of the total cost. Our model is a generalization of
418 those proposed in [Yano \(1987a\)](#); [Chu et al. \(1993\)](#); [Tang & Grubbström \(2003\)](#); [Hnaien et al. \(2009\)](#); [Fallah-Jamshidi
419 et al. \(2011\)](#).

420 The same problem was treated in [Ben-Ammar et al. \(2018\)](#), where a recursive procedure was introduced to account
421 for the dependence among levels, and a B&B algorithm was introduced to determine optimum solutions. However, the
422 limitation of that study was that each lead-time varied between one and a known upper limit, and it was observed that
423 the recursive function requires a recursively enumerable domain and depends on the distribution functions. The major
424 drawback of this recursive procedure is its large influence on computation time, which becomes far too time-intensive.
425 In this paper, an iterative approach ensures a significant reduction in the time required to calculate the cumulative
426 distribution function of each assembly date. To optimize the parameters, various additional techniques introduced in
427 [Ben-Ammar et al. \(2016\)](#) were used. They are based on hybrid GAs (HGAs) to find good planned order release dates.
428 The results show that the HGA obtained can very efficiently find good-quality approximate solutions, regardless of
429 the type of assembly system, the number of components at the last levels, or the variability of finished product-related
430 costs related.

431 Our approach, for a given demand and due date, determines the optimal order release dates for the components, and
432 is therefore readily adaptable to similar ATO environments. Furthermore, our analytical approach can be employed
433 to generalize the models proposed in [Ould-Louly et al. \(2008b,a\)](#); [Ould-Louly & Dolgui \(2002a, 2013\)](#); [Dolgui &
434 Prodhon \(2007\)](#); [Shojaie et al. \(2015\)](#), and can also be applied in disassembly systems under uncertainties ([Bentaha
435 et al., 2014](#)).

436 The current study was not designed to model multi-period planning for multi-level assembly systems. It also carries
437 several limitations. First, it only considers an ATO environment with one demand. Second, we only consider release
438 dates of the entire supply chain from the final assembly point of view, as we assume it is impossible to interact with
439 supply chain partners to optimize all release decisions globally at all intermediate levels. Third, we do not explicitly
440 treat the choice of suppliers. Fourth, we do not integrate supplier-related costs such as purchasing cost and ordering
441 cost. Research is underway to overcome these limitations. We intend to focus on developing mathematical formulations
442 for dynamic supplier selection strategies in multi-period supply planning for assembly systems under stochastic lead-
443 times. Furthermore, our future work will extend this model and the various techniques, in particular treating some new
444 case studies calculating the planned lead-times when a company has to deal with production and replenishment time
445 uncertainties.

446 This paper offers techniques for replenishment planning in dynamic ATO environments with stochastic lead-times.

447 To our knowledge, in only one previous study (Ben-Ammar et al., 2018) the authors develop a generalised proba-
 448 bilistic model to study the case of one-period planning for multi-level assembly systems. The newly-developed model
 449 and optimization algorithm can, in just a few seconds, compute optimal replenishment release dates for ATO supply
 450 networks with more than seven levels. All previous results have been limited to three levels. The proposed model
 451 can be used to reduce the impact of lead-time uncertainty in enterprises, especially by selecting appropriate planned
 452 lead-time parameters in their MRP systems. This is especially critical in manual assembly systems with several levels
 453 and components (typically automotive sub-modules or electrical appliances, for example). For small and medium sized
 454 problems, the models furnish optimal solutions. For more complex systems, the approach proposed in this paper can
 455 generate good-quality solutions within reasonable computation times.

456 From the practitioner's standpoint, the proposed approach can be successfully used for a number of applications.
 457 For example, we have been working with ZF Friedrichshafen AG, a German car parts maker, at its assembly plant
 458 for gear boxes located in Saint-Étienne, France. Based on statistical data, the company allocated, for all suppliers,
 459 safety coefficients greater than 1 and used them in calculation of planned lead-times. A coefficient is calculated for
 460 each supplier according to its previous delivery performances. To set its planned lead-time in the MRP system, its
 461 contractual lead-time is multiplied by the corresponding coefficient. This strategy is applied to anticipate delays and to
 462 assess supplier reliability better. The closer the coefficient is to 1, the more reliable the supplier is. The main limitation
 463 of this empirical strategy is its inability to provide good parameters since suppliers are considered independently, the
 464 coefficients are calculated empirically and inventory synchronization aspects and costs are neglected. The proposed
 465 model and algorithms offer better estimations of these coefficients by taking into account inventory and backlogging
 466 costs, the independence (synchronization) of suppliers via the assembly operations, and probability distributions of
 467 supplier lead-times.

468 8. Appendix

Proof of Proposition 4.1.

Components $c_{k,m}$ at the last level m are ordered from several suppliers and used to assemble components $c_{i,m-1}$ at level $m - 1$. Then, at the first step, the iterative process starts by calculating $\mathbb{F}_{i,m-1}(s)$ the cumulative distribution functions of $M_{i,m-1}$, i.e. the assembly dates for components at level $m - 1$. These assembly dates $M_{i,m-1}$ are positive random discrete variables with a finite number of possible values. Knowing that $L_{k,m} + X_{k,m}$, for $c_{k,m} \in S_{i,m-1}$, are independent, and that:

$$\mathbb{P}(M_{i,m-1} \leq s) = \mathbb{P}\left(\max_{c_{k,m} \in S_{i,m-1}} (X_{k,m} + L_{k,m} \leq s)\right) = \prod_{c_{k,m} \in S_{i,m-1}} \mathbb{P}(L_{k,m} + X_{k,m} \leq s)$$

Then:

$$\mathbb{F}_{i,m-1}(s) = \prod_{c_{k,m} \in S_{i,m-1}} \mathbb{F}_{k,m}(-X_{k,m} + s) \quad (15)$$

Then, at the second step, the iterative process starts by calculating $\mathbb{F}_{i,m-2}(s)$, the cumulative distribution function of $M_{i,m-2}$, i.e. the assembly dates for components at level $m - 2$. These assembly dates $M_{i,m-2}$ are positive random

discrete variables with a finite number of possible values. Knowing that:

$$\mathbb{P}(M_{i,m-2} \leq s) = \mathbb{P}\left(\max_{c_{k,m-1} \in \mathcal{S}_{i,m-2}} (X_{k,m-1} + L_{k,m-1} \leq s)\right)$$

and $\forall k = 1, \dots, N_{m-1}$, the random variables $(M_{k,m-1} + L_{k,m-1})$, are independent, then:

$$\begin{aligned} \mathbb{F}_{i,m-2}(s) &= \prod_{c_{k,m-1} \in \mathcal{S}_{i,m-2}} \mathbb{P}(M_{k,m-1} + L_{k,m-1} \leq s) \\ &= \prod_{c_{k,m-1} \in \mathcal{S}_{i,m-2}} \sum_{\substack{o_1 + w_1 = s \\ o_1 + w_1 \in \mathbb{N}}} \mathbb{P}(L_{k,m-1} = o_1) \mathbb{P}(M_{k,m-1} \leq w_1) \\ &= \prod_{c_{k,m-1} \in \mathcal{S}_{i,m-2}} \sum_{\substack{o_1 + w_1 = s \\ o_1 + w_1 \in \mathbb{N}}} \mathbb{P}(L_{k,m-1} = o_1) \mathbb{F}_{k,m-1}(w_1) \end{aligned}$$

469 Using expression (15), we obtain the cumulative distribution function of $M_{i,m-2}$:

$$\mathbb{F}_{i,m-2}(s) = \prod_{c_{k,m-1} \in \mathcal{S}_{i,m-2}} \sum_{\substack{o_1 + w_1 = s \\ o_1 + w_1 \in \mathbb{N}}} \left(\mathbb{P}(L_{k,m-1} = o_1) \prod_{c_{k,m} \in \mathcal{S}_{i,m-1}} \mathbb{F}_{k,m}(-X_{k,m} + w_1) \right) \quad (16)$$

At the third step, the iterative process starts by calculating $\mathbb{F}_{i,m-3}(s)$, the cumulative distribution function of $M_{i,m-3}$ the assembly dates for components at level $m-3$. These assembly dates $M_{i,m-3}$ are positive random discrete variables with a finite number of possible values. In the same way and using expression (16), we can deduce this cumulative distribution function:

$$\mathbb{F}_{i,m-3}(s) = \prod_{c_{k,m-2} \in \mathcal{S}_{i,m-3}} \sum_{\substack{o_2 + w_2 = s \\ o_2 + w_2 \in \mathbb{N}}} \mathbb{P}(L_{k,m-2} = o_2) \mathbb{F}_{i,m-2}(w_2)$$

470 Using the same iterative process, the cumulative distribution function of $M_{i,l}$, for $l \in \llbracket 0, m-4 \rrbracket$ are calculated in the
471 same way. ■

472

473 **Proof of Proposition 4.2.**

474 In Ben-Ammar et al. (2018), the general expression of the total expected cost, which is noted $\mathbb{E}(C(X, L))$ was given
475 and written as follows:

$$\begin{aligned} \mathbb{E}(C(X, L)) &= \sum_{i=1}^{N_1} h_{i,1} \mathbb{E}(M_{FP}) - \sum_{l=1}^{m-1} \left(\sum_{i=1}^{N_i} H_i \cdot \mathbb{E}(M_{i,l}) \right) - \sum_{l=1}^m \left(\sum_{i=1}^{N_i} h_{i,l} \mathbb{E}(L_{i,l}) \right) \\ &\quad - \sum_{i=1}^{N_m} h_{i,m} \mathbb{E}(X_{i,m}) + b \cdot (\mathbb{E}(M_{FP}^+) - T) + r \cdot (T - \mathbb{E}(M_{FP}^-)) \end{aligned} \quad (17)$$

476 Based on expression (2), the authors gave the expressions of $\mathbb{E}(M_{i,l})$, $\mathbb{E}(M_{FP})$, $\mathbb{E}(M_{FP}^+)$ and $\mathbb{E}(M_{FP}^-)$ (see Definition
477 4.1). They were calculated independently using the recursive function $Q^+(L_{i,j}, s, j)$. Here, the cumulative distribution
478 functions $\mathbb{F}(\cdot)$ of assembly dates (found by an iterative process; see Proposition 4.1) and the limits $t_{i,l}$ and $u_{i,l}$ of
479 lead-times $L_{i,l}$ are used to simplify the calculations.

480

For $l \in \llbracket 1, m \rrbracket$, each $L_{k,l}$ varies in $\llbracket t_{k,l}, u_{k,l} \rrbracket$. Let $\alpha_{i,l} = \max_{c_{k,l+1} \in \mathcal{S}_{i,l}} (\alpha_{k,l+1} + t_{k,l+1})$ with $\alpha_{i,m} = X_{i,m}$ and $\beta_{i,l} = \max_{c_{k,l+1} \in \mathcal{S}_{i,l}} (\beta_{k,l+1} + u_{k,l+1})$ with $\beta_{i,m} = X_{i,m}$. Thus, $M_{i,l}$ varies between $\alpha_{i,l}$ and $\beta_{i,l}$ (see Fig. 4), and expression (3) can be simplified as follows:

$$\mathbb{E}(M_{i,l}) = \beta_{i,l} - \sum_{s=\alpha_{i,l}}^{\beta_{i,l}-1} \mathbb{F}_{i,l}(s) \quad (18)$$

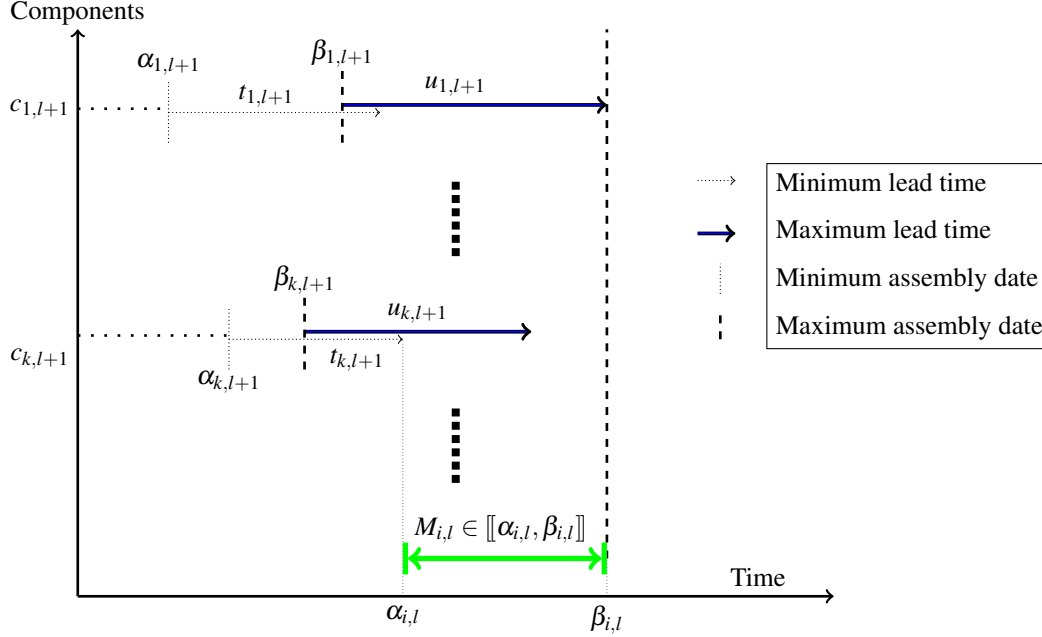


Figure 4: Time interval in which the assembly date $M_{i,l}$ varies.

By using the cumulative distribution function defined in expression (6), and knowing that M_{FP}^+ varies between T and $\beta_{0,0} = \max_{k \in \llbracket 1, N_m \rrbracket} (X_{k,m} + U_{k,m})$, $\mathbb{E}(M_{FP}^+)$, given in expression (5), can be written as follows:

$$\mathbb{E}(M_{FP}^+) = T + \sum_{s \geq T} (1 - \mathbb{F}_{0,0}(s)) = \beta_{0,0} - \sum_{s=T}^{\beta_{0,0}-1} \mathbb{F}_{0,0}(s) \quad (19)$$

Note that $\mathbb{E}(M_{FP}^-)$ is calculated in the same way and is equal to:

$$\mathbb{E}(M_{FP}^-) = T - \sum_{s=\alpha_{0,0}}^{T-1} \mathbb{F}_{0,0}(s) \quad (20)$$

481 where $\alpha_{0,0} = \max_{i \in \llbracket 1, N_i \rrbracket} (\alpha_{i,1} + t_{i,1})$.

482

The expression of $\mathbb{E}(M_{FP})$ is calculated using expressions (19-20) and is equal to:

$$\mathbb{E}(M_{FP}) = \mathbb{E}(M_{FP}^+) + \mathbb{E}(M_{FP}^-) - T = \beta_{0,0} - \sum_{s=T}^{\beta_{0,0}-1} \mathbb{F}_{0,0}(s) - \sum_{s=\alpha_{0,0}}^{T-1} \mathbb{F}_{0,0}(s) \quad (21)$$

483 Then, using expressions (18, 19, 20 and 21), the mathematical expectation of the total cost can be found directly. ■

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