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# Singular self-energy for itinerant electrons in a dilute Ising spin bath

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**Abstract.** We consider itinerant spinless electrons moving as defects in a dilute two-dimensional Ising spin system, leading to an effective interaction mediated by spin fluctuations. Coupled self-consistent equations are analyzed after expressing the spin and fermion operators in terms of Grassmann variables. The effective mass, density of states, and specific heat are evaluated. We found that at low temperature and low electron density the effective mass is reduced, whereas in the critical region it sharply diverges. At higher temperature, the fermions behave like a Fermi liquid with a small enhanced mass.

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## 1. Introduction

The interplay between spin fluctuations and itinerant holes or electrons is important in understanding the nature of quasiparticles in complex systems where cooperative phenomena are dominant. For example, in hole doped copper oxide material  $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$ , hole segregation was observed and interpreted as stripe modulation [1] and this spatial structure is assumed to be responsible for the suppression of superconductivity. In general mobile defects in spin environment have been studied theoretically [2, 3, 4], and itinerant electrons coupled to a spin bath are known to generate domain structures [5, 6, 7]. Quantum Monte Carlo methods were investigated in order to determine the magnetic properties of coupled lattices between spins and fermions in models for metal-insulator interfaces [8]. It is expected that in the case of coupled layers itinerant electrons will suppress the magnetic order, and spin fluctuations will induce correlation between electrons. In this paper we would like to analyze the coupling between spinless electrons and ferromagnetic Ising spins in two-dimensional lattices where itinerant electrons occupy empty sites and interact via indirect spin fluctuations. We would like to study in particular how spin fluctuations modify the effective mass.

The paper is organized as follow. In section 2, we define the Hamiltonian of the model, then we use a Grassmannian representation of the spin sector in presence of holes or defects to

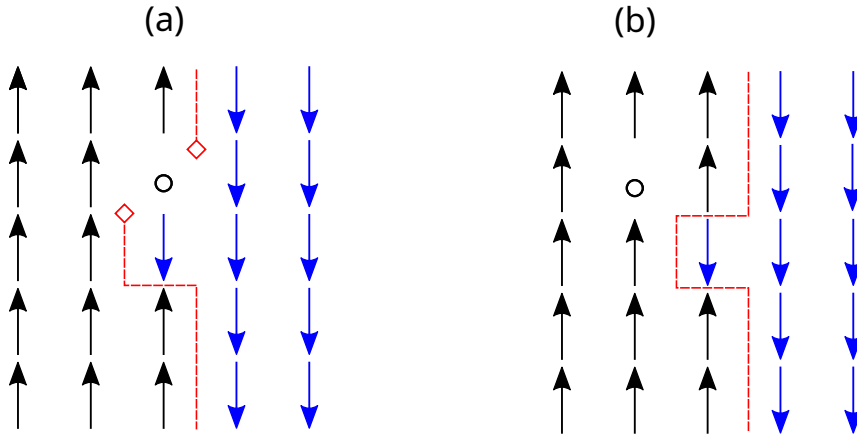


Figure 1: Spin configurations separating two regions of opposite polarity, when one hole is located along the domain wall (a) and outside (b). The energy  $E_a$  for the spin configuration (a) has a lower energy than the energy  $E_b$  in (b), since  $E_b - E_a = 4J$ .

obtain an effective fermionic action. Then the electronic part of the action is also transformed into a Grassmann integral representation at finite temperature. In section 3 we present the self-consistent equations for the model using an approximation of the Green function and self-energy. Finally in section 4 we study the physical solutions and evaluate the effective mass, density of states, and specific heat.

## 2. Model

We consider a two-dimensional lattice of size  $L^2 = N$  where each site  $\mathbf{r}$  is occupied either by a Ising spin  $\sigma_{\mathbf{r}} = \pm 1$  or an electron or hole with Fermi statistics. Spins are interacting with a ferromagnetic coupling  $J > 0$ . The electrons are represented by a tight-binding model and we do not consider their Coulomb interaction. A spin-hole configuration in a  $N = 5 \times 5$  system is represented in Figure 1 with a single hole defect and a domain wall (red dashed line) separating two regions of opposite spin polarity due for example to opposite boundary conditions. The indirect interaction between holes and spins can be understood by the fact that there exist hole configurations which lower the domain wall excess energy. These configurations have the hole attached to the domain wall, which gives in Figure 1(a) an energy gain of  $4J$  compared to configuration (b) where the hole sits strictly inside the region of spins up. We therefore expect an effective interaction between holes and spins where spin fluctuations play an essential role in the electron or hole dynamics.

The Hamiltonian describing the interaction between spins and electrons can be written in terms of fermionic operators ( $c_{\mathbf{r}}, c_{\mathbf{r}}^{\dagger}$ ) and fermion numbers  $n_{\mathbf{r}} = c_{\mathbf{r}}^{\dagger} c_{\mathbf{r}}$  as

$$H = -\frac{J}{2} \sum_{\mathbf{r}, \delta=\pm\mathbf{x}, \pm\mathbf{y}} \sigma_{\mathbf{r}} \sigma_{\mathbf{r}+\delta} (1 - n_{\mathbf{r}}) (1 - n_{\mathbf{r}+\delta}) - t \sum_{\mathbf{r}, \delta=\pm\mathbf{x}, \pm\mathbf{y}} c_{\mathbf{r}+\delta}^{\dagger} c_{\mathbf{r}}, \quad (1)$$

where  $t$  is the kinetic energy for the itinerant electrons. We first consider the fermionic action of

the Ising spin system in absence of holes,  $H_0 = -\frac{1}{2}J \sum_{\mathbf{r}, \delta=\pm\mathbf{x}, \pm\mathbf{y}} \sigma_{\mathbf{r}} \sigma_{\mathbf{r}+\delta}$ , and the representation of its partition function  $Z_0$  in terms of Grassmann variables  $a_{\mathbf{r}}$  and  $\bar{a}_{\mathbf{r}}$  [9, 10]

$$Z_0 = \text{Tr} e^{-\beta H_0} = \int \left( \prod_{\mathbf{r}} d\bar{a}_{\mathbf{r}} da_{\mathbf{r}} \right) e^{S_0}, \quad (2)$$

where the Grassmannian action  $S_0$  is defined by

$$S_0 = \sum_{\mathbf{r}} S_{\mathbf{r}},$$

$$S_{\mathbf{r}} = a_{\mathbf{r}} \bar{a}_{\mathbf{r}} + u(a_{\mathbf{r}} + \bar{a}_{\mathbf{r}})(a_{\mathbf{r}-\mathbf{x}} - \bar{a}_{\mathbf{r}-\mathbf{y}}) - u^2 a_{\mathbf{r}-\mathbf{x}} \bar{a}_{\mathbf{r}-\mathbf{y}} + 2 \ln \cosh(\beta J), \quad (3)$$

In this expression  $u = \tanh(\beta J) = \tanh(J/k_B T)$  and  $\mathbf{r}$  runs over the lattice. The action  $S_0$  is quadratic in Grassmann variables and was derived in detail by Plechko [11, 12, 10, 13] who studied various geometries of the lattice. We now consider a dilute system where empty sites are replaced by itinerant spinless electrons. In presence of vacancies (or empty sites) at different locations  $\mathbf{r}_i$ , the partition function is modified accordingly [14, 15, 16]

$$Z_h[\{\mathbf{r}_i\}] = \int \prod_{\mathbf{r}} d\bar{a}_{\mathbf{r}} da_{\mathbf{r}} [\delta_{n_{\mathbf{r}},0} e^{S_{\mathbf{r}}} + \delta_{n_{\mathbf{r}},1} a_{\mathbf{r}} \bar{a}_{\mathbf{r}}], \quad (4)$$

where  $n_{\mathbf{r}} = 1$  when a hole is present at site  $\mathbf{r} = \mathbf{r}_i$ , and  $n_{\mathbf{r}} = 0$  otherwise, when the site is occupied by a spin. This partition function can be rewritten as

$$Z_h[\{\mathbf{r}_i\}] = \int \prod_{\mathbf{r}} d\bar{a}_{\mathbf{r}} da_{\mathbf{r}} [1 + (a_{\mathbf{r}} \bar{a}_{\mathbf{r}} e^{-S_{\mathbf{r}}} - 1) n_{\mathbf{r}}] e^{S_0}. \quad (5)$$

We first diagonalize  $S_0$ , using the Fourier space transformation  $a_{\mathbf{r}} = L^{-1} \sum_{\mathbf{k}} a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}$ ,  $\bar{a}_{\mathbf{r}} = L^{-1} \sum_{\mathbf{k}} \bar{a}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}$ , and express the partition function  $Z_0$  as a product over decoupled momentum dependent factors, after relabeling  $k_x \rightarrow -k_x$ , see Appendix A:

$$S_0 = 2N \ln \cosh(\beta J)$$

$$+ \sum_{\mathbf{k}}' [g_{\mathbf{k}} a_{\mathbf{k}} \bar{a}_{\mathbf{k}} + \bar{g}_{\mathbf{k}} a_{-\mathbf{k}} \bar{a}_{-\mathbf{k}} - 2iu \sin k_x a_{\mathbf{k}} a_{-\mathbf{k}} + 2iu \sin k_y \bar{a}_{\mathbf{k}} \bar{a}_{-\mathbf{k}}],$$

$$g_{\mathbf{k}} = 1 - u(e^{ik_x} + e^{ik_y}) - u^2 e^{ik_x + ik_y}, \quad (6)$$

$$Z_0 = \cosh(\beta J)^{2N} \prod_{\mathbf{k}}' (g_{\mathbf{k}} \bar{g}_{\mathbf{k}} - 4u^2 \sin k_x \sin k_y)$$

$$= \cosh(\beta J)^{2N} \prod_{\mathbf{k}}' [(1 + u^2)^2 - 2u(1 - u^2)(\cos k_x + \cos k_y)].$$

The prime symbol corresponds to the set of momenta  $\mathbf{k}$  located in half of the Brillouin zone, the opposite momenta  $-\mathbf{k}$  completing the zone. One can write an effective action for (5) by introducing nilpotent commuting variables  $b_{\mathbf{r}}$  such that  $b_{\mathbf{r}}^2 = 0$  and

$$Z_h[\{\mathbf{r}_i\}] = \int \prod_{\mathbf{r}} d\bar{a}_{\mathbf{r}} da_{\mathbf{r}} db_{\mathbf{r}} \exp \left( S_0 + \sum_{\mathbf{r}} b_{\mathbf{r}} (1 - n_{\mathbf{r}}) + \sum_{\mathbf{r}} b_{\mathbf{r}} n_{\mathbf{r}} a_{\mathbf{r}} \bar{a}_{\mathbf{r}} e^{-S_{\mathbf{r}}} \right). \quad (7)$$

One should notice that  $b_{\mathbf{r}}$  can always be written as a product of two Grassmann variables. The term  $a_{\mathbf{r}}\bar{a}_{\mathbf{r}}e^{-S_{\mathbf{r}}} = a_{\mathbf{r}}\bar{a}_{\mathbf{r}} + u^2 a_{\mathbf{r}}a_{\mathbf{r}-\mathbf{x}}\bar{a}_{\mathbf{r}}\bar{a}_{\mathbf{r}-\mathbf{y}}$  will be approximated in the following by the quadratic part since the quartic part contains second order derivatives which will be assumed to be negligible for small momenta. From  $Z_h$  we can now construct the total partition function  $Z = \text{Tr} e^{-\beta H} = \int e^S$ , where the action  $S$  includes the Ising part, the electron kinetic energy, and the hole constraint in equation (1). It can be expressed as a Grassmannian functional integral by substituting the Fermi operators with dimensionless Grassmann variables  $c_{\mathbf{r}} \rightarrow c_{\mathbf{r}}(\tau)$  and  $c_{\mathbf{r}}^\dagger \rightarrow \bar{c}_{\mathbf{r}}(\tau)$  [17, 18], where  $\tau$  is an imaginary time variable taking values between 0 and  $\beta$

$$\begin{aligned} Z = & \int \prod_{\mathbf{r}} d\bar{a}_{\mathbf{r}} da_{\mathbf{r}} db_{\mathbf{r}} \int \prod_{\mathbf{r}, \tau} d\bar{c}_{\mathbf{r}}(\tau) dc_{\mathbf{r}}(\tau) \exp \left( S_0 + \sum_{\mathbf{r}} b_{\mathbf{r}} \left[ \frac{1}{2} - \beta^{-1} \int_0^\beta d\tau \bar{c}_{\mathbf{r}}(\tau) c_{\mathbf{r}}(\tau) \right] \right. \\ & + \sum_{\mathbf{r}} b_{\mathbf{r}} \left[ \frac{1}{2} + \beta^{-1} \int_0^\beta d\tau \bar{c}_{\mathbf{r}}(\tau) c_{\mathbf{r}}(\tau) \right] a_{\mathbf{r}} \bar{a}_{\mathbf{r}} \\ & \left. + \sum_{\mathbf{r}} \int_0^\beta d\tau \left[ -\bar{c}_{\mathbf{r}}(\tau) \partial_\tau c_{\mathbf{r}}(\tau) + t \sum_{\delta} \bar{c}_{\mathbf{r}+\delta}(\tau) c_{\mathbf{r}}(\tau) \right] \right). \end{aligned} \quad (8)$$

We have used the following prescription in the case where products of two Fermi operators on the same site are encountered:  $c_{\mathbf{r}}^\dagger c_{\mathbf{r}} = (c_{\mathbf{r}}^\dagger c_{\mathbf{r}} - c_{\mathbf{r}} c_{\mathbf{r}}^\dagger + 1)/2 \rightarrow \bar{c}_{\mathbf{r}}(\tau) c_{\mathbf{r}}(\tau) + 1/2$ . In the following section we study the extrema of this action using Lagrange multipliers.

### 3. Self-consistent equations

We consider the total action  $S$  in equation (8) with antisymmetric boundary conditions  $c_{\mathbf{r}}(0) = -c_{\mathbf{r}}(\beta)$  and  $\bar{c}_{\mathbf{r}}(0) = -\bar{c}_{\mathbf{r}}(\beta)$ . These operators can be expanded using the Matsubara frequencies  $\omega_n = (2n + 1)\pi/\beta$ ,  $n = 0, \pm 1, \pm 2, \dots$ , for which we have

$$c_{\mathbf{r}}(\tau) = \beta^{-1/2} \sum_{\omega_n} e^{-i\omega_n \tau} c_{\mathbf{r}}(\omega_n), \quad \bar{c}_{\mathbf{r}}(\tau) = \beta^{-1/2} \sum_{\omega_n} e^{i\omega_n \tau} \bar{c}_{\mathbf{r}}(\omega_n). \quad (9)$$

This allows us to write the total action as

$$\begin{aligned} S = & S_0 + \sum_{\mathbf{r}} b_{\mathbf{r}} \left[ \frac{1}{2} - \beta^{-1} \sum_{\omega_n} \bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n) \right] + \sum_{\mathbf{r}} b_{\mathbf{r}} \left[ \frac{1}{2} + \beta^{-1} \sum_{\omega_n} \bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n) \right] a_{\mathbf{r}} \bar{a}_{\mathbf{r}} \\ & + \sum_{\mathbf{r}} \sum_{\omega_n} \left[ (i\omega_n + \mu) \bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n) + t \sum_{\delta} \bar{c}_{\mathbf{r}+\delta}(\omega_n) c_{\mathbf{r}}(\omega_n) \right], \quad (10) \\ n_{\mathbf{r}} = & \frac{1}{2} + \beta^{-1} \sum_{\omega_n} \bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n), \end{aligned}$$

where we have introduced a chemical potential  $\mu$ . To simplify the problem, we would like to express the action in terms of order parameters satisfying self-consistent mean-field equations. The idea is to replace  $\bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n)$  by a Green function  $G_{\mathbf{r}}(\omega_n)$  with conjugate variable  $\Sigma_{\mathbf{r}}(\omega_n)$ ,

or self-energy. After integrating over  $b_{\mathbf{r}}$ , we can write the following effective action

$$\begin{aligned}
S_{\text{eff}} = & S_0 + \sum_{\mathbf{r}} \ln \left( \frac{1}{2} - \beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n) \right) + \sum_{\mathbf{r}} \frac{1 + 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)}{1 - 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)} a_{\mathbf{r}} \bar{a}_{\mathbf{r}} \\
& + \sum_{\mathbf{r}, \omega_n} \Sigma_{\mathbf{r}}(\omega_n) \left[ G_{\mathbf{r}}(\omega_n) - \bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n) \right] \\
& + \sum_{\mathbf{r}} \sum_{\omega_n} \left[ (i\omega_n + \mu) \bar{c}_{\mathbf{r}}(\omega_n) c_{\mathbf{r}}(\omega_n) + t \sum_{\delta} \bar{c}_{\mathbf{r}+\delta}(\omega_n) c_{\mathbf{r}}(\omega_n) \right]. \tag{11}
\end{aligned}$$

From this expression, we then compute the effective thermodynamical potential  $-\beta\Omega = \ln(\int e^{S_{\text{eff}}})$  which is given by

$$\begin{aligned}
-\beta\Omega = & 2N \ln \cosh(\beta J) + \sum_{\mathbf{r}} \ln \left( \frac{1}{2} - \beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n) \right) + \sum_{\mathbf{r}, \omega_n} \Sigma_{\mathbf{r}}(\omega_n) G_{\mathbf{r}}(\omega_n) \\
& + \frac{1}{2} \ln \det \left( A_0 + V \right)_{\mathbf{r}, \mathbf{r}'} + \sum_{\omega_n} \ln \det \left( [i\omega_n + \mu - \Sigma_{\mathbf{r}}(\omega_n)] \delta_{\mathbf{r}, \mathbf{r}'} + t \sum_{\delta} \delta_{\mathbf{r}', \mathbf{r}+\delta} \right)_{\mathbf{r}, \mathbf{r}'},
\end{aligned}$$

where  $A_0$  is the matrix corresponding to the Ising part in the real space:  $S_0 = \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}'} \Psi_{\mathbf{r}}^{\top} (A_0)_{\mathbf{r}, \mathbf{r}'} \Psi_{\mathbf{r}'}$ , and  $V$  the local potential due to the hole contribution

$$(V)_{\mathbf{r}, \mathbf{r}'} = \delta_{\mathbf{r}, \mathbf{r}'} \frac{1 + 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)}{1 - 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)} i\sigma_2, \tag{12}$$

in the basis of vectors  $\Psi_{\mathbf{r}} = (a_{\mathbf{r}}, \bar{a}_{\mathbf{r}})^{\top}$  (see Appendix B). If we extremize  $\Omega$  with respect to the parameters  $\Sigma_{\mathbf{r}}(\omega_n)$  and  $G_{\mathbf{r}}(\omega_n)$ , we obtain a set of self-consistent equations for the different order parameters, in particular

$$\begin{aligned}
G_{\mathbf{r}}(\omega_n) = & -\frac{\delta}{\delta \Sigma_{\mathbf{r}}(\omega_n)} \ln \det \left( [i\omega_n + \mu - \Sigma_{\mathbf{r}}(\omega_n)] \delta_{\mathbf{r}, \mathbf{r}'} + t \sum_{\delta} \delta_{\mathbf{r}', \mathbf{r}+\delta} \right)_{\mathbf{r}, \mathbf{r}'}, \\
\frac{1}{1 - n_{\mathbf{r}}} - \beta \Sigma_{\mathbf{r}}(\omega_n) = & \frac{\beta}{2} \frac{\delta}{\delta G_{\mathbf{r}}(\omega_n)} \ln \det \left( A_0 + V \right)_{\mathbf{r}, \mathbf{r}'}. \tag{13}
\end{aligned}$$

Since the system is invariant by translation, we may seek solutions of the form  $G_{\mathbf{r}}(\omega_n) = G(\omega_n)$ ,  $\Sigma_{\mathbf{r}}(\omega_n) = \Sigma$ , and  $n_{\mathbf{r}} = n_e$  the particle density. We can therefore simplify (13) and obtain under this approximation the following equations

$$\begin{aligned}
G(\omega_n) = & \frac{1}{N} \sum_{\mathbf{k}} \frac{1}{i\omega_n + \mu - \Sigma - \epsilon_{\mathbf{k}}}, \quad \epsilon_{\mathbf{k}} = 2t(2 - \cos k_x - \cos k_y), \\
\beta \Sigma = & \frac{1}{1 - n_e} - \frac{\mathcal{F}(v)}{(1 - n_e)^2}, \\
\mathcal{F}(v) \equiv & \frac{1}{N} \sum_{\mathbf{k}}' \frac{2v + g_{\mathbf{k}} + \bar{g}_{\mathbf{k}}}{(g_{\mathbf{k}} + v)(\bar{g}_{\mathbf{k}} + v) - 4u^2 \sin k_x \sin k_y}, \\
n_e = & \frac{1}{2} + \beta^{-1} \sum_{\omega_n} G(\omega_n),
\end{aligned}$$

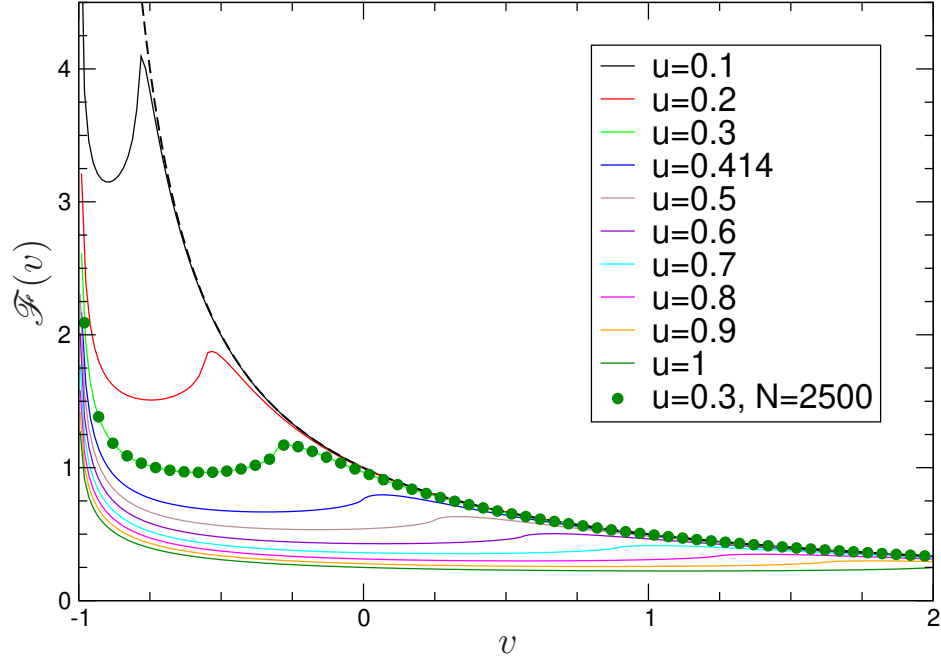


Figure 2: Plot of function  $\mathcal{F}(v)$  in the thermodynamical limit, equation (C.10), for several temperature values or  $u = \tanh(\beta J)$ . At  $v = 0$  the function becomes singular with respect to  $u$  for the critical value  $u = \sqrt{2} - 1 \simeq 0.414$ . The dashed line is the asymptotic value  $\mathcal{F}(v) = (1 + v)^{-1}$  in the high temperature regime, see equation (C.11). For comparison, dark green circles are values from the discrete formula defined in equation (14) with  $u = 0.3$  and  $N = 2500$  lattice points.

$$v = \frac{1 + 2\beta^{-1} \sum_{\omega_n} G(\omega_n)}{1 - 2\beta^{-1} \sum_{\omega_n} G(\omega_n)} = \frac{n_e}{1 - n_e}, \quad (14)$$

where we have shifted  $\mu \rightarrow \mu - 4t$  for convenience, so that for small momenta  $\epsilon_{\mathbf{k}} \simeq tk^2$ .  $\Sigma$  depends only on  $\mu$  and the temperature. The exact expression of  $\mathcal{F}(v)$  in the thermodynamical limit is given in Appendix C, equation (C.10), in terms of elliptic functions. In Figure 2 we have plotted  $\mathcal{F}(v)$  for several temperature values or  $u$ , as well as the asymptotic value at high temperature (dashed line) whose expression is given by equation (C.11). The sum over the Matsubara frequencies of the Green functions  $G(\omega_n)$  can be performed since  $\Sigma$  is independent of  $\omega_n$

$$\sum_{\omega_n} G(\omega_n) = \frac{\beta}{2N} \sum_{\mathbf{k}} \tanh \frac{\beta}{2} (\mu - \Sigma - \epsilon_{\mathbf{k}}). \quad (15)$$

The potential is finally given by

$$\begin{aligned} -\frac{\beta}{N} \Omega &= 2 \ln \cosh(\beta J) + \ln(1 - n_e) + \left(n_e - \frac{1}{2}\right) \beta \Sigma \\ &+ \frac{1}{N} \sum_{\mathbf{k}} \ln \cosh \left[ \frac{\beta}{2} (\mu - \Sigma - \epsilon_{\mathbf{k}}) \right] \end{aligned} \quad (16)$$

$$+\frac{1}{2N} \sum_{\mathbf{k}} \ln [(1+v)^2 + u^2(2+u^2) - 2u(1+v-u^2)(\cos k_x + \cos k_y) - 2u^2v \cos(k_x + k_y)].$$

#### 4. Effective mass and singular behavior of the self-energy

When the density is small  $n_e \simeq 0$ , we have the approximations  $v \simeq n_e$  and  $\Sigma \simeq k_B T [1 - \mathcal{F}(0)]$ .  $\mathcal{F}(0) \simeq 1/4$  when  $T \rightarrow 0$  and  $\mathcal{F}(0) \simeq 1 - 3u^4$  when  $T \rightarrow \infty$ . In the latter case,  $\Sigma \simeq 3J^4/k_B^3 T^3$  and is positive. We can notice that due to the dilution of the spins with electrons, the singular part of  $\Omega$  gives a critical temperature  $(\beta_c J)^{-1}$  satisfying  $\tanh(\beta_c J) = \sqrt{2+v} - 1 = u_c(v)$  and which is lower than the Ising value  $u_c(v) \geq u_c(0)$  deduced from the singular behavior of  $Z_0$  when  $\mathbf{k} \simeq 0$ . We can notice that this critical temperature vanishes when  $u_c(v) = 1$  at the density  $n_e = 2/3 \simeq 0.67$  given by the saddle point approximation. Efficient Monte Carlo algorithms give a lower value close to  $n_e = 0.59$  for the percolation threshold [19, 20] above which the critical transition disappears. When fermions are coupled separately with Ising lattices, the ferromagnetic coupling is known to decrease the critical temperature whereas in the antiferromagnetic case it increases it [8]. Near the Fermi surface, the normalized effective mass  $m^*$  with respect to the bare mass of the particles, which is defined as  $m_e = \hbar^2/2t$ , is equal to  $m^* = 1 - \partial\Sigma/\partial\mu$  [21, 22, 23, 24]. Using this definition, we first derive the self-energy in (14) with respect to  $\mu$

$$\beta \frac{\partial\Sigma}{\partial\mu} = \frac{n'_e}{(1-n_e)^4} [(1-n_e)^2 - 2(1-n_e)\mathcal{F}(v) - \mathcal{F}'(v)] = n'_e \Lambda, \quad (17)$$

with  $\mathcal{F}'(v) = d\mathcal{F}(v)/dv$  and the density of states  $n'_e = \partial n_e / \partial\mu$

$$n'_e \equiv \left(1 - \frac{\partial\Sigma}{\partial\mu}\right) \frac{\beta}{4N} \sum_{\mathbf{k}} \left[1 - \tanh^2 \frac{\beta}{2} (\mu - \Sigma - \epsilon_{\mathbf{k}})\right] \equiv \left(1 - \frac{\partial\Sigma}{\partial\mu}\right) \frac{\beta \tilde{n}'_e}{4}. \quad (18)$$

We can then express the effective mass as  $m^* = (1 + \tilde{n}'_e \Lambda/4)^{-1}$ , with  $0 < \tilde{n}'_e < 1$ . The value of the effective mass relatively to unity depends therefore on the sign of the function  $\Lambda$ . At high temperature where  $u \ll 1$  we have  $\Lambda \simeq -6u^4(1-n_e) < 0$ . This follows from the series expansion  $\mathcal{F}(v) \simeq (1+v)^{-1} - 3u^4(1+v)^{-4}$ , see equation (C.11). We then obtain the following approximation for the effective mass at high temperature, where  $u \ll 1$  and  $\tilde{n}'_e \simeq 1$

$$m^* \simeq 1 + \frac{3}{2} u^4 (1 - n_e) > 1, \quad (19)$$

which is always greater than unity. In the following we will take  $t = 1$  for the numerical evaluations. At low temperature, or  $u \lesssim 1$ , we find that  $\Lambda > 0$  for parameter  $u$  above a threshold value  $u = u^* > u_c$  which depends on  $n_e$ , see top inset of Figure 3. In this region  $m^* < 1$  and  $u^*$  goes to unity when the density approaches a critical value  $n_e^* \simeq 0.545$ , using  $N = 500^2$  lattice points. The low temperature correlated spin fluctuations reduce the effective mass, whereas they tend to increase it at higher temperatures. At criticality the mass diverges, as evidenced in Figure 3. This can be interpreted as the fact that large spin correlations lead to the localization of electrons. In the low temperature regime, the mass is less than unity and tends to decrease as  $n_e$  increases from zero with a minimum around  $n_e \simeq 0.425$ , see bottom inset of Figure 3. A qualitative criteria for



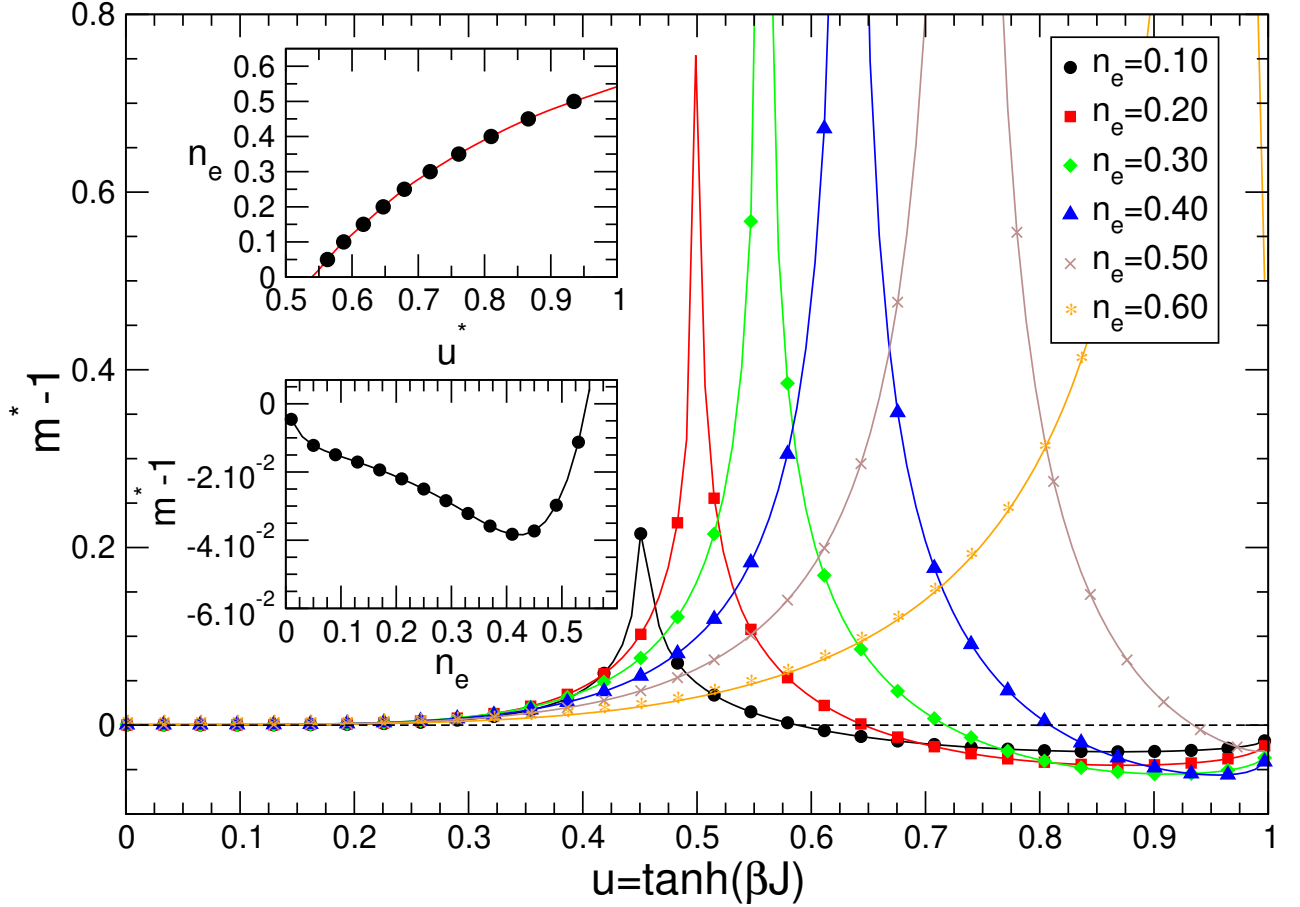


Figure 3: Effective mass  $m^*$  as function of  $u = \tanh(\beta J)$  for several values of  $n_e$  and  $t = 1$ . First inset: Threshold temperature or  $u^*$  defined as the temperature below which the mass becomes less than unity as function of  $n_e$ . Red line is an interpolation which shows that  $m^* > 1$  for all temperature above the critical value  $n_e^* \simeq 0.55$ . Second inset: Effective mass at low temperature ( $u = 0.999$ ) as function of  $n_e$ . We have chosen  $N = 500^2$  lattice sites for evaluating the chemical potential.

the existence of the crossover value  $u^*$  can be given by comparing the two characteristic lengths of the system, which are the average distance between the particles  $n_e^{-1/2}$  and the spin correlation length  $\xi$  defined by

$$\xi^{-2} = \frac{(1 + v - 2u - u^2)^2}{u\sqrt{(1 + v - u^2)(1 + v + 2uv - u^2)}} \quad (20)$$

This quantity is extracted from the constant term of the free energy singular part at low momenta  $\mathbf{k}$ , see equations (16) and (C.2), after expansion and diagonalization of the quadratic form, and which can be identified as the inverse square of the correlation length. We propose to characterize the transition at  $u^*$  by the fact that both lengths are of the same order  $\xi \sim n_e^{-1/2}$  which gives an

equation for  $u^*$  as function of  $n_e$ . We then obtain a similar behavior to that of the curve in the first inset of Figure 3, with a critical density value  $n_e^* \simeq 0.51$  close to the numerical result 0.545. Spin correlations in the low temperature regime are therefore efficient in reducing the effective particle mass as long as their typical length does not exceed the average distance between particles, or when the particle density is low enough. This can be compared with two-dimensional electron systems involving Coulomb interaction in silicon where an electron density increase leads instead to a reduction of the effective mass, whereas at lower electron density there is a sharp increase of the mass below a threshold value [21, 25, 23, 26].

The density of states  $n'_e$  is displayed in Figure 4 as function of  $\mu$  and for several values of  $u$ . At low temperature ( $u \simeq 1$ ) a peak in density develops which is due to the singular part of the self-energy. The peak actually coincides with the divergence of the effective mass of Figure 3, due to the fact that  $n'_e$  is proportional to  $m^*$ . The location of this peak moves to higher energy as the temperature decreases. In Figure 5 we have plotted the specific heat  $C_v/k_B = -T\partial^2\Omega/\partial^2T$  for different values of  $\mu$ . For low chemical potential or low electron density the specific heat is dominated by the spin fluctuations which is characterized by an exponential decay near zero temperature and logarithmic divergence near the singularity at  $u_c$ . As  $\mu$  (or density) increases the contribution from the electrons which is linear in  $T$  at low temperature becomes dominant whereas the singular peak vanishes at a value  $\mu < 6$ .

## 5. Conclusion

We have studied a two-dimensional system of itinerant Fermi particles embedded inside a dilute Ising system or spin bath using Grassmannian path integral technique. Particles and spins are interacting through the principle of volume exclusion which yields an effective interaction between the electrons. This interaction originates from the spin fluctuations. At low temperature, the spin fluctuations tend to lower the mass of the particles whereas in the high temperature regime the independent spin fluctuations lead to a small mass increase proportional to  $1/T^4$ , which makes the electronic sector a Fermi liquid. At very low temperature, we expect that there are still spin fluctuations as some of the electron kinetic energy  $t$  can be exchanged for spin fluctuations mediated by  $J$ . At the critical point of the dilute Ising system, the long range fluctuations lead to a sharp increase of the mass which can be considered as a localized phase. As a result the density of states and specific heat present a similar singular behavior as function of the energy and temperature respectively.

## Appendix A. Representation of the fermionic action in terms of fermionic oscillator

The Ising action  $S_0$ , equation (6), can be diagonalized in order to express the quadratic action as a fermionic oscillator. Let us consider the following change of Grassmannian variables for which

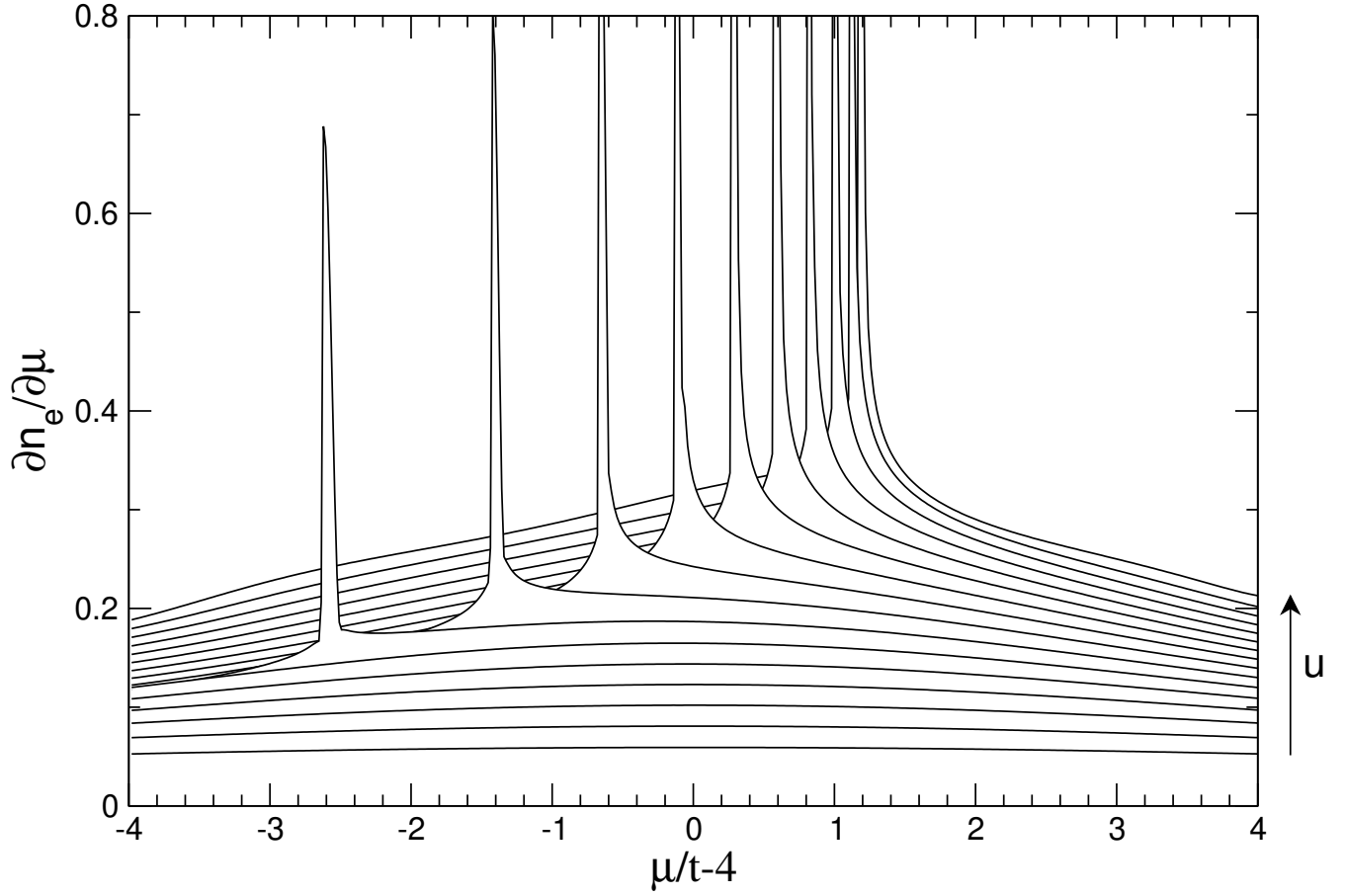


Figure 4: Density of states as function of  $\mu$  for several temperature values. From bottom to top,  $u$  increases ( $T$  decreases) from 0.2 to 0.95 (see arrow) by increments of 0.05. Each curve is shifted upward for clarity.

the Jacobian is unity

$$a_{\mathbf{k}} = 2iu\alpha \sin k_y \left( \frac{1}{\text{Re}g_{\mathbf{k}} + \sqrt{\Delta_{\mathbf{k}}}} b_{-\mathbf{k}} + \frac{1}{\text{Re}g_{\mathbf{k}} - \sqrt{\Delta_{\mathbf{k}}}} b_{\mathbf{k}} \right)$$

$$\bar{a}_{\mathbf{k}} = 2iu\alpha \sin k_x \left( \frac{1}{\text{Re}g_{\mathbf{k}} + \sqrt{\Delta_{\mathbf{k}}}} \bar{b}_{-\mathbf{k}} + \frac{1}{\text{Re}g_{\mathbf{k}} - \sqrt{\Delta_{\mathbf{k}}}} \bar{b}_{\mathbf{k}} \right)$$

$$a_{-\mathbf{k}} = \alpha (\bar{b}_{-\mathbf{k}} + \bar{b}_{\mathbf{k}}), \quad \bar{a}_{-\mathbf{k}} = \alpha (b_{-\mathbf{k}} + b_{\mathbf{k}})$$

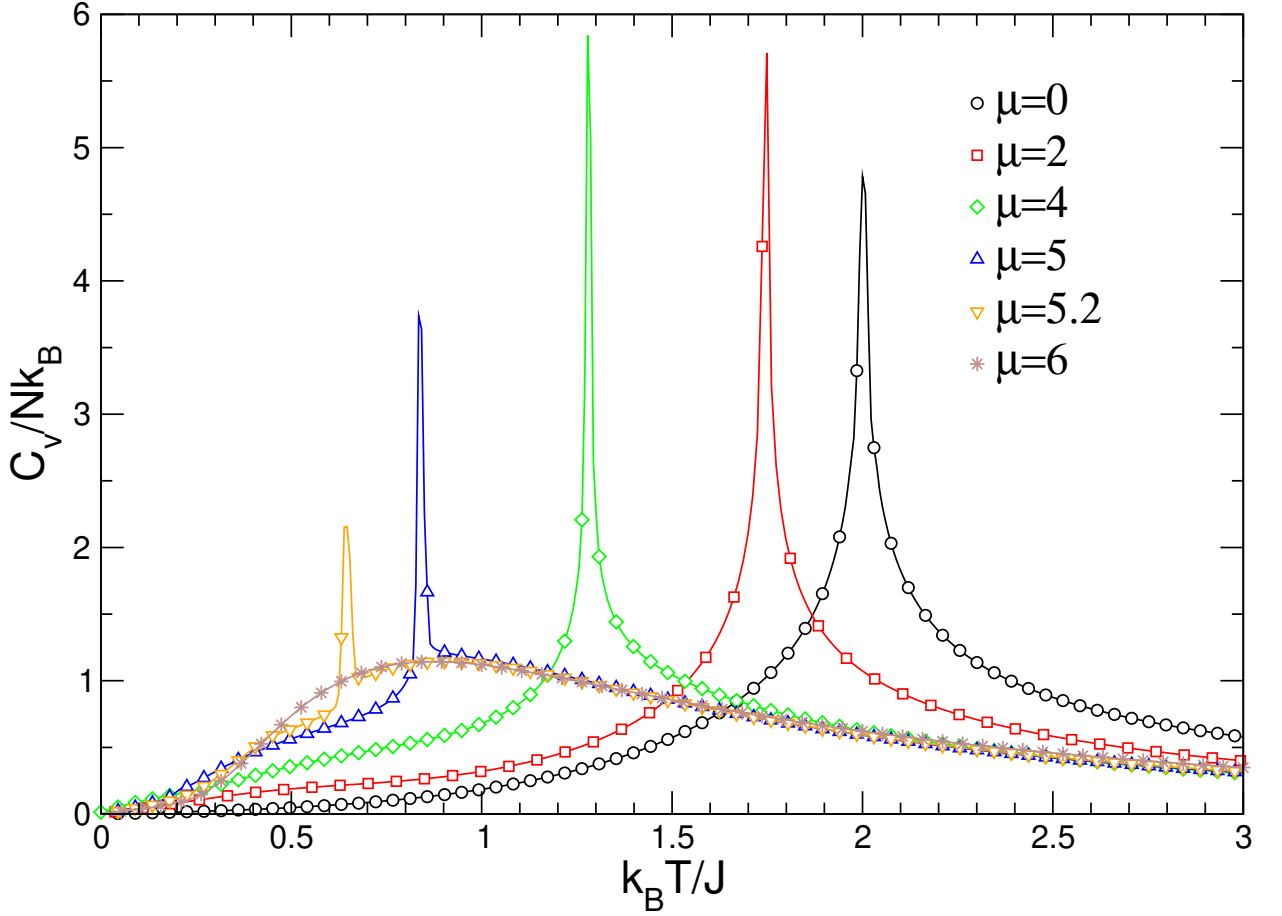


Figure 5: Specific heat for several value of the chemical potential.

$$\Delta_{\mathbf{k}} = (\text{Re}g_{\mathbf{k}})^2 - 4u^2 \sin k_x \sin k_y, \quad \alpha^2 = u \left( \frac{\sin k_x \sin k_y}{\Delta_{\mathbf{k}}} \right)^{1/2}. \quad (\text{A.1})$$

This transformation leads to an action that is simply the sum of two independent fermionic oscillators with frequencies  $\Omega_{\mathbf{k}}$  and  $\Omega_{-\mathbf{k}}$  in the reduced Brillouin zone

$$\begin{aligned} S_0 &= 2N \ln \cosh(\beta J) + \sum_{\mathbf{k}}' (\Omega_{\mathbf{k}} b_{\mathbf{k}} \bar{b}_{\mathbf{k}} + \Omega_{-\mathbf{k}} b_{-\mathbf{k}} \bar{b}_{-\mathbf{k}}), \\ \Omega_{\mathbf{k}} &= 2u \sqrt{\sin k_x \sin k_y} \frac{\sqrt{\Delta_{\mathbf{k}}} + i \text{Im}g_{\mathbf{k}}}{\sqrt{\Delta_{\mathbf{k}}} - \text{Re}g_{\mathbf{k}}}, \\ \Omega_{-\mathbf{k}} &= -2u \sqrt{\sin k_x \sin k_y} \frac{\sqrt{\Delta_{\mathbf{k}}} - i \text{Im}g_{\mathbf{k}}}{\sqrt{\Delta_{\mathbf{k}}} + \text{Re}g_{\mathbf{k}}}. \end{aligned} \quad (\text{A.2})$$

The partition function  $Z_0$  for the Ising model is therefore equivalent to

$$Z_0 = \int \prod_{\mathbf{k}}' d\bar{b}_{\mathbf{k}} db_{\mathbf{k}} db_{-\mathbf{k}} d\bar{b}_{-\mathbf{k}} e^{S_0} = \cosh(\beta J)^{2N} \prod_{\mathbf{k}}' \Omega_{\mathbf{k}} \Omega_{-\mathbf{k}}$$

$$\begin{aligned}
&= \cosh(\beta J)^{2N} \prod_{\mathbf{k}}' [ |g_{\mathbf{k}}|^2 - 4u^2 \sin(k_x) \sin(k_y) ] \\
&= \cosh(\beta J)^{2N} \prod_{\mathbf{k}}' [ (1+u^2)^2 - 2u(1-u^2)(\cos k_x + \cos k_y) ].
\end{aligned} \tag{A.3}$$

## Appendix B. Perturbation scheme for the Green function

The Ising unperturbed action  $S_0$ , equation (3), can be written, up to the constant term  $2N \ln \cosh(\beta J)$ , as

$$S_0 = \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}'} \Psi_{\mathbf{r}}^\top (A_0)_{\mathbf{r}, \mathbf{r}'} \Psi_{\mathbf{r}'}, \tag{B.1}$$

with the  $2 \times 2$  matrix elements

$$(A_0)_{\mathbf{r}, \mathbf{r}'} = \begin{pmatrix} u(\delta_{\mathbf{r}', \mathbf{r}-\mathbf{x}} - \delta_{\mathbf{r}', \mathbf{r}+\mathbf{x}}) & \delta_{\mathbf{r}', \mathbf{r}} - u\delta_{\mathbf{r}', \mathbf{r}+\mathbf{x}} - u\delta_{\mathbf{r}', \mathbf{r}-\mathbf{y}} - u^2\delta_{\mathbf{r}', \mathbf{r}+\mathbf{x}-\mathbf{y}} \\ -\delta_{\mathbf{r}', \mathbf{r}} + u\delta_{\mathbf{r}', \mathbf{r}-\mathbf{x}} + u\delta_{\mathbf{r}', \mathbf{r}+\mathbf{y}} + u^2\delta_{\mathbf{r}', \mathbf{r}-\mathbf{x}+\mathbf{y}} & u(\delta_{\mathbf{r}', \mathbf{r}+\mathbf{y}} - \delta_{\mathbf{r}', \mathbf{r}-\mathbf{y}}) \end{pmatrix},$$

and  $\Psi_{\mathbf{r}} = (a_{\mathbf{r}}, \bar{a}_{\mathbf{r}})^\top$ . The interaction part with the holes can be expressed as a local potential in this new basis

$$\sum_{\mathbf{r}} \frac{1 + 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)}{1 - 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)} a_{\mathbf{r}} \bar{a}_{\mathbf{r}} = \frac{1}{2} \sum_{\mathbf{r}} \Psi_{\mathbf{r}}^\top V_{\mathbf{r}} \Psi_{\mathbf{r}}, \tag{B.2}$$

with the  $2 \times 2$  local matrix

$$V_{\mathbf{r}} = \begin{pmatrix} 0 & v_{\mathbf{r}} \\ -v_{\mathbf{r}} & 0 \end{pmatrix} = v_{\mathbf{r}} i\sigma_2, \quad v_{\mathbf{r}} = \frac{1 + 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)}{1 - 2\beta^{-1} \sum_{\omega_n} G_{\mathbf{r}}(\omega_n)}. \tag{B.3}$$

The partial derivatives of  $\ln \det A$  that appears in the saddle point equations (13), with the global block matrix  $A = A_0 + V$  and  $(V)_{\mathbf{r}, \mathbf{r}'} = V_{\mathbf{r}} \delta_{\mathbf{r}, \mathbf{r}'}$ , can be expressed as a series expansion in powers of  $V$ . These derivatives with respect to  $G_{\mathbf{r}}(\omega_n)$  are both proportional to the derivative with respect to  $v_{\mathbf{r}}$  and we define the quantity

$$\begin{aligned}
\mathcal{G}_{\mathbf{r}} &\equiv \frac{\delta}{\delta v_{\mathbf{r}}} \ln \det A = \frac{\delta}{\delta v_{\mathbf{r}}} \text{tr} \ln A \\
&= \frac{\delta}{\delta v_{\mathbf{r}}} \text{tr} [\ln A_0 + \ln(1 + A_0^{-1}V)] = \frac{\delta}{\delta v_{\mathbf{r}}} \left[ \text{tr}(A_0^{-1}V) - \frac{1}{2} \text{tr}(A_0^{-1}V A_0^{-1}V) + \dots \right].
\end{aligned} \tag{B.4}$$

Let  $T_{\mathbf{r}}$  be the block matrix for which the diagonal element at position  $\mathbf{r}$  is equal to  $i\sigma_2$ , or  $(T_{\mathbf{r}})_{\mathbf{s}, \mathbf{s}'} = \delta_{\mathbf{s}, \mathbf{r}} \delta_{\mathbf{s}', \mathbf{r}} i\sigma_2$ . Then  $V = \sum_{\mathbf{r}} v_{\mathbf{r}} T_{\mathbf{r}}$ , and one has the following expansion

$$\mathcal{G}_{\mathbf{r}} = \text{tr}(A_0^{-1}T_{\mathbf{r}}) - \sum_{\mathbf{s}} v_{\mathbf{s}} \text{tr}(A_0^{-1}T_{\mathbf{r}} A_0^{-1}T_{\mathbf{s}}) - \sum_{\mathbf{s}, \mathbf{s}'} v_{\mathbf{s}} v_{\mathbf{s}'} \text{tr}(A_0^{-1}T_{\mathbf{r}} A_0^{-1}T_{\mathbf{s}} A_0^{-1}T_{\mathbf{s}'}) + \dots$$

Since the elements  $(A_0)_{\mathbf{r}, \mathbf{r}'}$  depend only on the difference  $\mathbf{r} - \mathbf{r}'$ , we define the following Fourier transform  $(A_0)_{\mathbf{r}, \mathbf{r}'} = \sum_{\mathbf{k}} (\tilde{A}_0)_{\mathbf{k}} e^{i\mathbf{k} \cdot (\mathbf{r}' - \mathbf{r})}$ , with  $\mathbf{k} = (-k_x, k_y)$  and

$$(\tilde{A}_0)_{\mathbf{k}} = \begin{pmatrix} -2iu \sin k_x & g_{\mathbf{k}} \\ -\bar{g}_{\mathbf{k}} & -2iu \sin k_y \end{pmatrix}. \tag{B.5}$$

The Fourier elements of the inverse matrix  $A_0^{-1}$  are then given by

$$\begin{aligned} (\tilde{A}_0^{-1})_{\mathbf{k}} &= \frac{1}{D_0(k_x, k_y)} \begin{pmatrix} -2iu \sin k_y & -g_{\mathbf{k}} \\ \bar{g}_{\mathbf{k}} & -2iu \sin k_x \end{pmatrix}, \\ D_0(k_x, k_y) &= (1 + u^2)^2 - 2u(1 - u^2)(\cos k_x + \cos k_y). \end{aligned} \quad (\text{B.6})$$

One obtains, after some algebra, the expression for the Fourier transform of the quantity  $\mathcal{G}_{\mathbf{r}}$

$$\tilde{\mathcal{G}}_{\mathbf{k}} = \sum_{\mathbf{k}'} \text{tr}\{(\tilde{A}_0^{-1})_{\mathbf{k}'} i\sigma_2\} - \tilde{v}_{\mathbf{k}} \sum_{\mathbf{k}'} \text{tr}\{(\tilde{A}_0^{-1})_{\mathbf{k}'} i\sigma_2 \text{tr}(\tilde{A}_0^{-1})_{\mathbf{k}'-\mathbf{k}} i\sigma_2\} + \dots \quad (\text{B.7})$$

### Appendix C. Expression of $\mathcal{F}(v)$ in term of complete elliptic integrals

The expression of  $\mathcal{F}(v)$  in equation (14) defined in terms of a double discrete sum converges, in the thermodynamical limit  $N \rightarrow \infty$ , to the following double integral

$$\mathcal{F}(v) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} dk_x dk_y \frac{1 + v - u(\cos k_x + \cos k_y) - u^2 \cos(k_x + k_y)}{\mathcal{D}(k_x, k_y)}, \quad (\text{C.1})$$

where the denominator  $\mathcal{D}(k_x, k_y)$  is equal to

$$\begin{aligned} \mathcal{D}(k_x, k_y) &= (1 + v)^2 + u^2(2 + u^2) - 2u(1 + v - u^2)(\cos k_x + \cos k_y) \\ &\quad - 2u^2v \cos(k_x + k_y). \end{aligned} \quad (\text{C.2})$$

We can rearrange the expression for  $\mathcal{F}(v)$  using symmetries such as  $\mathcal{D}(k_x, k_y) = \mathcal{D}(k_y, k_x)$  so that

$$\begin{aligned} \mathcal{F}(v) &= \frac{1}{2v} - \frac{1 - v^2 + u^2(2 + u^2)}{8\pi^2 v} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{dk_x dk_y}{\mathcal{D}(k_x, k_y)} \\ &\quad + \frac{u(1 - u^2)}{2\pi^2 v} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} dk_x dk_y \frac{\cos k_y}{\mathcal{D}(k_x, k_y)}. \end{aligned} \quad (\text{C.3})$$

The computation of  $\mathcal{F}(v)$  is reduced to evaluating two double integrals. The first integral gives

$$\begin{aligned} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{dk_x dk_y}{\mathcal{D}(k_x, k_y)} &\equiv \frac{K_1}{(1 + v)^2 + u^2(2 + u^2)} \\ K_1 &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{dk_x dk_y}{1 - \alpha(\cos k_x + \cos k_y) - \beta \cos(k_x + k_y)}, \end{aligned} \quad (\text{C.4})$$

with

$$\alpha = \frac{2u(1 + v - u^2)}{(1 + v)^2 + u^2(2 + u^2)}, \quad \beta = \frac{2u^2v}{(1 + v)^2 + u^2(2 + u^2)}. \quad (\text{C.5})$$

After integrating over  $k_x$ , we find that

$$\begin{aligned} K_1 &= \frac{2\pi}{\alpha} \int_{-\pi}^{\pi} \frac{dk_y}{\sqrt{(r_1 - \cos k_y)(r_2 - \cos k_y)}} \\ r_1 &= \frac{1 + \beta + \sqrt{\alpha^2 + 2\beta(1 + \beta)}}{\alpha}, \quad r_2 = \frac{1 + \beta - \sqrt{\alpha^2 + 2\beta(1 + \beta)}}{\alpha}, \end{aligned} \quad (\text{C.6})$$

with  $r_1 > r_2 > 1$ . A standard change of variable  $k_y \rightarrow \tan(k_y/2)$  leads to the evaluation of the following complete elliptic integral of the first kind [27]

$$K_1 = \frac{8\pi}{|\alpha| [(r_1^2 - 1)(r_2^2 - 1)]^{1/4}}, \quad q^2 = \frac{1}{2} \left( 1 - \frac{(r_1 r_2 - 1)}{\sqrt{(r_1^2 - 1)(r_2^2 - 1)}} \right). \quad (\text{C.7})$$

The second integral is given by

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} dk_x dk_y \frac{\cos k_y}{\mathcal{D}(k_x, k_y)} \equiv \frac{K_2}{(1+v)^2 + u^2(2+u^2)}$$

$$K_2 = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} dk_x dk_y \frac{\cos k_y}{1 - \alpha(\cos k_x + \cos k_y) - \beta \cos(k_x + k_y)}. \quad (\text{C.8})$$

As before, after integrating first over  $k_x$  and performing the previous change of variable on  $k_y$ , we obtain [27]

$$K_2 = \frac{8\pi}{|\alpha| [(r_1^2 - 1)(r_2^2 - 1)]^{1/4}} \frac{1 + A_0}{1 - A_0} \left[ K(q) - \Pi \left( -\frac{(1 - A_0)^2}{4A_0}, q \right) \right],$$

$$A_0^2 = \frac{(r_1^2 - 1)(r_2^2 - 1)}{[(r_1 + 1)(r_2 + 1)]^2}, \quad (\text{C.9})$$

where  $\Pi$  is the complete elliptic integral of the third kind. Finally, we can express  $\mathcal{F}(v)$  as function of  $K_1$  and  $K_2$

$$\mathcal{F}(v) = \frac{1}{2v} - \frac{1}{v} \frac{1 - v^2 + u^2(2 + u^2)}{(1 + v)^2 + u^2(2 + u^2)} \frac{K_1}{8\pi^2} + \frac{1}{v} \frac{u(1 - u^2)}{(1 + v)^2 + u^2(2 + u^2)} \frac{K_2}{2\pi^2}. \quad (\text{C.10})$$

In particular, in the high temperature regime  $T \rightarrow \infty$  or  $u \simeq 0$ , we find the following asymptotic limit

$$\mathcal{F}(v) \simeq \frac{1}{1 + v} - \frac{3u^4}{(1 + v)^4}. \quad (\text{C.11})$$

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