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# Event-triggered boundary control of constant-parameter reaction-diffusion PDEs: a small-gain approach

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**Abstract**—This paper deals with an event-triggered boundary control of constant-parameters reaction-diffusion PDE systems. The approach relies on the emulation of backstepping control along with a suitable triggering condition which establishes the time instants at which the control value needs to be sampled/updated. In this paper, it is stated that under the proposed event-triggered boundary control, there exists a minimal dwell-time (independent of the initial condition) between two triggering times and furthermore the well-posedness and global exponential stability are guaranteed. The analysis follows small-gain arguments and builds on recent papers on sampled-data control for this kind of PDE. A simulation example is presented to validate the theoretical results.

## I. INTRODUCTION

Control and estimation strategies must be implemented and validated into digital platforms. It is important to study carefully the issues concerning digital control such as sampling. This is because, if sampling is not addressed properly, the stability and estimation properties may be lost. For finite-dimensional systems, namely networked control systems modeled by ordinary differential equations (ODEs), digital control has been extensively developed and several schemes for discretization and for sampling in time continuous-time controllers have been investigated, e.g., by sampled-data control [11] and event-triggered control strategies [28], [10], [23], [20], [9], [24], [13], [21]. The latter has become popular and promising due to not only its efficient way of using communication and computational resources by updating the control value aperiodically (only when needed) but also due to its rigorous way of implementing continuous-time controllers into digital platforms.

In general, event-triggered control includes two main components: a feedback control law which stabilizes the system and an event-triggered mechanism which contains a triggering condition that determines the time instants at which the control needs to be updated. Two general approaches exist for the design: *Emulation* from which the controller is a priori pre-designed and only the event-triggered algorithm has to be designed (as in e.g. [28]) and *Co-design*, where the joint design of the control law and the event-triggering mechanism is performed simultaneously

(see e.g. [26]).

Nevertheless, for partial differential equations (PDEs) sampled-data and event-triggered control strategies without model reduction have not achieved a sufficient level of maturity as in the finite-dimensional case. It has not been sufficiently clear (from theoretical and practical point of view) how fast sampling the in-domain or the boundary continuous-time controllers should be for preserving both stability and convergence properties of PDE systems. Few approaches on sampled-data and event-triggered control of parabolic PDEs are considered in [8], [15], [25], [31], [12]. In the context of abstract formulation of distributed parameter systems, sampled-data control is investigated in [22] and [29]. For hyperbolic PDEs, sampled-data is studied in [3] and [14]. Some recent works have introduced event-triggered control strategies for linear hyperbolic PDEs under an emulation approach [4], [5], [6]. In [4] and [5], for instance, event-triggered boundary controllers for linear conservation laws using output feedback are studied by following Lyapunov techniques (inspired by [1]). In [6], the approach relies on the backstepping method for coupled system of balance laws (inspired by [30], [18]) which leads to a full-state feedback control which is sampled according to a dynamic triggering condition. Under such a triggering policy, it has been possible to prove the existence of a minimal dwell-time between triggering time instants and therefore avoiding the so-called Zeno phenomena.

In sampled-data control as well as in event-triggered control scenarios for PDEs, the effect of sampling (and therefore, the underlying actuation error) has to be carefully handled. In particular, for reaction-diffusion parabolic PDEs the situation of having such errors at the boundaries has been challenging and has become a central issue; especially when having Dirichlet boundary conditions due to the lack of an ISS-Lyapunov function for the stability analysis. In [15] this problem has been overcome by studying ISS properties directly from the nature of the PDE system (see also e.g. [17], [16]) while using modal decomposition and Fourier series analysis. Lyapunov-based approach has not been necessary to perform the stability analysis and to be able to come up with ISS properties and small gain arguments. Thus, it has been possible to establish the robustness with respect to the actuation error. This approach has allowed the derivation of an estimate of the diameter of the sampling period on which the control is updated in a sampled-and-hold fashion. The drawback, however, is that such a period turns out to be

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truly small, rendering the approach very conservative. With periodic implementation, one may produce unnecessary updates of the sampled controllers, which cause over utilization of computational and communication resources, as well as actuator changes that are more frequent than necessary.

This issue strongly motivates the study of event-triggered control for PDE systems. Therefore, inspired by [15], in this paper we propose an event-triggered boundary control based on the emulation of the backstepping boundary control. An event-triggering condition is derived and the stability analysis is performed by using small-gain arguments.

The main contributions are summed up as follows:

- We state that under the event-triggered control no Zeno solutions can appear. A uniform minimal dwell-time (independent of the initial condition) can be obtained.
- We guarantee the existence and uniqueness of solutions to the closed-loop system.
- We prove that under the event-triggered boundary control, the closed-loop system is globally exponentially stable in the  $L^2$ - norm sense.

The paper is organized as follows. In Section II, we introduce the class of reaction-diffusion parabolic systems, some preliminaries on stability and backstepping boundary control and preliminary notion of existence and uniqueness of solutions. Section III provides the event-triggered boundary control and the main results. Section IV provides a numerical example to illustrate the main results. Finally, conclusions and perspectives are given in Section V.

The proofs of various results are omitted due to space limitations, but they can be found in [7].

*Notations:*  $\mathbb{R}_+$  will denote the set of nonnegative real numbers. Let  $S \subseteq \mathbb{R}^n$  be an open set and let  $A \subseteq \mathbb{R}^n$  be a set that satisfies  $S \subseteq A \subseteq \bar{S}$ . By  $C^0(A; \Omega)$ , we denote the class of continuous functions on  $A$ , which take values in  $\Omega \subseteq \mathbb{R}$ . By  $C^k(A; \Omega)$ , where  $k \geq 1$  is an integer, we denote the class of functions on  $A$ , which takes values in  $\Omega$  and has continuous derivatives of order  $k$ . In other words, the functions of class  $C^k(A; \Omega)$  are the functions which have continuous derivatives of order  $k$  in  $S = \text{int}(A)$  that can be continued continuously to all points in  $\partial S \cap A$ .  $L^2(0, 1)$  denotes the equivalence class of Lebesgue measurable functions  $f : [0, 1] \rightarrow \mathbb{R}$  such that  $\|f\| = \left( \int_0^1 |f(x)|^2 dx \right)^{1/2} < \infty$ . Let  $u : \mathbb{R}_+ \times [0, 1] \rightarrow \mathbb{R}$  be given.  $u[t]$  denotes the profile of  $u$  at certain  $t \geq 0$ , i.e.  $(u[t])(x) = u(t, x)$ , for all  $x \in [0, 1]$ . For an interval  $I \subseteq \mathbb{R}_+$ , the space  $C^0(I; L^2(0, 1))$  is the space of continuous mappings  $I \ni t \rightarrow u[t] \in L^2(0, 1)$ .  $H^2(0, 1)$  denotes the Sobolev space of functions  $f \in L^2(0, 1)$  with square integrable (weak) first and second-order derivatives  $f'(\cdot), f''(\cdot) \in L^2(0, 1)$ .  $I_m(\cdot), J_m(\cdot)$  with  $m \in \mathbb{Z}$ , denote the modified Bessel and (nonmodified) Bessel functions of the first kind.

## II. PRELIMINARIES AND PROBLEM DESCRIPTION

Let us consider the following scalar reaction-diffusion system with constant coefficients:

$$u_t(t, x) = \theta u_{xx}(t, x) + \lambda u(t, x) \quad (1)$$

$$u(t, 0) = 0 \quad (2)$$

$$u(t, 1) = U(t) \quad (3)$$

and initial condition:

$$u(0, x) = u_0(x) \quad (4)$$

where  $\theta > 0$  and  $\lambda \in \mathbb{R}$ .  $u : [0, \infty) \times [0, 1] \rightarrow \mathbb{R}$  is the system state and  $U(t) \in \mathbb{R}$  is the control input. The control design relies on the Backstepping approach [27], [19] under which the following continuous-time controller (nominal boundary feedback) has been obtained:

$$U(t) = \int_0^1 K(1, y) u(t, y) dy \quad (5)$$

It has then been proved that the under continuous-time controller (5) with control gain  $K$  satisfying:

$$K(x, y) = -y\gamma \frac{I_1\left(\sqrt{\gamma(x^2 - y^2)}\right)}{\sqrt{\gamma(x^2 - y^2)}} \quad (6)$$

evolving in a triangular domain given by  $\mathcal{T} = \{(x, y) : 0 \leq y < x \leq 1\}$  and with  $\gamma = (\lambda + c)/\theta$  (where  $c \geq 0$  is a design parameter), the closed-loop system (1)-(4) is globally exponentially stable in  $L^2$ - norm sense.

### A. Event-triggered control and emulation of the backstepping design

We aim at stabilizing the closed-loop system on events while sampling the continuous-time controller (5) at certain sequence of time instants  $(t_j)_{j \in \mathbb{N}}$ , that will be characterized later on. The control value is held constant between two successive time instants and it is updated when some state-dependent condition is verified. In this scenario, we need to suitably modify the boundary condition in (1)-(3). The boundary value of the state is going to be given by:

$$u(t, 1) = U_d(t) \quad (7)$$

with

$$U_d(t) = \int_0^1 K(1, y) u(t_j, y) dy \quad (8)$$

for all  $t \in [t_j, t_{j+1})$ ,  $j \geq 0$ . Note that  $U_d(t) = U(t) + d(t)$  with  $U(t)$  given by (5) and  $d$  given by:

$$d(t) = \int_0^1 K(1, y) u(t_j, y) dy - \int_0^1 K(1, y) u(t, y) dy \quad (9)$$

Here,  $d$  (which will be fully characterized along with  $(t_j)_{j \in \mathbb{N}}$  in the next section) can be viewed as an actuation deviation between the nominal boundary feedback and the event-triggered boundary control <sup>1</sup>.

<sup>1</sup>In sampled-data control as in [15], such a deviation is called input holding error.

Hence, the control problem we aim at handling is the following:

$$u_t(t, x) = \theta u_{xx}(t, x) + \lambda u(t, x) \quad (10)$$

$$u(t, 0) = 0 \quad (11)$$

$$u(t, 1) = U_d(t) \quad (12)$$

for all  $t \in [t_j, t_{j+1})$ ,  $j \geq 0$ , and initial condition:

$$u(0, x) = u_0(x) \quad (13)$$

We will perform the emulation of the backstepping which requires also information of the target system. Indeed, let us recall that the backstepping method makes uses of an invertible Volterra transformation:

$$w(t, x) = u(t, x) - \int_0^x K(x, y)u(t, y)dy \quad (14)$$

with kernel  $K(x, y)$  satisfying (6) which maps the system (10)-(13) into the following target system:

$$w_t(t, x) = \theta w_{xx}(t, x) - cw(t, x) \quad (15)$$

$$w(t, 0) = 0 \quad (16)$$

$$w(t, 1) = d(t) \quad (17)$$

with initial condition:

$$w(0, x) = u_0(x) - \int_0^x K(x, y)u_0(y)dy \quad (18)$$

where  $c > 0$  can be chosen arbitrary.

**Remark 1:** It is worth recalling that the Volterra backstepping transformation (14) is invertible whose inverse is given as follows:

$$u(t, x) = w(t, x) + \int_0^x L(x, y)w(t, y)dy \quad (19)$$

where  $L$  satisfies:

$$L(x, y) = -y\gamma \frac{J_1\left(\sqrt{\gamma(x^2 - y^2)}\right)}{\sqrt{\gamma(x^2 - y^2)}} \quad (20)$$

with  $\gamma = (\lambda + c)/\theta$ .

### B. Well-posedness issues

The notion of solution for 1-D linear parabolic systems under boundary sampled-data control has been rigorously analyzed in [15]. In this paper, we follow the same framework.

**Proposition 1:** *There exists a unique solution  $u \in C^0([t_j, t_{j+1}]; L^2(0, 1))$  to the system (10)-(13) between two time instants  $t_j$  and  $t_{j+1}$  satisfying  $u \in C^1((t_j, t_{j+1}) \times [0, 1])$ ,  $u[t] \in C^2([0, 1])$  for all  $t \in (t_j, t_{j+1}]$  and initial data  $u[t_j] \in L^2(0, 1)$ .*

In what follows we assume that in open-loop, the system (10)-(13) is unstable or neutrally stable, i.e.,  $\lambda \geq \theta\pi^2$ . The analysis is similar (and far easier) for the case where the open-loop system is asymptotically stable, because in this case we can use the trivial feedback law with  $K(1, y) = 0$ .

## III. EVENT-TRIGGERED BOUNDARY CONTROL AND MAIN RESULTS

In this section we introduce the event-triggered boundary control and the main results: the existence of a minimal dwell-time which is independent of the initial condition, the well-posedness and the exponential stability of the closed-loop system under the event-triggered boundary control.

Let us first define the event-triggered boundary control considered in this paper. It encloses both a triggering condition (which determines the time instant at which the controller needs to be sampled/updated) and the backstepping boundary feedback (8). The proposed event-triggering condition is based on the evolution of the magnitude of the actuation deviation (9) and the evolution of the  $L^2$ - norm of the state.

**Definition 1:** *Let  $\beta > 0$  and let  $k(y) := K(1, y)$  with  $K$  being the kernel given in (6). The event-triggered boundary control is defined by considering the following components:*

I) *(The event-trigger) The times of the events  $t_j \geq 0$  with  $t_0 = 0$  form a finite or countable set of times which is determined by the following rules for some  $j \geq 0$ :*

a) if  $\{t \in \mathbb{R}_+ | t > t_j \wedge |d(t)| > \beta\|k\|\|u[t]\| + \beta\|k\|\|u[t_j]\|\} = \emptyset$  then the set of the times of the events is  $\{t_0, \dots, t_j\}$ .

b) if  $\{t \in \mathbb{R}_+ | t > t_j \wedge |d(t)| > \beta\|k\|\|u[t]\| + \beta\|k\|\|u[t_j]\|\} \neq \emptyset$ , then the next event time is given by:

$$t_{j+1} := \inf\{t \in \mathbb{R}_+ | t > t_j \wedge |d(t)| > \beta\|k\|\|u[t]\| + \beta\|k\|\|u[t_j]\|\} \quad (21)$$

II) *(the control action) The boundary feedback law*

$$U_d(t) = \int_0^1 k(y)u(t_j, y)dy, \quad \forall t \in [t_j, t_{j+1}) \quad (22)$$

### A. Avoidance of the Zeno phenomena

It is worth mentioning that guaranteeing the existence of a minimal dwell-time between two triggering times avoids the so-called Zeno phenomena that means infinite triggering times in a finite-time interval. It represents infeasible practical implementations into digital platforms because it would be required to sample infinitely fast. Before we tackle the result on existence of minimal dwell-time, let us first introduce the following intermediate result.

**Lemma 1:** *For the closed-loop system (10)-(12), the following estimate holds, for all  $t \in [t_j, t_{j+1}]$ ,  $j \geq 0$ :*

$$\sup_{t_j \leq s \leq t_{j+1}} (\|u[s]\|) \leq Q\|u[t_j]\| \quad (23)$$

where  $Q = e^{p/2(t_{j+1}-t_j)}(1 + \frac{\sqrt{3}}{3}\|k\| + \frac{\|k\|}{\sqrt{p}}) + \frac{\sqrt{3}}{3}\|k\|$  and  $p = -2\theta\pi^2 + 2\lambda + \frac{1}{3}\lambda^2$ .

**Theorem 1:** *Under the event-triggered boundary control (21)-(22), there exists a minimal dwell-time between two triggering times, i.e. there exists a constant  $\tau > 0$  (independent of the initial condition  $u_0$ ) such that  $t_{j+1} - t_j \geq \tau$ , for all  $j \geq 0$ .*

Since there is a minimal dwell-time (which is uniform and does not depend on either initial condition or on the state of the system), no Zeno solution can appear. Consequently, the following result on the existence of solutions in of the system system (10)-(13) with (21)-(22), holds for all  $t \in \mathbb{R}_+$ .

**Corollary 1:** *For every  $u_0 \in L^2(0,1)$ , there exist a unique solution  $u \in C^0(\mathbb{R}_+; L^2(0,1))$  to the system (10)-(13), (21),(22) satisfying  $u \in C^1(I \times [0, 1])$ ,  $u[t] \in C^2([0, 1])$  for all  $t > 0$  where  $I = \mathbb{R}_+ \setminus \{t_j \geq 0, k = 0, 1, 2, \dots\}$*

### B. Stability result

In this subsection, we are going to follow small-gain arguments and seek for an Input-to-State stability property with respect to the deviation  $d(t)$ .

**Lemma 2 (ISS of the target system):** *The target system (15)-(18) is ISS with respect to  $d(t)$ ; more precisely, the following estimate holds:*

$$\|w[t]\| \leq Ge^{-\sigma t}\|w[0]\| + \gamma \sup_{0 \leq s \leq t} (|d(s)|e^{-\sigma(t-s)}) \quad (24)$$

where  $\sigma \in (0, \mu_1)$  with  $\mu_1 = \pi^2\theta + c$ ,  $G := \sqrt{(1+b^{-1})}$ , for arbitrary  $b > 0$  and the gain  $\gamma$  is given as follows:

$$\gamma := \sqrt{(1+b)} \begin{cases} \left( \frac{\pi^2\theta+c}{\pi^2\theta+c-\sigma} \right) \frac{(\sinh(\frac{2\sqrt{c}}{\sqrt{\theta}}) - 2\frac{\sqrt{c}}{\sqrt{\theta}})}{2\sinh(\frac{\sqrt{c}}{\sqrt{\theta}})(\frac{c}{\theta})^{1/4}}, & \text{if } c \neq 0 \\ \frac{1}{\sqrt{3}} \left( \frac{\pi^2\theta}{\pi^2\theta-\sigma} \right), & \text{if } c = 0 \end{cases} \quad (25)$$

**Theorem 2:** *Let  $\tilde{L} := 1 + \left( \int_0^1 \left( \int_0^x |L(x,y)|^2 dy \right) dx \right)^{1/2}$  with  $L$  satisfying (20) and  $k(y) = K(1,y)$  with  $K$  satisfying (6). Let  $\gamma$  be given as in Lemma 2. Let  $\beta > 0$  be as in (21). If the following condition is fulfilled,*

$$\Phi_e := 2\beta\gamma\|k\|\tilde{L} < 1 \quad (26)$$

*then, the closed-loop system (10)-(13) with event-triggered boundary control (21)-(22) has a unique solution and is globally exponentially stable; i.e. there exist  $M, \sigma > 0$  such that for every  $u_0 \in L^2(0,1)$  the unique mapping  $u \in C^0(\mathbb{R}_+; L^2(0,1))$  satisfying  $u \in C^1(I \times [0, 1])$ ,  $u[t] \in C^2([0, 1])$  for all  $t > 0$  where  $I = \mathbb{R}_+ \setminus \{t_j \geq 0, k = 0, 1, 2, \dots\}$  satisfies:*

$$\|u[t]\| \leq Me^{-\sigma t}\|u[0]\|, \quad \text{for all } t \geq 0 \quad (27)$$

*Proof:* By Corollary 1, the existence and uniqueness of a solution to the system (10)-(13) with event-triggered boundary control (21)-(22) hold. Let us show that the system is globally exponential stable in the  $L^2$ -norm sense.

It follows from (21) that the following inequality holds for all  $t \in [t_j, t_{j+1})$ :

$$|d(t)| \leq \beta\|k\|\|u[t_j]\| + \beta\|k\|\|u[t]\| \quad (28)$$

Therefore, inequality (28) implies the following inequality for all  $t \geq 0$ :

$$\sup_{0 \leq s \leq t} (|d(s)|e^{\sigma s}) \leq 2\beta\|k\| \sup_{0 \leq s \leq t} (\|u[s]\|e^{\sigma s}) \quad (29)$$

On the other hand, by Lemma 2, we have

$$\|w[t]\|e^{\sigma t} \leq G\|w[0]\| + \gamma \sup_{0 \leq s \leq t} (|d(s)|e^{\sigma s}) \quad (30)$$

The following estimate is a consequence of (30):

$$\sup_{0 \leq s \leq t} (\|w[s]\|e^{\sigma s}) \leq G\|w[0]\| + \gamma \sup_{0 \leq s \leq t} (|d(s)|e^{\sigma s}) \quad (31)$$

Hence, combining (29) with (31), we obtain

$$\sup_{0 \leq s \leq t} (\|w[s]\|e^{\sigma s}) \leq G\|w[0]\| + 2\beta\gamma\|k\| \sup_{0 \leq s \leq t} (\|u[s]\|e^{\sigma s}) \quad (32)$$

and using the fact  $\|u[t]\| \leq \tilde{L}\|w[t]\|$ , we get

$$\sup_{0 \leq s \leq t} (\|w[s]\|e^{\sigma s}) \leq G\|w[0]\| + \Phi_e \sup_{0 \leq s \leq t} (\|w[s]\|e^{\sigma s}) \quad (33)$$

where

$$\Phi_e := 2\beta\gamma\|k\|\tilde{L} \quad (34)$$

Notice that, by virtue of (26), it holds that  $\Phi_e < 1$ . Thereby, using the estimate of the backstepping transformation, i.e.  $\|w[t]\| \leq \tilde{K}\|u[t]\|$  with  $\tilde{K} := 1 + \left( \int_0^1 \left( \int_0^x |K(x,y)|^2 dy \right) dx \right)^{1/2}$  and  $K$  satisfying (6), we obtain from (33) and (34) the following estimate for the solution to the closed-loop system (10)-(13) with event-triggered control (21)-(22):

$$\sup_{0 \leq s \leq t} (\|u[s]\|e^{\sigma s}) \leq G(1 - \Phi_e)^{-1}\tilde{K}\tilde{L}\|u[0]\| \quad (35)$$

which leads to

$$\|u[t]\| \leq Me^{-\sigma t}\|u[0]\| \quad (36)$$

with  $M := G(1 - \Phi_e)^{-1}\tilde{K}\tilde{L}$ . It concludes the proof.  $\blacksquare$

## IV. NUMERICAL SIMULATIONS

We consider the reaction-diffusion system with  $\theta = c = 1$ ,  $\lambda = \pi^2$  and initial condition  $u_0(x) = \sum_{n=1}^3 \frac{\sqrt{2}}{n} \sin(n\pi x) + 3(x^2 - x^3)$ ,  $x \in [0, 1]$ . For numerical simulations, the state of the system has been discretized by divided differences on a uniform grid with the step  $h = 0.01$  for the space variable. The discretization with respect to time was done using the implicit Euler

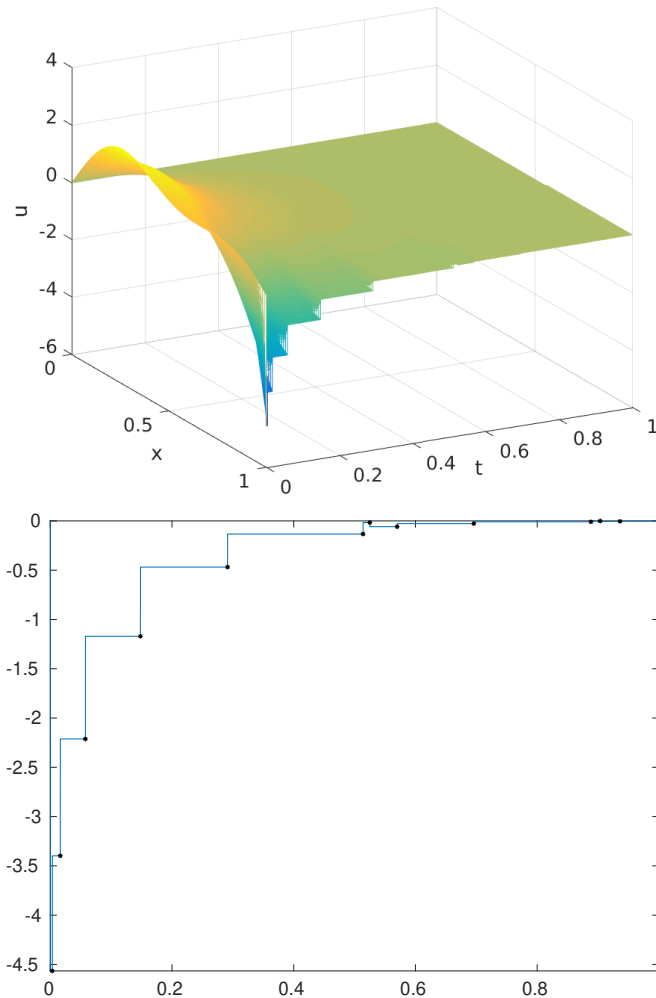


Fig. 1. Numerical solution of the closed-loop system (10)-(13) with event-triggered control (21)-(22) and time-evolution of the event-triggered boundary control (21)-(22).

scheme with step size  $\Delta t = h^2$ .

We stabilize the system on events under the event-triggered boundary control (21)-(22) where the parameter  $\beta$  is selected such that condition (26) in Theorem 2 is verified. In addition,  $\tilde{L} = 1.8407$ ,  $\|k\| = 5.61$  and  $\gamma = 0.574$  which is computed according to the information provided in Lemma 2. We choose e.g.  $\beta = 0.07$  yielding  $\Phi_e = 0.83 < 1$ . In this case, 12 events (updating times of the control) are obtained. Figure 1 shows the numerical solution of the closed-loop system (10)-(13) with event-triggered control (21)-(22) as well as the time-evolution of control function under the event-triggered case.

## V. CONCLUSION

In this paper, we have proposed an event-triggered boundary control to stabilize (on events) a reaction-diffusion PDE system with Dirichlet boundary condition. A suitable state-dependent event-triggering condition is considered. It determines when the control has to be updated. The existence of a minimal dwell-time which is independent

of the initial condition is guaranteed. There is no Zeno behavior and thus the well-posedness and the stability of the closed-loop system are guaranteed as well.

In future work, we may consider observer-based event-triggered control and possibly sampling output measurements on events as well. It may suggest that another event-triggered strategy shall be considered to be combined with the one for actuation. We expect also to address periodic event-triggered strategies inspired by some recent result from finite-dimensional systems [2]. For that, we may use the obtained dwell-time as a period or to come up with a maybe less conservative period. In either cases, the period would be utilized to monitor periodically the triggering condition while the actuation is still on events. This would represent even a more realistic approach toward digital realizations while reducing the consumption of computational resources.

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