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# Approximation of the disturbance dynamics by Extended State Observer Using an Artificial Delay

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**Abstract:** A disturbance estimator based on the design of an extended state observer (ESO) often considers that the time-derivative of the perturbation (or higher order time-derivatives) is very small and can be taken equal to zero in the observer design (standard extended system). In this paper, a better approximation of the disturbance dynamics is proposed using a backward difference method. A new extended system is designed based on this approximation. Any observer that makes the error dynamics exponentially stable for the standard extended system can then be used to estimate the state of the new extended system. The efficiency of the method is illustrated through an example.

*Keywords:* Extended state observer, backward approximation, time-delay.

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## 1. INTRODUCTION

Disturbances either external or internal (parameter uncertainty) are always present in practice and can severely deteriorate the performance of a system if they are not taken into account properly. Since disturbances can not always be measured, various methods were designed to estimate them and then use the estimation to design a controller. This field of control theory is widely driven by industrial applications and conducted independently in different areas. Some applications in aeronautics Xu (2017), robotics Talole et al. (2010) and motor control Sun et al. (2013) prove the strong usefulness of disturbance-observer-based control in practice. Fault-tolerant control also relies on disturbance observer Chen et al. (2016a), Zhang et al. (2017). The literature on disturbances and state estimations can be divided into three classes of methods that estimate i) the disturbance only; ii) the state only; and/or iii) both the disturbance and the state. Note that in this paper, our focus is on the disturbance estimation and not on the controller design. More information about disturbance-observer-based control can be found in survey paper Chen et al. (2016b) or in the book Guo and Zhao (2016).

The first class of methods is usually called a disturbance observer (DOB). It dates back to the 80s with Ohishi et al. (1983) and was initially designed to reject unknown load torque in speed control Umeno and Hori (1991), Umeno et al. (1993). It is a frequency domain approach, where a low pass filter and an inverse of the plant are needed to design the DOB. A special care in tuning the

low pass filter is needed in order to choose the correct bandwidth to get a good estimation of the disturbance in presence of noise Sariyildiz and Ohnishi (2014). The special case of a non-minimum phase plant was treated in Chen et al. (2004). The DOB has also been extended to MIMO systems Güvenç et al. (2009) and to nonlinear robotic manipulators Chen (2004). In Kim et al. (2010), a nonlinear system with high order disturbance is also considered. A detailed review of DOB can be found in Shim et al. (2016). The main drawback of these methods is that they require the full-state knowledge.

The second class of methods (state observation only) relies on the design of an unknown input observer (UIO). The UIO is an observer that is able to estimate the state of a system in presence of an unknown input (can be a disturbance) but without explicitly estimating this input. The observer size is the same as the size of the original system (full observer) or less (reduced observer). The first articles on this topic go back to the 70s with Basile and Marro (1969) and Guidorzi and Marro (1971) using a geometric approach. After these seminal works, a vast number of methods were developed to design a reduced-order observer Wang et al. (1975) or a full-order observer Darouach et al. (1994). More references can be found in Corless and Tu (1998). Some UIO have also been designed for nonlinear systems Chen and Zheng (2006), state-affine systems Hammouri and Tmar (2010) or switched systems Hou et al. (2017). A study of UIO for fault detection and diagnosis (FDD) is given in Liu et al. (2016). With the similar objective of observing the state of a system in presence of an unknown input, sliding-mode observers

(SMO) have also been developed Fridman et al. (2011). A detailed comparison between these two approaches is given in Edwards (2004). As a by product of the state estimation, some UIO can provide an estimation of the perturbation as in Barbot et al. (2009) but this is not the main objective that is why applications are mostly in parameter identification, fault detection and isolation (FDI) or cryptography.

The third class of methods is an extended state observer method (ESO). The idea relies on the design of an observer on an extended system where the disturbance is part of the state. It was presented first by Johnson in Johnson (1971). In these results, the external disturbance is modeled by an LTI system. Some less general papers were presented almost simultaneously by Hostetter and Meditch (1973) and Meditch and Hostetter (1974) (“ $k$ -observers”). An application to position control was presented in Ohishi et al. (1987) in the framework of discrete control. The above results only deal with linear systems without parameter uncertainty. A further advance of the ESO was to lump the external disturbance with parameter uncertainties (total disturbance) and to estimate both at the same time Han (2009), Peng and Wang (2018). In the context of uncertain feedback linearization, a high-gain ESO was used for the control of nonlinear systems Freidovich and Khalil (2008). In the results mentioned above, the first time-derivative of the disturbance or higher time-derivatives are usually assumed to be equal to 0 in the design of the observer<sup>1</sup>. This is usually a very rough approximation because this is scarcely the case in practice.

The aim of this article is to improve the standard ESO method in order to get better estimations of both the state and the disturbance. Our contribution is to introduce an artificial delay in the observer design that allows to get a better disturbance approximation and as a consequence to reduce the estimation error of the state estimation.

This paper is organized as follows. The general result is given in Section 2. An example and some simulation results are given in Section 3. Finally, conclusion and future works are proposed in Section 4.

## 2. MAIN RESULT

Consider the system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dd(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

with  $x(t) \in \mathbb{R}^{n \times 1}$ ,  $u(t) \in \mathbb{R}^{m \times 1}$ ,  $y(t) \in \mathbb{R}^{p \times 1}$ ,  $d(t) \in \mathbb{R}^{q \times 1}$  and  $A, B, C$  and  $D$  are constant matrices of appropriate dimension. The signal  $u$  is a known input. Note that  $Dd$  can gather parameter uncertainties and external disturbances. In the sequel, the following assumptions are made.

*Assumption 1.* The disturbance  $d$  is twice differentiable and its time-derivatives are bounded. Thus, there exists  $d_1, d_2 > 0$  such that for  $t \geq 0$

$$\|\dot{d}(t)\| \leq d_1$$

and

$$\|\ddot{d}(t)\| \leq d_2.$$

<sup>1</sup> These articles do not assume that the disturbance is constant they only use this approximation in the observer design.

Considering the new state  $X = [x, d]^T$ , system (1) becomes

$$\begin{cases} \dot{X}(t) = \bar{A}X(t) + \bar{B}u(t) + \Gamma_1(t) \\ Y(t) = \bar{C}X(t) = y(t) \end{cases} \quad (2)$$

with

$$\bar{A} = \begin{bmatrix} A & D \\ 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = [C \ 0]$$

and  $\Gamma_1(t) = [0, \dot{d}(t)]^T$ . It is assumed that

*Assumption 2.* The extended system (2) is observable.

Thanks to Assumption 2, one knows that one can find a gain  $L$  such that the observation error  $e(t) = \hat{X}(t) - X(t)$  where  $\hat{X}$  is obtained thanks to the Luenberger observer of the form

$$\dot{\hat{X}}(t) = \bar{A}\hat{X}(t) + \bar{B}u(t) + L[\bar{C}\hat{X}(t) - y(t)] \quad (3)$$

is exponentially stable for the nominal system ( $\Gamma_1 = 0$ ): there exist  $k, \lambda, r_0 > 0$  such that

$$\|e(t)\| \leq k\|e(0)\|e^{-\lambda t}. \quad (4)$$

Observer (3) is called an Extended State Observer (ESO).

*Remark 3.* In (3), it is underlying that the dynamics of the perturbation  $d$  is assumed to be zero. Note that it is possible to approximate higher order time-derivative of the disturbance by choosing a different extended state in system (2) such as  $X = [x, d, \dot{d}, \dots, d^{(n)}]^T$ . For purpose of clarity only the case  $X = [x, d]^T$  is presented in the paper but the results also hold for higher-order approximation.

The observation error  $e(t)$  has the following dynamics

$$\dot{e}(t) = (\bar{A} + L\bar{C})e(t) - \Gamma_1(t). \quad (5)$$

The comparison method in (Khalil, 2002, Lemma 9.4) ensures that in presence of perturbation  $\Gamma_1$  the observation error verifies

$$\|e(t)\| \leq \alpha_1\|e(0)\|e^{-\beta_1 t} + \gamma_1 d_1 \quad (6)$$

with  $\alpha_1, \beta_1, \gamma_1 > 0$  and  $d_1$  defined in Assumption 1. Note that the bound of the convergence radius  $\gamma_1 d_1$  is proportional to the bound of the time derivative of the disturbance  $d_1$  which means that if the perturbation is fast varying, the convergence ball will be larger.

Instead of using the approximation that the time-derivative of the perturbation is zero (or higher-order time-derivative as mentioned in Remark 3), it is proposed to use the backward difference approximation in order to estimate  $\dot{d}$ :

$$\dot{d}(t) = \frac{d(t) - d(t-h)}{h} - R(t, h) \quad (7)$$

where  $h > 0$  is a constant delay that can be arbitrarily chosen (tuning parameter<sup>2</sup>) and  $R(t, h)$  is the remainder of the Taylor approximation. Note that  $R$  verifies the following inequality

$$|R(t, h)| \leq \frac{d_2}{2}h. \quad (8)$$

Using (7), system (2) can be rewritten as

$$\begin{cases} \dot{X}(t) = \bar{A}X(t) + \bar{B}u(t) + \frac{1}{h}D_2[X(t) - X(t-h)] + \Gamma_2(t) \\ Y(t) = \bar{C}X(t) = y(t) \end{cases} \quad (9)$$

<sup>2</sup> Some hints for the tuning of  $h$  will be given at the end of this section.

with  $\Gamma_2(t) \in \mathbb{R}^{(n+q) \times 1}$  and  $\Gamma_2(t) = [0, -R(t, h)]^T$  and

$$D_2 = \begin{bmatrix} 0 & 0 \\ 0 & I_q \end{bmatrix}.$$

One can then design an observer on this new extended system (9) as follows

$$\begin{aligned} \dot{\hat{X}}(t) = & \bar{A}\hat{X}(t) + \bar{B}u(t) + \frac{1}{h}D_2[\hat{X}(t) - \hat{X}(t-h)] \\ & + L[\bar{C}\hat{X}(t) - y(t)] \end{aligned} \quad (10)$$

where  $L$  is the same gain as in (3). In this case, the observation error  $e(t) = \hat{X}(t) - X(t)$  has the following dynamics

$$\dot{e}(t) = (\bar{A} + L\bar{C})e(t) + \frac{1}{h}D_2[e(t) - e(t-h)] - \Gamma_2(t). \quad (11)$$

*Remark 4.* The practical implementation of observer (10) will be made in discrete time. As a consequence, only a finite number of samples delayed states values will be stored in a buffer. However, to keep a reasonable size for the buffer, the discretization step will have to be chosen according to the length of the artificial delay  $h$ .

*Remark 5.* Instead of a Luenberger observer (10), other observers could be used. The Luenberger observer was chosen for simplicity and clarity.

The stability condition for (11) is given in the next theorem.

*Theorem 6.* Consider the system (9) with the observer (10). There exist  $h^*, \alpha_2, \beta_2, \gamma_2 > 0$  such that for all  $h > h^*$ , the observation error verifies

$$\|e(t)\| \leq \alpha_2 \|e(0)\| e^{-\beta_2 t} + \gamma_2 d_2 h. \quad (12)$$

**Proof.** Let first consider the nominal case where  $\Gamma_2 = 0$ . Let us consider  $V(t) = e(t)^T P e(t)$  where  $P > 0$  is the solution of the Lyapunov equation

$$(\bar{A} + L\bar{C})P + P(\bar{A} + L\bar{C})^T = -Q \quad (13)$$

with  $Q$  a positive definite matrix<sup>3</sup>. Taking the time derivative of  $V$  along the trajectories of (11) leads to

$$\dot{V}(t) = -e^T(t)Qe(t) + \frac{2}{h}e^T(t)PD_2[e(t) - e(t-h)] \quad (14)$$

Thus, one can get

$$\dot{V}(t) \leq -c_3 \|e(t)\|^2 + \frac{c_4}{h} \|e(t)\| (\|e(t)\| + \|e(t-h)\|). \quad (15)$$

with  $c_3, c_4 > 0$ . As in the standard Razumikhin reasoning Fridman (2014), the following condition is assumed: for a given  $\kappa > 1$ , the inequality

$$V(t-s) \leq \kappa V(t), \quad \forall s \in [0, h]. \quad (16)$$

Since  $\lambda_{\min}(P)\|e(t)\|^2 \leq e^T(t)Pe(t) \leq \lambda_{\max}(P)\|e(t)\|^2$  where  $\lambda_{\min}$  (resp.  $\lambda_{\max}(P)$ ) denotes the smallest (resp. the largest) eigenvalue of  $P$  then from (16), one deduces that

$$\|e(t-h)\| \leq c_5 \|e(t)\| \quad (17)$$

with  $c_5 = \sqrt{\kappa \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ . Using the above inequality, one obtains

$$\dot{V}(t) \leq -\left(c_3 - \frac{c_4}{h} - \frac{c_4 c_5}{h}\right) \|e(t)\|^2. \quad (18)$$

Defining  $c_6 = c_3 - \frac{c_4}{h} - \frac{c_4 c_5}{h}$ , one has

$$\dot{V}(t, e(t)) \leq -c_6 \|e(t)\|^2 \quad (19)$$

Provided that  $h$  is sufficiently large, it is possible to guarantee that  $c_6 > 0$ . The comparison method in (Khalil, 2002, Lemma 9.4) ensures that in presence of perturbation  $\Gamma_2 \neq 0$  the observation error verifies (12).

*Remark 7.* It is important to keep in mind that  $h$  is an artificial delay that can be tuned by the designer. This is different from the case where the delay is intrinsic to the system as in Wang et al. (2002) for example. To preserve stability,  $h$  has to be sufficiently large, however to reduce the radius bound  $\gamma_2 d_2 h$ ,  $h$  needs to be small. The other way around, in practice, is to increase the observer gain. Indeed, increasing the observer gain will make  $c_3$  in (18) larger this will allow to keep  $h$  small enough to maintain the desired level of precision. Removing this limitation is a way of improvement for future works.

### 3. AN EXAMPLE

Consider the model of a DC drive

$$\begin{cases} \frac{di(t)}{dt} = -\frac{R}{L}i(t) - \frac{K_v}{L}\omega(t) + \frac{1}{L}u(t) \\ \frac{d\omega(t)}{dt} = \frac{K_\tau}{J}i(t) - \frac{f}{J}\omega(t) - \frac{1}{J}\tau(t) \\ y(t) = [i(t), \omega(t)]^T \end{cases} \quad (20)$$

where  $i$  is the armature current,  $\omega$  is the motor speed,  $u$  is the armature input voltage and  $\tau$  is an external disturbance torque. The viscous friction is modeled by  $-f\omega$ . The definition and the numerical values of the parameters are given in Table 1. The system (20) can be rewritten in the form of (1) with  $x = [x_1, x_2]^T = [i, \omega]^T$  and

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{K_v}{L} \\ \frac{K_\tau}{J} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and  $d(t) = -\frac{f}{J}\omega(t) - \frac{1}{J}\tau(t)$ . In addition, we introduce some parameter uncertainties on the coefficients. We consider that only a nominal value, denoted by the subscript 0, is known for each parameter. For example, one has  $f = f_0 + \Delta f_0$ . The nominal values are given in Table 1. As a result, one has

$$\begin{cases} \dot{x}(t) = A_0 x(t) + B_0 u(t) + D_0 d_0(t) \\ y(t) = C x(t) \end{cases} \quad (21)$$

with

$$A_0 = \begin{bmatrix} -\frac{R_0}{L_0} & -\frac{K_{v0}}{L_0} \\ \frac{K_{\tau 0}}{J_0} & 0 \end{bmatrix}, B_0 = \begin{bmatrix} \frac{1}{L_0} \\ 0 \end{bmatrix}, D_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$d_0(t) = [\phi_1(t), \phi_2(t)]^T$$

with  $\phi_1(t) = \left(\frac{R_0}{L_0} - \frac{R}{L}\right)i + \left(\frac{K_{v0}}{L_0} - \frac{K_v}{L}\right)\omega + \left(\frac{1}{L} - \frac{1}{L_0}\right)u$

and  $\phi_2(t) = d(t) + \left(\frac{K_\tau}{J} - \frac{K_{\tau 0}}{J_0}\right)i$ . Note that the current, the speed and their time-derivatives are bounded in practice so Assumptions 1 holds. The input voltage  $u(t)$  is chosen equal to  $U_n = 110$  V. The initial condition of the plant is  $x(0) = [0, 0]^T$ . The disturbance torque  $\tau(t)$  is defined by a succession of constant, ramp and sine signals (Figure 1). The dimension of the extended state will be 4 because we observe  $d_1$  and  $d_2$ . Note that the extended system is observable so Assumption 2 is verified.

<sup>3</sup> The existence of matrix  $Q$  is guaranteed by Assumption 2

Definition	Real Value	Nominal Value	Unit
Viscous friction coefficient	$f = 0.008$	$f_0 = 0.007$	Nms
Armature resistance	$R = 0.55$	$R_0 = 0.6$	$\Omega$
Armature inductance	$L = 6$	$L_0 = 6.2$	mH
Voltage constant	$K_v = 0.52$	$K_{v0} = 0.6$	Vc-Nm/A
Torque constant	$K_\tau = 0.52$	$K_{\tau0} = 0.5$	Vc-Nm/A
Motor inertia	$J = 0.1$	$J_0 = 0.08$	Kgm <sup>2</sup>

Table 1. Parameters of the DC drive and pump

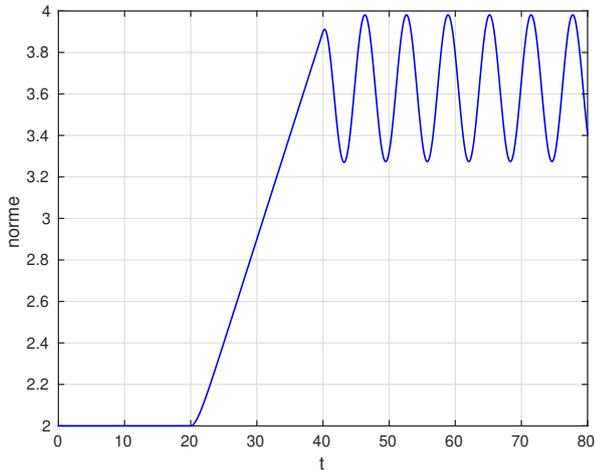


Fig. 1. Disturbance torque applied to the system  $\tau$

### 3.1 Luenberger Observer

In the sequel we are going to compare observers (3) and (10) for different values of  $h$ . This observer are design with the nominal values  $A_0$ ,  $B_0$  and  $D_0$ . To perform a fair comparison, both observers are designed with the same gain  $L$  and the same initial condition  $\hat{X}(0) = [0, 0, 0, 0]^T$ . The gain  $L$  is chosen in order to have eigenvalues in  $-10$ ,  $-20$ ,  $-30$  and  $-40$ .

In Figure 2, the norm of the state observation error is displayed for the Luenberger observer designed on the standard extended system (3) and for the Luenberger observer designed on the new extended system (10) (for three artificial delays). One can see that when the perturbation is constant, between 0 s and 20 s, the three observers converge to the exact extended state (the perturbation is exactly estimated). When the perturbation is a ramp, between 20 s and 40 s, the observer designed on the standard extended system exhibits a constant error whereas observers designed on the new extended system are able to estimate exactly the disturbance. Finally, when the perturbation is a sine signal, none of the observers can cancel the observation error. However, it can be noticed that the observers designed on the new extended system gives a better estimation, especially for a small delay  $h$ .

Reminding that  $d_1$  and  $d_2$  are defined in Assumption 1, one can analyze the simulation result with the theoretical results obtained above. The simulation for the observer designed on the standard extended system is in accordance with equation (6) because for a constant disturbance  $d_1 = 0$  so the convergence radius ( $R_1 = \gamma_1 d_1$ ) is equal to zero. The results for the observer (10) designed on the new

extended system are coherent with Theorem 6 because for a constant disturbance and a ramp  $d_2 = 0$  so the convergence radius ( $R_2 = \gamma_2 d_2 h$ ) is equal to 0. For a sine disturbance  $d_2 \neq 0$ , the estimation is better for a small delay since the convergence radius is proportional to  $h$ . However, note that decreasing the delay around  $h = 0.1$  s makes the transient very oscillating and decreasing more the delay makes the closed-loop system unstable as expected by the condition  $h > h^*$  given in Theorem 6.

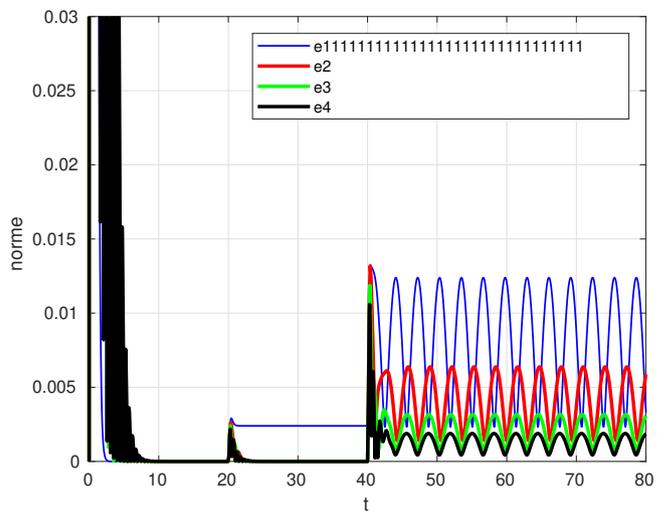


Fig. 2. Comparison of the estimation error for the Luenberger observer designed on the standard extended system (**std. Ext. Sys**) and the new extended system with an artificial delay (**new Ext. Sys**). The state is  $x = [i, \omega]^T$ .

In order, to assess the accuracy estimation of the external perturbation  $\tau$ , we used the real values of the parameter in the observer so that one gets  $\phi_1(t) = 0$  and  $\phi_2(t) = d(t)$ . In addition, we imposed  $f = f_0 = 0$  to have  $\phi_2(t) = -\frac{\tau(t)}{J}$ . The results are displayed on Figures 3 and 4 (with measurement noise). It can be seen that the disturbance is indeed better estimated with the observer designed on the new extended system with an artificial delay. The estimation is all the better for small delays. In order to get Figure 4, we have added noises on the state measurements. The noises are white noises with an amplitude of 5% of the maximum values of the current (resp. the velocity). Thanks to Figure 4, one can conclude that the method is robust to measurement noise. Obviously, in presence of noise the artificial delay cannot be reduced too much because it will amplify noises.

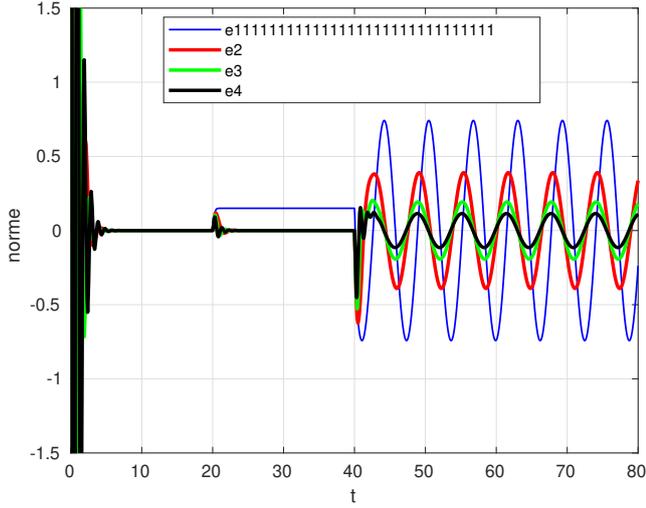


Fig. 3. Comparison of the estimation of the external perturbation for the Luenberger observer designed on the standard extended system (**std. Ext. Sys**) and the new extended system with an artificial delay (**new Ext. Sys**).

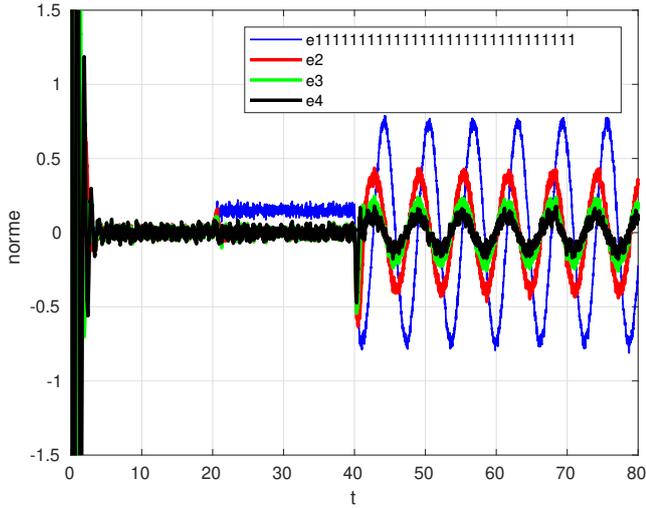


Fig. 4. Comparison of the estimation of the external perturbation for the Luenberger observer designed on the standard extended system (**std. Ext. Sys**) and the new extended system with an artificial delay (**new Ext. Sys**) with **measurement noise**.

### 3.2 Sliding Mode Observer

As it was mentioned in Remark 5, it is possible to apply the same method but using a different observer. In order to illustrate this statement, a first order sliding mode observer as been design. The observer equations are the same as (3) and (10) but substituting the correction term  $L[\bar{C}\hat{X}(t) - y(t)]$  by  $G_n\nu(t)$  where

$G_n = \begin{bmatrix} L \\ -I_2 \end{bmatrix}$  and  $\nu = \rho \cdot \text{sign}(y - \bar{C}\hat{X})$ . The parameters

have been chosen a follows  $L = \begin{bmatrix} 0 & -5 \\ -5 & 0 \end{bmatrix}$  and  $\rho = 10$ .

More details on the method can be found in Shtessel et al. (2014). From Figure 5, it is clear that the sliding mode observer gives similar result as for the Luenberger observer : the smaller  $h$  the more accurate the estimation.

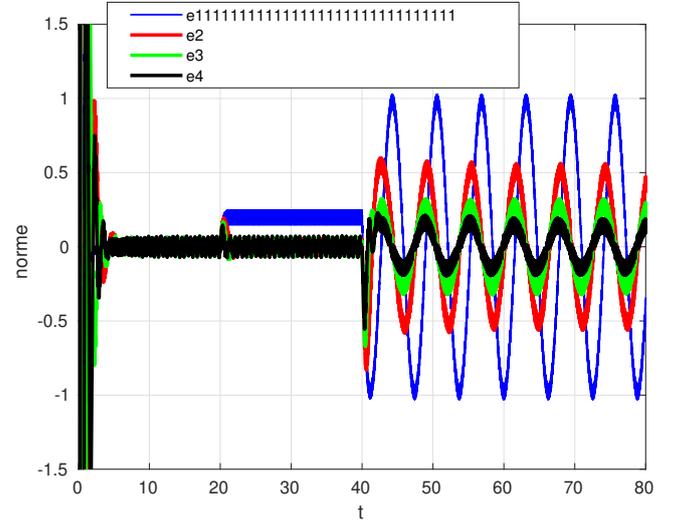


Fig. 5. Comparison of the estimation of the external perturbation for the **sliding mode observer** designed on the standard extended system (**std. Ext. Sys**) and the new extended system with an artificial delay (**new Ext. Sys**) without **measurement noise**.

## 4. CONCLUSION

A new extended system that allows to improve the observation precision with respect to the the standard extended system has been proposed for the design of ESO. The idea is based on the approximation of the disturbance dynamics by a backward difference method which involves an extra delayed term in the observer. The convergence proof of the estimation error is worked out thanks to a Lyapunov-Razumikhin analysis. It is shown that the quality of the estimation is closely related to the size of the artificial delay. Simulation results illustrate the efficiency of this new design. Improving the estimation of the disturbance dynamics using higher order approximation and designing a controller that takes the estimated disturbance into account are considered for further developments.

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