



**HAL**  
open science

# Prospective modelling of Long-Term Care biometric assumptions with multiple portfolios

Guillaume Biessy

► **To cite this version:**

Guillaume Biessy. Prospective modelling of Long-Term Care biometric assumptions with multiple portfolios. 2020. hal-02928250

**HAL Id: hal-02928250**

**<https://hal.science/hal-02928250>**

Preprint submitted on 2 Sep 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Prospective modelling of Long-Term Care biometric assumptions with multiple portfolios

Guillaume Biessy\*, PhD, SCOR<sup>†</sup> and Sorbonne Université<sup>‡</sup>

09/02/2020

## Abstract

The pricing of private Long-Term Care (LTC) products is a real challenge for actuaries. Indeed LTC is a relatively young risk as the first products launched in the late 1970s. The average LTC product is typically sold at the age of 60 while most claims occur after 85. Therefore insurers need to develop a prospective view of the risk to model their high-duration liabilities. This is made difficult by the many definitions of the LTC claim trigger which complexify the aggregation of several data sources. Thus the derivation of prospective biometric assumptions based on experience data is seen as an ambitious goal by many companies.

In this paper, we derive prospective best estimate assumptions for LTC relying on 8 insurance portfolios. We model the observed transitions (autonomous deaths, entries in LTC and deaths in LTC) in a Poisson Generalized Linear Mixed Model (GLMM) framework as functions of age, gender, calendar year, portfolio and time already spent in LTC (for the mortality in LTC). Inference of parameters relies on Penalized Quasi-Likelihood (PQL) and the Separation of Anisotropic Penalties (SAP) algorithm. We decompose each risk as a sum of marginal impact of those variables and their interactions. We then split the risk into a common pattern shared across portfolios and an idiosyncratic adjustment. Finally, by forecasting the insured population we show that the impact of using a prospective approach on insurer liabilities is material.

**Keywords :** Long-Term Care Insurance; Actuarial Modelling; Generalized Linear Mixed Models; P-Splines; Forecasting

## 1 Introduction

The first long-term care products launched in the late 1970s in the US and in the mid 1980s in France. Those products brought with them numerous challenges for actuaries as the majority of those products offer lifetime coverage in exchange of a level premium whose value is fixed at subscribing. While it is sometimes possible for insurer to adjust this premium, this may be subject to the approval of a controlling authority, for example in the US or in Israel (which is one of the most dynamic market for Long-Term Care), or limited to small adjustments (in France). Moreover, increasing the insurance premium may result in lapses among younger policyholders, which itself reduces the efficiency of the premium increase and may lead to a vicious circle. It may also impact the company reputation in the eye of the insureds, which can be detrimental for other lines of

---

\*guillaume.biessy78@gmail.com

<sup>†</sup>SCOR, 5 avenue Kléber, 75795 Paris Cedex 16, France

<sup>‡</sup>Sorbonne Université, CNRS, Laboratoire de Probabilités, Statistique et Modélisation, LPSM, 4 place Jussieu, 75005 Paris, France

business. Therefore it is paramount for insurers to offer a premium at the age of subscribing which reflects future liabilities as accurately as possible. In the case of French individual LTC insurance products, the average age of subscribing is around 60 while the average age of entry in LTC is close to 90, which means that the insurer must be able to derive biometric assumptions that reflect the risk 30 years in the future. This task is made even more complex by several factors. First the available data is scarce as for example in France there is no public data available to insurance companies. Then the underlying risk is complex, especially the mortality of disabled insureds that depends on their age but also the time they already spent in the LTC state, with much higher mortality during the first few months in LTC, see for example Biessy (2017). Second, the aggregation of data from different portfolios is made difficult by the coexistence of many definitions for the LTC triggers in addition to differences on the underwriting and claim management policies of insurers.

Several attempts have been made to derive prospective assumptions for countries where public data is available for example from the Italian National Institute of Social Security as in Levantesi and Menzietti (2012) or the U.S. Health and Retirement Study data which was used by Fong et al. (2015) and Li et al. (2017). Some authors propose alternative approaches based solely on national mortality data such as Gouriéroux and Lu (2014), or that do not require the number of deaths in LTC such as Kogure et al. (2019) for Japan.

In this article, we propose a different and ambitious approach where we rely on longitudinal data coming from 8 insurance portfolios on a period of 21 years. To overcome the heterogeneity linked to the many definitions of the LTC claims trigger, we rely on a mixed model framework where we separate between a common pattern shared across portfolios and an idiosyncratic pattern. The idiosyncratic pattern only consists of random effects and may therefore be considered as a nuisance component in the estimation of the common pattern. The mixed model framework allows us to treat every dimension of the problem in a similar way: age of the insured, calendar year, gender, portfolio and time spent in the LTC state. We express our model as a sum of marginal impact of those variables and interaction between them. It allows for greater interpretability in the model but also for more control over the behaviour of extrapolation and forecast as we show in the rest of the paper. Mixed models have already been used to forecast mortality rates in the past as in Currie et al. (2006) and more recently Carballo et al. (2020). In our work we integrate additional dimensions such as smoothing and extrapolation over the age and duration in LTC dimensions as well as qualitative variables such as Gender and Portfolio. The R statistical software, see R Core Team (2020), was used to generate all results presented in this paper.

The paper is organized as follows. Section 2 introduces the insurance portfolio data and the global modelling framework used within the paper. Section 3 illustrates the modelling philosophy that guides our work through a simple example. Sections 4 and 5 present two approaches to derive biometric assumptions, the first by building an estimate for each portfolio separately and the second by pooling all data together and considering the portfolio as a regular dependent variable. Finally, the penultimate section 6 forecasts the insured population and assesses the impact of using a prospective approach while Section 7 concludes the article.

## 2 Data and modelling

### 2.1 Data

We rely on data coming from 8 medium to large French Long-Term Care portfolios. Most of them rely on a definition for Long-Term Care based either on Activities of Daily Living or the AGGIR

grid which is used by the government for the attribution of the French public aid: the APA. The level of coverage varies from mild to severe LTC. Moreover, for the same definition and level of coverage, differences either in the wording of the general conditions of the product or in the lenience of the insurer claim acceptance policy cause significant variations in the number of accepted claims from one insurance company to another. External factors such as the wealth, area of living or occupation class among the target population also play a role. Finally, some products include a waiting period and people dying during this period are considered as autonomous deaths rather than disabled deaths in the insurer database. All those factors result in dramatic differences in the occurrence of LTC claims from one portfolio to another that cannot be linked to a single cause.

In this study we consider an observation period from the 1st of January 1998 to the 31st of December 2018. Let us note that some of the portfolios were launched after 1998 while some data extraction was done prior to the end of 2018. Besides, in order to account for the presence of an elimination period for most portfolios, we remove the experience for the first 3 years after subscribing. Figure 1 provides a summary of the number of observed autonomous deaths and entries in LTC that reveals significant differences in the relative size of the portfolio (after data cleaning and processing took place). The data pool for this study contains more than 60,000 claims in LTC and 110,000 autonomous deaths. Portfolios are not named to preserve confidentiality of results. For the 8th portfolio, it was not possible to separate autonomous deaths from lapses events, however the information about entries and sojourn in LTC is included in our study.

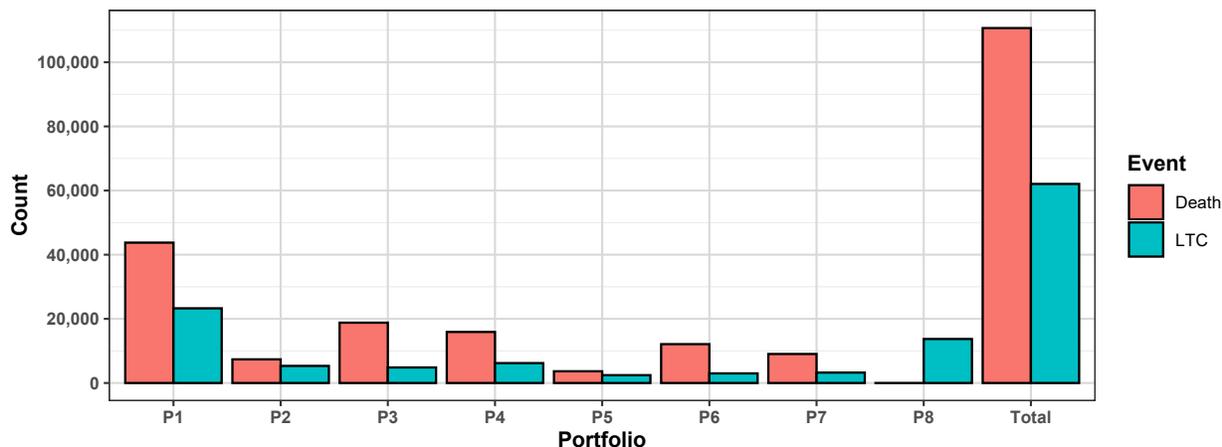


Figure 1: Number of observed autonomous deaths and entries in LTC by portfolio

## 2.2 Multi-state model with transition intensities

To model the Long-Term Care risk, we rely on a multi-state model with four states represented on Figure 2 and defined by its continuous-time transition intensities:

- $\lambda^{01}(x, y, g, p)$  the mortality for healthy insured
- $\lambda^{02}(x, y, g, p)$  the incidence in LTC for healthy insured
- $\lambda^{12}(x, y, z, g, p)$  the mortality for disabled insured

where  $x$  corresponds to the attained age of the insured,  $y$  is the calendar year,  $z$  is the time already spent in the LTC state (defined only for disabled insured),  $g$  is the gender and finally  $p$  is the

portfolio.

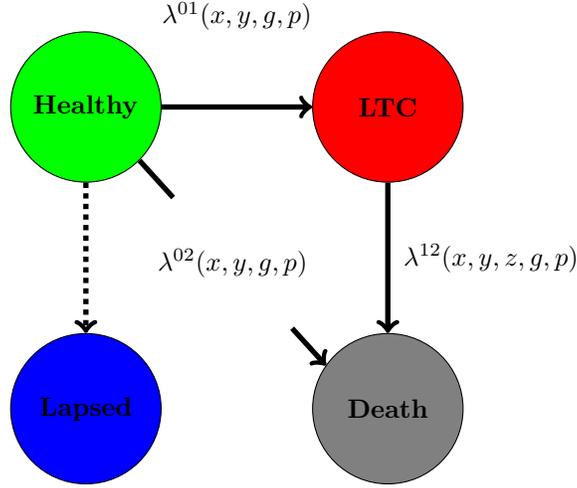


Figure 2: The multi-state model used for Long-Term Care modelling

We first assume that each of the 3 transition intensity is piecewise constant over intervals of the form  $[k, k + 1[$ , where  $k \in \mathbb{N}$  for both age  $x$  and calendar year  $y$ . As regards  $z$  the time spent in LTC, we assume the mortality in LTC to be constant over interval of one year for  $k \in \mathbb{N}^*$  and over intervals of one month of the form  $[k/12, (k + 1)/12[$  for  $0 \leq k \leq 11$  for the first year spent in LTC. Indeed, experience from LTC portfolios shows a significant decrease in the mortality level over the course of the first year spent in LTC and a constant transition intensity assumption over the whole year would be far too restrictive.

We then suppose that the number of observed events (autonomous deaths or entries in LTC)  $N^{0j}(x, y, g, p)$  follows a Poisson distribution whose parameter for the combination of variables  $(x, y, g, p)$  and  $j \in \{1, 2\}$  is the product of the discretized transition intensity  $\mu^{0j}(x, y, g, p)$  for this combination of variables and the central exposure  $e_c^i(x, y, g, p)$ , defined as the total number of years of observation for the insured population in the autonomous state. Therefore  $\log \mathbb{E}[N^{0j}(x, y, g, p)] = \log \mu^{0j}(x, y, g, p) + \log e_c^0(x, y, g, p)$ . A similar assumption is made for deaths in LTC so that  $\log \mathbb{E}[N^{21}(x, y, z, g, p)] = \log \mu^{21}(x, y, z, g, p) + \log e_c^2(x, y, z, g, p)$ .

Finally we assume that  $\mu^{ij}(x, y, g, p) \simeq \lambda^{ij}(x + 1/2, y + 1/2, g, p)$  *i.e.* the discretized transition intensity over the interval  $[x, x + 1[ \cap [y, y + 1[$  is close to the continuous time transition intensity at the middle point of the interval for all  $(x, y) \in \mathbb{N}$ . Considering those middle points allows us to define a distance between discretized transition intensities and consequently apply smoothing methodologies. In the case of mortality in LTC we get  $\mu^{12}(x, y, z, g, p) \simeq \lambda^{12}(x + 1/2, y + 1/2, z + 1/2, g, p)$  for  $z \in \mathbb{N}^*$  and  $\mu^{12}(x, y, k/12, g, p) \simeq \lambda^{12}(x + 1/2, y + 1/2, k/12 + 1/24, g, p)$  for  $k \in \{1, \dots, 11\}$ . Let us notice that in this particular case the distance between the lower bound of the intervals  $\{0, 1/12, \dots, 11/12, 1, 2, \dots\}$  is either  $1/12$  in the case of the first 12 intervals and  $1$  for the subsequent intervals while the distance between their middle points  $\{0 + 1/24, 1/12 + 1/24, \dots, 11/12 + 1/24, 1 + 1/2, 2 + 1/2, \dots\}$  is the same except for the distance between the 12th and 13th middle point which is  $7/12$ . It is therefore important to explicitly refer to middle points in this case.

At last, let us point out that lapse events are considered but not explicitly modelled. We assume

them to be independent from other type of events hence in the Poisson framework we introduced they are treated as regular independent censoring events.

## 2.3 Generalized Linear Mixed Model framework

In the multi-state model defined above, we require a model for the logarithm of each transition intensity, either  $\log \mu^{0j}(x, y, g, p)$  for  $j \in \{1, 2\}$  or  $\log \mu^{21}(x, y, z, g, p)$ . We consider mixed models of the form  $\eta = X\beta + Z\alpha$  where  $\beta$  is a vector of fixed effects and  $\alpha \sim \mathcal{N}(0, G_\theta)$  is a vector of random effects with variance-covariance matrix  $G_\theta$  where  $\theta$  is a vector of variance metaparameters. As we assume the number of observed events to follow a Poisson distribution, along with the logarithm link and the mixed model specification, our work hence lies within the scope of Generalized Linear Mixed Models (GLMM).

### 2.3.1 Inference in mixed models

Inference of parameters is done by maximizing the Penalized Quasi-Likelihood (PQL) as introduced in Breslow and Clayton (1993). Inference however cannot be done directly. Indeed, the likelihood for such a GLMM setting depends on an unknown weight matrix which corresponds in the Poisson GLMM to the number of predicted events. Furthermore, the parameters  $\alpha$  and  $\beta$  may be expressed with closed formulas that depends on  $\theta$ . On the other hand,  $\theta$  may be selected by minimizing the Residual Maximum Likelihood (REML) which depends on both  $\alpha$  and  $\beta$ . We therefore rely on the SAP algorithm as presented in Rodríguez-Álvarez et al. (2015) which is a recursive algorithm. The main structure of this algorithm is as follows: starting from initial parameters  $\alpha$ ,  $\beta$  and  $\theta$ , we determine the associated weight matrix  $W$  for the GLMM. Then, given  $W$  we iterate between determination of  $\alpha$  and  $\beta$  on one hand, and determination of  $\theta$  from  $\alpha$  and  $\beta$  using REML on the other hand, until convergence of  $\theta$  is achieved. We then update the value of  $W$  and iterate the previous step until convergence of the likelihood is achieved. The SAP algorithm therefore consists in two nested recursive loops, the inner loop being due to the presence of the metaparameters  $\theta$  that need to be estimated along  $\beta$  and  $\alpha$  and the outer loop being the same as in regular GLMs. Another advantage of the SAP algorithm is that computation of REML, which is usually the most computationally intensive part of the algorithm, can be made much quicker in the case where  $G_\theta$  is a diagonal matrix, which applies to the models we consider in the paper.

### 2.3.2 Smoothing and mixed models

The link between P-splines smoothing and mixed models has been addressed by several authors, see for example in the case of a non-Gaussian distribution, Lin and Zhang (1999). A key result is that should we consider a GLM  $\eta = B\theta$  where  $B$  is a B-splines basis and minimize the penalized log-likelihood  $\ell(\theta) + \lambda/2 \theta' P \theta$  where  $P = D_q' D_q$  is a penalization matrix and  $D_q$  is the matrix of differences of order  $q$ , then there is a mixed model decomposition  $B\theta = X\beta + Z\alpha$  and  $\alpha \sim \mathcal{N}(0, \sigma^2 \Sigma)$  for some matrices  $X, Z, \Sigma$  where  $\Sigma$  is a diagonal matrix and  $\sigma^2 = 1/\lambda$ . Furthermore, one may choose  $X = (1|x^1| \dots |x^{q-1})$  which is a basis for the space of parameters that are not affected by the penalization and  $x$  is the vector containing the values of the smoothing variable. To determine  $Z$  and  $\Sigma$  one may then use the Single Value Decomposition of  $P$ . For further details about this decomposition, the interested reader may refer to Chapter 2 of Lee Hwang (2010).

One of the advantages of treating smoothing as a mixed model problem is that the smoothing parameter  $\lambda$  may be selected by maximizing the restricted maximum-likelihood (REML), which seems to have better behaviour than AIC, BIC or GCV. Furthermore, the SAP algorithm is very

fast and scales well when several smoothing parameters are included. Indeed, using REML, the smoothing parameter may be selected at the same time of model parameters, whereas for the other criteria mentioned above, inference of parameters must be performed independently for each combination of smoothing parameters tested, making those methods untractable when more than 2 or 3 smoothing parameters are involved.

### 2.3.3 Time series and mixed models

The inference of parameters for ARIMA(p,d,q) time series may also be expressed as a mixed model. Let us consider a process  $(z_y)_{y \in \mathbb{N}}$  with linear drift. We may write  $z_y = \mu(y - \bar{y}) + S^d w_y$  where, using matrix notations  $\Psi w_y = \Theta \epsilon_y$  for all  $y$  and  $S$  is the lower triangular matrix with coefficients  $S_{i,j} = \delta_{i \leq j}$ . Then  $(z_y)_{y \in \mathbb{N}} = X\beta + Z\alpha$  where  $X = (1, \dots, n_y)' - (n_y + 1)/2 \mathbf{1}'$  corresponds to the deterministic drift with  $\beta = \mu$  and  $Z = S^d \Psi^{-1} \Theta$  with  $\alpha = (\epsilon_y)_{y \in \mathbb{N}}$  corresponds to the stationary component. ARIMA time series may thus be expressed as mixed models and their parameters estimated using penalized quasi-likelihood, as in the smoothing case above. This apply to all kind of ARIMA process, however, as we rely on the SAP algorithm, we need the variance-covariance matrix of the random effect to be diagonal which corresponds to ARIMA(0,d,0) models, in other words integrated white noise models. This may seem overly restrictive yet in our application there is no strong incentive to consider models with a more complex correlation structure. In addition, it is possible to include several time series in the model. In what follows we consider two kind of time series:

- A random walk  $S\epsilon_{y,1}$  where  $\epsilon_{y,1} \sim \mathcal{N}(0, \sigma_{\epsilon,1}^2 \mathbf{I})$ . It is used to model fluctuations that persist over future years an therefore to model a stochastic long-term component for the rates.
- A white noise  $\epsilon_{y,2} \sim \mathcal{N}(0, \sigma_{\epsilon,2}^2 \mathbf{I})$  used to account for yearly fluctuations in the rates.

In addition, a linear drift may be included in the model as a fixed effect  $b \times (y - \bar{y})$  with a single slope parameter  $b$ . The drift represents the deterministic part of the risk long-term trend.

### 2.3.4 Qualitative variables in mixed models

As regards qualitative variables such as gender and portfolio, they may be included in the model as either fixed or random effects, the difference being that in the second case the penalization applies to the parameters, reducing the effective dimension of the model, while in the first case it does not. Using random rather than fixed effects avoids overfitting in the model, especially in the case we consider interactions between qualitative variables.

### 2.3.5 Variable interactions in mixed models

Although we discussed how marginal impact of variables may be included in the model, either as smooth effect for age and duration in LTC, as time series for period and as additional random effects for gender and portfolio, we need to model interactions between two or more of those variables for a more realistic model. Let us assume we wish to model the interaction effect of two variables  $v_1$  and  $v_2$  on the risk such that the impact of  $v_i$  is  $X_i \beta_i + Z_i \alpha_i$  with variance-covariance matrix  $G_{\theta_i}$  such as  $\alpha_i \sim \mathcal{N}(0, G_{\theta_i})$ . Let us further note  $X_i = (\mathbf{1}_i | x_i)$ . Then the interaction between  $v_1$  and  $v_2$  has mixed model decomposition  $X_{1,2} \beta_{1,2} + Z_{1,2} \alpha_{1,2}$  where  $X_{1,2} = (x_2 \otimes x_1)$ ,  $Z_{1,2} = (x_2 \otimes Z_1 | Z_2 \otimes x_1 | Z_2 \otimes Z_1)$  and  $\alpha_{1,2} \sim \mathcal{N}(0, G_{\theta_{1,2}})$ , where the expression of  $G_{\theta_{1,2}}$  may be found in Rodriguez-Álvarez et al. (2015). The particular case where  $\theta_{1,2} = (\theta_1, \theta_2)$  i.e. the marginal impacts of  $v_1$  and  $v_2$  and their interaction use the same penalization is the default choice in many papers see for example Currie et al.

(2006) or Rodríguez-Álvarez et al. (2015). It is however possible to consider different penalizations for marginal and interaction terms, an idea that was developed in Chapter 4 of Lee Hwang (2010). In the next section, we discuss further the benefits of this strategy on a simple example before we apply it on a much bigger scale.

In the case where we wish to model the interaction of 3 variables, using similar notations we obtain the mixed model decomposition  $X_{1,2,3}\beta_{1,2,3} + Z_{1,2,3}\alpha_{1,2,3}$  where  $X_{1,2,3} = (x_3 \otimes x_2 \otimes x_1)$  and  $Z_{1,2,3} = (x_3 \otimes x_2 \otimes Z_1 | x_3 \otimes Z_2 \otimes x_1 | Z_3 \otimes x_2 \otimes x_1 | x_3 \otimes Z_2 \otimes Z_1 | Z_3 \otimes x_2 \otimes Z_1 | Z_3 \otimes Z_2 \otimes x_1 | Z_3 \otimes Z_2 \otimes Z_1)$  and  $\alpha_{1,2,3} \sim \mathcal{N}(0, G_{\theta_{1,2,3}})$ . While in this paper we only consider interactions between 2 to 3 variables, similar expressions may be easily derived for any number of dimensions. Let us point out that the nature of the variable is of no consequence here, it is indeed possible to consider an interaction between a smooth and a time series or and interaction of either or both with qualitative variables.

### 3 The GLMM ANOVA-type approach

In this section we present the modelling philosophy at the core of this paper, which is based on the decomposition of the risk between marginal impact of variables and their 2 or 3-way interactions. Because this decomposition reminds of the ANalysis Of VAriance (ANOVA) methodology we follow several authors such as Wahba et al. (1995) and Lee Hwang (2010) in naming it ANOVA or ANOVA-type approach. Let us illustrate this approach and its added value for the actuary through a simple example where we consider the problem of estimating the autonomous mortality  $\lambda^{01}(x, g)$  for portfolio 2, one of the smallest portfolios in our dataset, as a function of age and gender, assuming it does not depend on calendar year. We present 4 successive solutions to tackle this issue.

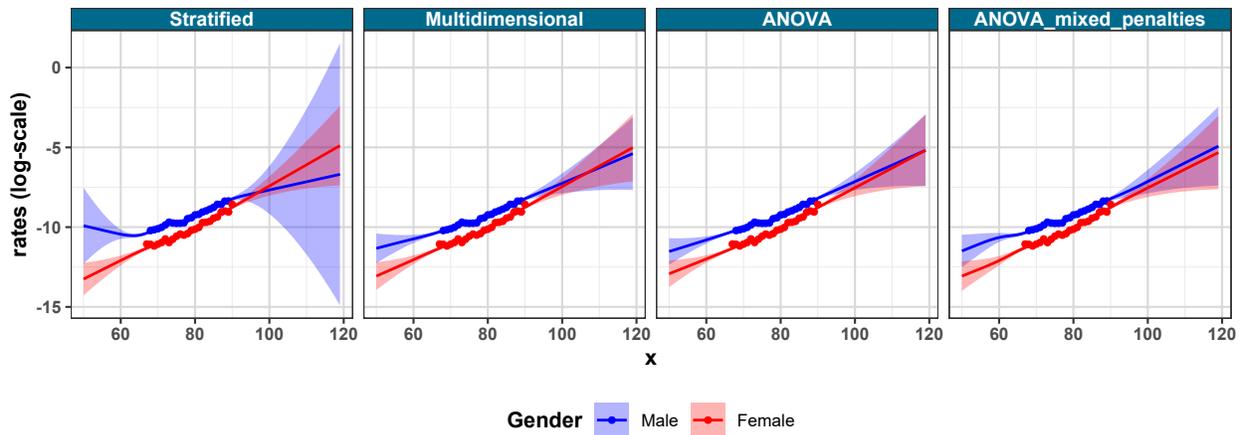


Figure 3: All 4 models fitted on autonomous mortality. Dots: empirical rates without smoothing. Straight lines: smoothed rates. Ribbon: confidence intervals.

#### 3.1 Stratified approach

The first approach we consider is a stratified analysis on the gender variable. We split the data according to Gender and our 2-dimension problem is decomposed into 2 smaller 1-dimension smoothing problems. We consider an order 2 penalty for smoothing, therefore a mixed model decomposition allows us to explicitly separate the smooth curve into a straight line and a centered

smooth:  $\log \mu^{01}(x, g) = f_x^g = a^g \times (x - \bar{x}) + s^g(x)$  where, for  $g \in \{1, 2\}$ ,  $a^g$  are slope parameters and  $s^g(x)$  are centered smooth built from a basis of cubic splines. Let us stress out the use of upper subscripts in our notation to indicate stratification. The left-end graph of Figure 3 represents the result of this model which has been extrapolated from 50 to 120. One may observe a crossover between the male and female curves at age 95. Besides, the extrapolated male mortality oddly decreases between 50 and 60.

### 3.2 Multidimensional smoothing approach

Our second solution considers gender as a regular dimension and use a 2-dimension approach with smoothing on the age dimension and gender modelled by a random effect. The associated model is  $\log \mu^{01}(x, g) = a \times (x - \bar{x}) + s_1(x) + \delta_g + a_g \times (x - \bar{x}) + s_2(x, g)$  where  $a$  and  $a_g$ , for  $g \in \{1, 2\}$  are slope parameters,  $s_1(x)$  is a smooth centered function of age built from a basis of cubic splines,  $\delta_g$  are centered coefficients corresponding to the marginal impact of gender and finally  $s_2(x, g)$  is a centered function of age and gender, smooth on the age dimension.

A penalization  $\lambda_x$  is used to smooth both coefficients  $s_1(x)$  and  $s_2(x, g)$  on the age dimension and a second penalization  $\lambda_g$  is used for the gender dimension in  $\delta_g$  and  $s_2(x, g)$ . The second graph of Figure 3 represents the result of the model. Compared to the previous approach, male confidence intervals are much narrower thanks to the mutualization of male and female data. A crossover between male and female mortality still occurs at age 105. The left part of Figure 4 represents the age and gender interaction term for this model.

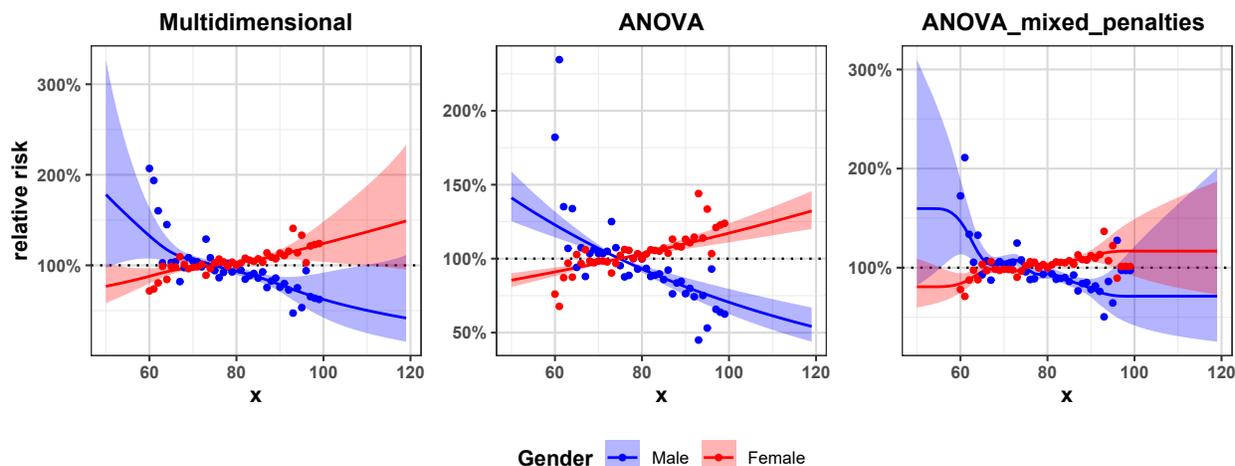


Figure 4: Age and Gender interaction terms in the last 3 models

### 3.3 ANOVA-type smoothing approach

Our third solution presents at first sight very little difference compared to the previous one. Indeed, the model we consider is the same as in the multidimensional smoothing approach, the only difference being that instead of common penalizations  $\lambda_x$  for both  $s_1(x)$  and  $s_2(x, g)$  and  $\lambda_g$  for both  $\delta_g$  and  $s_2(x, g)$  we consider 4 different penalizations  $\lambda_x^1, \lambda_x^2, \lambda_g^1, \lambda_g^2$  corresponding to the 4 aforementioned terms. In practice this offers greater flexibility to the model. and the third graph of Figure 3 thus shows overall very little difference with the second graph. However, the interaction term represented by the middle graph of 4 is different from the left graph. In the multidimensional approach, the

penalization is chosen for both the marginal age smooth and the age-gender interaction. Such penalization is thus a compromise between the optimal amount of smoothness required in both patterns. In our third solution however, an optimal penalization is picked for each of those patterns. In practice, the model selects a much stronger penalization for the age-gender interaction which in this case is almost a straight line. In this case the gender difference in mortality diverges linearly rather than quadratically and the crossover between male and female curves move to 120.

Hence, the crossover between male and female mortality is still present at very high ages. We therefore suggest to go one step further and instead of considering a penalization of order 2 for both the unisex smooth component and the gender difference by age, we replace the latter by an order 1 penalization. This change has little impact for ages where data is available yet it plays a key role in the extrapolation. Indeed, an order 1 penalization results in an asymptotically constant gender difference at very high ages as can be seen on the interaction effect on the right-end graph of Figure 4 and consequently the male and female mortality curves asymptotically become parallel as on right graph of 3, thus effectively removing any risk of crossover in the extrapolated part. The resulting model is a simplified version of the previous model where the gender-specific slopes  $a^g \times (x - \bar{x})$  have been removed.

### 3.4 Further benefits of the approach

The previous example demonstrated the benefits of applying an ANOVA-type approach to a relatively simple issue. By using a multidimensional rather than stratified approach, we improve the quality of the estimates through mutualization of the datasets, especially the male estimate where there was less data overall and even less data at lower ages. The use of an ANOVA-type approach further improved the estimate by selecting an optimal penalization for both the marginal and interaction terms rather than a compromise penalization between those terms. Finally, by using an order 1 penalization for age in the interaction term, we enforce an asymptotically parallel extrapolation of male and female curves avoiding an inconsistent crossover between those curves.

In what follows this approach will be extended to a bigger scale. Indeed, when working with multiple portfolios, it allows pooling of data not only across genders but also portfolios, which is all the more relevant as some portfolio lack data for specific age bands or even globally. Furthermore, in the mixed model framework, extrapolation is equivalent to forecasting, and the results we obtain can be transposed to provide consistent forecast across genders and portfolios. Finally, with the use of more complex models, including possibly thousands of parameters, the use of random instead of fixed effects automatically scales down through penalization the less significant features of the model, avoiding overfitting.

## 4 Stratified portfolio analysis

In this section, we perform a stratified analysis on the portfolio variable for all 3 transition intensities in the multistate model. It yields prospective biometric assumptions for each portfolio solely based on their own data. We focus on transitions originating from the autonomous state *i.e.* autonomous deaths and entries in LTC, as those transitions depend on the same variables and then move to the disabled mortality which offers an additional challenge due to the presence of a fifth dimension on the form of time spent in LTC.

## 4.1 Autonomous mortality and incidence in LTC

We assume additive models of the form  $\log \mu^{0j}(x, y, g, p) = \alpha_{x,g}^p + \kappa_y^p$ . Thus, the age and period effects are completely separated, as opposed to models widely spread in the field of Longevity modelling such as the Lee and Carter (1992) or CBD (Cairns et al., 2006) models. Indeed, modelling the interaction between the age and period component would prove very hazardous as we rely on longitudinal data with a possible correlation between both variables as individual at high ages are only observed during the most recent years. We further consider the models  $\alpha_{x,g}^p = a^p \times (x - \bar{x}) + s_1^p(x) + \delta_g^p + s_2^p(x, g)$  for the age effect and  $\kappa_y^p = b^p(y - \bar{y}) + S\epsilon_{y,1}^p + \epsilon_{y,2}^p$  for the period effect where  $a^p$  and  $b^p$  are slope parameters,  $s_1^p(x)$  are smooth centered functions of age,  $\delta_g^p$  are centered gender coefficients,  $s_2^p(x, g)$  are centered function of age and gender, smooth on the age dimension and finally  $\epsilon_{y,k}^p \sim \mathcal{N}(0, \sigma_{\epsilon,k}^p \mathbf{I})$  are *iid* white noise for  $k \in \{1, 2\}$ .

The age effect in this model is similar to the one presented in the previous section. The period effect however deserves some additional explanations. It consists on 3 components:  $b^p(y - \bar{y})$  is a deterministic drift,  $S\epsilon_{y,1}^p$  is a random walk that represents the stochastic long-term evolution of the rates while  $\epsilon_{y,2}^p$  is a white noise that represents yearly fluctuations in the rates. It would be possible to rely on a simplified model where either  $S\epsilon_{y,1}^p$  or  $\epsilon_{y,2}^p$  is removed, however this would have significant drawbacks. On one hand, removing the irandom walk would boil down to fitting a regression model to the period effect, which is not adapted for many portfolios. Moreover, for those portfolios it would result in a big jump the the period effect between the last year of observation and the first year of forecast. On the other hand, removing the white noise would imput all fluctuations to the stochastic improvement component which would thus have a much greater volatility. Indeed, it is not uncommon for insurance portfolios to experience outstanding years for the number of deaths or claims. However, most of the time, this phenomenon is limited to one year and often compensated by a lower death of claim count on the subsequent year. A model with only a stochastic improvement would consider such phenomenon as two consecutive extreme variations of opposite directions. The volatility of the integrated white noise process would therefore be considered very high which would lead to unrealistically large confidence intervals for the forecast, as for the random walk having two consecutive variations in the same or in opposite directions has the same probability. Hence we believe that a realistic prospective model for the period effect should include both components. At last, we chose not to consider an interaction between the period effect and gender as from prior attempts this was scaled down to nearly 0 by the penalization.

Figure 5 (resp. Figure 6) represents for each portfolio the age and period patterns associated with autonomous mortality (resp. incidence in LTC), as well as the slope of the period pattern which may be interpreted as the average one-year change in period mortality (resp. incidence in LTC) level. Remind that for portfolio 8, lapses and autonomous deaths cannot be identified and therefore this portfolio is not considered for the derivation of autonomous mortality. The age pattern of both autonomous mortality and incidence in LTC is very regular and increase exponentially with age. It is consistent across portfolios. The autonomous mortality is significantly higher for males than females while for incidence in LTC the rates are very close and the female incidence rates starts slightly lower and increase slightly faster than for males. The period pattern is a little chaotic, especially in the case of smaller portfolios. Nevertheless, thanks to the penalization, some of those fluctuations have been imputed to the low volume of data hence the estimate in plain line is less volatile than the observed level of risk for the period (see for example portfolios 2 and 5). Nevertheless, confidence intervals around the average one-year change are very wide and 0 belongs to most of those intervals. The stratified approach thus concludes that for most portfolio there is not enough evidence in the data to support the presence of a non-null period trend for autonomous mortality or incidence in

LTC, except in portfolios 1 and 8 which are the two biggest portfolios.

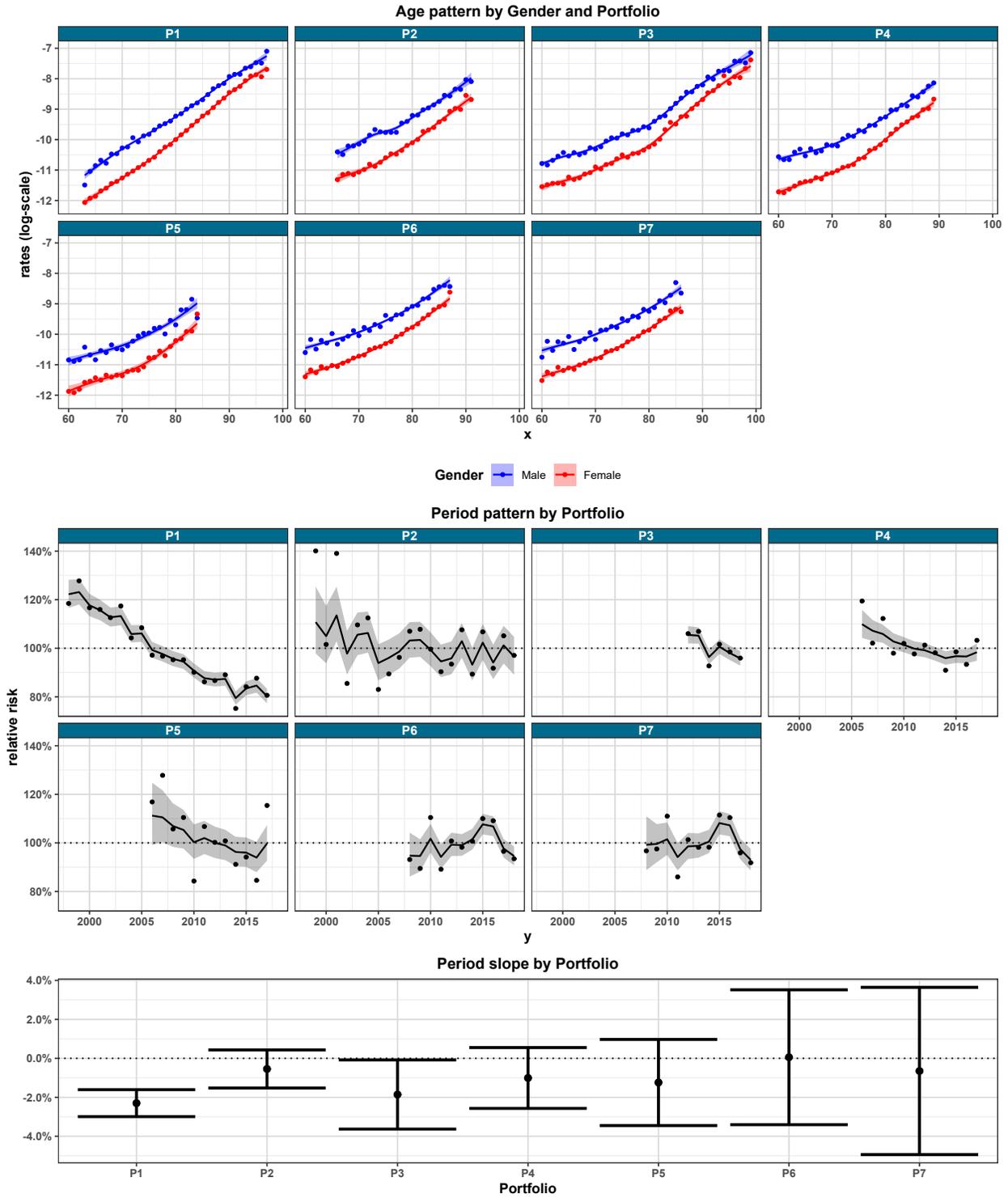


Figure 5: Stratified fit of autonomous mortality: age and period patterns

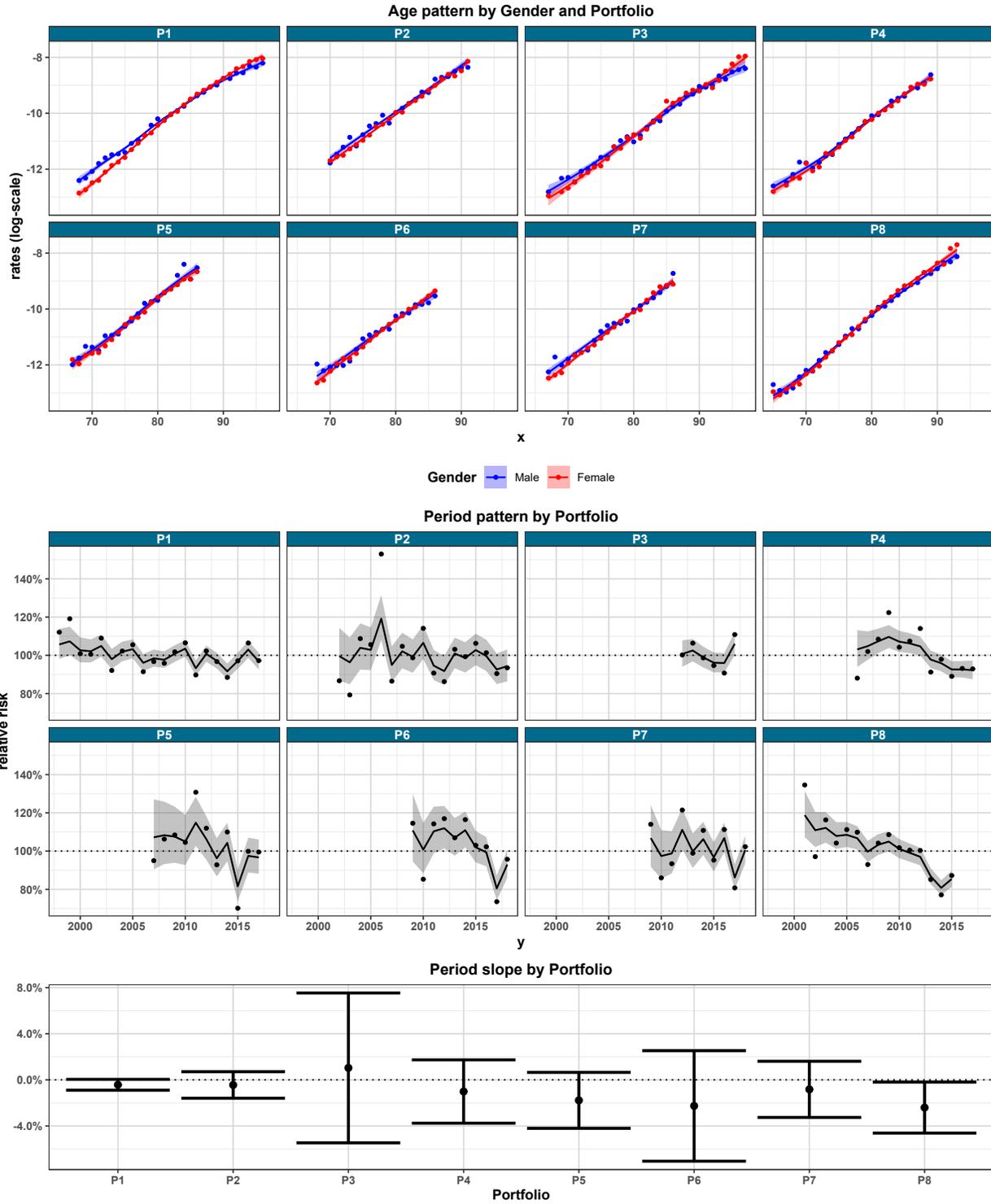


Figure 6: Stratified fit of incidence in LTC: age and period patterns

## 4.2 Disabled mortality

For the modelling of disabled mortality, we account for the time spent in LTC  $z$  also called duration in LTC. A simple model would only include an additive effect of duration in LTC in addition to the age and period effect. However, previous studies highlight an interaction effect between age and duration in LTC. Indeed, younger claimant in LTC are more likely than older claimants to become disabled because of Cancer, a pathology associated with a significantly higher mortality level, meaning that while mortality during the first few months in LTC is higher for all claimants, this effect is even stronger for younger claimants. The interested reader can refer to Chapter 4 of Biessy (2016) to find out more about the link between the pattern of disabled mortality and the underlying pathologies. To account for this interaction between age and duration in LTC, we therefore consider the following GLMM model:  $\log \mu^{21}(x, y, z, g, p) = \alpha_{x,z,g}^p + \kappa_y^p$ . The age-duration pattern is  $\alpha_{x,z,g}^p = a^p(x - \bar{x}) + s_1^p(x) + \delta_g^p + s_2^p(x, g) + s_3^p(z) + s_4^p(z, g) + s_5^p(x, z) + s_6^p(x, z, g)$  for each portfolio. The period pattern is as in the previous model  $\kappa_y^p = b^p(y - \bar{y}) + S\epsilon_{y,1}^p + S\epsilon_{y,2}^p$ . Here  $a^p$  and  $b^p$  are slope parameters, the functions  $s_1^p$  to  $s_6^p$  are smooth functions on the  $x$  and  $z$  dimension and finally  $\epsilon_{y,k}^p \sim \mathcal{N}(0, \sigma_{\epsilon,k}^p \mathbf{I})$  are *iid* white noise for  $k \in \{1, 2\}$ .

Let us point out that the age-duration pattern only includes a single fixed effect  $a^p(x - \bar{x})$  which is due to a few assumptions on our end. First, as for the previous models, we assume that the male and female age patterns are asymptotically parallel. Then, as regards the time spent in LTC component, we also rely on an order 1 penalization which means that we assume the impact of duration in LTC to be asymptotically constant for long LTC claims. Indeed, the aforementioned study imputed the pattern of duration in LTC to the heterogeneity in mortality caused by pathologies and this heterogeneity subsides as the claims associated with higher mortality are also the first to disappear from the population. Besides, the age by duration in LTC interaction term also uses order 1 penalty on both dimensions as the higher mortality observed during the first months in LTC fades off at higher ages. Finally, we consider the full interaction with the gender variable as the causes for LTC claims are different for males and females (more cancer and neurological diseases for males and more dementia for female) therefore there could also be significant differences in gender patterns.

Figures 7 and 8 represents the age-duration and period patterns associated with disabled mortality as well as the slope of the period pattern which may be interpreted as the average one-year change in period disabled mortality. The age pattern for portfolios with enough data is U-shaped. The disabled mortality starts at a much higher level than autonomous mortality but does not increase as much at higher ages. All portfolios present an initial peak of mortality for the first few months following claim inception. For the portfolios that include a deferment period, no data is available during the first one to three months, therefore this pattern is partially occulted. Most portfolios also present a clear age by duration in LTC interaction with a much higher mortality for claims made around age 70 in the early months of LTC. Except for a significant difference in the level of mortality, male and female duration and interaction patterns are extremely similar. Finally, fluctuations in the period pattern are mainly caused by the lack of data as only portfolios 1 and 5 exhibits non linear fluctuations. A stratified portfolio analysis would conclude in the absence of sufficient evidence for a period trend for all portfolios except for portfolio 6. Handling each portfolio separately therefore proves not to be an effective solution to neither detect trends nor forecast them. In the next section, we move to an aggregated approach where we regroup data from all portfolio in order to provide more robust estimates.

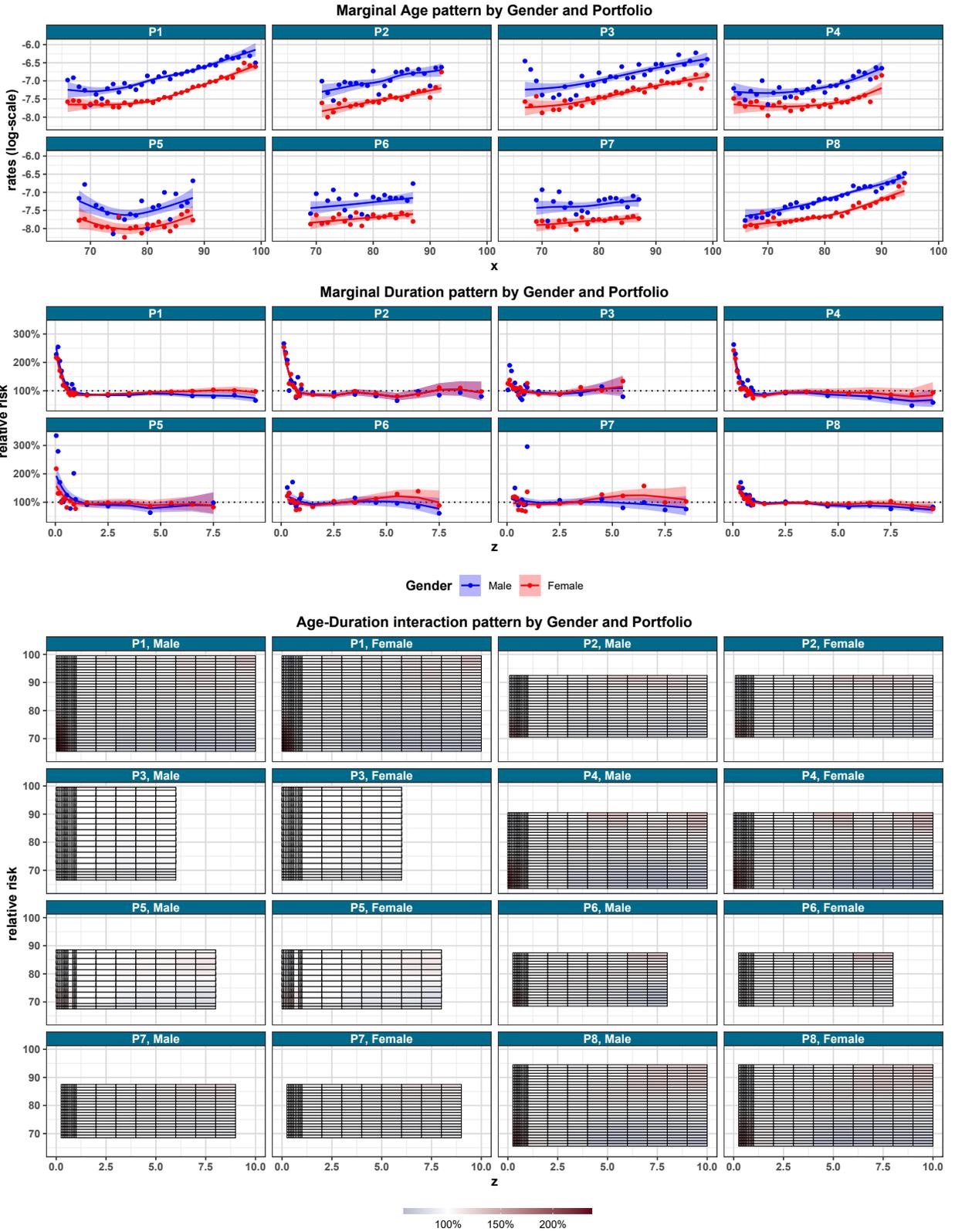


Figure 7: Stratified fit of mortality in LTC: age and duration patterns

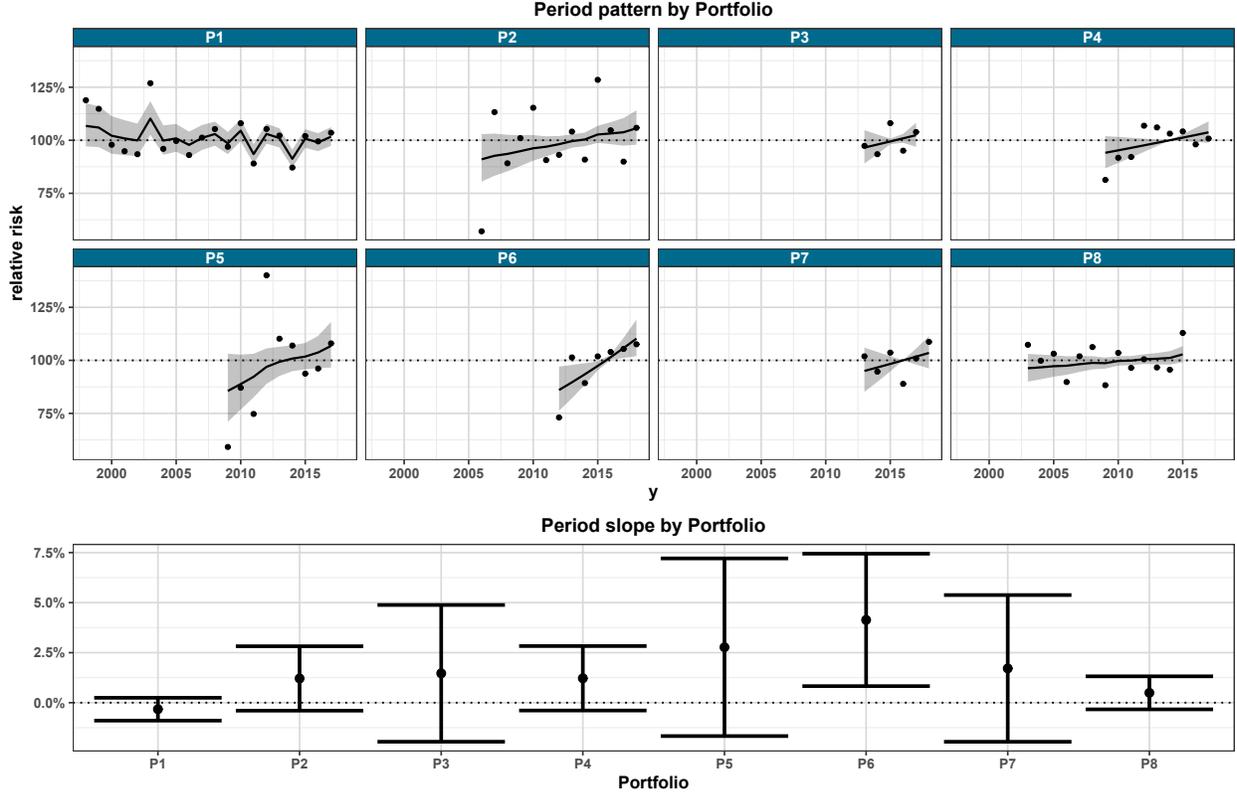


Figure 8: Stratified fit of mortality in LTC: period pattern and slope

## 5 Multiportfolio analysis

In this section, we move from a stratified to an aggregated analysis where all portfolio data is pooled together. As in the previous section we start with autonomous mortality and incidence in LTC and then continue with disabled mortality.

### 5.1 Autonomous mortality and incidence in LTC

The key idea of our approach is to consider a model that contains both a common pattern, shared across all portfolios and an idiosyncratic pattern which is different for every portfolio. We extend this logic to both the age and period effect. The shared pattern contains a slope  $a \times (x - \bar{x})$  for age and another one  $b \times (y - \bar{y})$  for period. The idiosyncratic pattern however does not contain any slope neither for age nor for period. The main consequence is that for age and period where data is not available, the model follows the shape of the common pattern. In addition, the extrapolation and forecast of all portfolios will be asymptotically parallel lines. The model may be summarized as  $\log \mu^{0j}(x, y, g, p) = \alpha_{x,g}^1 + \kappa_y^1 + \alpha_{x,g,p}^2 + \kappa_{y,p}^2$  where

- $\alpha_{x,g}^1 = a \times (x - \bar{x}) + s_1(x) + \delta_g + s_2(x, g)$
- $\kappa_y^1 = b \times (y - \bar{y}) + S\epsilon_{y,1} + \epsilon_{y,2}$
- $\alpha_{x,g,p}^2 = \delta_p + s_3(x, p) + \delta_{g,p} + s_4(x, g, p)$
- $\kappa_{y,p}^2 = S\epsilon_{y,p,3} + \epsilon_{y,p,4}$

- $a$  and  $b$  are slope parameters,  $s_1$  to  $s_4$  are smooth centered functions of age and gender,  $\delta_g$ ,  $\delta_p$  and  $\delta_{g,p}$  are centered gender and portfolio coefficients and finally  $\epsilon_{y,k} \sim \mathcal{N}(0, \sigma_{\epsilon,k}^2 \mathbf{I})$  (resp.  $\epsilon_{y,p,l} \sim \mathcal{N}(0, \sigma_{\epsilon,l}^2 \mathbf{I}_p \otimes \mathbf{I}_y)$ ) are *iid* white noise for  $k, l \in \{1, 2\}$ .

Figure 9 represents the common age and period patterns for autonomous mortality and incidence in LTC, while Figure 10 and 11 represents the associated idiosyncratic patterns. Finally, Figure 14 (resp. 15) in the Appendix section represents the final pattern obtained by summing the common and idiosyncratic patterns. An interesting byproduct of the model, also included at the bottom of Figures 10 and 11, is the  $\delta_p$  coefficient which may be interpreted as a summary indicator of the relative level of risk of each portfolio which accounts for differences linked to age and period distribution across portfolios. Large differences between portfolios may be observed. The autonomous mortality of portfolio 6 is almost twice the mortality of portfolio 1 and the incidence in LTC of portfolio 5 is almost 3 times the incidence of portfolio of portfolio 3. The average improvement in the common pattern, represented on the right side of Figure 9, is for autonomous mortality -1.52 % and for incidence in LTC it is -1.01 % per year. As 0 is not included in the confidence intervals, the assumption that either improvement is equal to 0 is rejected based on this model.

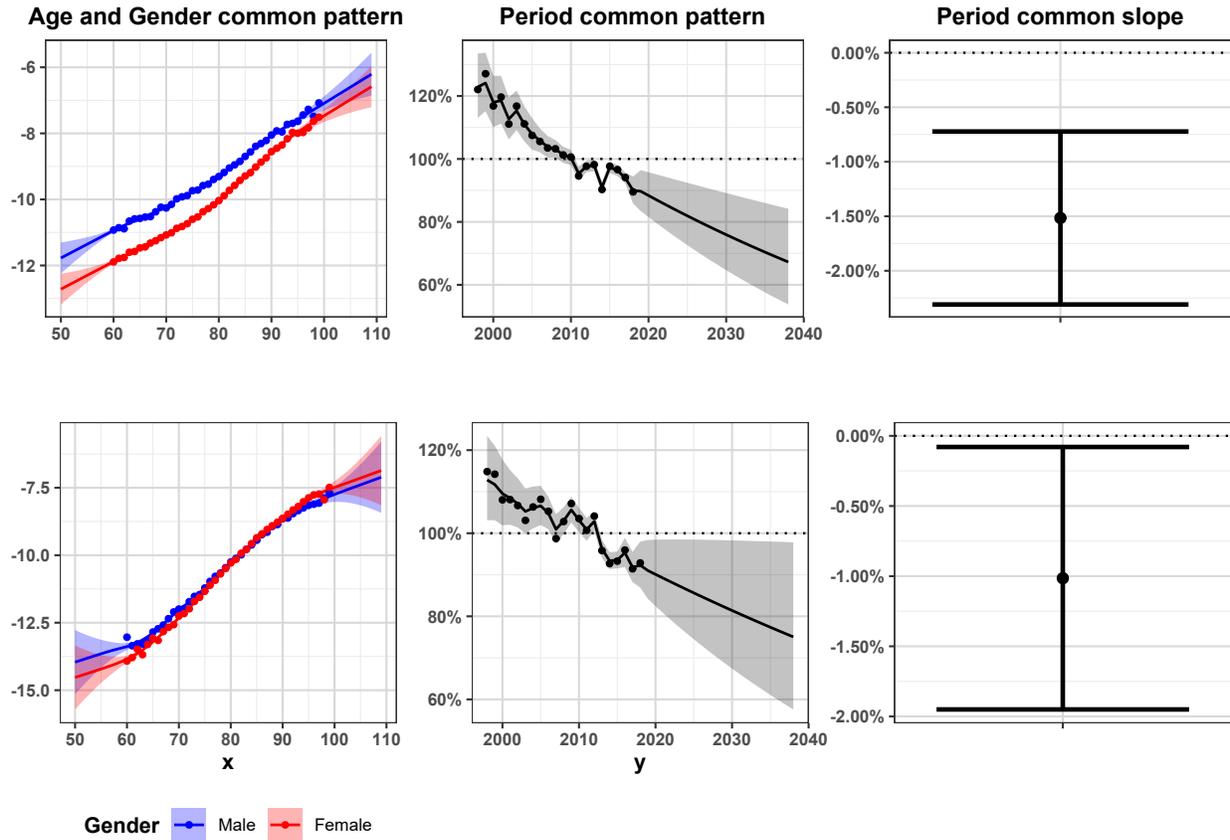


Figure 9: Common age and period patterns for autonomous mortality (top) and incidence in LTC (bottom)

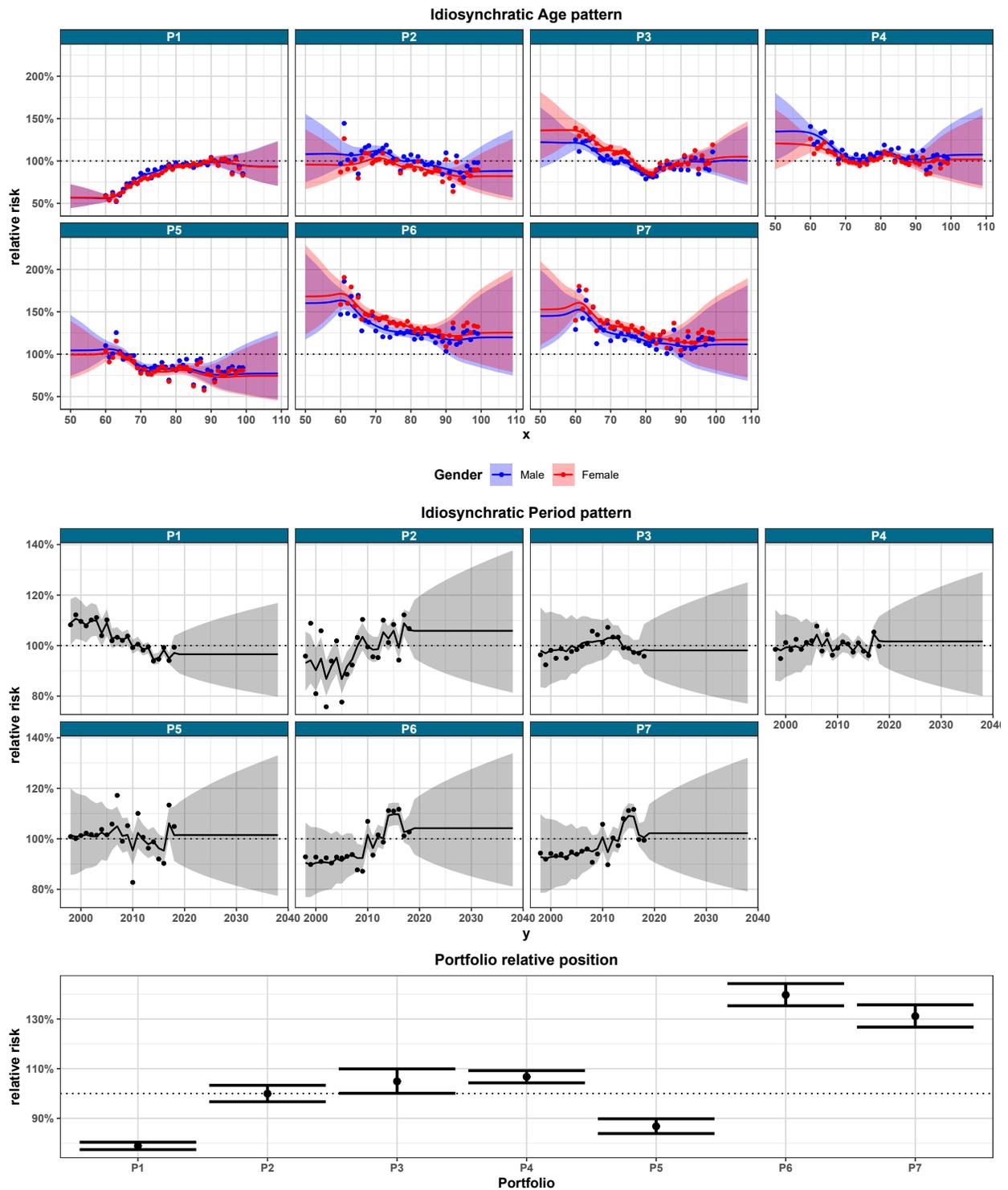


Figure 10: Idiosyncratic age and period patterns and portfolio relative position for autonomous mortality

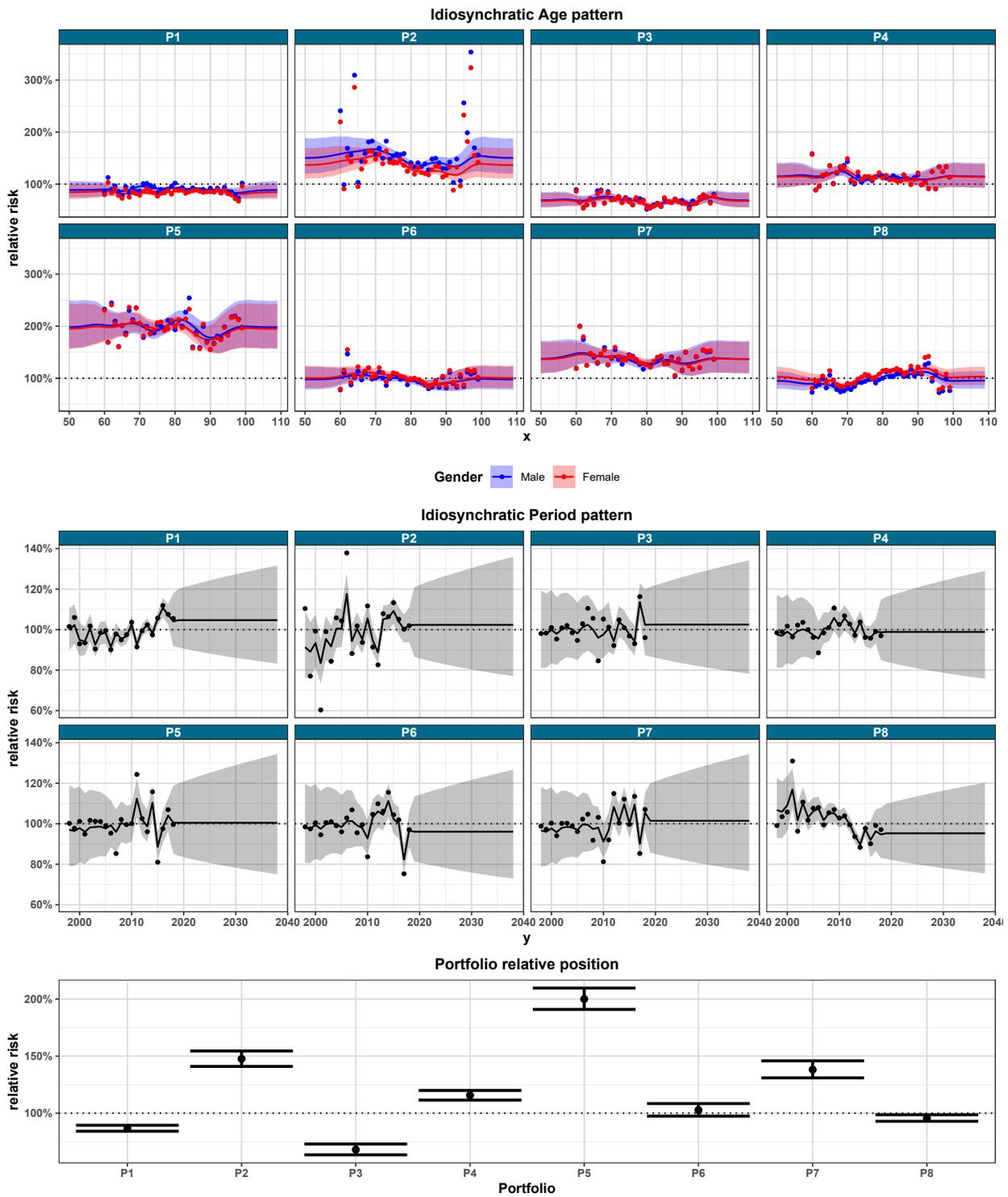


Figure 11: Idiosyncratic age and period patterns and portfolio relative position for incidence in LTC

## 5.2 Disabled mortality

For disabled mortality, we follow the same approach and assume  $\log \mu^{21}(x, y, g, p) = \alpha_{x,z,g}^1 + \kappa_y^1 + \alpha_{x,z,g,p}^2 + \kappa_{y,p}^2$  where

- $\alpha_{x,z,g}^1 = a \times (x - \bar{x}) + s_1(x) + \delta_g + s_2(x, g) + s_3(z) + s_4(z, g) + s_5(x, z) + s_6(x, z, g)$
- $\kappa_y^1 = b \times (y - \bar{y}) + S\epsilon_{y,1} + \epsilon_{y,2}$
- $\alpha_{x,z,g,p}^2 = \delta_p + s_7(x, p) + \delta_{g,p} + s_8(x, g, p) + s_9(z, p) + s_{10}(z, g, p) + s_{11}(x, z, p) + s_{12}(x, z, g, p)$
- $\kappa_{y,p}^2 = S\epsilon_{y,p,3} + \epsilon_{y,p,4}$
- $a$  and  $b$  are slope parameters,  $s_1$  to  $s_{12}$  are smooth centered functions of age and duration as well as centered functions of gender and portfolio,  $\delta_g$ ,  $\delta_p$  and  $\delta_{g,p}$  are centered gender and portfolio coefficients and finally  $\epsilon_{y,k} \sim \mathcal{N}(0, \sigma_{\epsilon,k}^2 \mathbf{I})$  (resp.  $\epsilon_{y,p,l} \sim \mathcal{N}(0, \sigma_{\epsilon,l}^2 \mathbf{I}_p \otimes \mathbf{I}_y)$ ) are *iid* white noise for  $k, l \in \{1, 2\}$ .

At this point, let us raise some numerical concerns about the above model. In previous models, we chose the number of subdivisions for the splines basis to be 4 times the lower distance between two consecutive observations, which means for the age dimension we introduce one subdivision (and therefore one spline) every 4 years and for the duration in LTC dimension this would be one spline every 4 months. Therefore the number of splines, assuming order 2 penalties, is  $[(109 - 50)/4] + 3 = 18$  for the age dimension and  $[12 * (9 + 1/2 - 1/24)/4] + 3 = 32$  for the duration in LTC dimension. The associated number of parameters for each of the smooth ranges from only  $18 - 1 = 17$  for  $s_1$  and 32 for  $s_3$  to  $18 \times 32 \times 8 = 4,608$  for  $s_{11}$  and  $18 \times 32 \times 2 \times 8 = 9,216$  for  $s_{12}$ . In our implementation of the SAP algorithm, we use Generalized Linear Array Models as introduced in Currie et al. (2006). The most computationally intensive part of the algorithm is the inversion of the matrix  $Z'WZ + \phi G_\theta^{-1}$  of dimensions  $p \times p$  where  $p$  is the number of parameters with nearly cubic complexity in  $p$ . Indeed, we observe that the computation time increases dramatically where more than 3,000 parameters are included. Worst, sometimes the matrix inversion fails or the algorithm does not converge. Inference in the above model is therefore not numerically doable unless we bring some drastic modifications. Therefore we first remove the coefficients  $s_{11}$  and  $s_{12}$  from the model. Indeed, those coefficients represent 3 and 4-way interactions that are likely to be scaled down to 0 by the model. Then we modify the coefficients  $s_5$  to  $s_{10}$  so that they only use half the number of subdivisions used for  $s_1$  to  $s_4$ , following an idea from Lee Hwang (2010). This halves the number of parameters in  $s_7$ ,  $s_8$ ,  $s_9$  and  $s_{10}$  and divides by 4 the number of parameters in  $s_5$  and  $s_6$ . The total number of parameters in the model therefore drops from over 20,000 to less than 3,000.

Figure 12 represents the common marginal age and duration patterns, the common age-duration interaction for disabled mortality and the common period pattern. Figure 13 represents the idiosyncratic age, duration and period pattern. Finally, the combined pattern obtained by summing the common and idiosyncratic patterns is displayed on Figure 16 in the Appendix section. The average improvement in the common pattern for disabled mortality is 0.24 %. As 0 is included in the confidence intervals, one cannot reject the assumption that there is no period improvement.

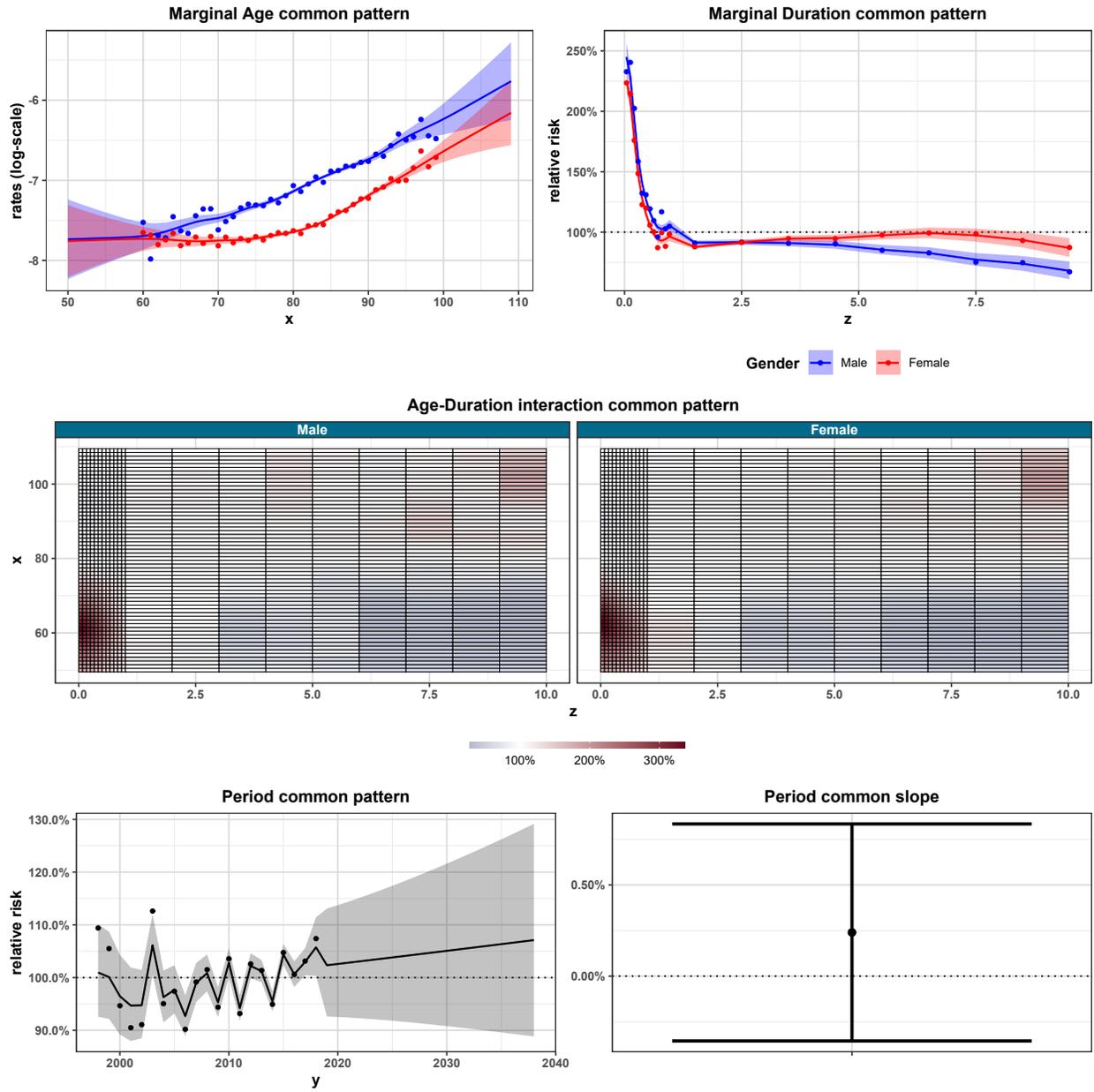


Figure 12: Common age, duration and period patterns for disabled mortality

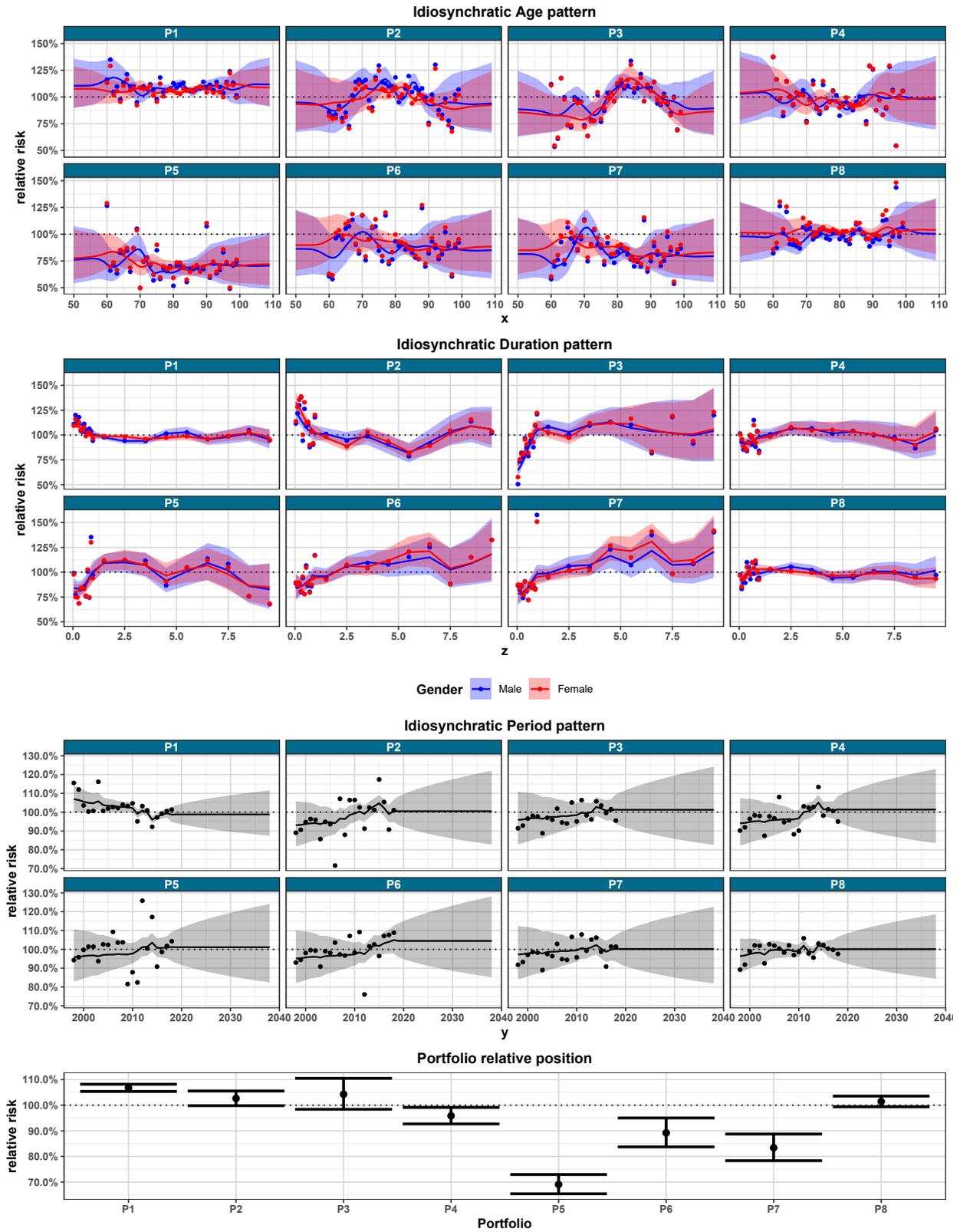


Figure 13: Idiosyncratic age, duration and period patterns and portfolio relative position for disabled mortality

Table 1: Comparison of stratified and multiportfolio fitted models

Assumption	Model	$n_{\text{obs}}$	$n_{\text{par}}$	deviance	ed	AIC	BIC	GCV
$\mu^{01}$	stratified	5,434	959	5,642	119	5,880	6,665	4.64
	multiportfolio		1,089	5,376	139	5,655	6,574	4.54
$\mu^{02}$	stratified	6,512	1,096	6,581	115	6,810	7,589	3.22
	multiportfolio		1,225	6,228	157	6,541	7,602	3.10
$\mu^{21}$	stratified	75,560	14,488	53,503	232	53,967	56,111	0.73
	multiportfolio		2,806	53,179	287	53,754	56,406	0.73

### 5.3 Comparison with the stratified analysis

Table 1 summarizes information about the stratified and multiportfolio approaches for each of the transition intensities. We observe that the multiportfolio approach tends to be more accurate at the cost of slightly higher effective dimension of the model ( $ed$  in the table). For the AIC and GCV criteria, it is the most efficient approach for all 3 transition intensities while for the BIC the stratified approach has a better trade-off between accuracy and parsimony. Nevertheless, the interest of the multiportfolio approach does not lie on its ability to better fit the data but rather on its ability to create consistent assumptions where data is not available. An alternative approach would be to either use some version of the credibility theory, see Norberg (2014), or a relational model, for example as in Hannerz (2001). Both previous approaches rely on a reference assumption that would in the absence of an external reference be based on one or several of the portfolios. The common pattern in the multiportfolio approach may also be interpreted as a reference. However, it has the advantage to be based on all available data, to make minimal assumptions about the relative level of risk in each portfolio. Furthermore, the reference is inferred alongside other parameters of the model.

## 6 Insured population forecast

### 6.1 Motivation

In this penultimate section, we rely on the previously described models to forecast insured trajectories in the future with a double objective. Our first aim is to assess the impact of adopting a prospective approach. We use two benchmarks: a constant forecast approach where using the previous model we set the mortality level for all future years to the one of 2019 which is the first year of forecast and an alternative approach where we ignore the period dimension and therefore remove all period terms from the models. The rationale behind the inclusion of this second benchmark is that there is a strong correlation between the age and period dimensions in longitudinal studies based on run-off insurance portfolios as the average age of the portfolio increases over time. In an approach that ignores the period dimension, part of the period mortality slope might therefore be captured by the age pattern slope. Our second aim is to compare the forecast of the stratified and multiportfolio approaches developed in the previous sections. Therefore, we consider 6 set of assumptions:

- The Stratified Prospective (SP) approach presented previously,
- The Multiportfolio Prospective (MP) approach presented previously,
- A Stratified Constant Forecast (SCF) approach obtained by using the previous stratified

approach above but using the 2019 forecast for all future years

- A Multiportfolio Constant Forecast (MCF) approach obtained by using the previous multi-portfolio approach above but using the 2019 forecast for all future years
- A Stratified Non Prospective (SNP) approach obtained by fitting a simplified version of the previous stratified model where all period terms are removed
- A Multiportfolio Non Prospective (MNP) approach obtained by fitting a simplified version of the previous stratified model where all period terms are removed

## 6.2 Summary indicators

To compare each set of assumption, we rely on a limited set of summary indicators that are computed for each portfolio then globally:

- $a_{\text{death}}$ : the average age at death, based on all insured neither dead nor lapsed at the start of the forecast
- $p_{\text{LTC}}$ : the probability for an autonomous insured at the start of the forecast to make an LTC claim during the forecast
- $a_{\text{LTC}}$ : the average age at entry in LTC, based on all autonomous insured at the start of the forecast
- $d_{\text{LTC}}$ : the average duration of LTC claims, based on all claim still open at the start of the forecast and all claims made during forecast
- $t_{\text{aut}}$ : the average time spent in the autonomous state since the start of the forecast, counting 0 for insured already in LTC at the start of the forecast
- $t_{\text{LTC}}$ : the average time spent in the LTC state since the start of the forecast, counting 0 for insured that will never go through LTC
- $S_{\text{LTC}} = t_{\text{LTC}} / (t_{\text{aut}} + t_{\text{LTC}})$ : the share of the insured lifespan spent in the LTC state since the start of the forecast

## 6.3 Forecast methodology

To proceed with the forecast, we only require two additional assumptions. We first assume constant transition intensities over one-year or one-month intervals as in the modelling section to be consistent with the way the assumptions were derived. We then assume independance between the insured deaths and entries in LTC. The populaton we forecast consists of all autonomous or disabled insured alive but not lapsed at the end of the observation period, which is either the end of year of 2017 or 2018 depending on the portfolio. We only consider portfolios 1 to 7 as we have no data regarding the autonomous mortality of portfolio 8. The insured population we forecast therefore includes a little more than 750,000 individuals. We do not model any future lapse events that as we treated those as independent censoring events. While it would be theoretically possible to find closed formulas for each of the summary indicators above, in order to avoid integration over 2 or 3 dimensions (age, period and time spent in LTC), we rather rely on a Monte Carlo approach. Due to the high population size, we only perform 20 simulations for each of the 6 set of assumptions. Thanks to mutualisation of the risk within each portfolio, the indicators are pretty stable across simulations and the estimation error from the Monte Carlo approach is not material.

Table 2: Indicators for each set of assumptions, aggregated over all portfolios

<b>Assumption</b>	$a_{\text{death}}$	$p_{\text{LTC}}$	$a_{\text{LTC}}$	$d_{\text{LTC}}$	$t_{\text{aut}}$	$t_{\text{LTC}}$	$S_{\text{LTC}}$
<b>SP</b>	89.99	45.9%	90.93	2.28	33.68	1.00	2.9%
<b>MP</b>	90.87	42.6%	91.24	2.87	34.37	1.18	3.3%
<b>SCF</b>	89.55	39.9%	90.01	3.33	32.94	1.30	3.8%
<b>MCF</b>	89.60	38.2%	89.90	3.16	33.11	1.17	3.4%
<b>SNP</b>	89.31	38.2%	89.77	3.44	32.71	1.29	3.8%
<b>MNP</b>	89.26	37.2%	89.71	3.22	32.78	1.16	3.4%

#### 6.4 Comparison of assumptions

Table 2 contains the summary indicators computed across all portfolios for each of the 6 sets of assumptions while table 4 provides details at the portfolio level. To make comparison easier we further display in Table 3 the relative difference between the indicators for each set of assumptions and Multiportfolio Prospective assumptions indicator. We first compare prospective assumptions to non-prospective assumptions. We observe that for all indicators, there is only little difference between the Constant Forecast (CF) and Non Prospective (NP) approaches, whether Stratified or Multiportfolio, compared to the difference between those two approaches and the Prospective approach. The NP approaches did not capture a sizeable part of the period pattern through the age pattern as we previously suggested it could. Furthermore, as the output of this approach is an average over the period, it is even further away from the prospective approach than the CF approach which is based on the first year of forecast.

By looking at the age at death  $a_{\text{death}}$  and the time spent in the autonomous state  $t_{\text{aut}}$ , we conclude that all NP approaches slightly underestimate both indicators as they do not account for the negative period slope for autonomous mortality. They also underestimate significantly the probability to enter the LTC state  $p_{\text{LTC}}$ . Indeed, despite a negative slope for incidence rates, the negative slope for autonomous mortality means that insured are more likely to reach very high ages where LTC likely occurs. As claims at higher ages are shorter, this also explain why the duration of claims  $d_{\text{LTC}}$  is longer in NP approaches. The average time spent in LTC  $t_{\text{LTC}}$  which depends on both the probability of LTC and the average duration of LTC claims is higher for the SCF and SNP approaches but not their multiportfolio counterparts MCF and MNP. Finally, the share of lifespan spent in LTC  $S_{\text{LTC}}$  is significantly higher for the SCF and SNP approaches but only marginally for MCF and MNP compared to the prospective approaches.

Regarding the difference between stratified and multiporfolio based approaches, it appears very large, whether we look at the prospective, non-prospective or constant forecast assumptions. While looking in table 4 at the  $S_{\text{LTC}}$  indicator, which provides a good summary of the weight of LTC in insured lifespans, we notice outstanding differences between the results of the stratified prospective approach and the results of other approaches on portfolio 4 to 6, while the stratified non-prospective approaches give outstanding results for portfolio 7. As could be expected, the stratified analysis leads to divergeing estimates across portfolios, especially in the case a prospective model, while the multiportfolio approach gives more homogeneous results.

Table 3: Relative difference between aggregated indicators and the indicator with MP assumptions

Assumption	$a_{\text{death}}$	$p_{\text{LTC}}$	$a_{\text{LTC}}$	$d_{\text{LTC}}$	$t_{\text{aut}}$	$t_{\text{LTC}}$	$S_{\text{LTC}}$
SP	-1.0%	7.7%	-0.3%	-20.6%	-2.0%	-15.7%	-13.6%
MP	<b>0.0%</b>	<b>0.0%</b>	<b>0.0%</b>	<b>0.0%</b>	<b>0.0%</b>	<b>0.0%</b>	<b>0.0%</b>
SCF	-1.5%	-6.3%	-1.3%	16.1%	-4.2%	9.7%	13.9%
MCF	-1.4%	-10.3%	-1.5%	10.0%	-3.7%	-0.8%	2.9%
SNP	-1.7%	-10.2%	-1.6%	19.9%	-4.8%	8.7%	13.7%
MNP	-1.8%	-12.7%	-1.7%	12.2%	-4.6%	-1.5%	3.2%

## 7 Discussion

In this paper, we introduce a modelling approach for the construction of prospective biometric assumptions in order to assess liabilities associated with Long-Term Care insurance risks. To do so, we rely on a Poisson Generalized Linear Mixed Model and data coming from 8 insurance portfolios. Among the main issues associated with this data are the many definitions of the LTC trigger that in addition with other differences makes for large differences in claim experience from one portfolio to another. In addition, as longitudinal data, it is limited on both the period and age dimensions, making extrapolation and forecast key issues. The flexibility of the mixed model framework allows us to decompose the risk as a sum of marginal impact of variables and interactions between those variables and select only those that we want to model. We apply it to first build biometric assumptions for each of the portfolio separately in a stratified approach, then extend the model to pool all data together and build multiportfolio estimate that contains a common pattern shared across all portfolio and an idiosyncratic part for each portfolio. Using *ad hoc* penalization functions, we are able to perform extrapolation and forecast in a consistent way across genders and portfolios.

We then forecast the insured population from the portfolios, using the biometric assumptions previously derived, to compare the stratified and multiportfolio approaches and also to compare those approaches to their non-prospective counterparts. Through simulations, we compute some indicators on the population forecast and conclude in a material difference between the stratified and multiportfolio approaches on one hand and the prospective and non-prospective approach on the other hand. Indeed, in the prospective approaches the distribution of claims shift toward higher ages and those claims are both shorter and more frequent. Furthermore, the stratified approach produces diverging forecast across portfolios while the multiportfolio approach brings consistency in the different estimates.

While we chose to compare our results by a forecast of the in-force portfolios, the difference between prospective approaches and non-prospective approaches would be even bigger in the pricing of new policies that would have a younger issue age. Besides, our forecast is based on the best-estimate assumptions we derived, however our methodology also provides confidence intervals on the forecast. It could therefore be used to define a 99.5 % extreme scenario for one or several of the biometric assumptions and assess the impact on insurer liabilities and ultimately the Solvency Capital Requirement associated with the Long-Term Care risk. In this work, we model each of the 3 biometric laws (autonomous mortality, disabled mortality and incidence in LTC) separately and do not account for possible correlation between them. While this is of limited impact in the derivation of best estimate assumptions, if the focus is on building consistent extreme scenario as in the SCR derivation example, then those correlations need to be taken into account. One way to do so would

be to use a multivariate ARIMA model, as for example in Levantesi and Menzietti (2012). The SAP model, that relies on the diagonal structure of the variance-covariance matrix for the random effects, would need to be adapted.

## **Acknowledgements**

We are thankful to our colleagues at SCOR for their support in putting together the database used in this study and to the SCOR company for letting us publish those results.

## **Funding**

This article was written as part of the author functions within the SCOR company. No external funding to declare.

## **Conflicts of interest**

None to declare

## **Availability of data and material**

Due to the sensitive nature of the data used in this paper and the competitive advantage they might represent for private insurance and reinsurance activities, the data is not available publically.

## **Code availability**

The code used to generate the results we present in the paper is available upon request. The development of a package for the R software, see R Core Team (2020), to make this code widely available is underway.

## **Authors contributions**

Not applicable

# Appendix

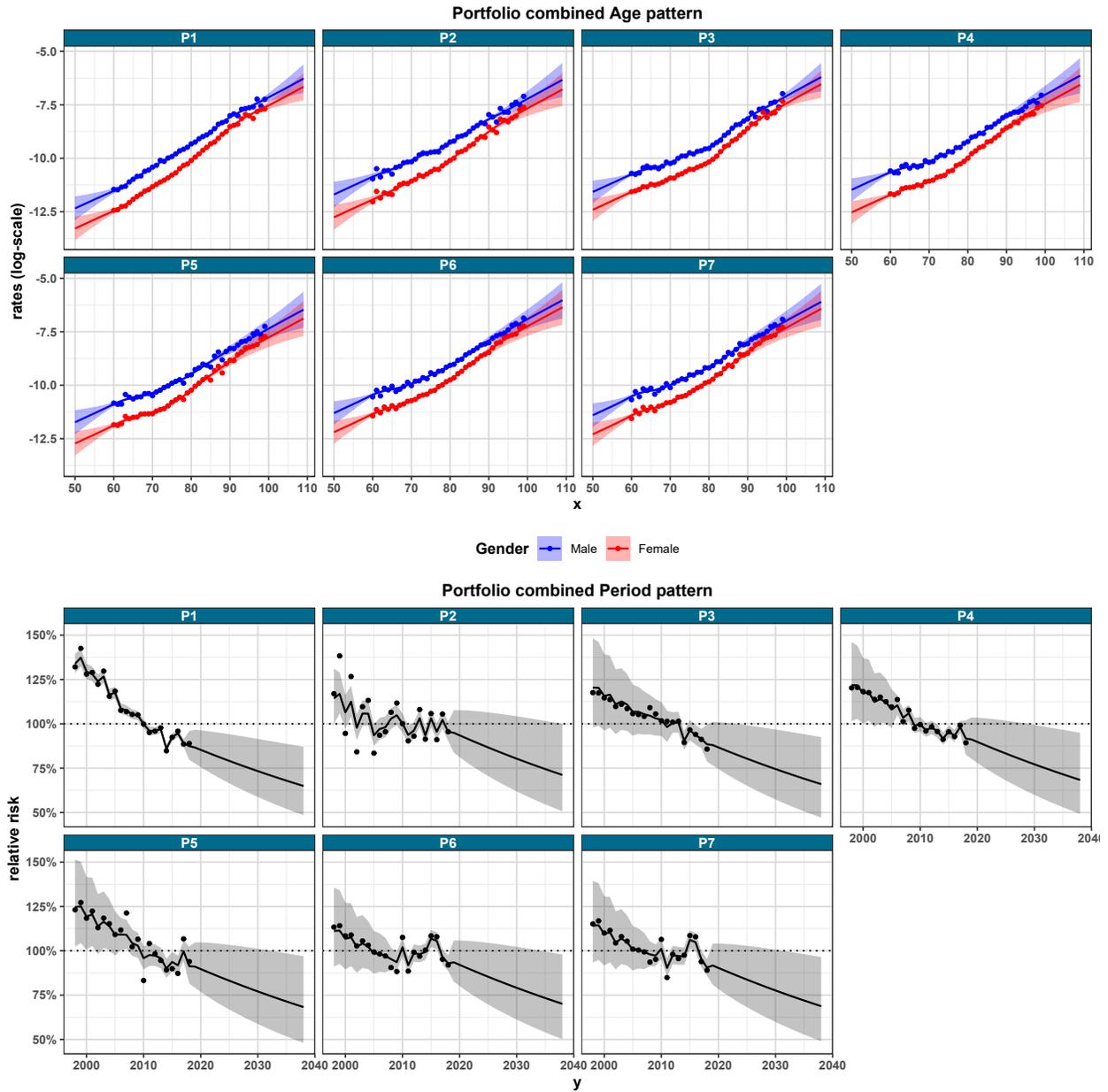


Figure 14: Combined age and period pattern for autonomous mortality

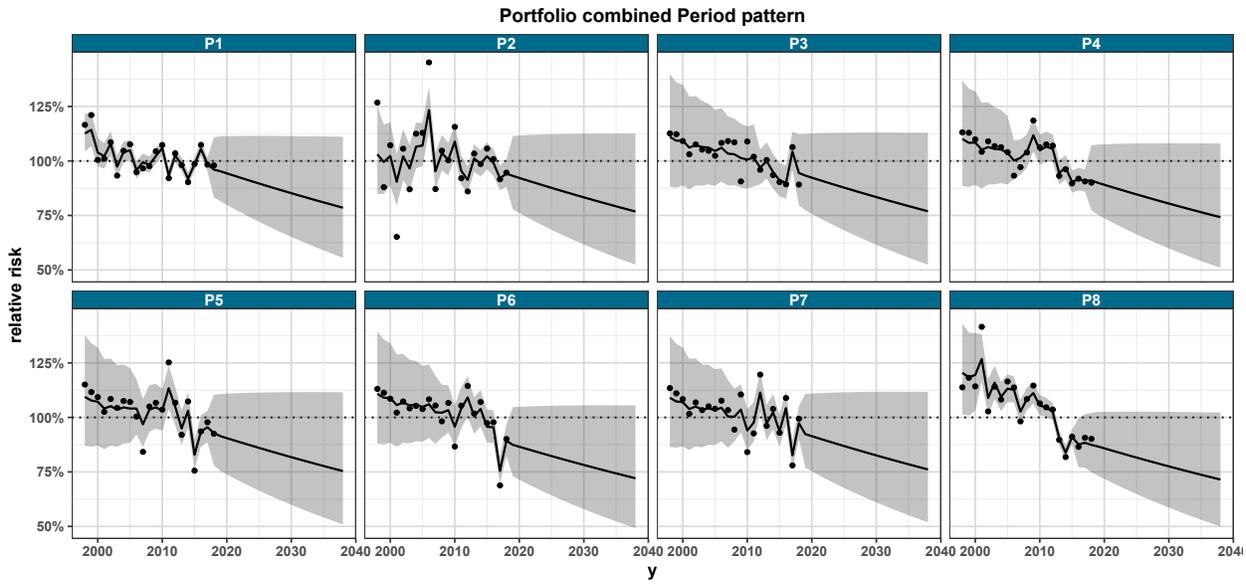
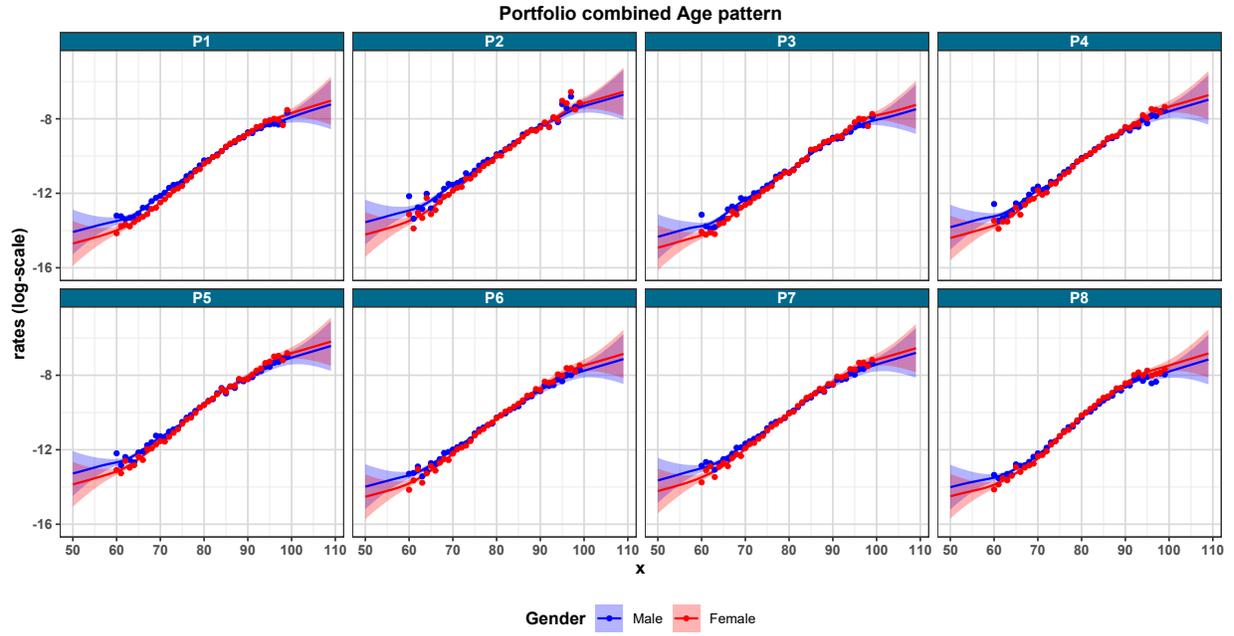


Figure 15: Combined age and period pattern for incidence in LTC

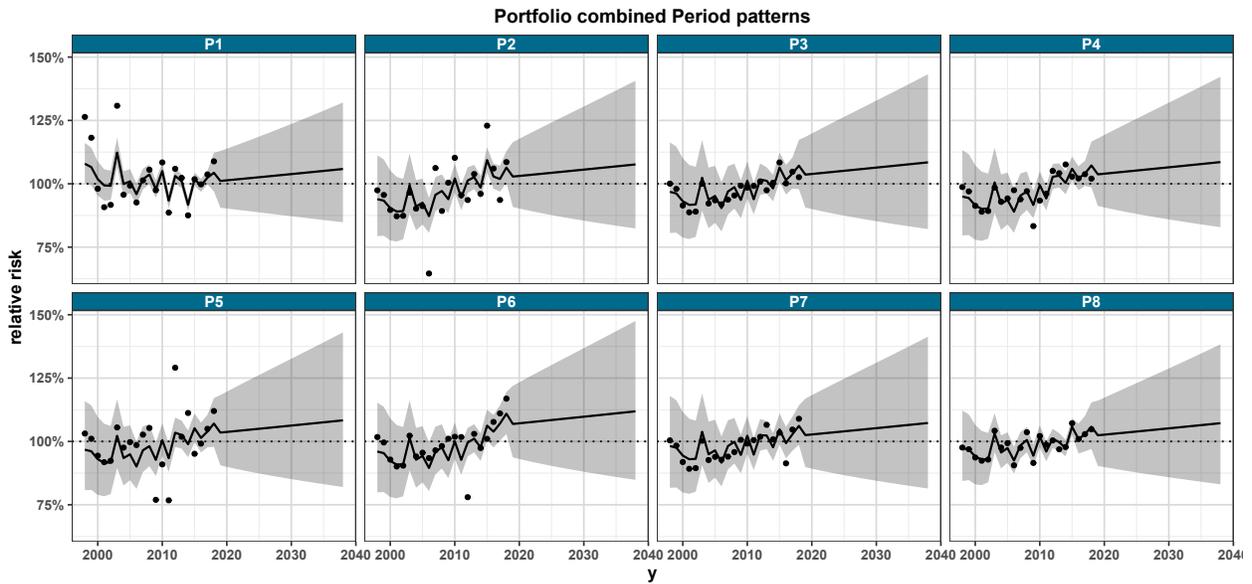
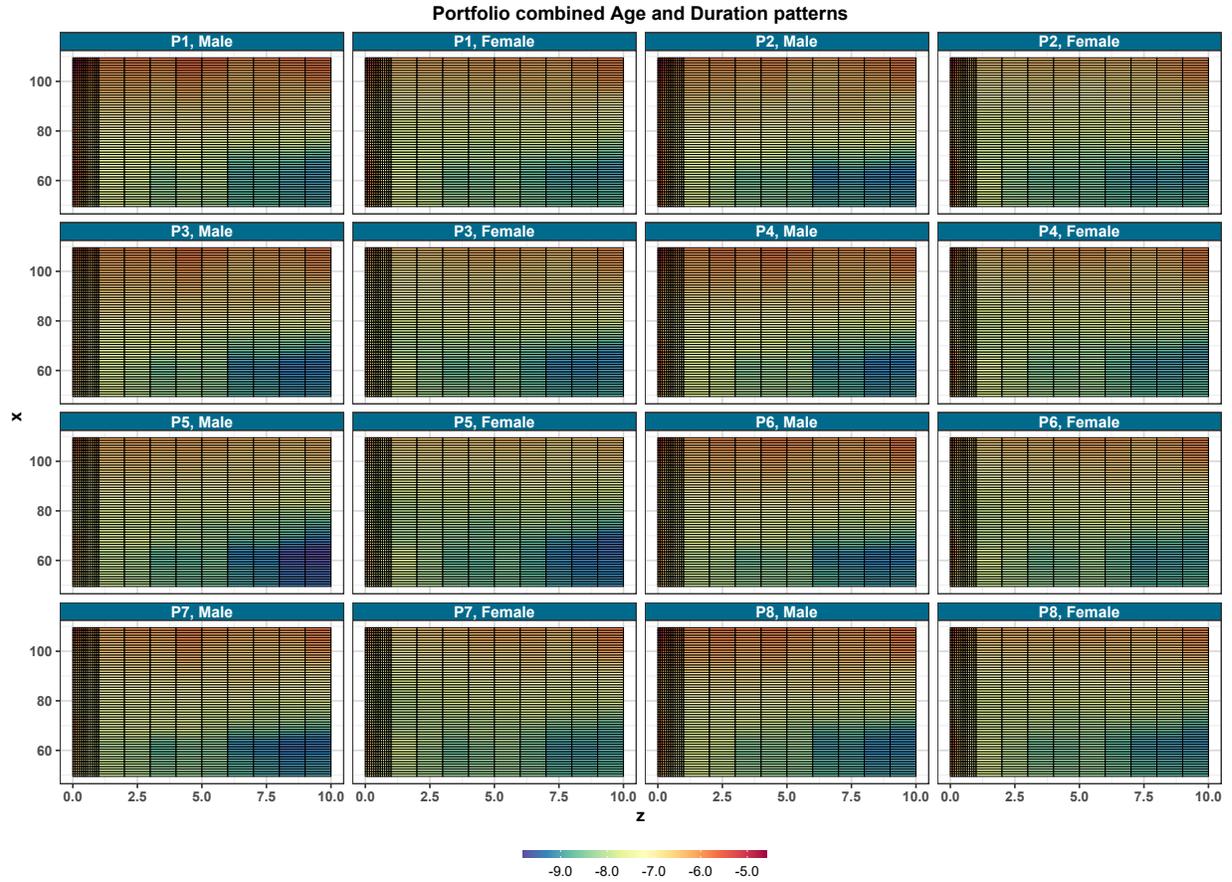


Figure 16: Combined age, duration and period pattern for disabled mortality

Table 4: Indicators for each portfolio and each set of assumptions

Portfolio	Assumption	$a_{\text{death}}$	$P_{\text{LTC}}$	$a_{\text{LTC}}$	$d_{\text{LTC}}$	$t_{\text{aut}}$	$t_{\text{LTC}}$	$S_{\text{LTC}}$
P1	SP	94.55	49.0%	94.01	2.94	28.72	1.28	4.3%
	MP	94.39	45.5%	93.82	2.80	28.72	1.11	3.7%
	SCF	94.12	45.8%	93.44	2.97	28.35	1.21	4.1%
	MCF	94.12	43.7%	93.46	2.91	28.44	1.12	3.8%
	SNP	93.73	43.7%	93.20	2.97	28.02	1.15	3.9%
	MNP	93.74	43.8%	93.23	2.95	28.04	1.15	3.9%
P2	SP	92.80	56.7%	92.06	3.12	26.32	1.59	5.7%
	MP	93.28	56.3%	92.54	3.12	26.82	1.57	5.5%
	SCF	92.81	56.4%	91.86	3.39	26.18	1.74	6.2%
	MCF	93.04	54.4%	92.17	3.28	26.54	1.61	5.7%
	SNP	92.81	56.1%	91.89	3.44	26.16	1.76	6.3%
	MNP	92.84	53.8%	92.01	3.21	26.40	1.55	5.6%
P3	SP	89.79	50.5%	91.36	1.71	40.25	0.85	2.1%
	MP	91.00	35.9%	92.31	2.37	41.47	0.84	2.0%
	SCF	88.96	33.8%	90.33	2.57	39.41	0.86	2.1%
	MCF	89.18	30.7%	90.33	2.69	39.67	0.82	2.0%
	SNP	88.68	30.5%	90.32	2.68	39.18	0.81	2.0%
	MNP	88.70	29.9%	90.13	2.70	39.21	0.80	2.0%
P4	SP	89.87	42.4%	90.62	2.26	30.66	0.91	2.9%
	MP	90.43	46.5%	90.69	2.74	30.91	1.23	3.8%
	SCF	89.21	41.8%	89.71	2.79	29.78	1.13	3.6%
	MCF	89.47	42.5%	89.62	2.96	29.95	1.22	3.9%
	SNP	89.08	42.0%	89.46	2.83	29.63	1.15	3.7%
	MNP	89.26	41.4%	89.41	3.13	29.71	1.26	4.1%
P5	SP	89.18	56.8%	88.93	2.40	30.93	1.29	4.0%
	MP	90.11	64.5%	88.93	3.90	30.71	2.45	7.4%
	SCF	88.77	58.4%	87.73	3.87	29.61	2.20	6.9%
	MCF	89.29	60.9%	87.95	4.22	29.83	2.51	7.8%
	SNP	88.57	62.0%	87.42	4.10	29.14	2.48	7.8%
	MNP	89.10	60.1%	87.74	4.38	29.57	2.57	8.0%
P6	SP	88.60	25.3%	89.72	2.21	28.51	0.53	1.8%
	MP	89.89	38.9%	90.60	2.96	29.20	1.13	3.7%
	SCF	88.97	33.4%	89.57	4.14	28.04	1.36	4.6%
	MCF	88.71	34.3%	89.28	3.29	28.05	1.11	3.8%
	SNP	88.81	30.9%	88.91	4.33	27.93	1.32	4.5%
	MNP	88.52	31.8%	89.04	3.37	27.91	1.05	3.6%
P7	SP	89.34	49.5%	89.68	3.50	28.03	1.69	5.7%
	MP	89.77	48.4%	89.76	3.41	28.55	1.61	5.3%
	SCF	89.41	49.5%	89.08	4.87	27.42	2.38	8.0%
	MCF	88.72	44.1%	88.62	3.71	27.51	1.60	5.5%
	SNP	89.06	48.6%	88.88	4.87	27.11	2.33	7.9%
	MNP	88.44	43.2%	88.45	3.69	27.27	1.55	5.4%

## References

- Guillaume Biessy. *Semi-Markov modeling of the loss of autonomy among elderly people: application to long-term care insurance*. PhD thesis, Paris Saclay, 2016.
- Guillaume Biessy. Continuous-time semi-markov inference of biometric laws associated with a long-term care insurance portfolio. *ASTIN Bulletin: The Journal of the IAA*, 47(2):527–561, 2017.
- Norman E Breslow and David G Clayton. Approximate inference in generalized linear mixed models. *Journal of the American statistical Association*, 88(421):9–25, 1993.
- Andrew JG Cairns, David Blake, and Kevin Dowd. A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration. *Journal of Risk and Insurance*, 73(4):687–718, 2006.
- Alba Carballo, Maria Durban, Göran Kauermann, and Dae-Jin Lee. A general framework for prediction in penalized regression. *Statistical Modelling*, page 1471082X19896867, 2020.
- Iain D Currie, Maria Durban, and Paul HC Eilers. Generalized linear array models with applications to multidimensional smoothing. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 68(2):259–280, 2006.
- Joelle H Fong, Adam W Shao, and Michael Sherris. Multistate actuarial models of functional disability. *North American Actuarial Journal*, 19(1):41–59, 2015.
- Christian Gouriéroux and Yang Lu. Long-term care and longevity. *Available at SSRN 2347735*, 2014.
- Harald Hannerz. An extension of relational methods in mortality estimation. *Demographic Research*, 4:337–368, 2001.
- Atsuyuki Kogure, Takahiro Fushimi, and Shinichi Kamiya. Mortality forecasts for long-term care subpopulations with longevity risk: A bayesian approach. *North American Actuarial Journal*, pages 1–11, 2019.
- Ronald D Lee and Lawrence R Carter. Modeling and forecasting US mortality. *Journal of the American statistical association*, 87(419):659–671, 1992.
- Dae-Jin Lee Hwang. Smoothing mixed models for spatial and spatio-temporal data. 2010.
- Susanna Levantesi and Massimiliano Menzietti. Managing longevity and disability risks in life annuities with long term care. *Insurance: Mathematics and Economics*, 50(3):391–401, 2012.
- Zixi Li, Adam W Shao, and Michael Sherris. The impact of systematic trend and uncertainty on mortality and disability in a multistate latent factor model for transition rates. *North American Actuarial Journal*, 21(4):594–610, 2017.
- Xihong Lin and Daowen Zhang. Inference in generalized additive mixed models by using smoothing splines. *Journal of the royal statistical society: Series b (statistical methodology)*, 61(2):381–400, 1999.
- Ragnar Norberg. Credibility theory. *Wiley StatsRef: Statistics Reference Online*, pages 1–15, 2014.
- R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2020. URL <https://www.R-project.org/>.

María Xosé Rodríguez-Álvarez, Dae-Jin Lee, Thomas Kneib, María Durbán, and Paul Eilers. Fast smoothing parameter separation in multidimensional generalized p-splines: the SAP algorithm. *Statistics and Computing*, 25(5):941–957, 2015.

Grace Wahba, Yuedong Wang, Chong Gu, Ronald Klein, Barbara Klein, et al. Smoothing spline ANOVA for exponential families, with application to the wisconsin epidemiological study of diabetic retinopathy: the 1994 neyman memorial lecture. *The Annals of Statistics*, 23(6):1865–1895, 1995.